Tetraquarks $\bar{b}\bar{b}ud$, $I(J^P) = 0(1^-)$ and $\bar{b}\bar{c}ud$ with $I(J^P) = 0(0^+), 0(1^+)$ from Lattice QCD Static Potentials

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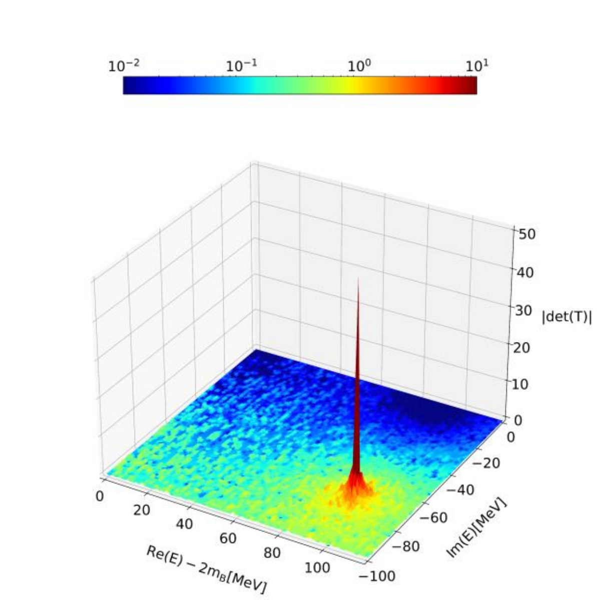


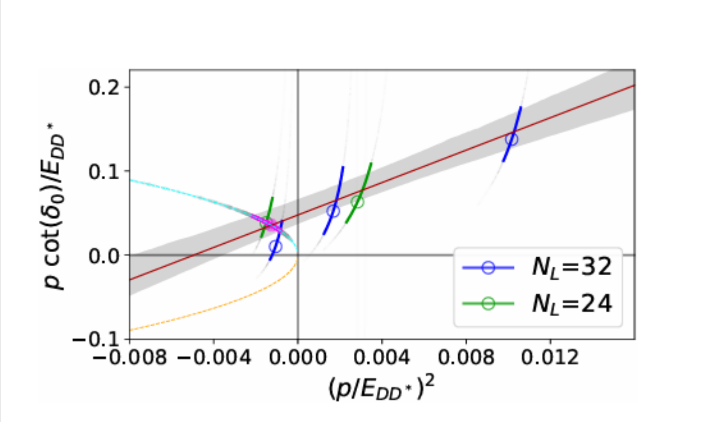
Lattice 2024
Liverpool, 07/30/2024

Outline

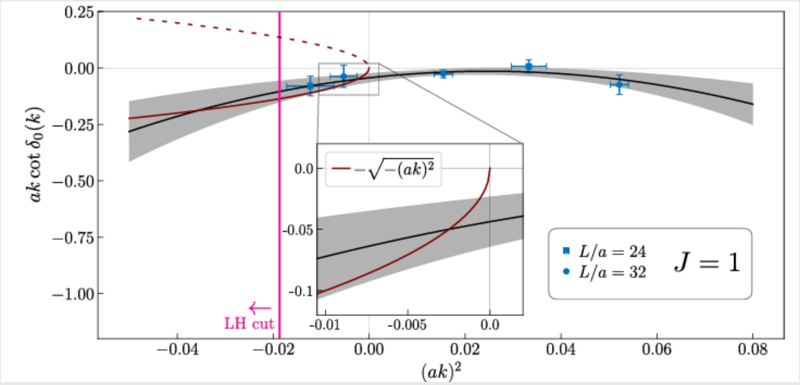
1) **Introduction**
2) **Tetraquark $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^-)$**
3) **Tetraquark $\bar{b}\bar{c}ud$ with $I(J^P) = 0(0^+), 0(1^+)$**

Why are Tetraquarks interesting?

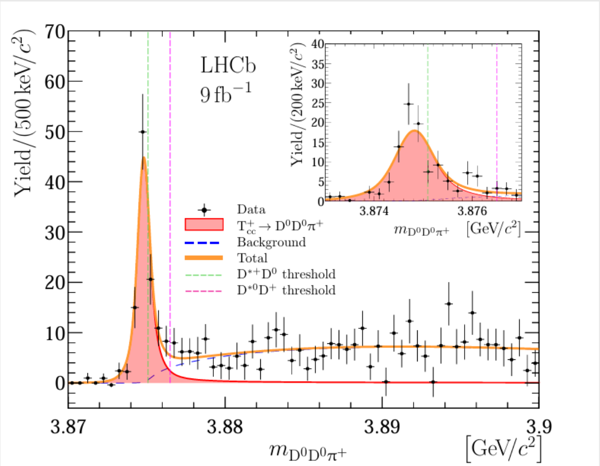
- In 2013 the X(3872) was discovered which could not be explained by ordinary mesons
- Experimentally established candidate T_{cc}^{+} with quantum numbers $\mathrm{I}(\mathrm{J}^{\mathrm{P}}) = 0(1^{+})$
- Antiheavy-Antiheavy-Light-Light tetraquark bound states T_{bb} and T_{bc} were studied with lattice QCD and finite volume scattering
- A $\bar{\mathrm{b}}\bar{\mathrm{b}}\mathrm{ud}$ tetraquark resonance with $\mathrm{I}(\mathrm{J}^{\mathrm{P}}) = 0(1^{-})$ was predicted using lattice QCD potentials and scattering theory [**Bicudo et al, 1704.02383**]



T_{cc}^{+} scattering phase shift from **2202.10110**



T_{bc} scattering phase shift from **2312.02925**



T_{cc}^{+} peak in $D^0 D^0 \pi^+$ cross section taken from **2109.01038**

Static Potentials from Lattice QCD

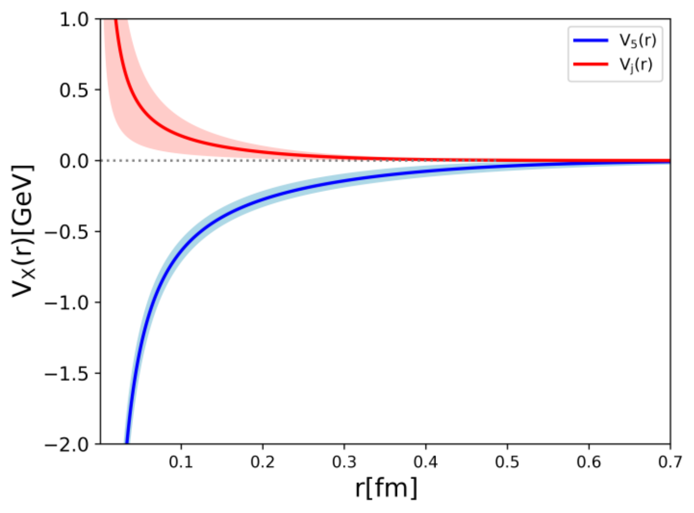
- Static potentials are extracted via the asymptotic behavior of a temporal correlation function at fixed spatial separation r

$$C_{BB}(t) = \left\langle \Omega \middle| O_{BB}(t,r) O^{\dagger}_{BB}(0,r) \middle| \Omega \right\rangle \sim \exp(-V_X(r)t), \qquad t \to \infty$$

$$O_{BB}(t,r) = (CL_q)_{\alpha\beta}(CS_Q)_{\gamma\delta}(\bar{Q}^a{}_{\gamma}(t,r) q^a{}_{\alpha}(t,r))(\bar{Q}^b{}_{\delta}(t,0) q^b{}_{\beta}(t,0))$$

[Bicudo et al, 1510.03441]

- Different combinations of light-quark spin $\mathrm{L_q}$ and heavy-quark spin $\mathrm{S_Q}$ correspond to different operators $\mathrm{O_{BB}}$



- One attractive potential V_5 and one repulsive potential V_j
- Parametrization of potentials:

$$V_X(r) = \pm \frac{\alpha_X}{r} e^{-\left(\frac{r}{d_X}\right)^2}, \qquad X \in \{5, j\}$$

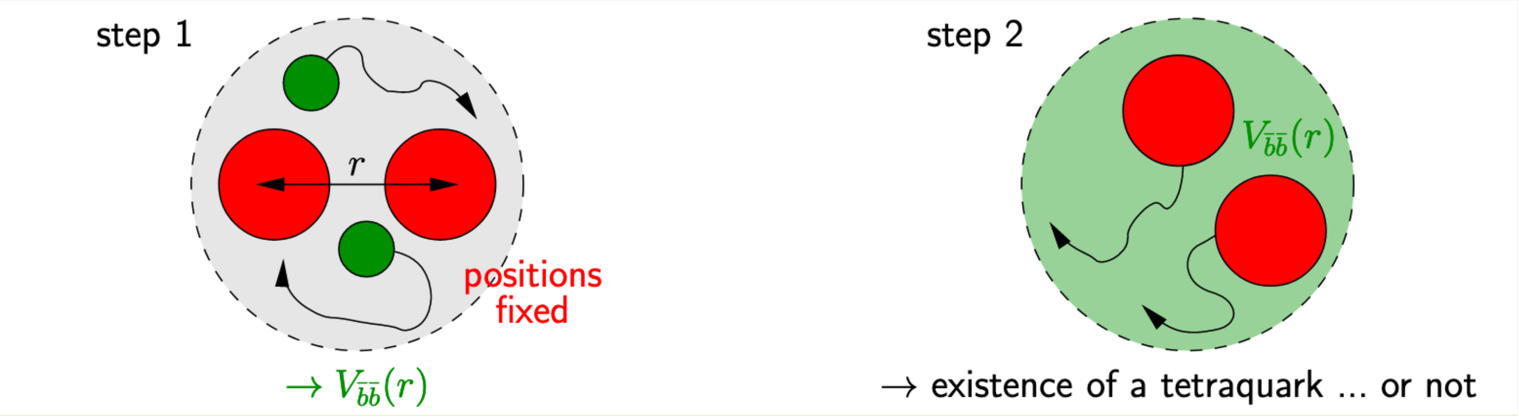
α_X: strength, d_X: depth

Our Approach: Lattice QCD + Born Oppenheimer

- **Main idea** : divide study into two independent steps

  1) Compute static potentials between heavy $\bar{b}$-quarks in the presence of light quarks using methods from lattice QCD (see P. Bicudo's talk on "Antistatic-antistatic-light-light tetraquark potentials with u, d and s quarks from lattice QCD")

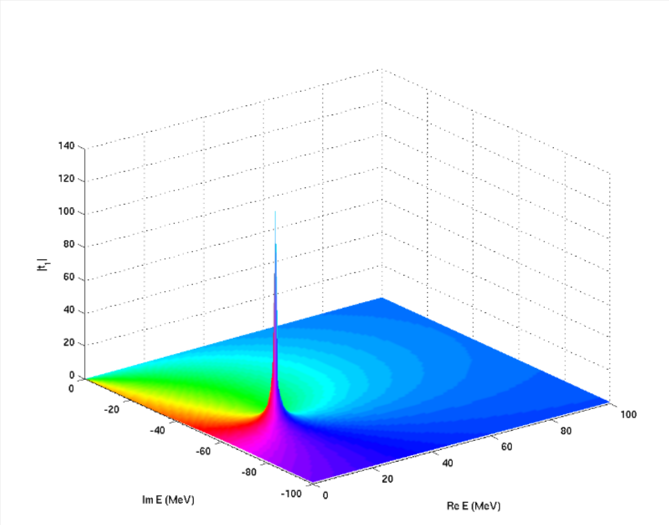
  2) Insert potentials into a $\bar{b}\bar{b}$ Schroedinger equation and look for poles in the scattering amplitude to detect bound states or resonances (content of this talk)



Tetraquark $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^-)$

Previous Work

- In **Bicudo et al,1704.02383** a resonance pole was found at $\mathrm{Re(E)} = 17^{+4}_{-4}\ \mathrm{MeV}$ above the BB threshold using lattice QCD Static potentials and methods from scattering theory
- Resonance has orbital angular momentum $\mathrm{L} = 1$ (p-wave) and $\mathrm{I}(\mathrm{J}^{\mathrm{P}}) = 0(1^{-})$
- In **Bicudo et al,1704.02383** heavy quark spin effects were neglected (Degenerate masses of B and B* mesons)



Pole of scattering amplitude from Bicudo et al, **1704.02383**

Including Heavy-Quark Spin Effects

- Different quark bilinears can be mapped to linear combinations of the pseudoscalar B- and vector B^* mesons using **Fierz identities**
- Heavy spin effects introduce mass splitting $m_{B^*} - m_B \approx 45$ MeV
- Coupling of different meson-meson combination leads to 16 x 16 Schroedinger equation
- 16 x 16 equation can be block diagonalized by considering the total spin S of the system
- Most promising candidate is 2 x 2 equation with total spin S=0 (contains **1704.02383**)

$$\left[\begin{pmatrix} 2m_B & 0 \\ 0 & 2m_{B^*} \end{pmatrix} - \frac{\hbar^2}{2\mu}\Delta\, \mathbb{1}_{2\times 2} + V_{2\times 2}(r)\right] \vec{\psi}_{2\times 2}(\vec{r}) = E\vec{\psi}_{2\times 2}(\vec{r})$$

$$V_{2\times 2}(r) = \frac{1}{4}\begin{pmatrix} V_5(r) + 3V_j(r) & \sqrt{3}\,(V_5(r) - V_j(r)) \\ \sqrt{3}\,(V_5(r) - V_j(r)) & 3V_5(r) + V_j(r) \end{pmatrix}$$

$$\vec{\psi}_{2\times 2}(\vec{r}) \mathrel{\widehat{=}} \begin{pmatrix} \mathrm{BB} \\ \frac{1}{\sqrt{3}} B_j^* B_j^* \end{pmatrix}$$

Two-Channel Scattering Formalism

- For general Energies $E > 2m_B$ the system can be cast as a scattering problem with two channel momenta k and k*
- Decompose scattering problem into an incident partial wave $\vec{\psi}_0(\vec{r})$ and an emergent spherical wave $\vec{\chi}(\vec{r})$

$$\vec{\psi}(\vec{r}) = \sum_{L,m} \begin{pmatrix} A_{BB} j_L(kr) + \chi_{BB}(r)/r \\ A_{B^*B^*} j_L(k^*r) + \chi_{B^*B^*}(r)/r \end{pmatrix} Y_{L,m}(\theta, \phi)$$

- Projecting out L=1 contribution leads to the radial equation

$$\left[\begin{pmatrix} 2m_B & 0 \\ 0 & 2m_{B^*} \end{pmatrix} - \frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2} - \frac{2}{r^2}\right) + V_{2\times2} - E\right] \begin{pmatrix} \chi_{BB}(r) \\ \chi_{B^*B^*}(r) \end{pmatrix} = -\begin{pmatrix} \left(\frac{V_5+3V_j}{4}\right) r A_{BB} j_1(kr) + \frac{\sqrt{3}}{4}(V_5 - V_j) r A_{B^*B^*} j_1(k^*r) \\ \frac{\sqrt{3}}{4}(V_5 - V_j) r A_{BB} j_1(kr) + \left(\frac{3V_5+V_j}{4}\right) r A_{B^*B^*} j_1(k^*r) \end{pmatrix}$$

Boundary Conditions and T Matrix

- We impose scattering boundary conditions for large r on each emergent wave component for pure BB and pure B*B* incident wave configurations

$$\chi_\alpha(r) \propto i r t_{BB;\alpha} \, h_\ell^{(1)}(k_\alpha r) \qquad \text{for} \;\; (A_{BB}, A_{B^*B^*}) = (1,0)$$

$$\chi_\alpha(r) \propto i r t_{B^*B^*;\alpha} \, h_\ell^{(1)}(k_\alpha r) \qquad \text{for} \;\; (A_{BB}, A_{B^*B^*}) = (0,1)$$

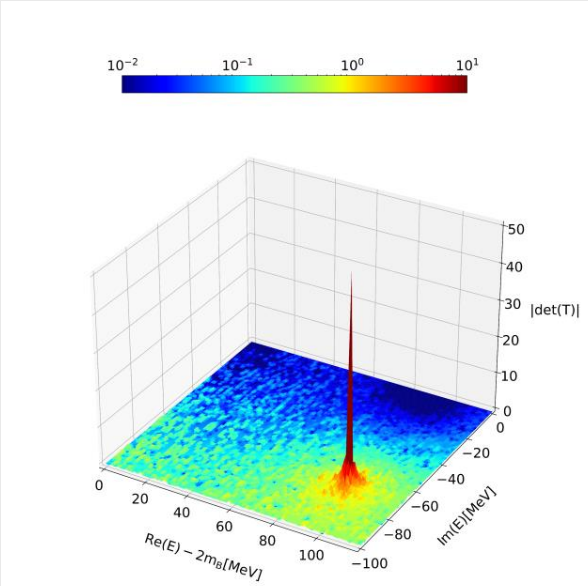
- 2 x 2 T matrix defined via $t_{\beta;\alpha}$ components

$$\mathbf{T} = \begin{pmatrix} t_{BB;BB} & t_{BB;B^*B^*} \\ t_{B^*B^*;BB} & t_{B^*B^*;B^*B^*} \end{pmatrix}$$

- **Bound states** correspond to real axis T matrix poles on the first Riemann sheet
- **Resonances** correspond to T matrix poles with nonzero imaginary energy

Pole Search in the Complex Plane

- Schroedinger equation solved for complex energies with standard Runge-Kutta 4th + shooting method
- Poles correspond to roots of 1/det(**T**)
- Resonance found with $\mathrm{Re}(E) - 2m_B = 94.4^{+8.6}_{-9.8}$ MeV and $\Gamma = 140^{+30}_{-38}$ MeV
- Resonance is located 4 MeV above the B*B* threshold



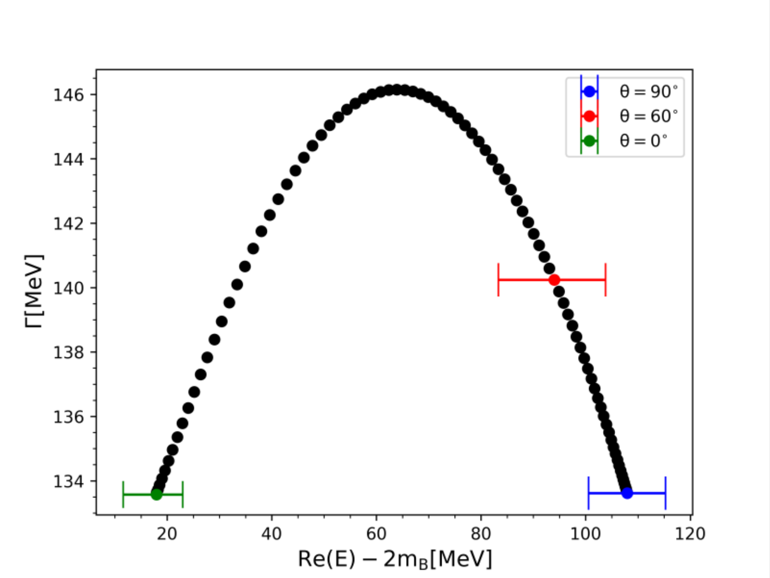
- **Change of 76MeV in real part of energy compared to resonance found in Bicudo et al , 1704.02383 is explained on the following slides**

Varying the Potential Matrix

- Potential matrix has a rotation matrix like structure with rotation angle $\theta = \pi/3$

$$V_{2\times 2} = \begin{pmatrix} \cos^2(\theta)V_5 + \sin^2(\theta)V_j & \sin(\theta)\cos(\theta)(V_5 - V_j) \\ \sin(\theta)\cos(\theta)(V_5 - V_j) & \sin^2(\theta)V_5 + \cos^2(\theta)V_j \end{pmatrix}$$

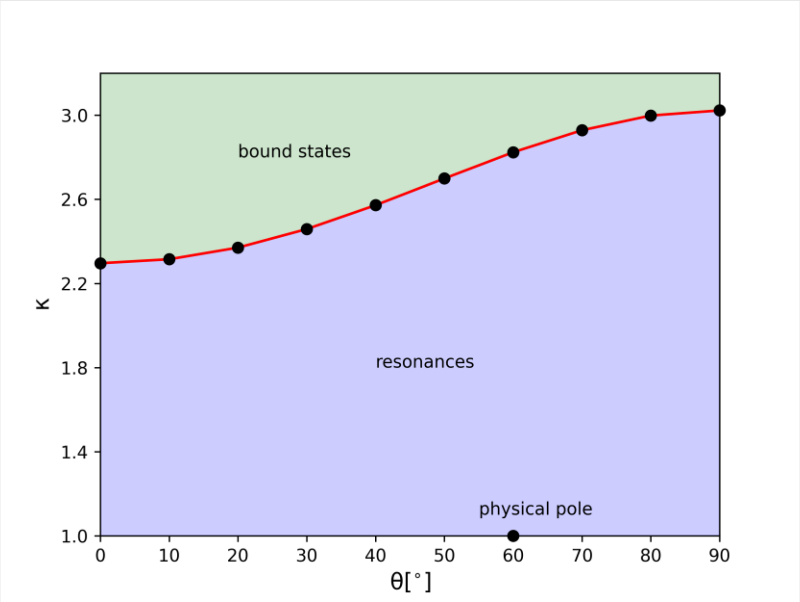
[J. Hoffmann, M. Wagner, unpublished ongoing work]



- For $\theta = 0$ and $\theta = \pi/2$ poles are found with same width but differ by $2m_{B*}$ in the real energy
- For $\theta \in [0°; 36°)$ width increases, reaches a maximum at $\theta = 36°$, decreases for $\theta \in (36°; 90°]$
- Pole trajectory is a result of splitting of the attractive potential between the channels
- Maximum in width naively expected at $\theta = \pi/4$

Varying the Heavy-Quark Mass and the Mixing Angle

- Mass splitting decreases for heavier than physical $m_b = \kappa m_{b,phys}$ according to the HQET relation $m_{B*} - m_B \propto 1/\kappa$
- Decreasing mass splitting is an effective attractive contribution to the potential
- It is possible to vary θ and κ simultaneously
- For each $\theta \in \{0°, 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, 90°\}$ the parameter κ was varied



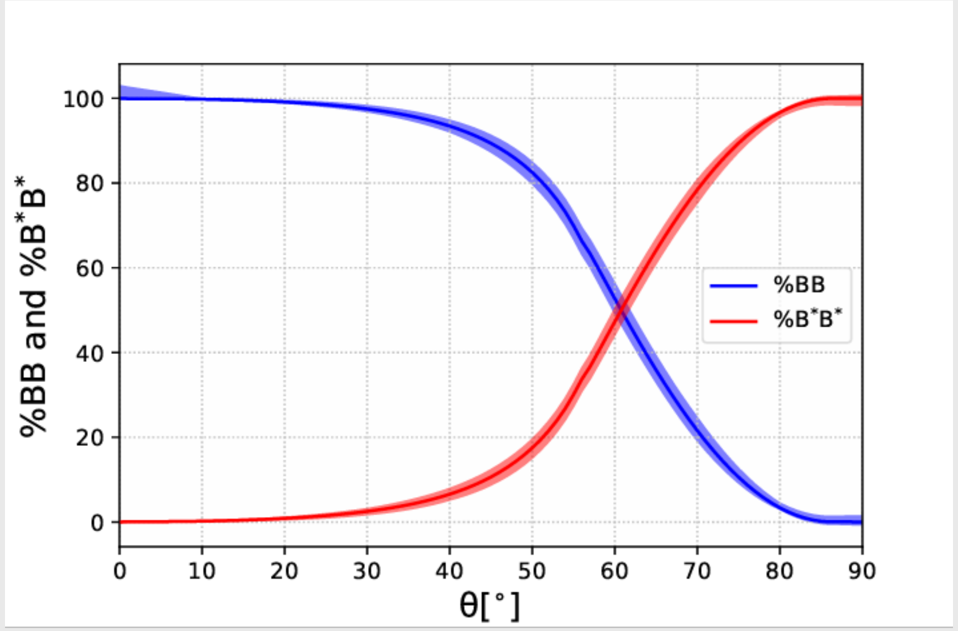
- **Bound states start to appear for large enough κ**
- **Slope of boundary correlated with splitting of attractive potential**

Meson Composition

- Meson channel percentages given by wave function

$$\%X = \frac{X}{\%BB + \%B^*B^*}, \qquad X = \int_0^{r_{\max}} |\chi_X(r)|^2 \, dr, \qquad X \in \{BB, B^*B^*\}$$

[Hoffmann et al, 2211.15765]



- Competition between the ore attractive B*B* channel and the lower mass in the BB channel
- At $\theta = \pi/3$ the composition is approximately 50%-50%
- Transition point is shifted by mass splitting to larger angles θ

Finite Volume Scattering Study

- Full dynamical lattice QCD study consists of two steps
  1) Calculation of ground state and excited finite volume energy levels
  2) Solution of two-particle quantization condition (QC2)
- Full lattice QCD study for the $I(J^P) = 0(1^-)$ resonance bears following complications
  - Coupled channel scattering of BB and B*B*
    (see related talk by **Travis White** on coupled channel DD* and D*D* scattering)
  - Left hand cut effects due to one- and two pion exchange in B*B* channel
    (see related talks by **A. Raposo** and **S. Aoki**)
  - Effects from three particle $BB\pi$ channel (50 MeV above B*B* threshold)
    (see related talks by **Stephe Sharpe** and **Sebastian Dawid** on $\eta\pi\pi + \bar{K}K\pi$ and $DD\pi$ scattering)
- $I(J^P) = 0(1^-)$ resonance : another application of three particle formalism?

Tetraquark $\bar{b}\bar{c}ud$ with $I(J^P) = 0(0^+), 0(1^+)$

Coupled Channel Scattering for $\bar{b}\bar{c}ud$, $J^P \in \{0^+, 1^+\}$

- Two heavy flavors $Q \in \{b, c\}$ can be combined symmetrically or antisymmetrically

- Tetraquark with $J^P = 0^+$ results from antisymmetric heavy flavor

- Two-channel equation for $J^P = 0^+$ with mesonic channels BD, B*D*, three-channel equation for $J^P = 1^+$ with mesonic channels B*D, BD*, B*D*

$$\left[\begin{pmatrix} m_B + m_D & 0 \\ 0 & m_{B^*} + m_{D^*} \end{pmatrix} - \frac{1}{2\mu_{bc}}\left(\frac{d^2}{dr^2} - \frac{L(L+1)}{r^2}\right) \otimes \mathbb{1}_{2\times 2} + H_{\text{int}}\right] \begin{pmatrix} \chi_{BD}(r) \\ \chi_{B^*D^*}(r) \end{pmatrix}$$
$$= -\frac{r}{4} \begin{pmatrix} (V_5 + 3V_j)\ \alpha_{BD} j_L(k_{BD^*} r) + \sqrt{3}(V_5 - V_j)\ \alpha_{B^*D^*} j_L(k_{B^*D^*} r) \\ \sqrt{3}\,(V_5 - V_j)\ \alpha_{BD} j_L(k_{BD} r) + (3V_5 + V_j)\ \alpha_{B^*D^*} j_L(k_{B^*D^*} r) \end{pmatrix}$$

$$\left[\begin{pmatrix} m_{B^*} + m_D & 0 & 0 \\ 0 & m_B + m_{D^*} & 0 \\ 0 & 0 & m_{B^*} + m_{D^*} \end{pmatrix} - \frac{1}{2\mu_{bc}}\left(\frac{d^2}{dr^2} - \frac{L(L+1)}{r^2}\right) \otimes \mathbb{1}_{3\times 3} + H_{\text{int}}\right] \begin{pmatrix} \chi_{B^*D}(r) \\ \chi_{BD^*}(r) \\ \chi_{B^*D^*}(r) \end{pmatrix}$$
$$= -\frac{r}{4} \begin{pmatrix} (V_5 + 3V_j)\ \alpha_{B^*D} j_L(k_{B^*D} r) + (V_j - V_5)\ \alpha_{BD^*} j_L(k_{BD^*} r) + \sqrt{2}(V_5 - V_j)\ \alpha_{B^*D^*} j_L(k_{B^*D^*} r) \\ (V_j - V_5)\ \alpha_{B^*D} j_L(k_{B^*D} r) + (V_5 + 3V_j)\ \alpha_{BD^*} j_L(k_{BD^*} r) + \sqrt{2}(V_j - V_5)\ \alpha_{B^*D^*} j_L(k_{B^*D^*} r) \\ \sqrt{2}\,(V_5 - V_j)\ \alpha_{B^*D} j_L(k_{B^*D} r) + \sqrt{2}(V_j - V_5)\ \alpha_{BD^*} j_L(k_{BD^*} r) + 2(V_5 + V_j)\ \alpha_{B^*D^*} j_L(k_{B^*D^*} r) \end{pmatrix}$$

Coupled Channel Scattering for $\bar{b}\bar{c}ud$, $J^P \in \{0^+, 1^+\}$

- Naive treatment with one channel equation leads to a bound state with binding energy $\Delta E \approx -0.9$ MeV (consistent with weakly bound state predictions from [1-2])
- For treatment with heavy spin effects :
  - Virtual bound state is found for $J^P = 0^+$ at 106MeV below BD threshold on (-,+) Riemann sheet
  - Virtual bound state is found for $J^P = 1^+$ at 100MeV below B*D threshold on (-,+,+) Riemann sheet

[1] Alexandrou et al, 2312.02925

[2] Padmanath et al, 2307.14128

Summary

Tetraquark $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^-)$:

- Resonance was found 4 MeV above the B*B* threshold
- Resonance has nearly equal BB and B*B* percentages

Tetraquark $\bar{b}\bar{c}ud$ with $I(J^P) = 0(0^+), 0(1^+)$:

- Virtual bound state found for $J^P = 0^+$ at 106 MeV below BD threshold
- Virtual bound state found for $J^P = 1^+$ at 100 MeV below B*D threshold

➡ **Attractive potentials appear to be too weak** (**see P. Bicudo's talk**)

Questions?

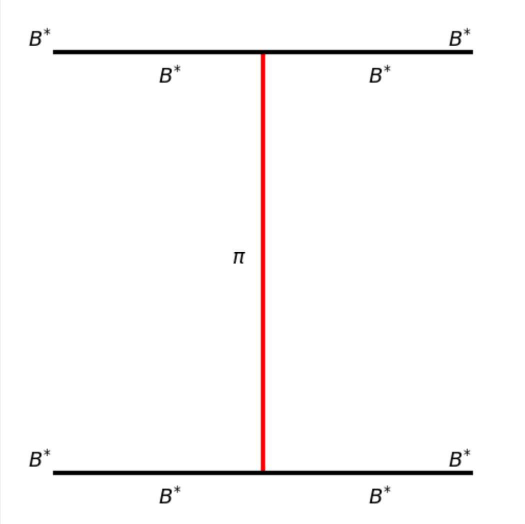
Backup

Left Hand Cut Problem

- One pion exchange invalidates QC2 below t- and u-channel branch points
- QC2 can be replaced by modified two particle quantization condition

$$\det[S(P_j, L)^{-1} + \xi^\dagger \mathcal{K}_{\text{os}}(P_j)\xi + 2(g_{B^*B^*\pi})^2 \mathcal{T}(P_j)] = 0$$

[Hansen et al, 2311.18793]



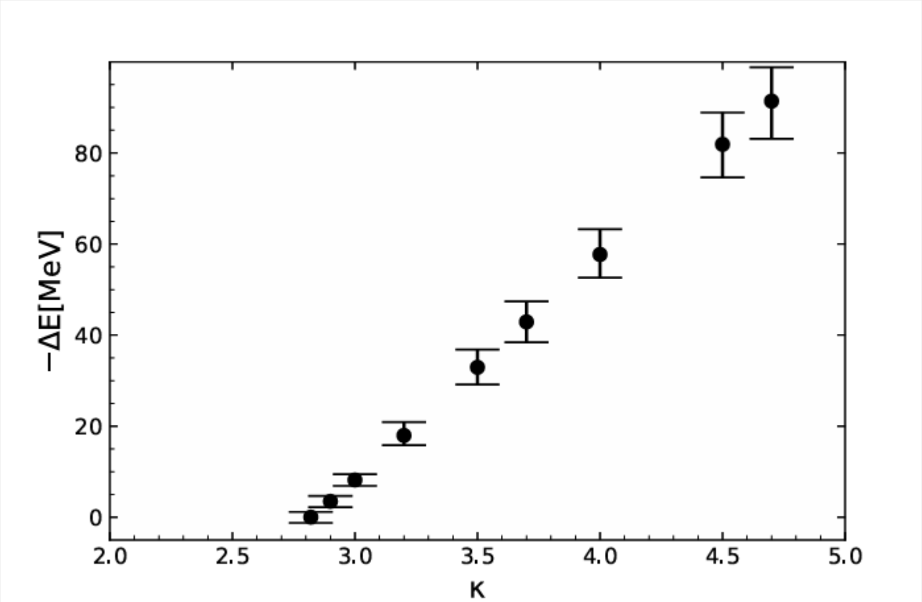
- Possible Lagrangian respecting all symmetries is

$$\mathcal{L}_{B^*B^*\pi} = g_{B^*B^*\pi}\epsilon^{\mu\nu\alpha\beta}\partial_\mu B_\nu^{*+}\partial_\alpha B_\beta^{*0}\pi^-$$

- Does not rely on specific Effective field theory models
- Scattering amplitude must be extracted via Integral equations

Varying Only the Heavy-Quark Mass

- We vary not only the potential matrix but also the heavy quark mass
- Mass splitting decreases for heavier than physical $m_b = \kappa m_{b,phys}$ according to the HQET relation $m_{B*} - m_B \propto 1/\kappa$
- Decreasing mass splitting is an effective attractive contribution to the potential



- Resonances found for $\kappa \in [1.0; 2.82)$
- For $\kappa > 2.82$ bound states exist