Hybrid static potentials and gluelumps on $N_f = 3 + 1$ ensembles

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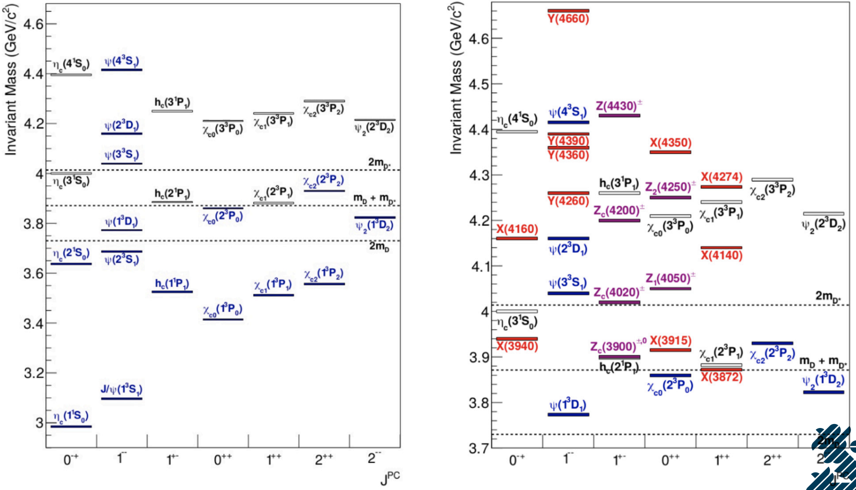
Motivation: charmonium spectrum



2003 and nowadays (above open charm threshold)

Hybrid static potentials

- *Hybrid mesons* are states consisting of a quark-antiquark pair bound by an excited gluon field.
- Λ_η^ϵ, cylindrical symmetry group $D_{\infty h} = D_\infty \times Z_2$ (D_{4h})
- angular momentum about molecular axis: $\Lambda = 0, 1, 2, 3, \ldots \equiv \Sigma, \Pi, \Delta, \Phi, \ldots$
- $\eta = +, - \equiv g, u$ under parity and charge conjugation $\mathcal{P} \circ \mathcal{C}$
- for $\Lambda = 0$ additional parity index $\epsilon = +, -$
- $R \to \infty$, multiplets associated with excitations of a *relativistic string*
- $R \to 0$, multiplets associated with *gluelumps*, which are energy levels of QCD in the presence of a color-octet source ($\mathbf{3} \otimes \mathbf{3}^* = \mathbf{1} \oplus \mathbf{8}$)

Gluelumps

- A system containing a valence gluon connected by an adjoint string to an adjoint source is called *gluelump*.
  - adjoint source $\rightarrow$ infinitely heavy gluon $\rightarrow$ *gluino*?
  - no physical object, not measured experimentally
- Why consider gluelumps?
  - gluelump masses define decay of field strength correlators in QCD vacuum
    $G_{gluelump}(x,y) = \langle F^a_{\mu\nu}(x) F^b_{\lambda\sigma}(y) \rangle$
  - inverse mass of lowest gluelump equals the gluon correlation length T_g
  - testing ground for models of low energy QCD (MIT bag model, flux tube model, pNRQCD$\rightarrow$BOEFT)
  - needed for getting insights in non-perturbative features of QCD (heavy quarkonia)

Laplace trial states

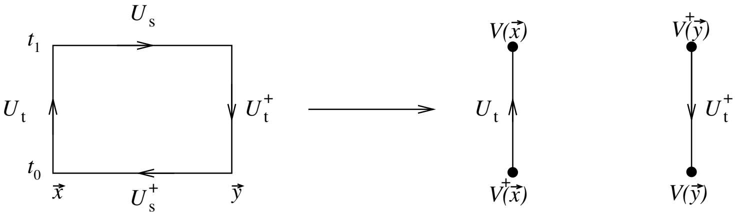
- N_v eigenvectors $v(\vec{x})$ of the 3D covariant Laplace operator
- spatial Wilson line: $U'_s(\vec{x},\vec{y}) = G(\vec{x})U_s(\vec{x},\vec{y})G^\dagger(\vec{y})$

$$v'(\vec{x})v'^\dagger(\vec{y}) = G(\vec{x})v(\vec{x})v^\dagger(\vec{y})G^\dagger(\vec{y})$$

- Wilson loop of size $(R = |\vec{r}| = |\vec{x}-\vec{y}|) \times (T = |t_1 - t_0|)$

$$\Sigma_g^+(R,T) = \langle \mathrm{tr}\,[U_t(\vec{x};t_0,t_1)U_s(\vec{x},\vec{y};t_1)U_t^\dagger(\vec{y};t_0,t_1)U_s^\dagger(\vec{x},\vec{y};t_0)]\rangle$$

$$\to \langle \sum_{i,j}^{N_v} \mathrm{tr}\,[U_t(\vec{x})\rho_j(t_1)v_j(\vec{x},t_1)v_j^\dagger(\vec{y},t_1)U_t^\dagger(\vec{y})\rho_i(t_0)v_i(\vec{y},t_0)v_i^\dagger(\vec{x},t_0)]\rangle$$



Laplacian static-hybrid operators

- Gaussian profile functions: $\rho_i^{(k)}(\lambda_i) = \exp(-\lambda_i^2/2\sigma_k^2)$
- we can realize gluonic excitations via covariant derivatives
  $\nabla_{\vec{k}} v(\vec{x}) = \frac{1}{2}[U_k(\vec{x})v(\vec{x}+\hat{k}) - U_k^\dagger(\vec{x}-\hat{k})v(\vec{x}-\hat{k})]$
- $\Sigma_{u/g}^{\mp}(R,T) = \sum_{\vec{x},t_0,i,j,\vec{k}||\vec{r}=\vec{y}-\vec{x}}$
  $\Big\langle \mathrm{tr}\, \Big[U_t(\vec{x};t_0,t_1)\rho(\lambda_j)\{[\nabla_{\vec{k}}v_j](\vec{x},t_1)v_j^\dagger(\vec{y},t_1) \pm v_j(\vec{x},t_1)[\nabla_{\vec{k}}v_j]^\dagger(\vec{y},t_1)\}$
  $U_t^\dagger(\vec{y};t_0,t_1)\rho(\lambda_i)\{[\nabla_{\vec{k}}v_i](\vec{y},t_0)v_i^\dagger(\vec{x},t_0) \pm v_i(\vec{y},t_0)[\nabla_{\vec{k}}v_i]^\dagger(\vec{x},t_0)\}\Big]\Big\rangle$
- $\Pi_{u/g}(R,T) = \Pi_{\mp}(R,T) = \sum_{\vec{x},t_0,i,j,\vec{k}\perp\vec{r}=\vec{y}-\vec{x}}$
  $\Big\langle \mathrm{tr}\, \Big[U_t(\vec{x};t_0,t_1)\rho(\lambda_j)\{[\nabla_{\vec{k}}v_j](\vec{x},t_1)v_j^\dagger(\vec{y},t_1) \pm v_j(\vec{x},t_1)[\nabla_{\vec{k}}v_j]^\dagger(\vec{y},t_1)\}$
  $U_t^\dagger(\vec{y};t_0,t_1)\rho(\lambda_i)\{[\nabla_{\vec{k}}v_i](\vec{y},t_0)v_i^\dagger(\vec{x},t_0) \pm v_i(\vec{y},t_0)[\nabla_{\vec{k}}v_i]^\dagger(\vec{x},t_0)\}$
- $\ldots \nabla_i \nabla_j$

Gluelump operators

- 35 different 3D loop shapes $W_s(\vec{x}, t)$
- fix parity by considering the sum (P=+1) or difference (P=-1) of each loop with its parity twin
- fix charge conjugation by using $W_s^{P,C=\pm} = W_s^P \pm W_s^{P\,\dagger}$
- elements of the cubic group can act on $W_s^{PC}(\vec{x}, t)$, resulting in 24 loops which form a basis which generates the regular (or permutation) representation of the group
- linear combinations of the 24 loops project to the five irreps $\Lambda \in \{A1, A2, E, T1, T2\}$
- projection coefficients differ from those for glueballs, since symmetries have to hold locally

Gluelump correlator

$$C_{\Lambda^{PC}}(t_1 - t_0) =$$

$$\sum_{\vec{x},t_0} \{\mathrm{tr}\,[W_{\Lambda^{PC}}(\vec{x},t_0)U_t(\vec{x},t_0,t_1)W^{\dagger}_{\Lambda^{PC}}(\vec{x},t_1)U^{\dagger}_t(\vec{x},t_0,t_1)]$$

$$-\mathrm{tr}\,[W_{\Lambda^{PC}}(\vec{x},t_0)]\mathrm{tr}\,[W^{\dagger}_{\Lambda^{PC}}(\vec{x},t_1)]/3\}$$

effective masses contain a self-energy contribution $m_{self}(a)$ from adjoint color sources, which differs from the self-energy in the (hybrid) static potentials $V_{self}(a)$, in leading order PT

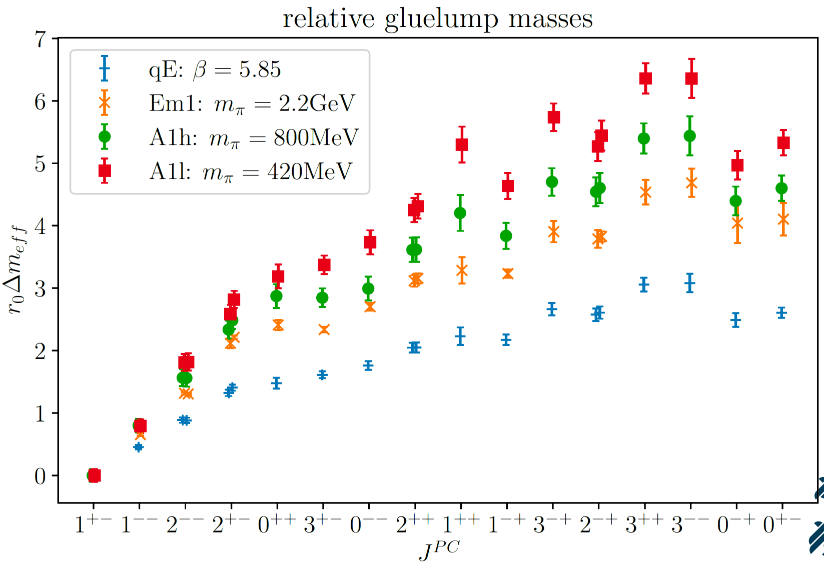
$$m_{self}(a) = \frac{C_A}{C_F}\frac{V_{self}(a)}{2} = \frac{N^2}{N^2-1}V_{self}(a) > V_{self}(a),$$

with $C_A = N$ and $C_F = (N^2-1)/2N$ the quadratic Casimir invariants of the adjoint and fund. representations of $SU(N)$

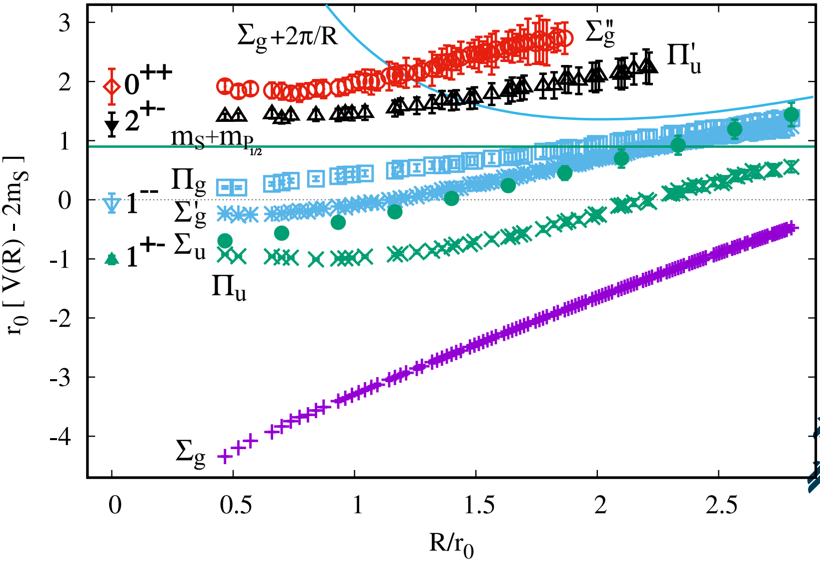
Lattice ensembles

ens.	qE	Em1	A1h	A1	A2
volume	48×24^3	48×24^3	96×32^3	96×32^3	128×48^3
β	5.85	5.3	3.24	3.24	3.24
N_f	-	0+2	3+1	3+1	3+1
κ_l	-	-	0.13392	0.134407	0.134396
κ_c	-	0.13270	0.12834	0.12784	0.12798
m_π	-	2.2(1) GeV	788(6) MeV	406(3) MeV	409(1) MeV
r_0/a	4.2612(42)	4.2866(24)	7.278(27)	9.023(63)	8.998(54)
t_0/a^2	-	1.8477(3)	5.078(14)	7.438(32)	7.434(20)
a [fm]	0.0662(12)	0.0658(10)	0.0690(11)	0.05359(15)	0.05355(13)
N_{cfg}	6480	7098	4000	4000	2000
N^l_{vec}	-	-	200	100	100
N^l_{per}	-	-	2000	4000	2000
N^c_{vec}	-	200	200	200	-
N^c_{per}	-	1000	2000	4000	-

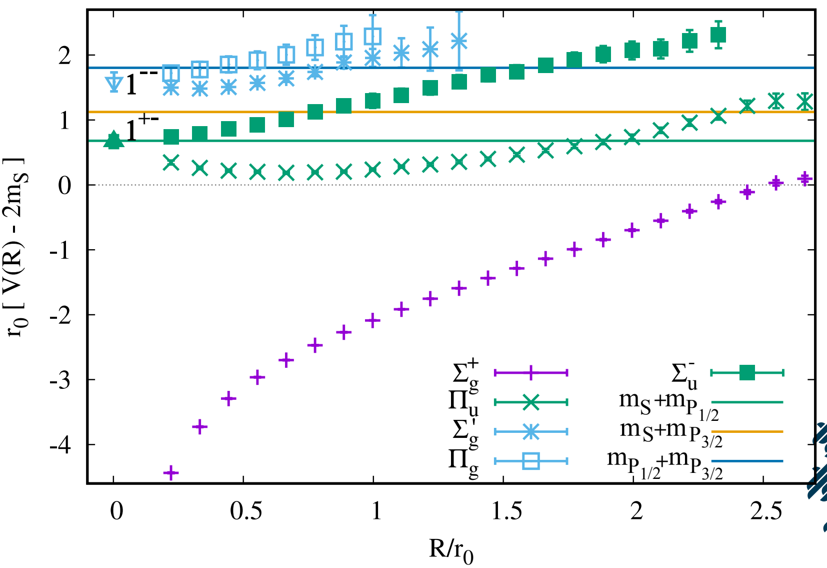
for $N_f = 3+1$ ensembles we use open b.c., Lüscher-Weisz gauge action
Wilson quarks with a non-perturbatively determined clover coefficient in a massive O(a) improvement scheme ➜ *Fritzsch, 2018*


relative gluelump masses
qE: $\beta = 5.85$
Em1: $m_\pi = 2.2$GeV
A1h: $m_\pi = 800$MeV
A1l: $m_\pi = 420$MeV
$r_0 \Delta m_{eff}$
0
1
2
3
4
5
6
7
1^{+-} 1^{--} 2^{--} 2^{+-} 0^{++} 3^{+-} 0^{--} 2^{++} 1^{++} 1^{-+} 3^{-+} 2^{-+} 3^{++} 3^{--} 0^{-+} 0^{+-}
J^{PC}

Static-hybrid states, $N_f = 2$, $m_\pi = 2.2$GeV



Preliminary!!! $N_f = 3+1$, $m_\pi = 420$ MeV



Conclusions & Outlook

- ✓ hybrid static potentials (hybrid meson masses) from Laplacian trial states with covariant derivatives of eigenvectors as gluonic excitations
- ✓ effective gluelump masses from extended Wilson loop basis correlators ($R \to 0$ limit)
- ✓ inputs for Born-Oppenheimer Effective Field Theories
- 🔧 BOEFT allows systematically improvable approximations to various properties of hadrons (XYZ)
- 🔧 more (hybrid) static(-light) and multi-quark potentials
- 🔧 hybrid string breaking study non-trivial (various avoided and broad level crossings)
- ? questions, discussion, many possible applications...

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