Beautiful exotics in a non-perturbatively tuned Lattice NRQCD setup

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Two types of beautiful exotics

1. The B_s cousins of the $D^*_{s0}(2317)$ and $D_{s1}(2460)$
   - $D^*_{s0}(2317)$ and $D_{s1}(2460)$ observed as narrow peaks at the B-factories
   - Striking example of heavy-quark hadrons not well described by naive quark models
   - Their B_s cousins are not yet seen in experiment and can be predicted
2. The $I(J^P) = 0(1^+)$ $ud\bar{b}\bar{b}$ tetraquark, the T_{bb}
   - Most concrete pure-tetraquark candidate phenomenologically and from the lattice
   - Cousin of the T_{cc} but likely has quite different physics
     T_{bb} **bound by** ≈ 100 **MeV**, T_{cc} by 360 KeV
   - In the diquark picture:
     - "Good" light diquark $(u^T C\gamma_5 d)$ - lighter diquark increases binding
     - Color-Coulomb heavy antidiquark $(\bar{b}C\gamma_i\bar{b}^T)$ - deeper binding as heavy mass gets heavier

NRQCD action

Typical tadpole-improved NRQCD action (here we will use n=4)

`Lepage et al., PRD 46, 4052-4067 (1992)`

$$
\begin{aligned}
H_0 &= -\frac{1}{2aM_0}\Delta^2, \\
H_I &= \left(-c_1\frac{1}{8(aM_0)^2} - c_6\frac{1}{16n(aM_0)^2}\right)(\Delta^2)^2 + c_2\frac{i}{8(aM_0)^2}(\tilde{\Delta}\cdot\tilde{E} - \tilde{E}\cdot\tilde{\Delta}) + c_5\frac{\Delta^4}{24(aM_0)} \\
H_D &= -c_3\frac{1}{8(aM_0)^2}\sigma\cdot\left(\tilde{\Delta}\times\tilde{E} - \tilde{E}\times\tilde{\Delta}\right) - c_4\frac{1}{8(aM_0)}\sigma\cdot\tilde{B} \\
\delta H &= H_I + H_D.
\end{aligned}
$$

Propagators generated through symmetric evolution equation

$$
G(x,t+1) = \left(1-\frac{\delta H}{2}\right)\left(1-\frac{H_0}{2n}\right)^n \tilde{U}_t(x,t_0)^\dagger \left(1-\frac{H_0}{2n}\right)^n \left(1-\frac{\delta H}{2}\right) G(x,t).
$$

- We also tune a $\mathcal{O}(v^6)$ action with tree-level coefficients for the higher order terms

Neural net (RHQ and) NRQCD tuning and setup

R.J. Hudspith, DM, PRD 106, 034508 (2022)
R.J. Hudspith, DM, PRD 107, 114510 (2023)

- Calculate runs with a random distribution for the action parameters
- Let the neural network make parameter predictions
- Due to additive mass we must only consider splittings $\rightarrow$ we subtract the η_B from all states
- Perform tuning at $SU(3)_f$-symmetric point
- Gauge-fixed wall sources
- Tuning precision is about 1%

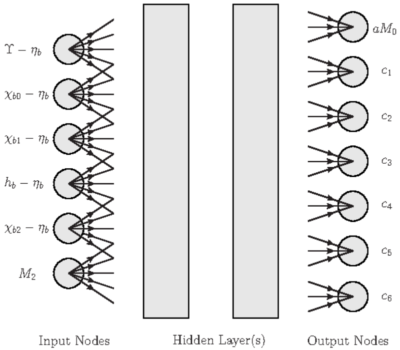
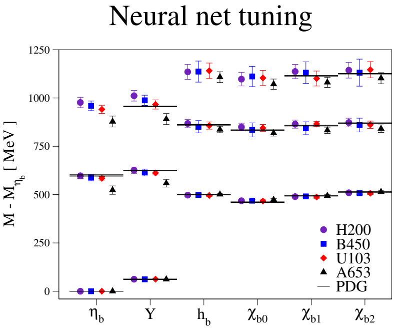
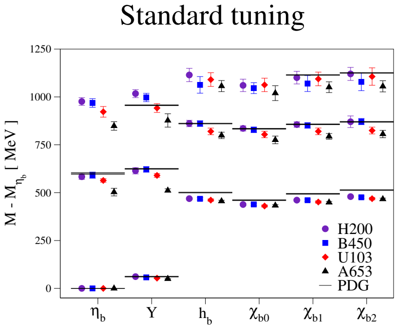


Figure: Schematic picture of our NRQCD setup

NRQCD Neural Net Tuning: Stable s- and p-wave bottomonia





- Higher S- and P-wave states serve as a check whether our tuning leads to reasonable results
- Main results from the lattice spacing of U103; H200 used to estimate systematics

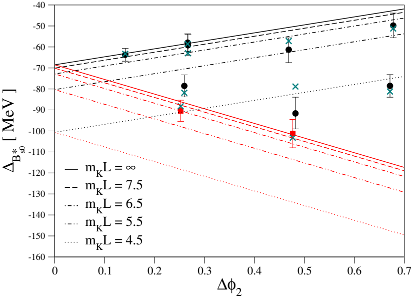
B_s: Chiral – infinite volume extrapolation

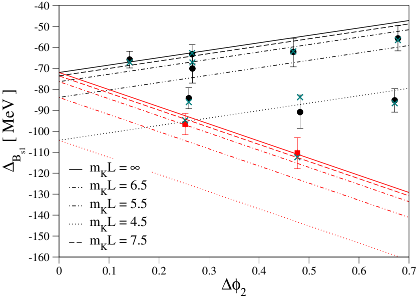
- We explore the previously predicted $J^P = 0^+$ and 1^+ bound states
- We use simple single-hadron interpolators with gauge-fixed wall sources and smeared sinks
- Mainly the CLS $\mathrm{Tr}M$ = const trajectory and 2 m_S = const ensembles

Combined extrapolation:

$$\Delta_{B^*_{s0}/B_{s1}}(\Delta\phi_2, m_K L, a) = \Delta_{B^*_{s0}/B_{s1}}(0, \infty, a)\left(1 + A\Delta\phi_2 + Be^{-m_K L}\right)$$

$$\Delta\phi_2 = \phi_2^{\mathrm{Lat}} - \phi_2^{\mathrm{Phys}} \qquad ; \qquad \phi_2 = 8t_0 m_\pi^2$$





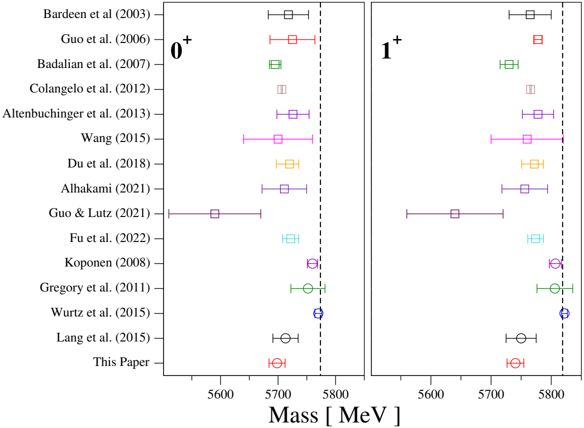
Systematic uncertainties and final result

Resulting binding energies:

$$\Delta_{B^*_{s0}}(0,\infty,0) = -75.4(3.0)_{\text{Stat.}}(13.7)_{\text{a}}\ [\text{MeV}],$$
$$\Delta_{B_{s1}}(0,\infty,0) = -78.7(3.7)_{\text{Stat.}}(13.4)_{\text{a}}\ [\text{MeV}].$$

- Small uncertainty from statistics + combined extrapolation
- Largest systematics from usage of NRQCD/discretization effects
- Central value shifted by applying half the mass difference between H200 and U103
- All other explored uncertainties (finite volume shapes, modified quark-mass dependence, etc.) small

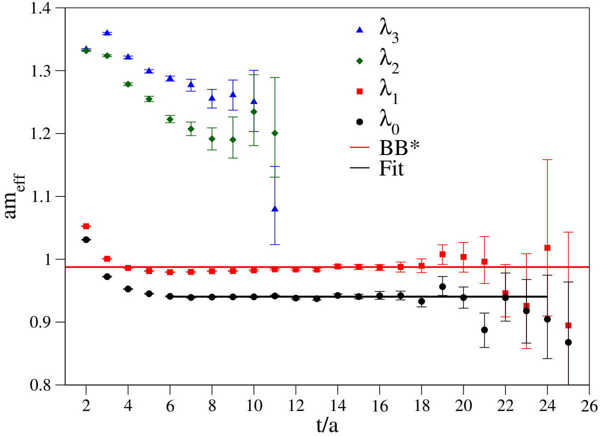
Comparison to the literature



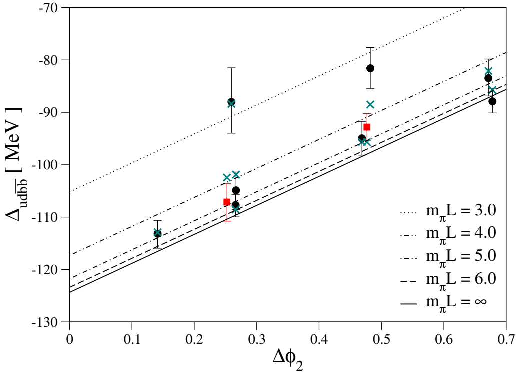
- Results agree well with models based on unitarized χPT
- Improved uncertainty estimate over older Lattice calculations

T_{bb} – Basis and effective masses (on N101)

$$D = ({u_a}^T C\gamma_5 d_b)(\bar{b}_a C\gamma_i \bar{b}_b^T), \quad E = ({u_a}^T C\gamma_t\gamma_5 d_b)(\bar{b}_a C\gamma_i\gamma_t \bar{b}_b^T),$$
$$M = (\bar{b}\gamma_5 u)(\bar{b}\gamma_i d) - [u \leftrightarrow d], \quad N = (\bar{b} I u)(\bar{b}\gamma_5\gamma_i d) - [u \leftrightarrow d].$$



Combined mass and volume extrapolations



- Ansatz for a deeply-bound state:

$$\Delta_{ud\bar{b}\bar{b}}(\Delta\phi_2, m_\pi L, a) = \Delta_{ud\bar{b}\bar{b}}(0, \infty, a)(1 + A\Delta\phi_2 + Be^{-m_\pi L}).$$

- Strong $e^{-m_\pi L}$ volume effects and deeper binding at lighter pion mass.

Varying the NRQCD tuning

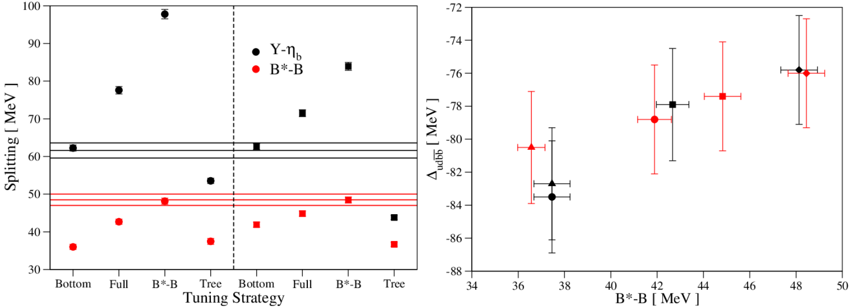
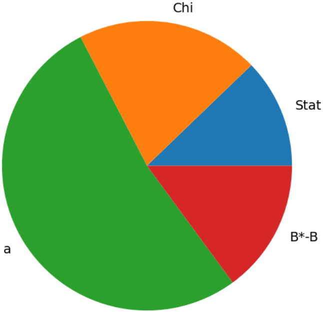


Figure: Alternative tuning strategies with/without B-mesons and higher-order terms (left). Clear correlation of the $B^* - B$ splitting with the T_{bb} binding. (right)

- Simultaneously reproducing both hyperfine splittings seems impossible
- Tree-level performs poor; For our strategies higher order terms help.
- Shallower T_{bb} binding with increased $B^* - B$ splitting.

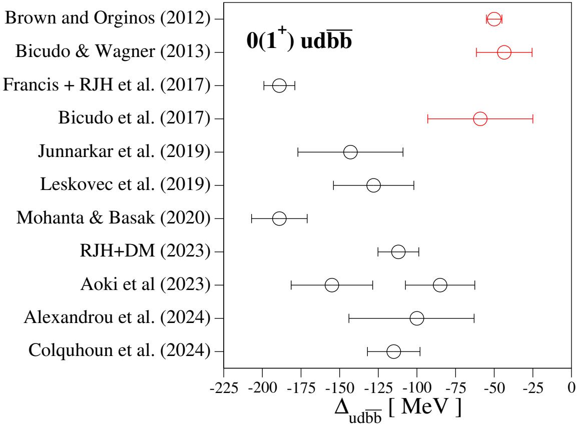
T_{bb} – quantifying systematics



$$\Delta_{ud\bar{b}\bar{b}}(0,\infty,0) = -112.0(2.7)_{\text{Stat.}}(4.5)_{\chi}(11.6)_{a}(3.3)_{B^*-B}$$

- $(..)_a$ uncertainty from comparison of the results for two lattice spacings (H200 vs. U103)
- Two leading systematic uncertainties come from discretization effects/ the use of Lattice NRQCD!

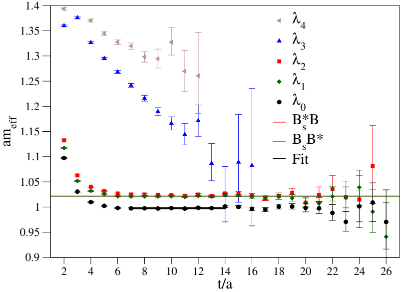
Overview of Lattice $I(J^P) = 0(1^+)$ T_{bb} determinations



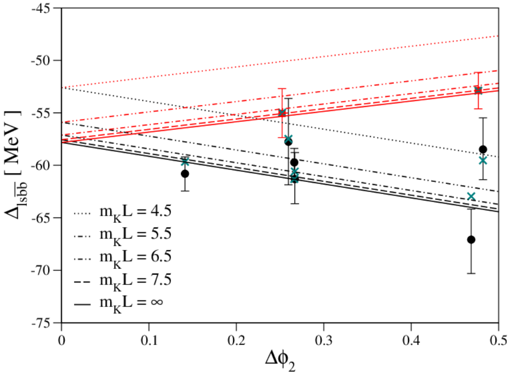
- Red: Static b-quarks; Black: Lattice NRQCD b quarks
- Interesting playground for understanding systematic uncertainties!

T_{bbs} – Basis and effective masses

$$M = (\bar{b}\gamma_5 u)(\bar{b}\gamma_i s), \quad N = (\bar{b}Iu)(\bar{b}\gamma_5\gamma_i s)$$
$$O = (\bar{b}\gamma_5 s)(\bar{b}\gamma_i u), \quad P = (\bar{b}Is)(\bar{b}\gamma_5\gamma_i u)$$
$$Q = \epsilon_{ijk}(\bar{b}\gamma_j u)(\bar{b}\gamma_k s).$$



T_{bbs} – chiral and infinite volume extrapolation

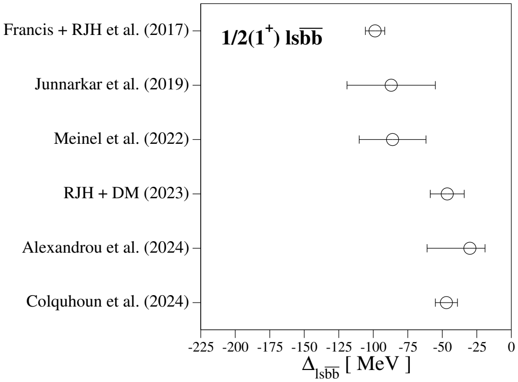


- Chiral/infinite-volume Ansatz:

$$\Delta_{\ell s\bar{b}\bar{b}}(\Delta\phi_2, m_K L, a) = \Delta_{\ell s\bar{b}\bar{b}}(0, \infty, a)\left(1 + A\Delta\phi_2 + Be^{-m_K L}\right)$$

- Large $e^{-m_K L}$ volume effects.
- Consistent with light-diquark picture.

Overview of lattice T_{bbs} determinations



- Close/overlapping EM threshold $BB_s\gamma$, still possible that it is narrow and decays weakly

Conclusions and Outlook

- Positive-parity heavy-light mesons
  - NRQCD calculation with full uncertainty estimate for B_0^* and B_{s1} $\rightarrow$ refined predictions for LHCb, BelleII
- Explicitly exotic heavy-quark tetraquarks
  - Lattice QCD is good at determining deeply-bound states and can rule out phenomenological models for states not yet observed in experiment
  - The calculations are systematically-improvable and we are seeing convergence for the easiest-to-compute quantities such as the T_{bb}
  - The smoking-gun tetraquark state T_{bb} is very difficult to see in current experiments; it is worth exploring weaker-bound candidates such as T_{bc}
  - More and more indications that the multi-quark exotic spectrum at heavy masses is diverse
- Systematics from using Lattice NRQCD is now limiting our predictions for these states.
- Calculation could be further improved with RHQ action (work in progress)

Input used for the tuning

Consider only quark-line connected parts of simple meson operators

$$O(x) = (\bar{b}\Gamma(x)b)(x),$$

State	PDG mass [GeV]	$\Gamma(x)$
$\eta_b(1S)$	9.3987(20)	γ_5
$\Upsilon(1S)$	9.4603(3)	γ_i
$\chi_{b0}(1P)$	9.8594(5)	$\sigma \cdot \Delta$
$\chi_{b1}(1P)$	9.8928(4)	$\sigma_j\Delta_i - \sigma_i\Delta_j \ (i \neq j)$
$\chi_{b2}(1P)$	9.9122(4)	$\sigma_j\Delta_i + \sigma_i\Delta_j \ (i \neq j)$
$h_b(1P)$	9.8993(8)	Δ_i

Table: Table of lattice operators used and their continuum analogs.

Comparison of b and c parameters - c_E and c_B

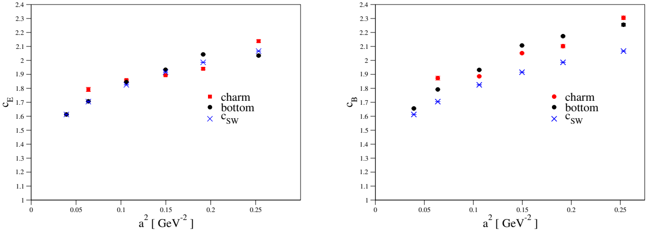


Figure: RHQ clover terms c_E and c_B for **bottom** and **charm**

As a rule of thumb $c_E \approx c_{SW}$, $c_B > c_E$. No big difference between bottom and charm!

Comparison of b and c parameters - κ, r_s, ν

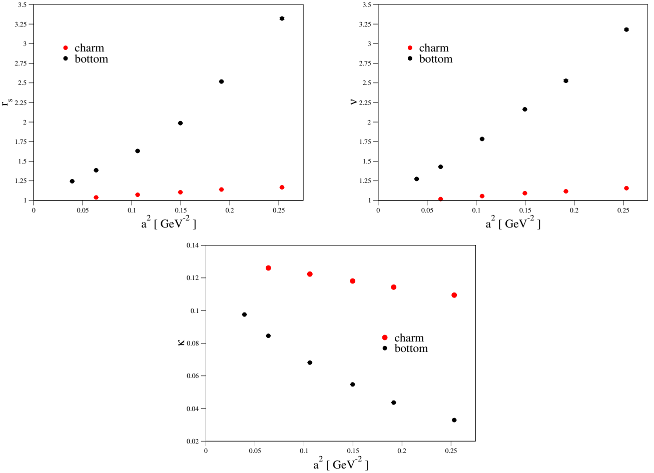


Figure: RHQ action terms r_s, ν, κ for **bottom** and **charm**

CLS ensembles used for heavy-light mesons

R.J. Hudspith, DM, PRD 107, 114510 (2023)

Ensemble	Mass trajectory	$L^3 \times L_T$	$N_{\text{Conf}} \times N_{\text{Prop}}$
U103	$\text{Tr}[M] = C$	$24^3 \times 128$	1000×23
H101	$\text{Tr}[M] = C$	$32^3 \times 96$	500×12
U102	$\text{Tr}[M] = C$	$24^3 \times 128$	732×18
H102	$\text{Tr}[M] = C$	$32^3 \times 96$	500×16
U101	$\text{Tr}[M] = C$	$24^3 \times 128$	600×18
H105	$\text{Tr}[M] = C$	$32^3 \times 96$	500×16
N101	$\text{Tr}[M] = C$	$48^3 \times 128$	537×18
C101	$\text{Tr}[M] = C$	$48^3 \times 96$	400×16
H107	$\widetilde{m_s} = \widetilde{m_s}^{\text{Phys.}}$	$32^3 \times 96$	500×16
H106	$\widetilde{m_s} = \widetilde{m_s}^{\text{Phys.}}$	$32^3 \times 96$	500×16
H200	$\text{Tr}[M] = C$	$32^3 \times 96$	500×28