Scalar and tensor charmonium resonances from lattice QCD

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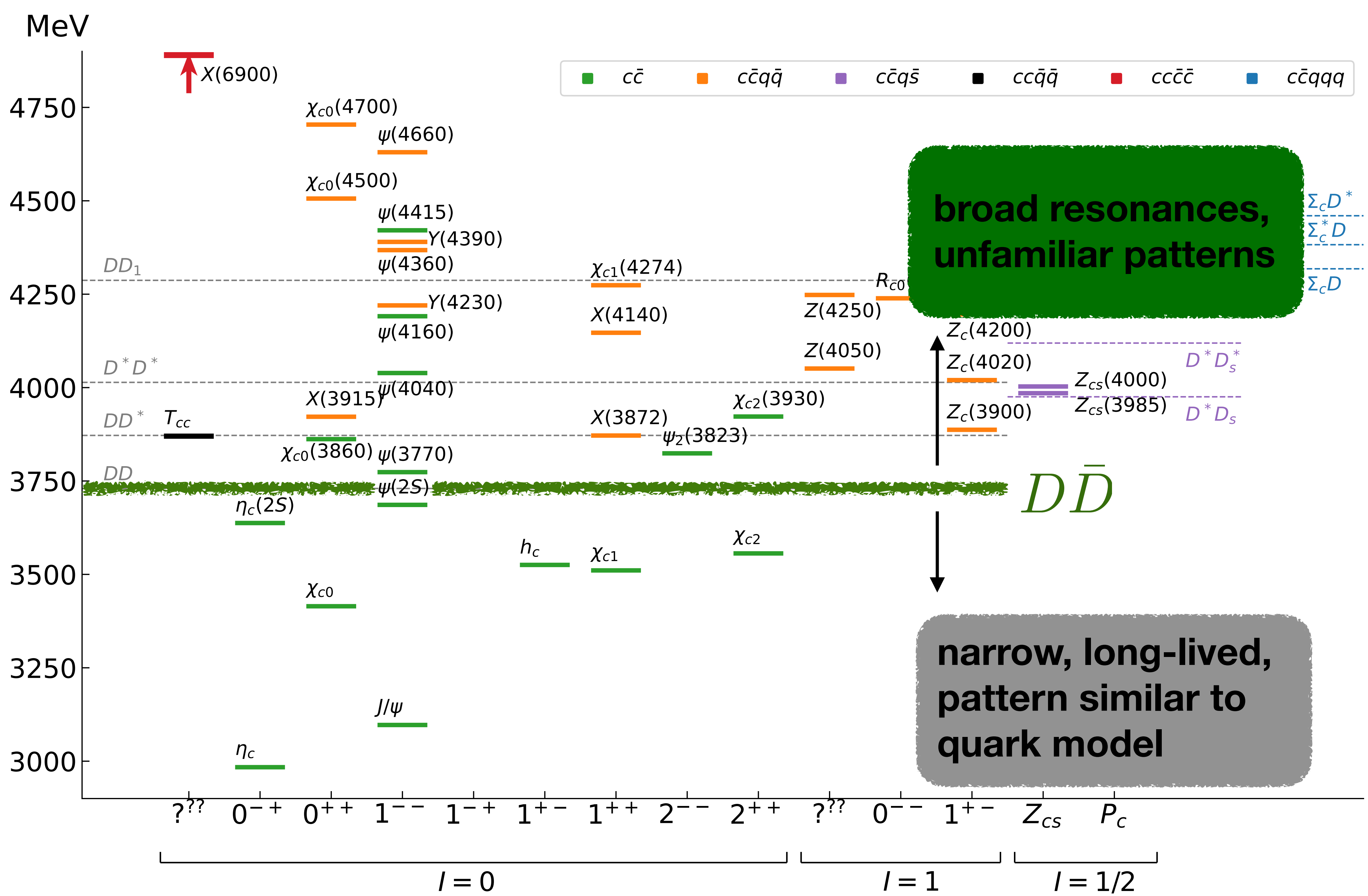
hadspec
hadspec.org

Lattice 2024 Liverpool
28th July – 3rd August 2024

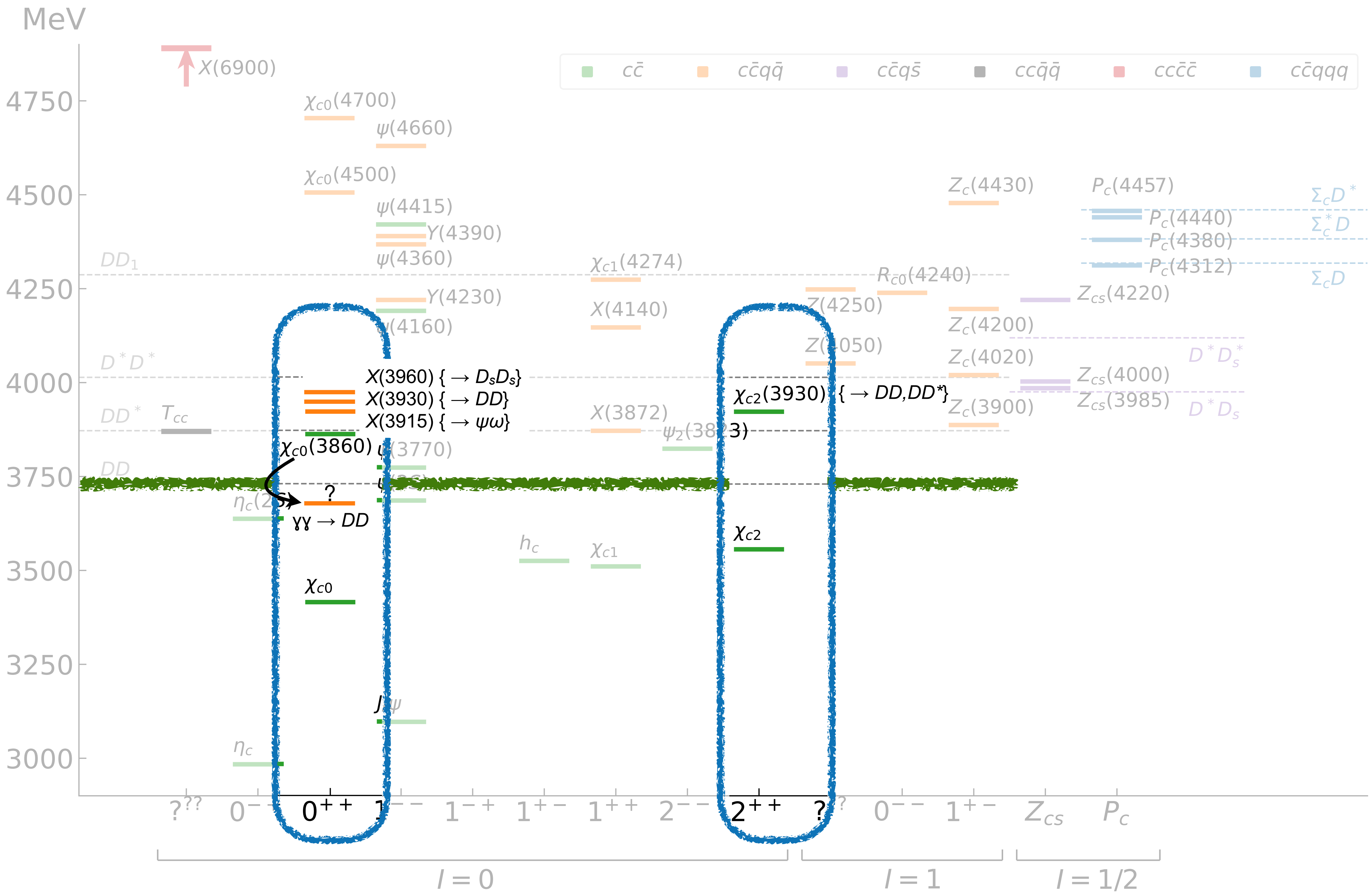
based on work:
PRL Editors' choice: arXiv: 2309.14070 (7 pages)
PRD Editors' choice: arXiv: 2309.14071 (55 pages)

UNIVERSITY OF CAMBRIDGE

THE ROYAL SOCIETY



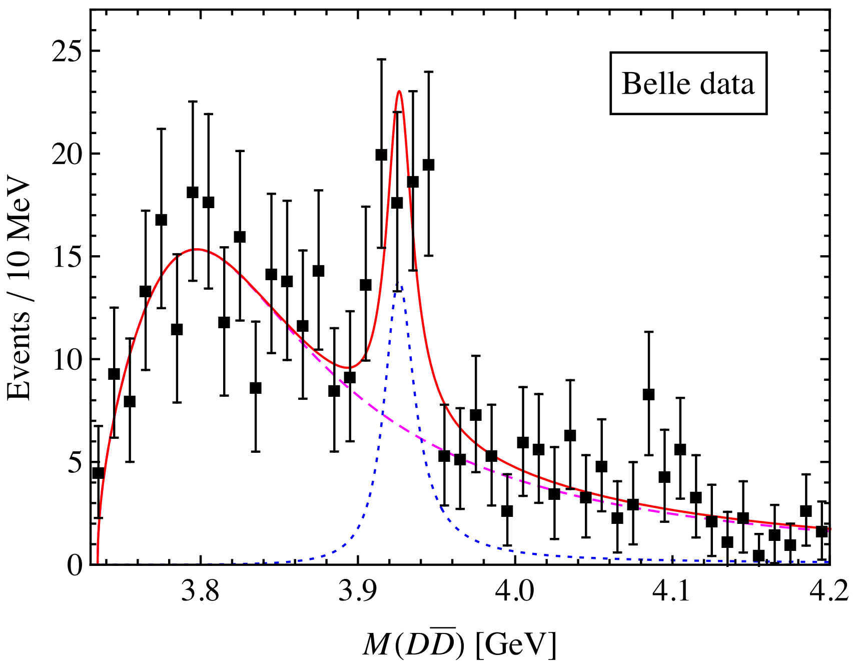
JPAC arXiv:2112.13436



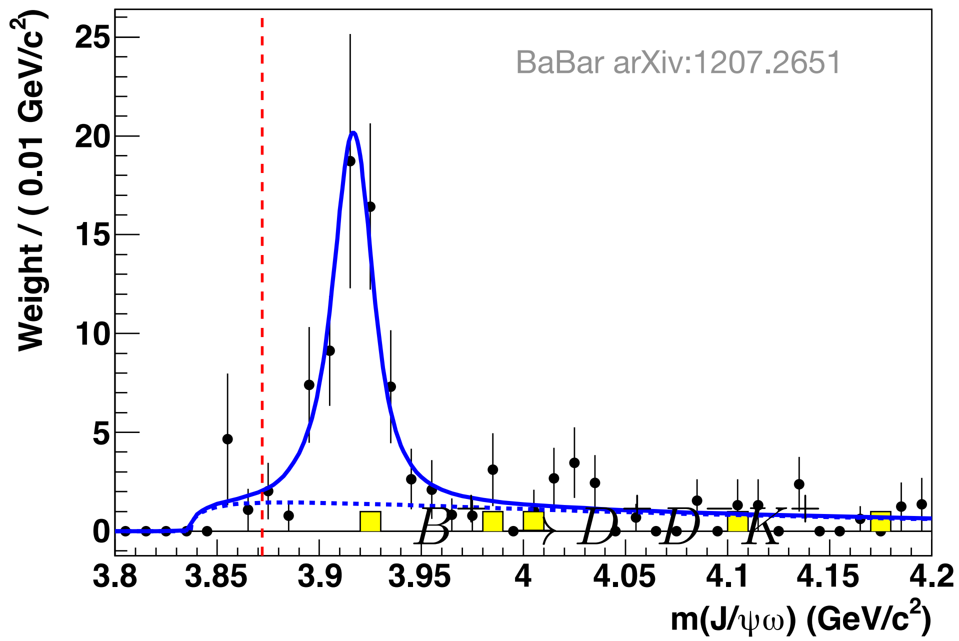
MeV
4750
4500
4250
4000
3750
3500
3250
3000
X(6900)
$\chi_{c0}(4700)$
$\psi(4660)$
$\chi_{c0}(4500)$
$\psi(4415)$
Y(4390)
$\psi(4360)$
Y(4230)
$\psi(4160)$
X(3960) {→ D_sD_s}
X(3930) {→ DD}
X(3915) {→ $\psi\omega$}
$\chi_{c0}(3860)$
$\psi(3770)$
?
γγ → DD
χ_{c0}
J/ψ
η_c
$\eta_c(2S)$
T_{cc}
DD_1
D^*D^*
DD^*
DD
$\chi_{c1}(4274)$
X(4140)
X(3872)
$\psi_2(3823)$
h_c
χ_{c1}
$\chi_{c2}(3930)$ {→ DD,DD*}
χ_{c2}
Z(4250)
Z(4050)
$R_{c0}(4240)$
$Z_c(4430)$
$Z_c(4200)$
$Z_c(4020)$
$Z_c(3900)$
$Z_{cs}(4220)$
$Z_{cs}(4000)$
$Z_{cs}(3985)$
$D^*D_s^*$
D^*D_s
$P_c(4457)$
$P_c(4440)$
$P_c(4380)$
$P_c(4312)$
Σ_cD^*
Σ_c^*D
Σ_cD
$c\bar{c}$
$c\bar{c}q\bar{q}$
$c\bar{c}q\bar{s}$
$cc\bar{q}\bar{q}$
$cc\bar{c}\bar{c}$
$c\bar{c}qqq$
$?^{??}$
0^{-+}
0^{++}
1^{--}
1^{-+}
1^{+-}
1^{++}
2^{--}
2^{++}
$?^{?}$
0^{--}
1^{+-}
Z_{cs}
P_c
$I = 0$
$I = 1$
$I = 1/2$

Experimental results

$\gamma\gamma \to D\bar{D}$



$J/\psi\omega$

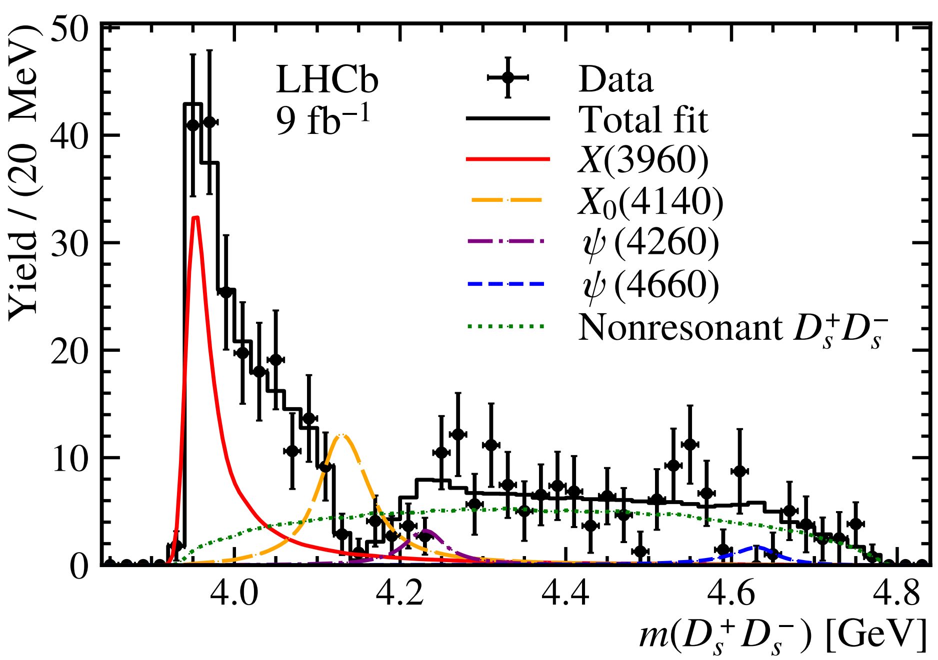


$$m = (3919.4 \pm 2.2 \pm 1.6)\ \text{MeV}$$
$$\Gamma = (13 \pm 6 \pm 3)\ \text{MeV}$$
$$J^P = 0^+$$

Just a few examples
Many many more
(References in the longer paper)

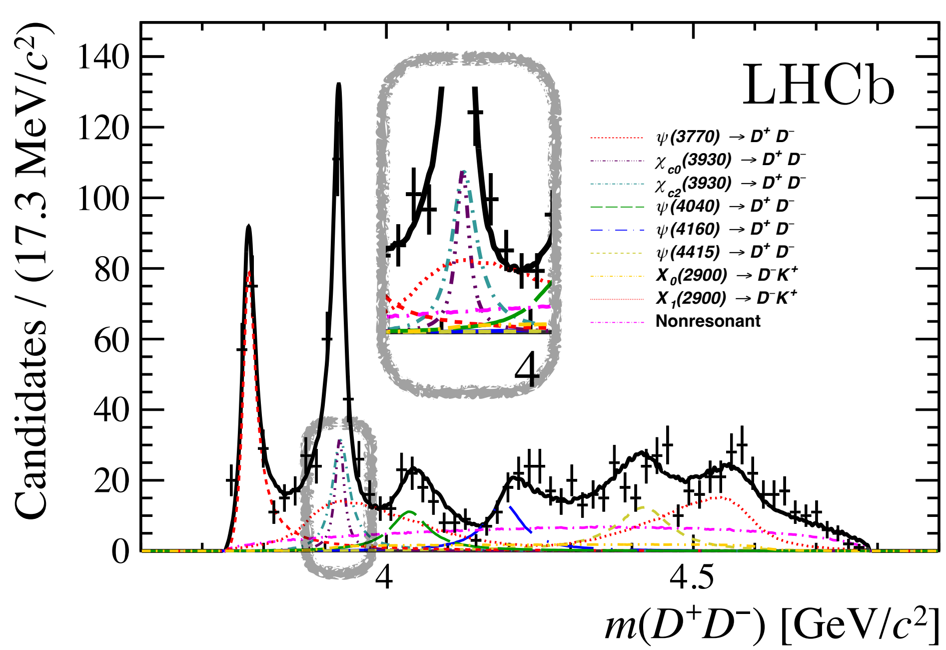
$B^+ \to D_s^+ D_s^- K^+$

arXiv:2210.15153
LHCb



$B^+ \to D^+ D^- K^+$

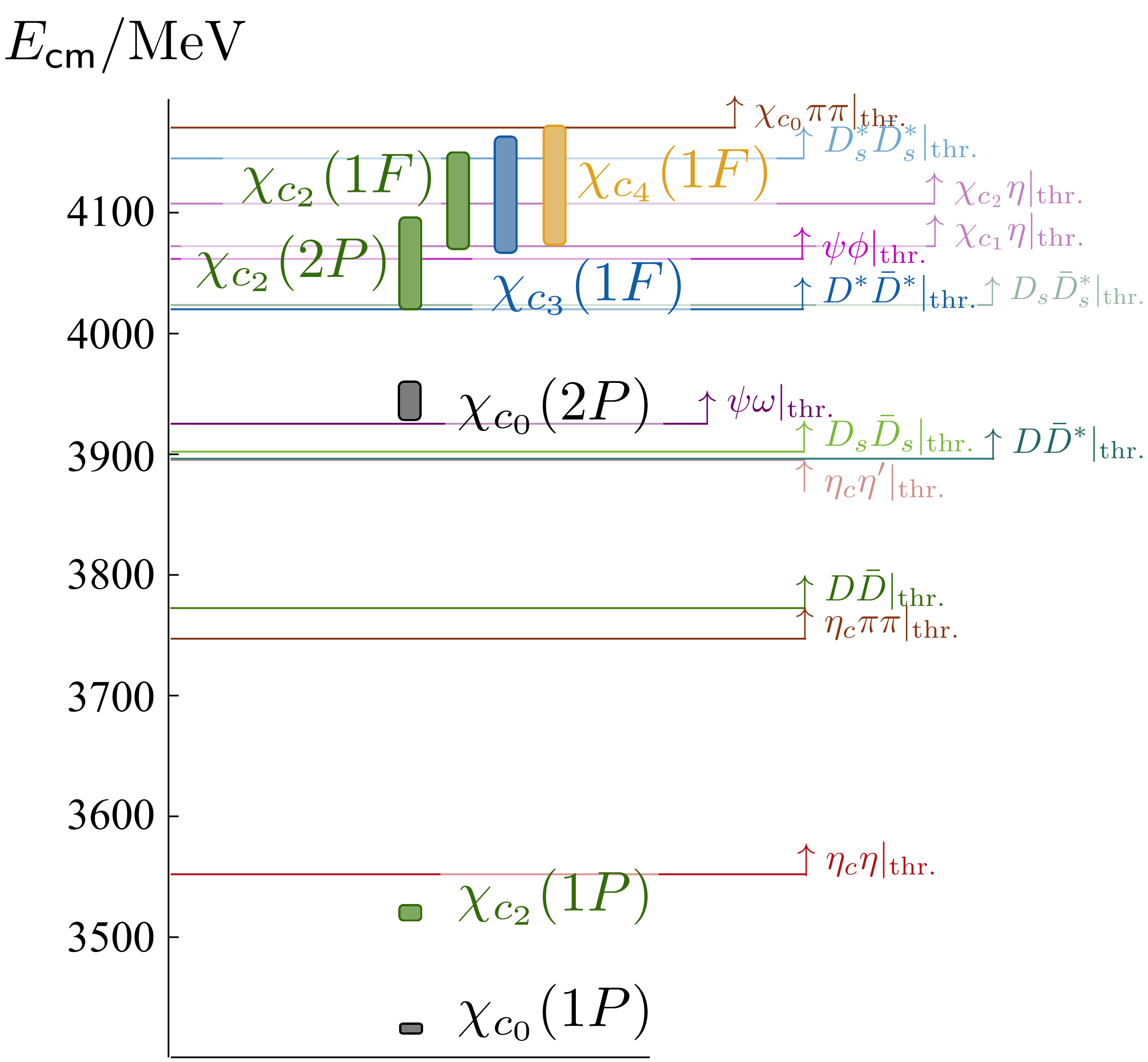
arXiv:2009.00026



overlapping 0++ and 2++ resonances around 3925 MeV

no need for a low 0++ resonance

Previously:



spectra from qqbar operators only,
Liu et al JHEP 1207 (2012) 126

"HadSpec" lattices

anisotropic (3.5 finer spacing in time)
Wilson-Clover

L/a_s=16, 20, 24
m_π = 391 MeV

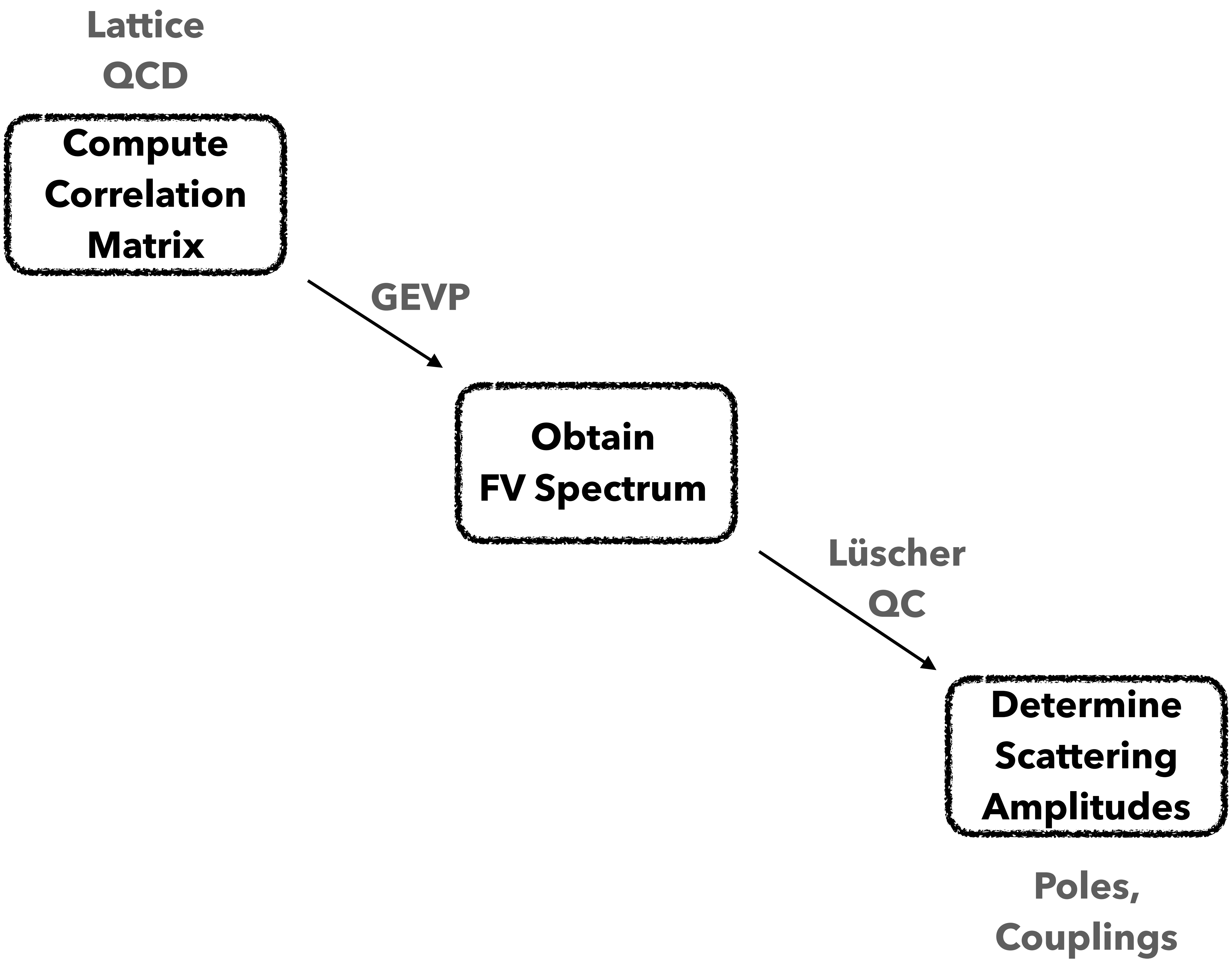
rest and moving frames

N_f = 2+1 flavours
all light+strange annihilations included
no charm annihilation

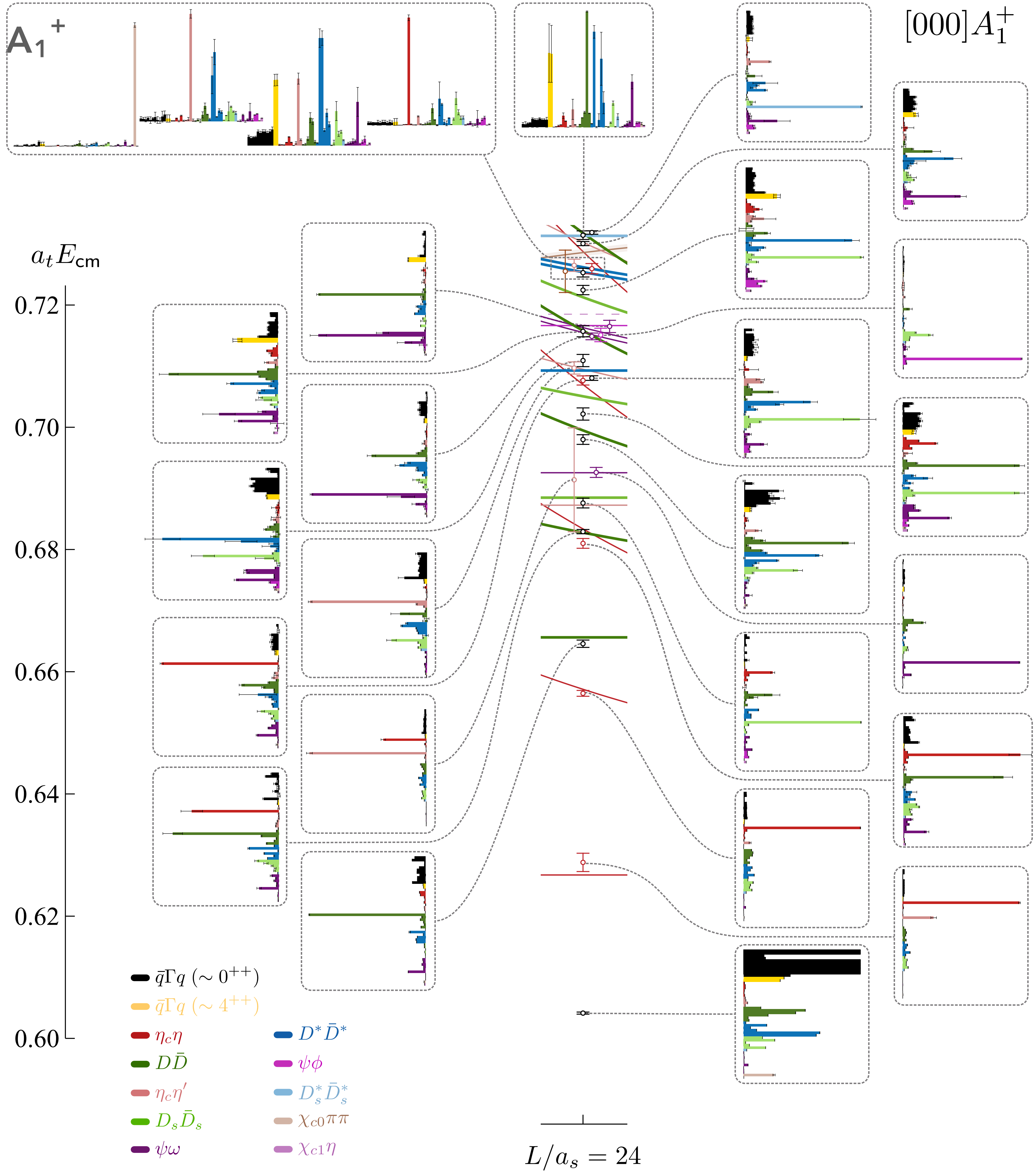
using *distillation* (Peardon *et al* 2009)
many channels, many wick contractions

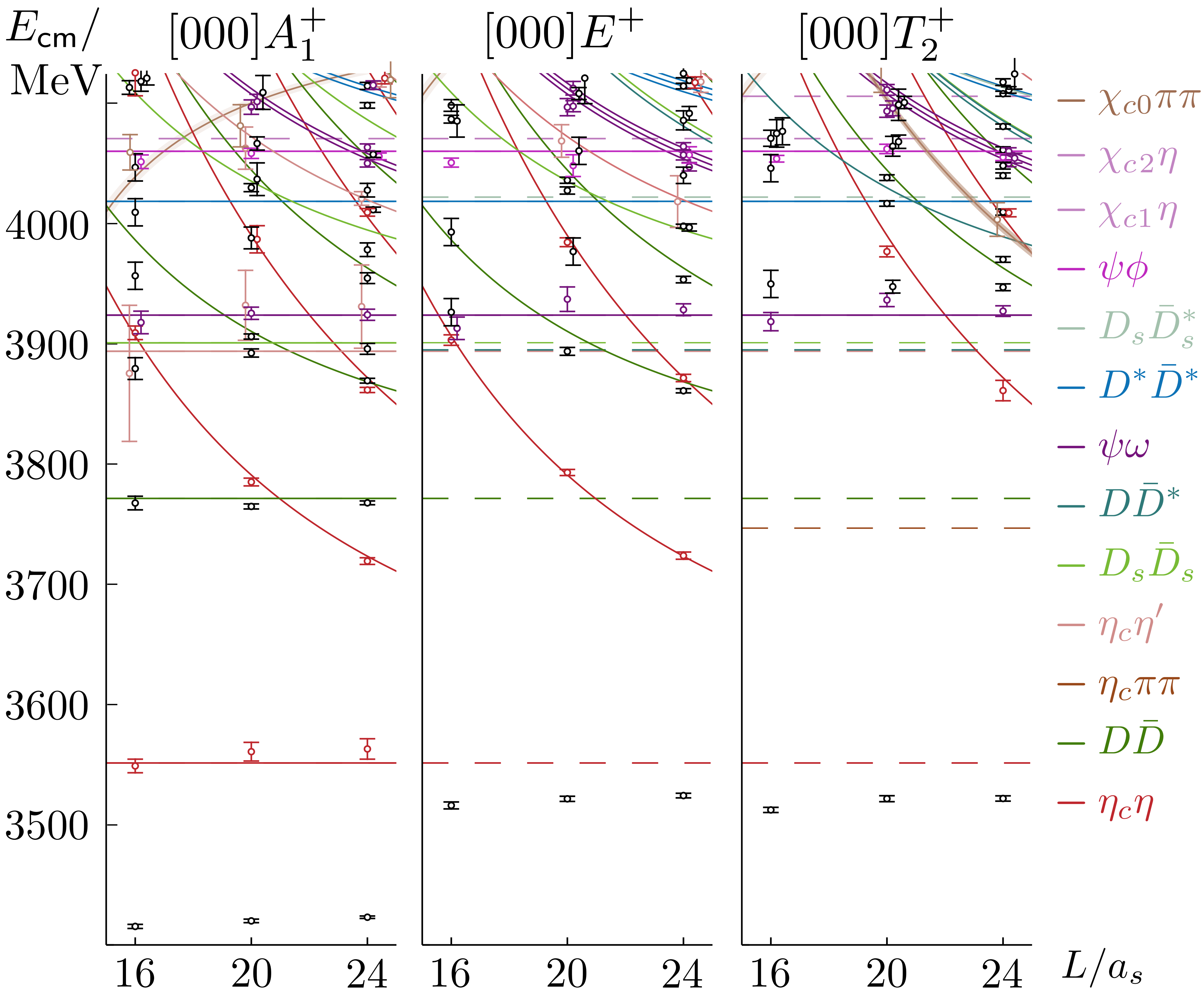
This study: Meson-meson + qqbar ops

- compute a large correlation matrix
- solve generalised eigenvalue problem to extract energies



Review: Briceño, Dudek, Young,
https://doi.org/10.1103/RevModPhys.90.025001


$[000]A_1^+$
$a_t E_{cm}$
0.72
0.70
0.68
0.66
0.64
0.62
0.60
$L/a_s = 24$
$\bar{q}\Gamma q$ ($\sim 0^{++}$)
$\bar{q}\Gamma q$ ($\sim 4^{++}$)
$\eta_c\eta$
$D\bar{D}$
$\eta_c\eta'$
$D_s\bar{D}_s$
$\psi\omega$
$D^*\bar{D}^*$
$\psi\phi$
$D_s^*\bar{D}_s^*$
$\chi_{c0}\pi\pi$
$\chi_{c1}\eta$


E_{cm}/ MeV
[000] A_1^+
[000] E^+
[000] T_2^+
3500
3600
3700
3800
3900
4000
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$

"0++" "2++" "2++ & 3++"

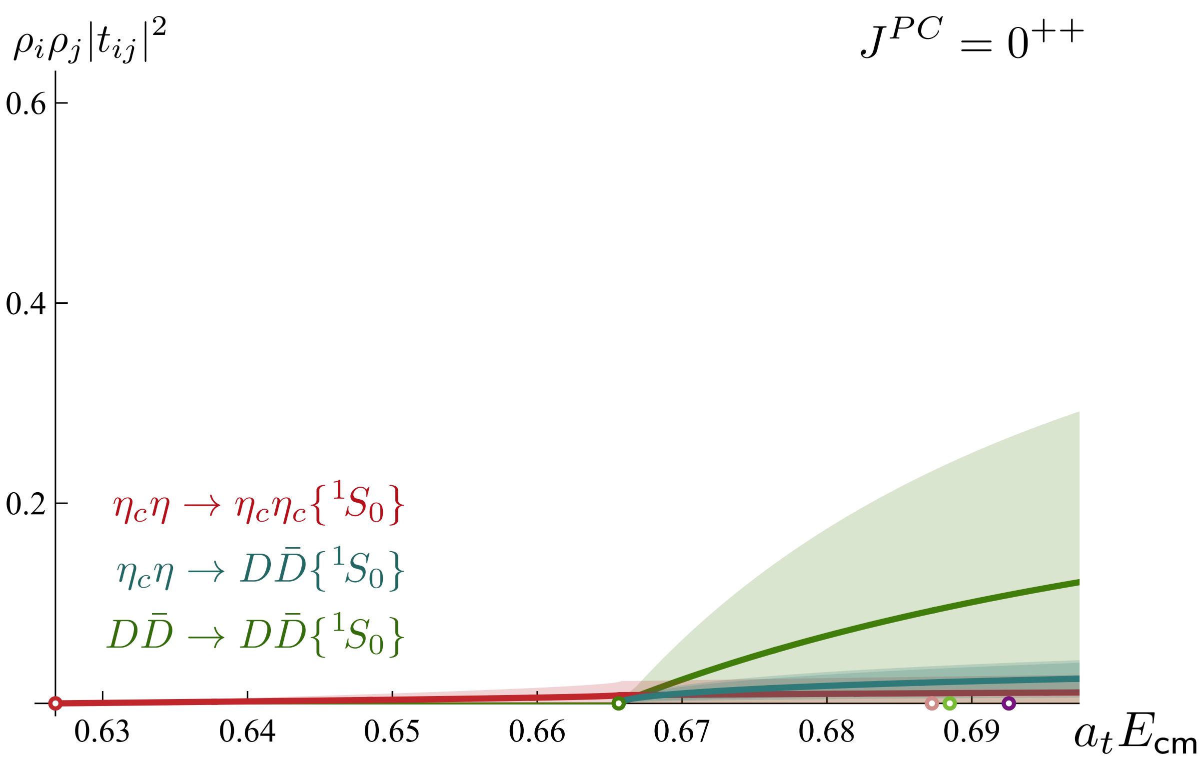
$$\boldsymbol{S} = \mathbf{1} + 2i\boldsymbol{\rho}^{\frac{1}{2}} \cdot \boldsymbol{t} \cdot \boldsymbol{\rho}^{\frac{1}{2}}$$

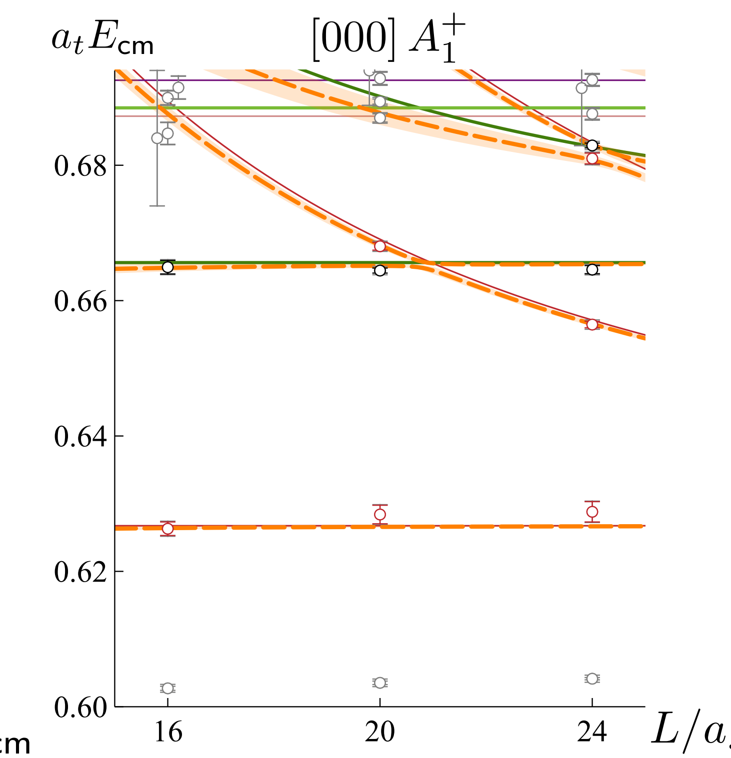
$$\boldsymbol{t}^{-1} = \boldsymbol{K}^{-1} + \boldsymbol{I}$$

$$\mathrm{Im} I_{ij} = -\rho_i = 2k_i/\sqrt{s}$$

$$\det[\mathbf{1} + i\boldsymbol{\rho} \cdot \boldsymbol{t}\,(\mathbf{1} + i\boldsymbol{\mathcal{M}}(L))] = 0$$

$$\boldsymbol{K} = \begin{bmatrix} \gamma_{\eta_c\eta \to \eta_c\eta} & \gamma_{\eta_c\eta \to D\bar{D}} \\ \gamma_{\eta_c\eta \to D\bar{D}} & \gamma_{D\bar{D} \to D\bar{D}} \end{bmatrix}$$



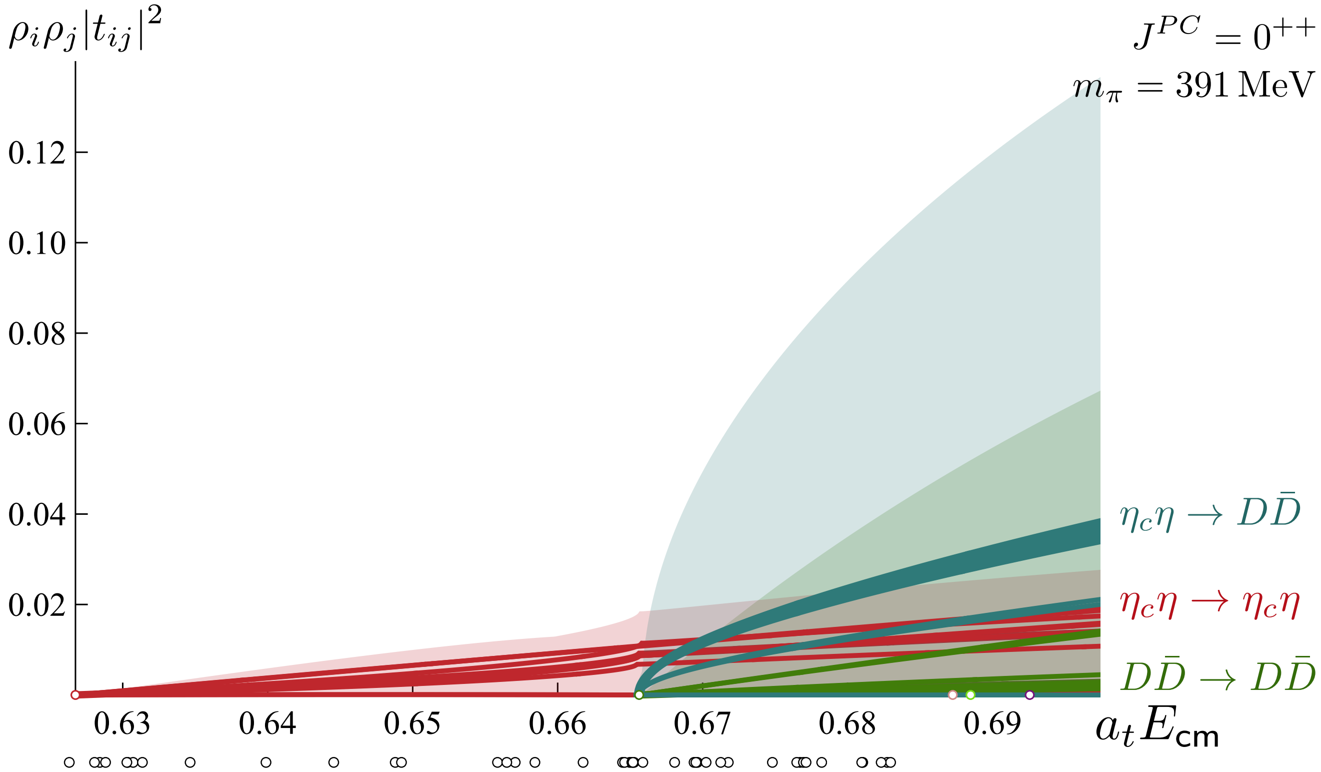


using rest-frame only

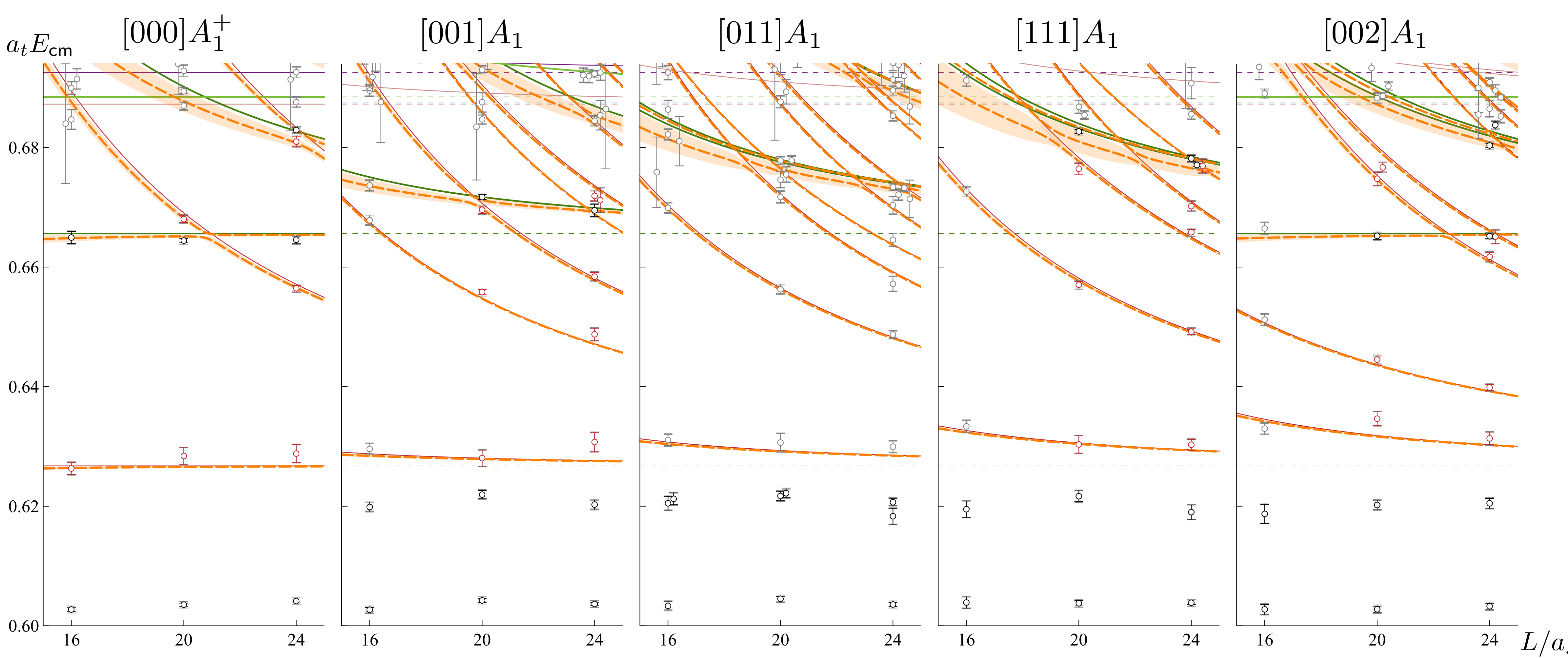
$$\begin{aligned} \gamma_{\eta_c\eta \to \eta_c\eta} &= (0.34 \pm 0.23 \pm 0.09) \\ \gamma_{\eta_c\eta \to D\bar{D}} &= (0.58 \pm 0.29 \pm 0.05) \\ \gamma_{D\bar{D} \to D\bar{D}} &= (1.39 \pm 1.19 \pm 0.24) \end{aligned} \qquad \begin{bmatrix} 1.00 & 0.77 & -0.24 \\ & 1.00 & -0.22 \\ & & 1.00 \end{bmatrix}$$

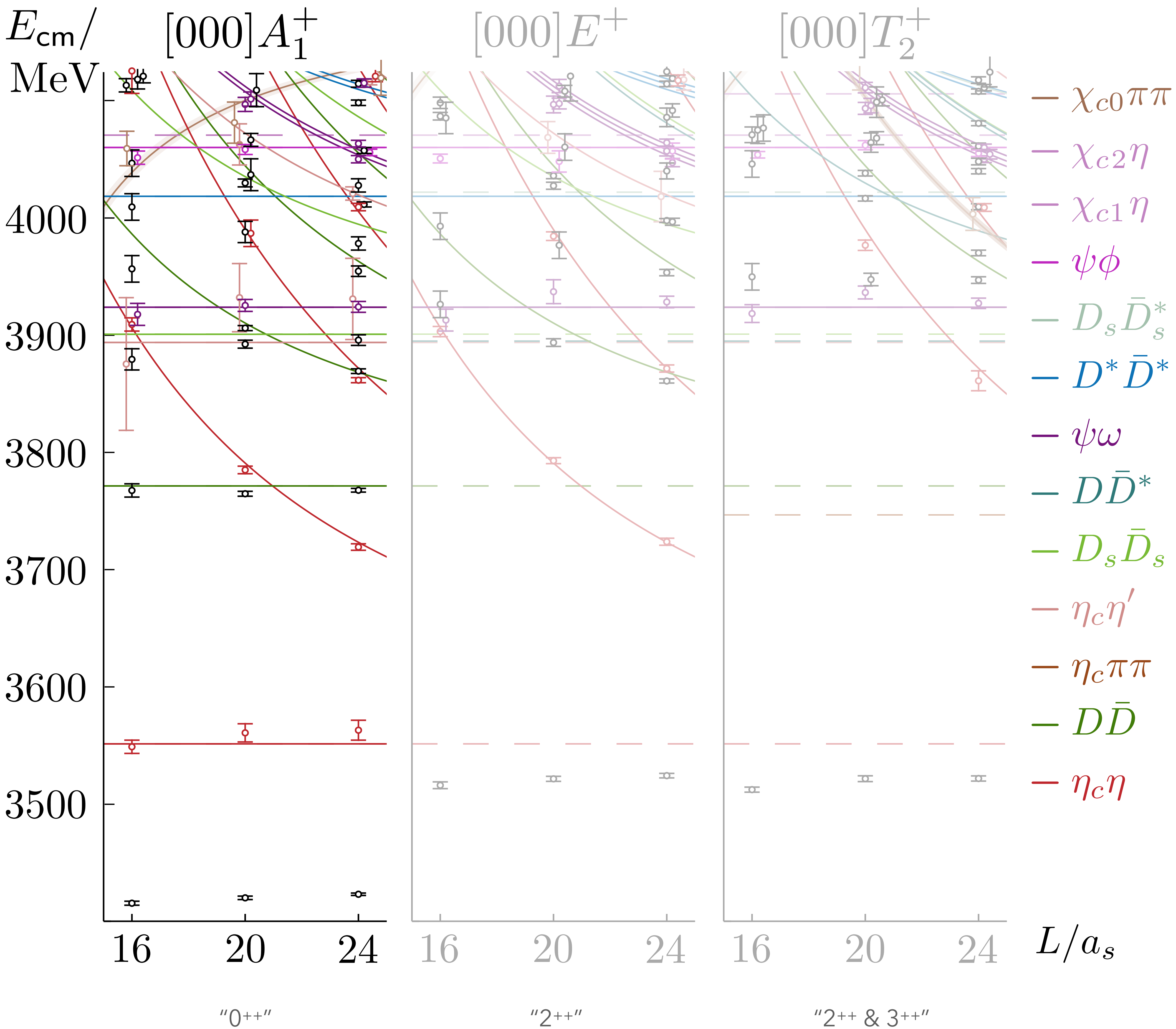
$$\chi^2/N_{\mathrm{dof}} = \frac{5.65}{10-3} = 0.81$$

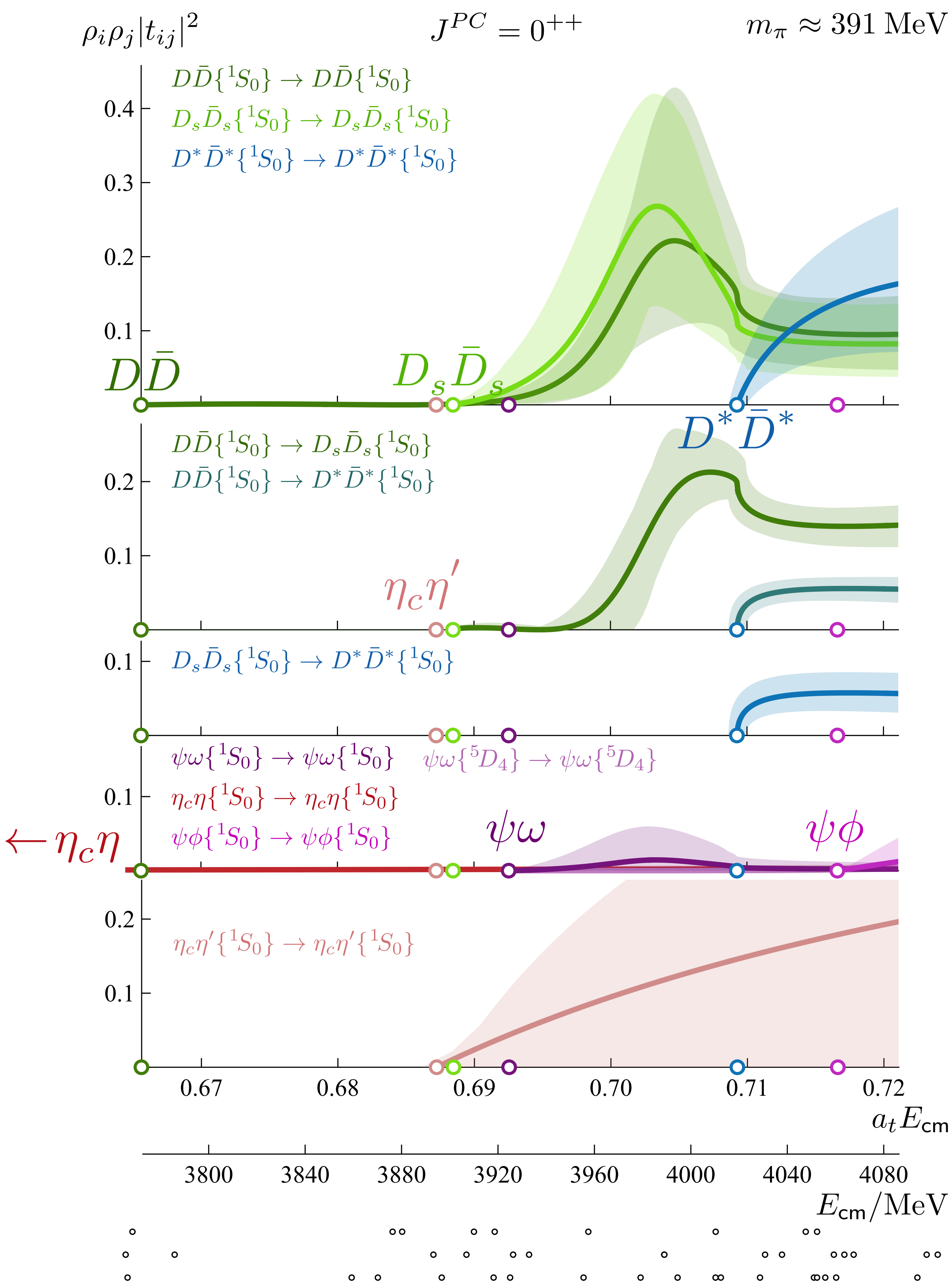
(smaller range)



using zero and non-zero total momentum




E_{cm}/ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
"0++"
"2++"
"2++ & 3++"



three channels open close together:
$\eta_c\eta'$, $D_s\bar{D}_s$, $\psi\omega$

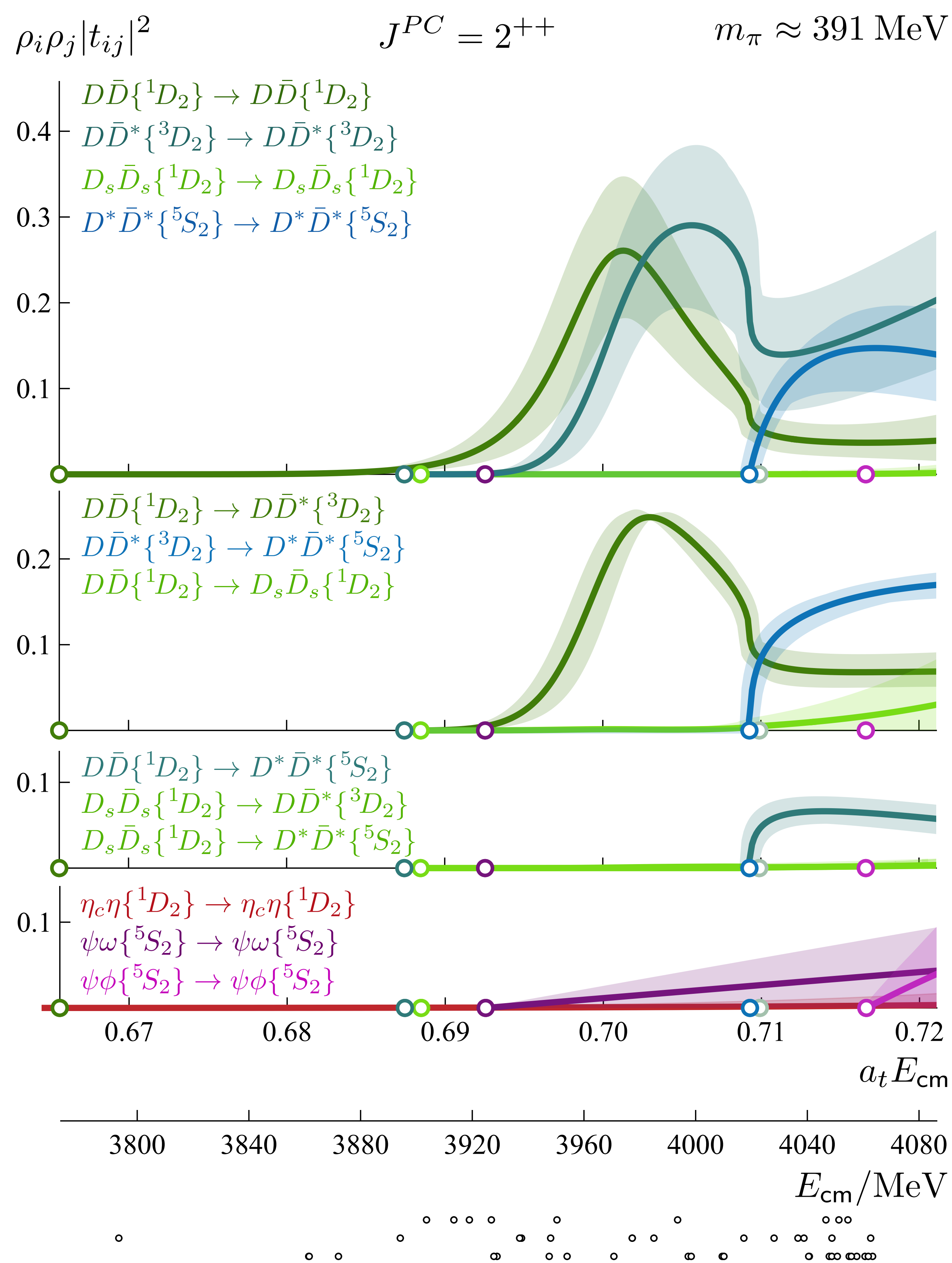
operator overlaps suggest $D^*\bar{D}^*$ is important

$\psi\phi$ has been seen to be important in some places

consider 7-channel system

$$K_{ij} = \frac{g_i g_j}{m^2 - s} + \gamma_{ij}$$

K-matrix pole terms become necessary to obtain a good quality of fit



7-channels, mixture of *S* and *D*

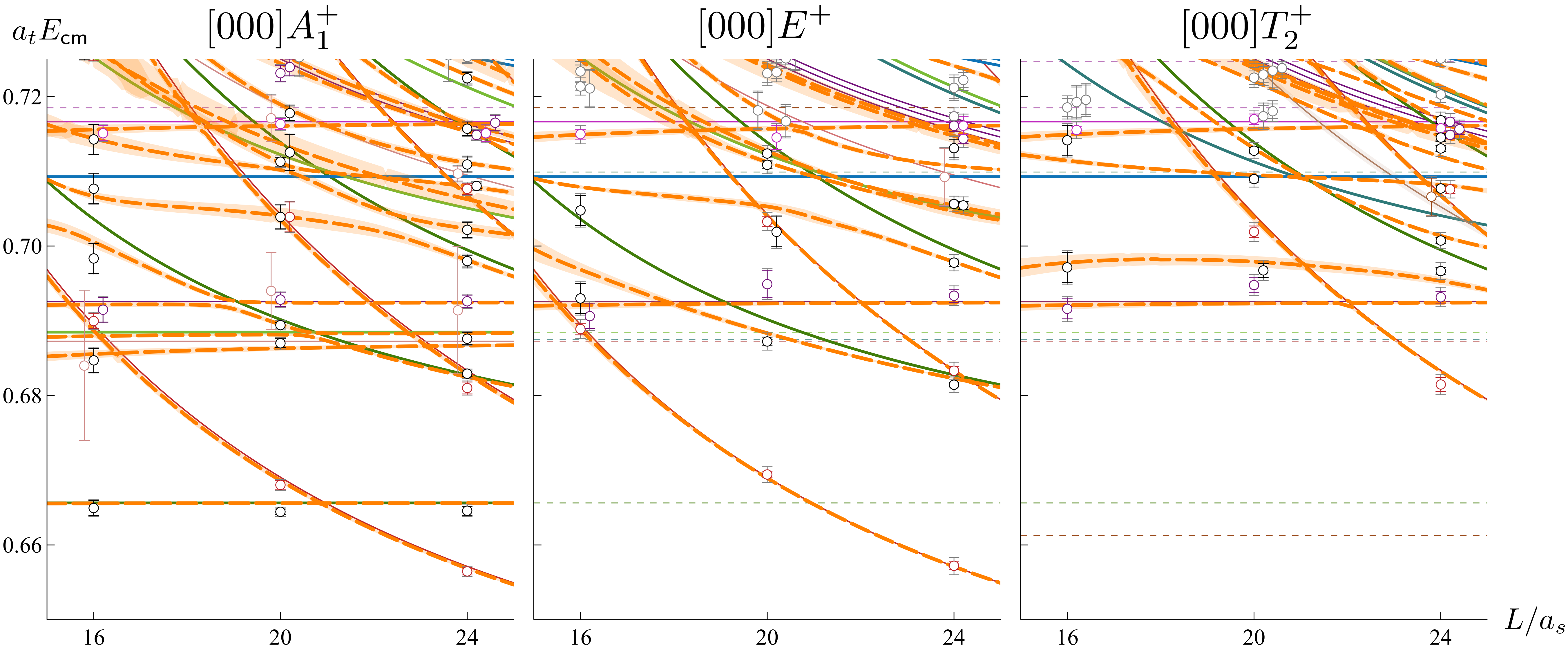
$D\bar{D}, D_s\bar{D}_s\{^1D_2\}$ $D\bar{D}^*\{^3D_2\}$ $D^*\bar{D}^*\{^5S_2\}$

$\eta_c\eta\{^1D_2\}$ $\psi\omega, \psi\phi\{^5S_2\}$

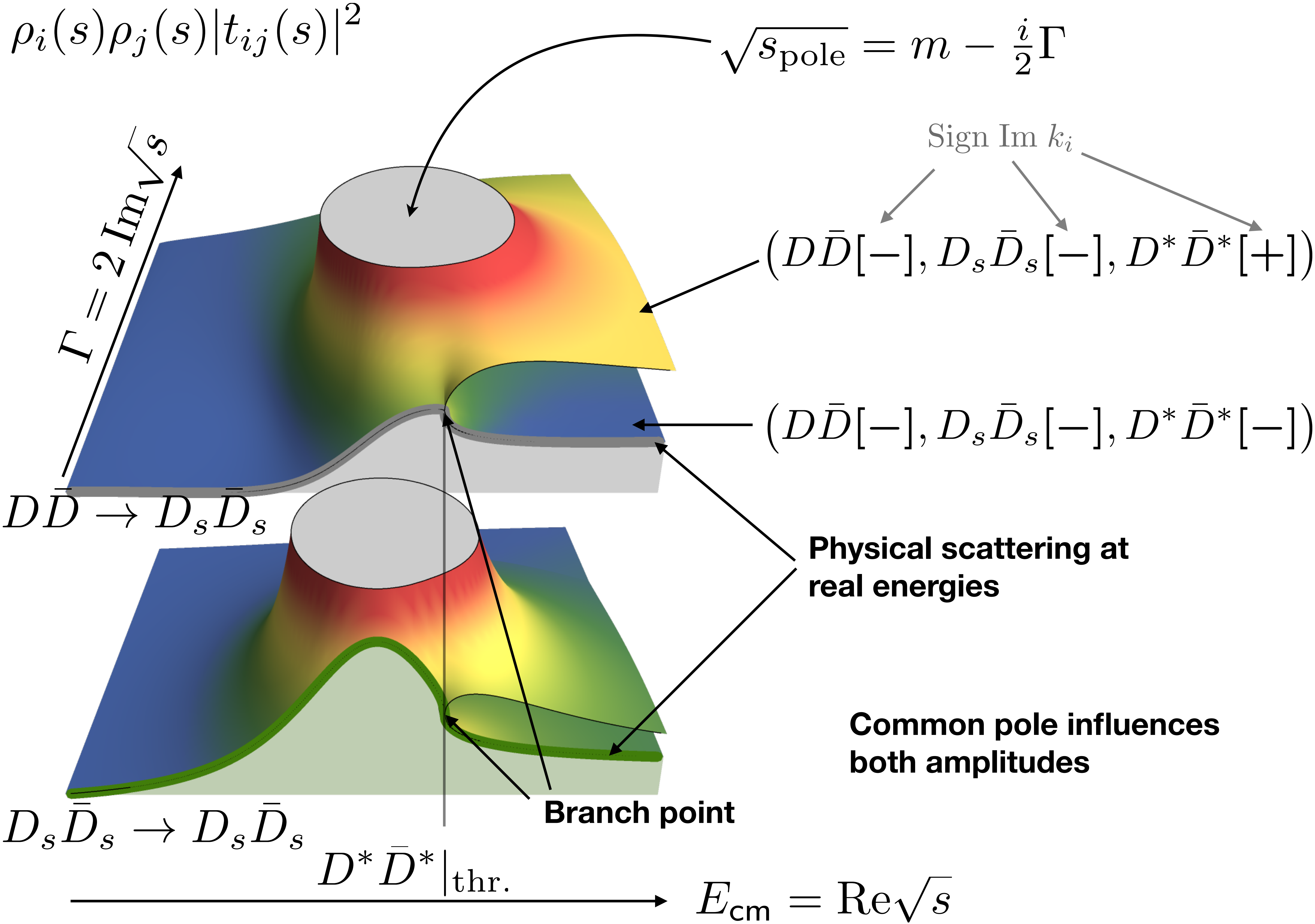
peaks at a similar energy

very small DsDs amplitudes -
some phase space suppression

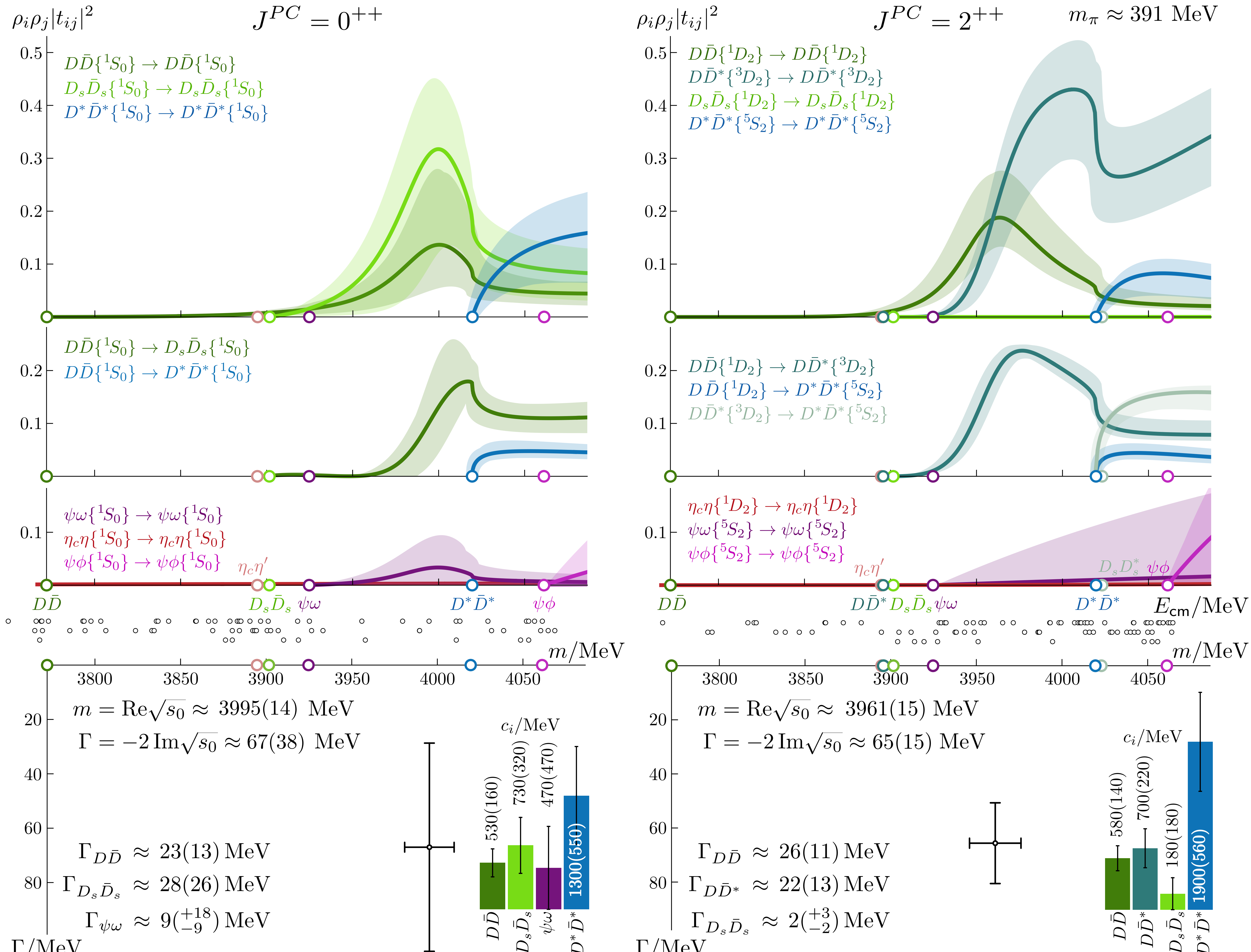
DD* is large -
similar phase space to DsDs



$$\det[\mathbf{1} + i\boldsymbol{\rho}\cdot\boldsymbol{t}\,(\mathbf{1} + i\boldsymbol{\mathcal{M}}(L))] = 0$$



$\rho_i(s)\rho_j(s)|t_{ij}(s)|^2$
$\sqrt{s_{\text{pole}}} = m - \frac{i}{2}\Gamma$
Sign Im k_i
$(D\bar{D}[-], D_s\bar{D}_s[-], D^*\bar{D}^*[+])$
$(D\bar{D}[-], D_s\bar{D}_s[-], D^*\bar{D}^*[-])$
$\Gamma = 2\,\text{Im}\sqrt{s}$
$D\bar{D} \to D_s\bar{D}_s$
$D_s\bar{D}_s \to D_s\bar{D}_s$
Physical scattering at real energies
Common pole influences both amplitudes
Branch point
$D^*\bar{D}^*|_{\text{thr.}}$
$E_{\text{cm}} = \text{Re}\sqrt{s}$



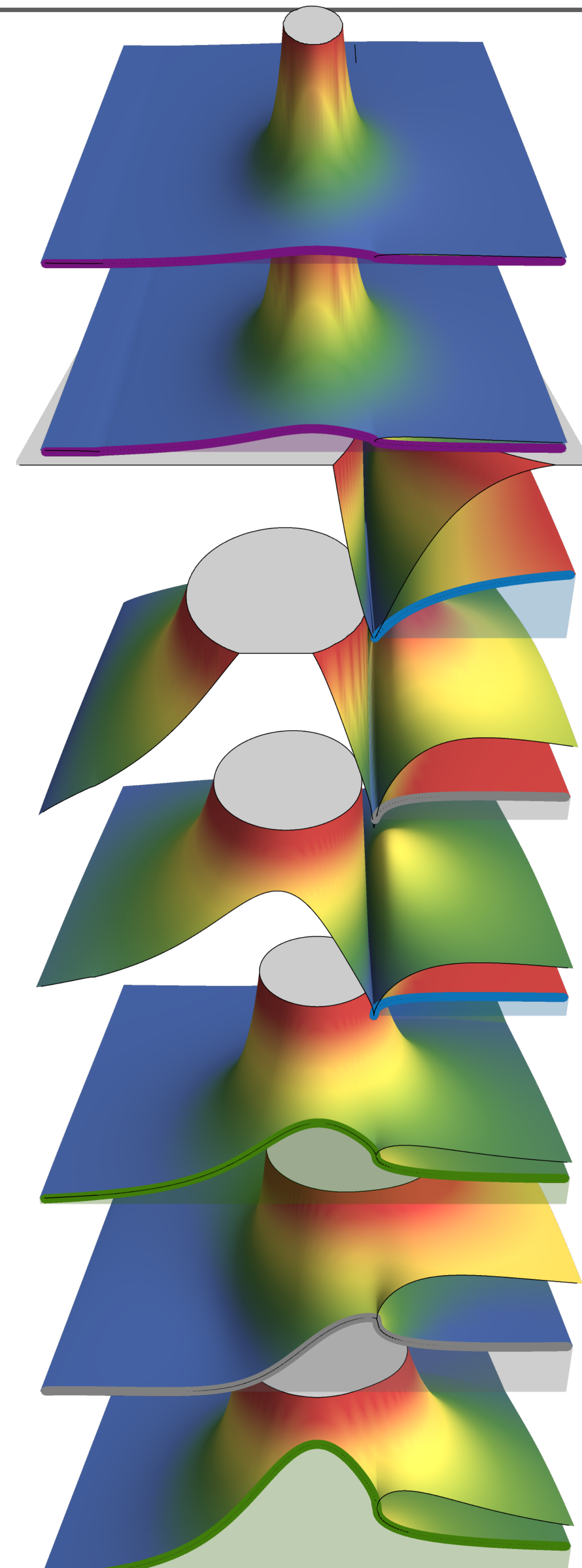
$\rho_i \rho_j |t_{ij}|^2$
$J^{PC} = 0^{++}$
$D\bar{D}\{^1S_0\} \to D\bar{D}\{^1S_0\}$
$D_s\bar{D}_s\{^1S_0\} \to D_s\bar{D}_s\{^1S_0\}$
$D^*\bar{D}^*\{^1S_0\} \to D^*\bar{D}^*\{^1S_0\}$
$D\bar{D}\{^1S_0\} \to D_s\bar{D}_s\{^1S_0\}$
$D\bar{D}\{^1S_0\} \to D^*\bar{D}^*\{^1S_0\}$
$\psi\omega\{^1S_0\} \to \psi\omega\{^1S_0\}$
$\eta_c\eta\{^1S_0\} \to \eta_c\eta\{^1S_0\}$
$\psi\phi\{^1S_0\} \to \psi\phi\{^1S_0\}$
$\eta_c\eta'$
$D\bar{D}$ $D_s\bar{D}_s$ $\psi\omega$ $D^*\bar{D}^*$ $\psi\phi$
m/MeV
3800 3850 3900 3950 4000 4050
$m = \mathrm{Re}\sqrt{s_0} \approx 3995(14)$ MeV
$\Gamma = -2\,\mathrm{Im}\sqrt{s_0} \approx 67(38)$ MeV
$\Gamma_{D\bar{D}} \approx 23(13)$ MeV
$\Gamma_{D_s\bar{D}_s} \approx 28(26)$ MeV
$\Gamma_{\psi\omega} \approx 9(^{+18}_{-9})$ MeV
c_i/MeV
530(160) 730(320) 470(470) 1300(550)
$D\bar{D}$ $D_s\bar{D}_s$ $\psi\omega$ $D^*\bar{D}^*$
Γ/MeV
$\rho_i \rho_j |t_{ij}|^2$
$J^{PC} = 2^{++}$
$m_\pi \approx 391$ MeV
$D\bar{D}\{^1D_2\} \to D\bar{D}\{^1D_2\}$
$D\bar{D}^*\{^3D_2\} \to D\bar{D}^*\{^3D_2\}$
$D_s\bar{D}_s\{^1D_2\} \to D_s\bar{D}_s\{^1D_2\}$
$D^*\bar{D}^*\{^5S_2\} \to D^*\bar{D}^*\{^5S_2\}$
$D\bar{D}\{^1D_2\} \to D\bar{D}^*\{^3D_2\}$
$D\bar{D}\{^1D_2\} \to D^*\bar{D}^*\{^5S_2\}$
$D\bar{D}^*\{^3D_2\} \to D^*\bar{D}^*\{^5S_2\}$
$\eta_c\eta\{^1D_2\} \to \eta_c\eta\{^1D_2\}$
$\psi\omega\{^5S_2\} \to \psi\omega\{^5S_2\}$
$\psi\phi\{^5S_2\} \to \psi\phi\{^5S_2\}$
$\eta_c\eta'$ $D_sD_s^*$ $\psi\phi$
$D\bar{D}$ $D\bar{D}^*$ $D_s\bar{D}_s$ $\psi\omega$ $D^*\bar{D}^*$
E_{cm}/MeV
m/MeV
3800 3850 3900 3950 4000 4050
$m = \mathrm{Re}\sqrt{s_0} \approx 3961(15)$ MeV
$\Gamma = -2\,\mathrm{Im}\sqrt{s_0} \approx 65(15)$ MeV
$\Gamma_{D\bar{D}} \approx 26(11)$ MeV
$\Gamma_{D\bar{D}^*} \approx 22(13)$ MeV
$\Gamma_{D_s\bar{D}_s} \approx 2(^{+3}_{-2})$ MeV
c_i/MeV
580(140) 700(220) 180(180) 1900(560)
$D\bar{D}$ $D\bar{D}^*$ $D_s\bar{D}_s$ $D^*\bar{D}^*$
Γ/MeV

Scalar and tensor charmonium scattering amplitudes have been determined

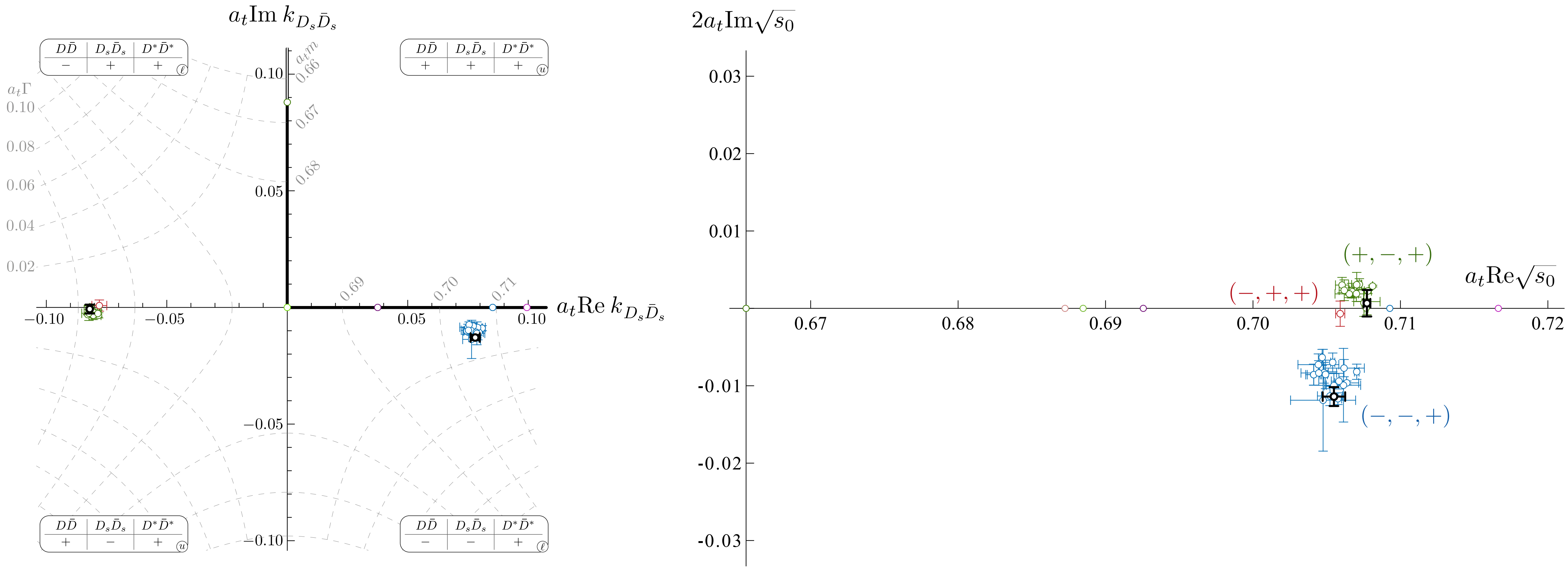
- at m_π=391 MeV, the **level counting is not** obviously **different from the quark model**
- large **coupled-channel** effects in OZI **connected D-meson channels**
- OZI **disconnected** channels look **small everywhere**
- we have extracted a **complete** unitary **S-matrix** and this naturally **connects** features seen in **different channels** and simplifies the overall picture
- also a clear, as yet unobserved, 3++ resonance is present in DDbar*
- we **do not find** a **near-threshold DDbar** state (between 3700 and 3860 MeV)
- these methods can also be applied to the X(3872) 1++ channel

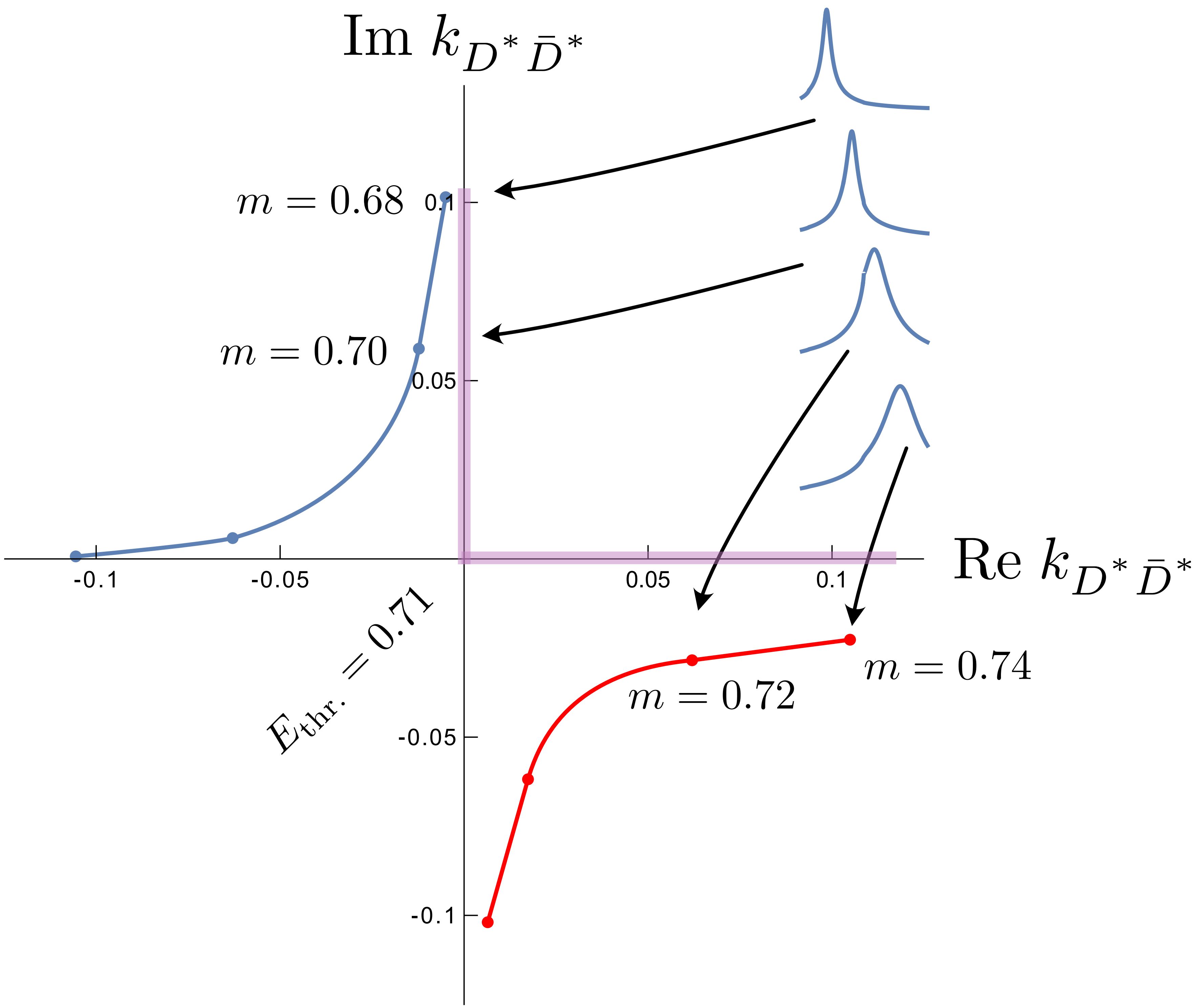
$$\rho_i(s)\rho_j(s)|t_{ij}(s)|^2$$

one resonance pole
– many different amplitudes

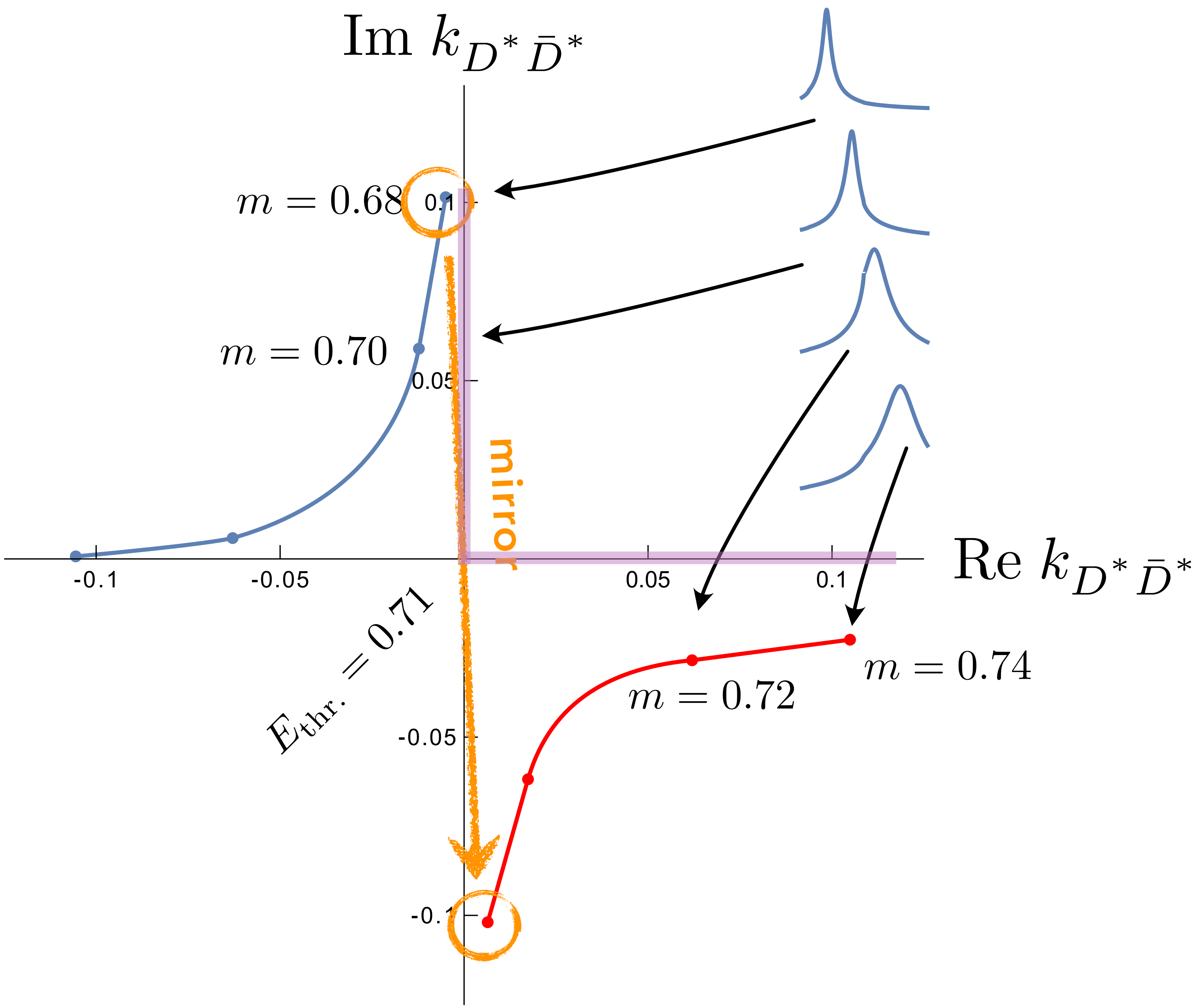


poles - scalar

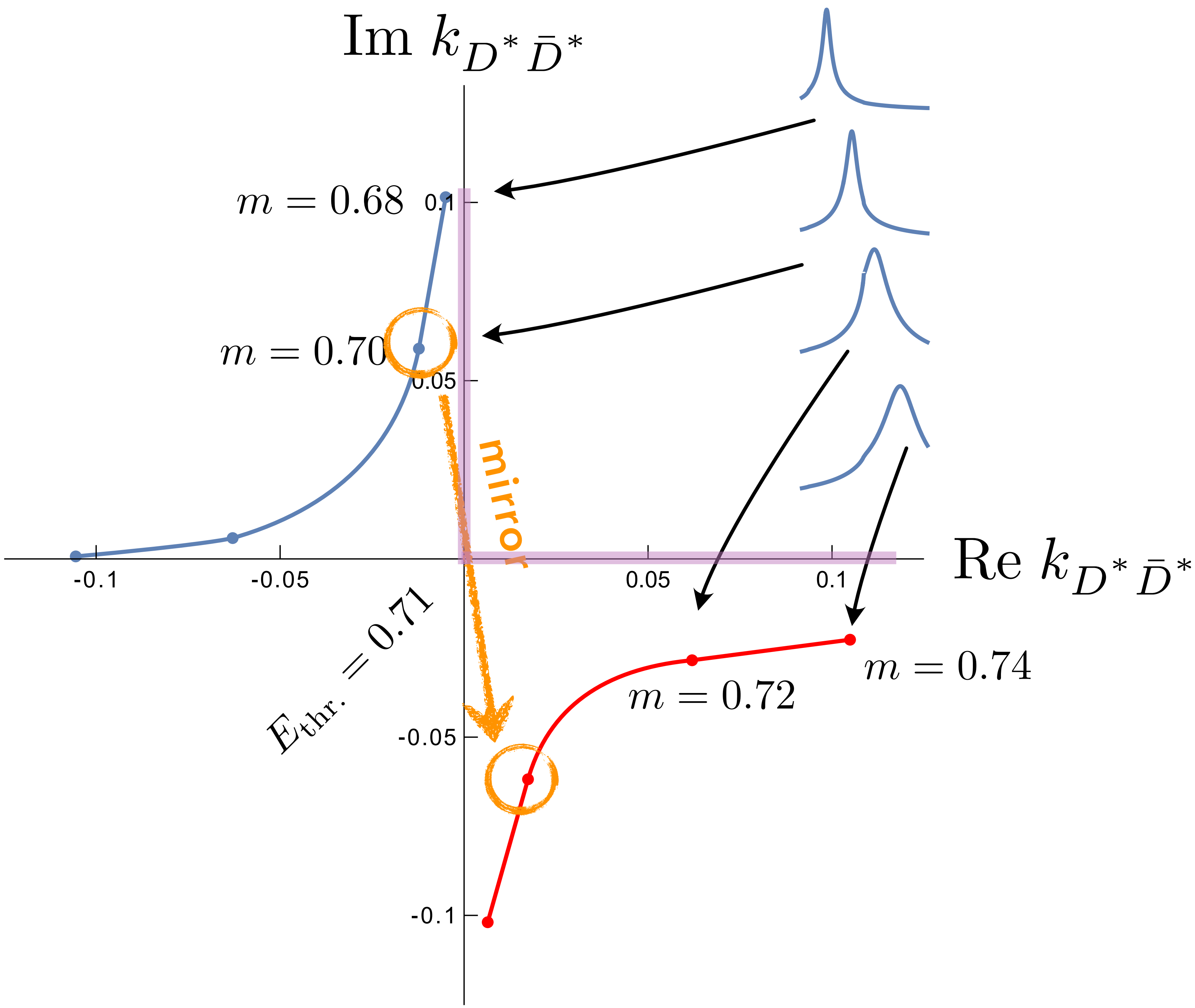




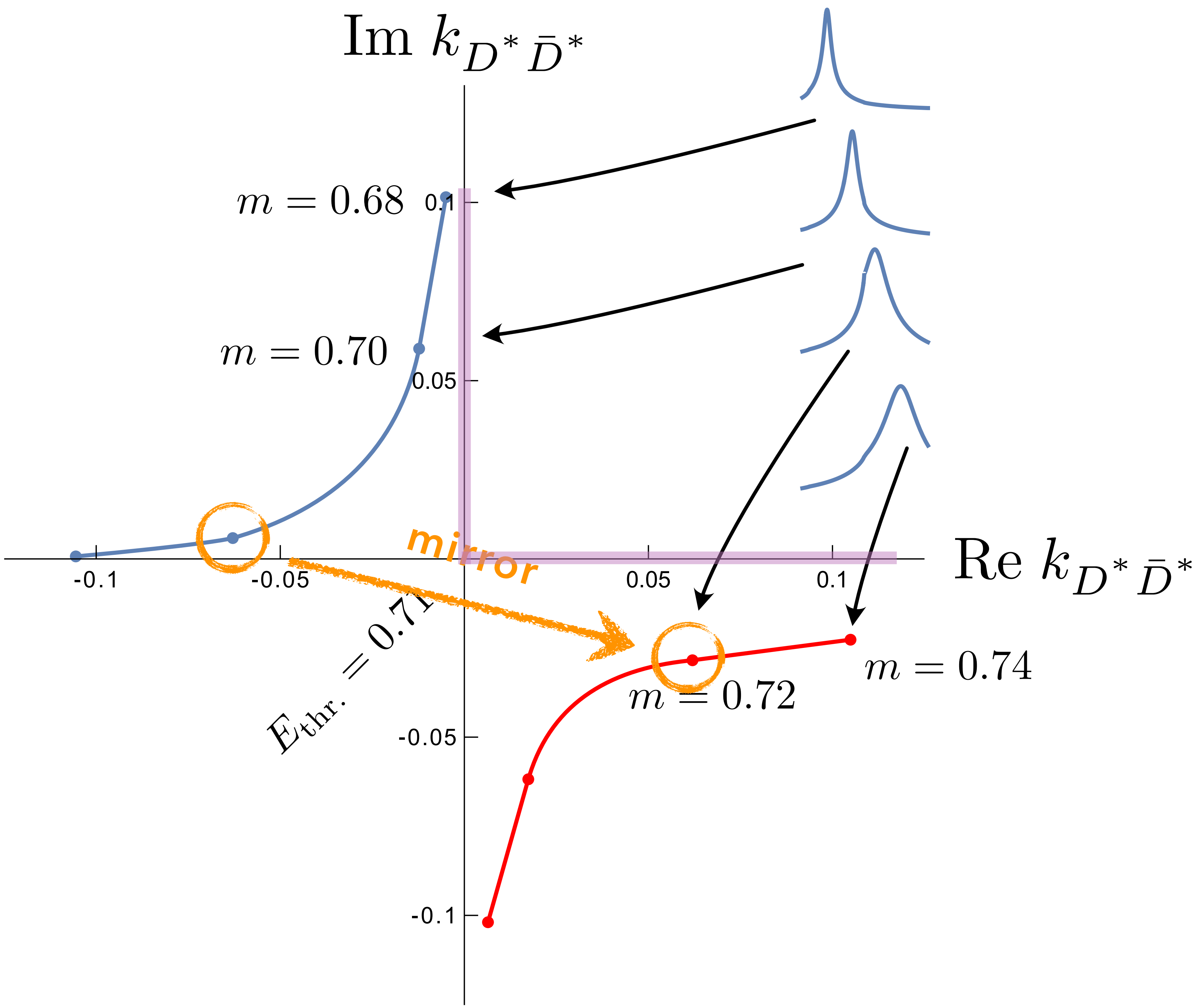
$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g_{D\bar{D}}^2 \rho_{D\bar{D}} - i g_{D^*\bar{D}^*}^2 \rho_{D^*\bar{D}^*}}$$



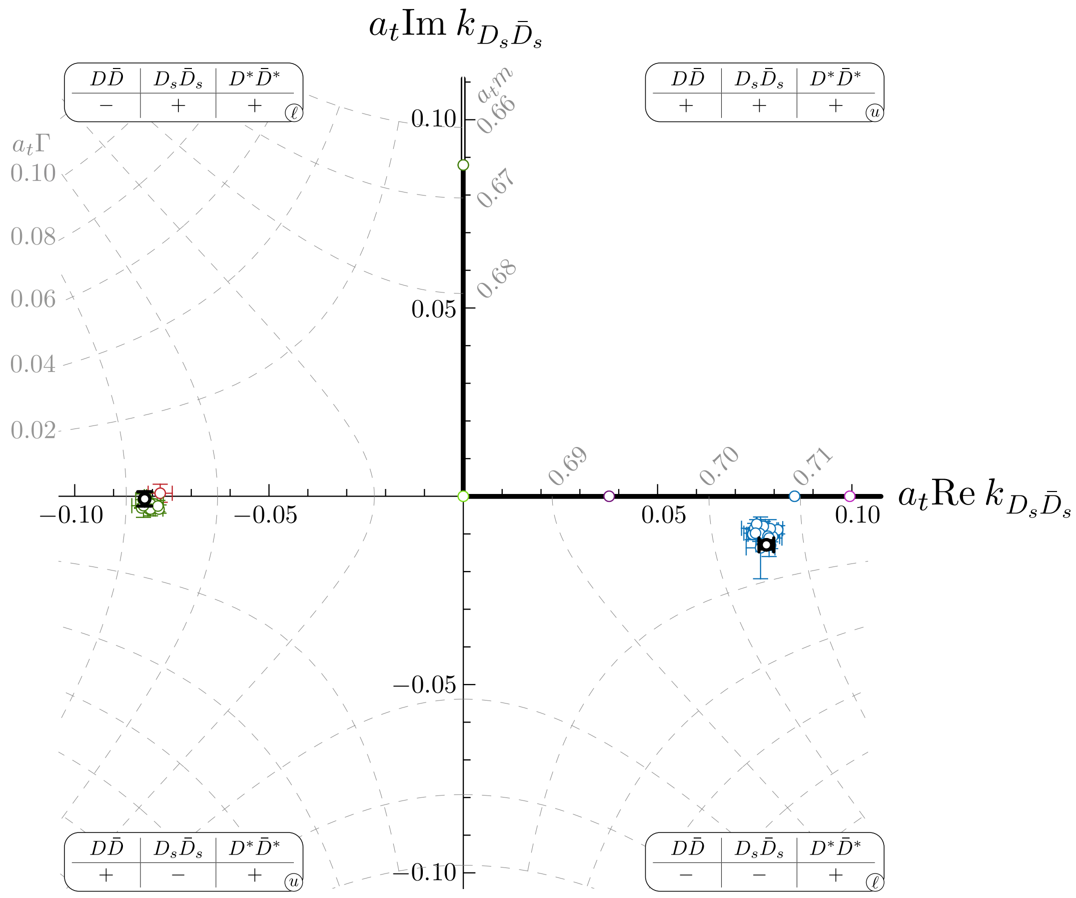
$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g_{D\bar{D}}^2 \rho_{D\bar{D}} - i g_{D^*\bar{D}^*}^2 \rho_{D^*\bar{D}^*}}$$

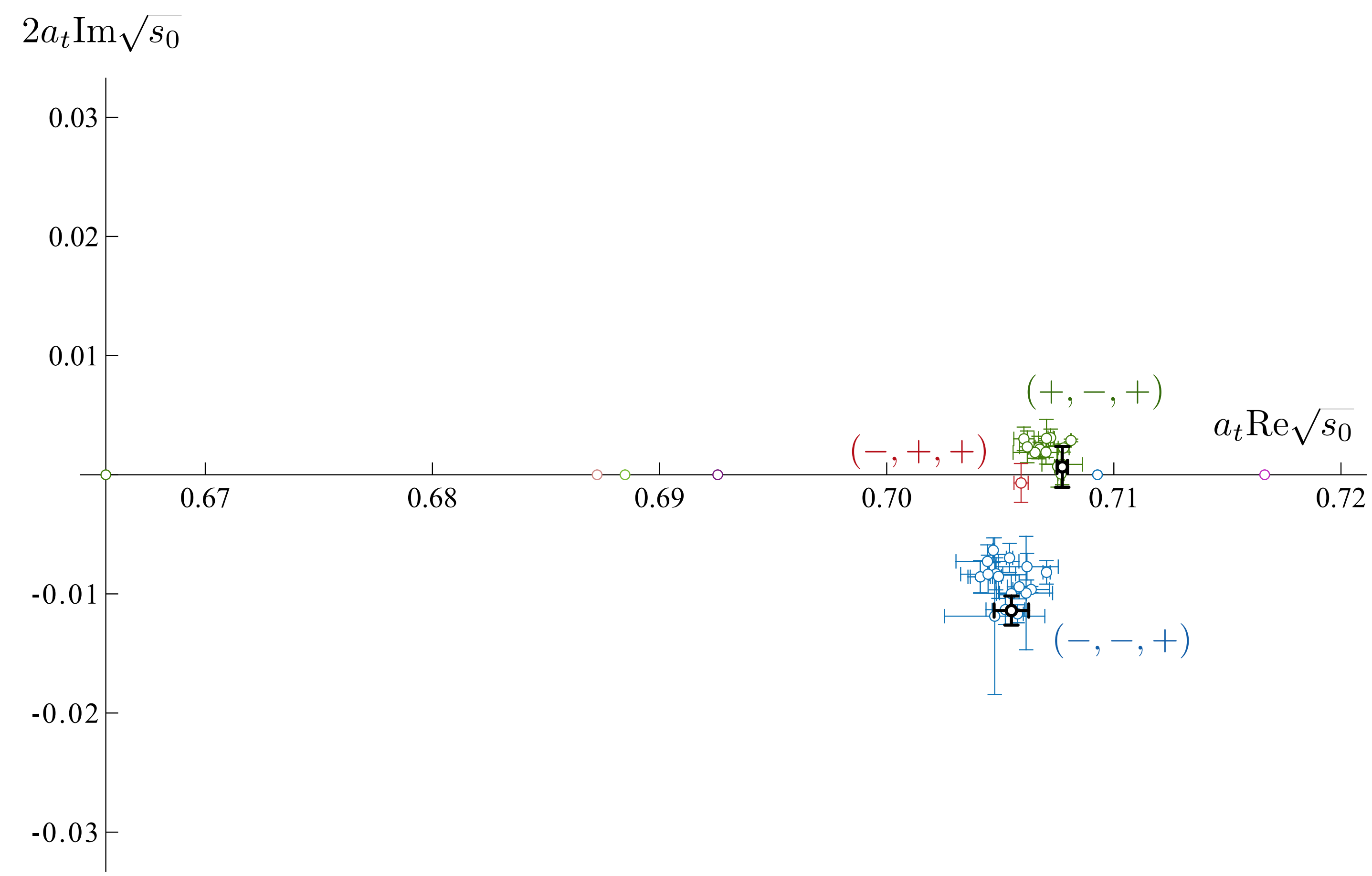


$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g^2_{D\bar{D}} \rho_{D\bar{D}} - i g^2_{D^*\bar{D}^*} \rho_{D^*\bar{D}^*}}$$



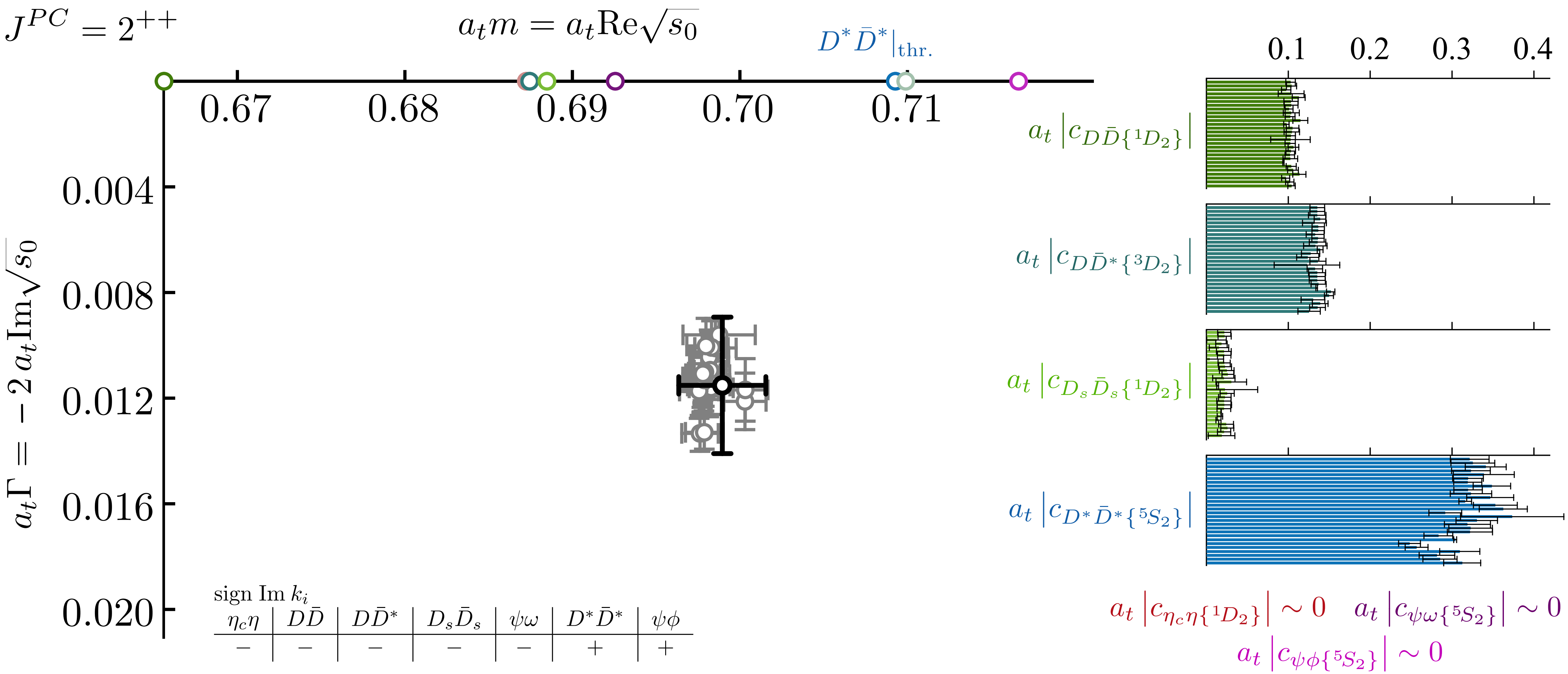
$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g_{D\bar{D}}^2 \rho_{D\bar{D}} - i g_{D^*\bar{D}^*}^2 \rho_{D^*\bar{D}^*}}$$





the “green” cluster of poles are just mirror poles

- amplitude is **dominated by a single resonance pole** in this energy region

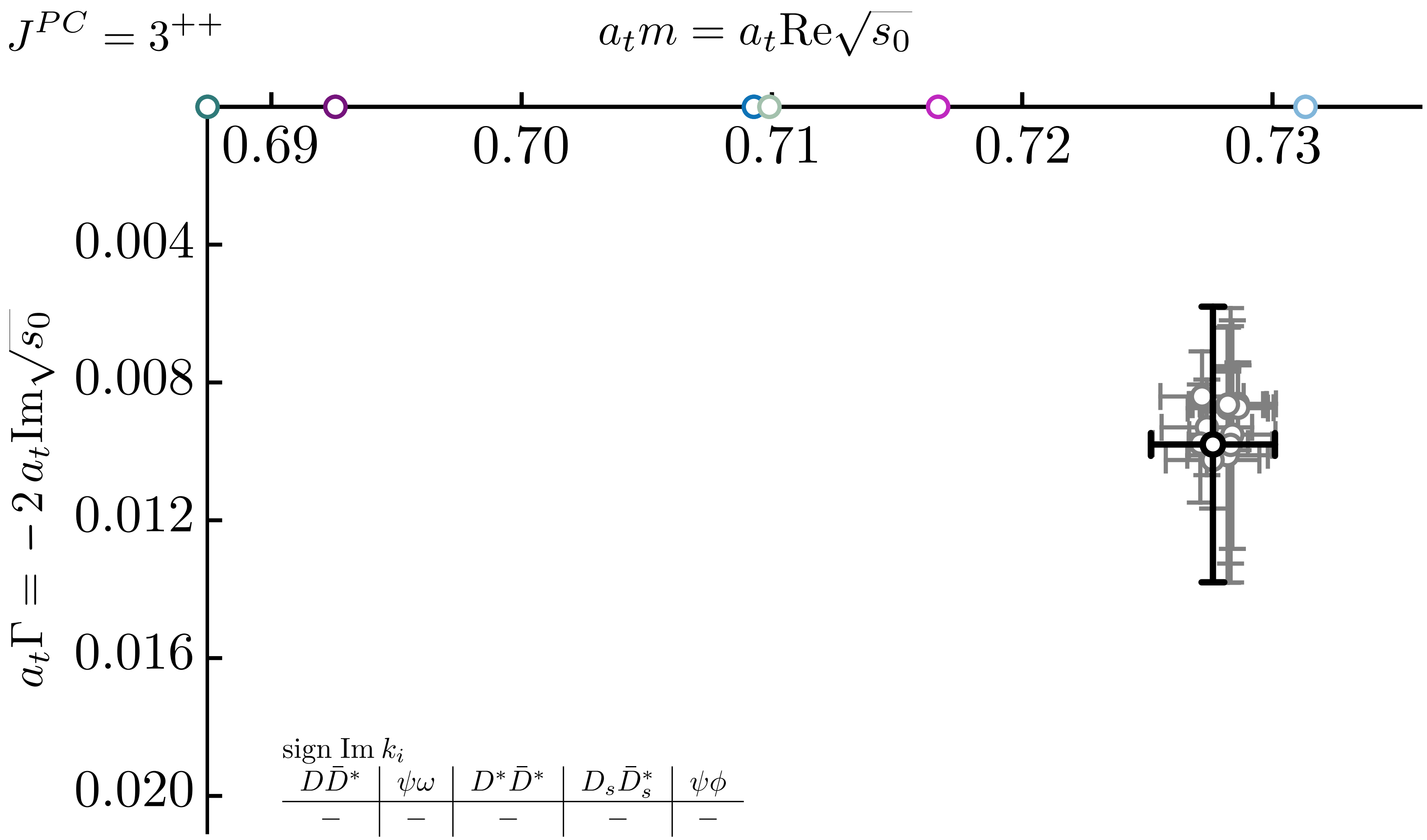


additional poles were found

- don't appear to be important

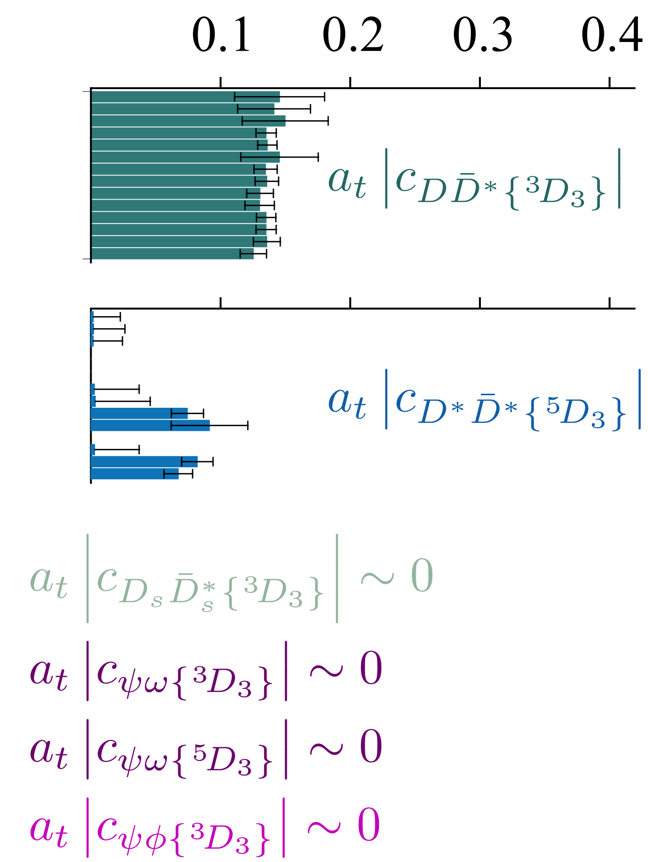
"coupling-ratio" phenomena seen in K-matrix pole parameters

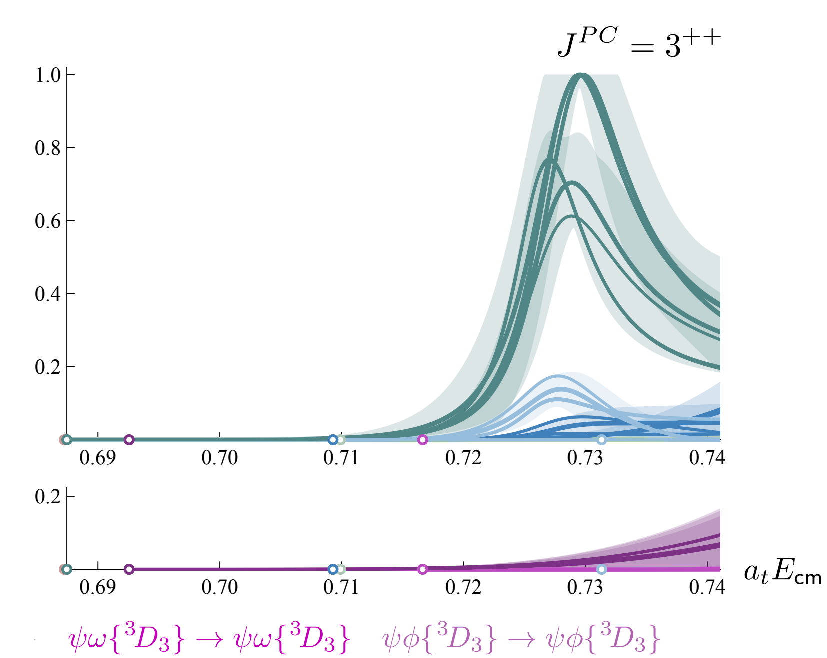
- possible to rescale K-matrix g_i factors and obtain similar amplitudes
- t-matrix couplings are found to be well-determined

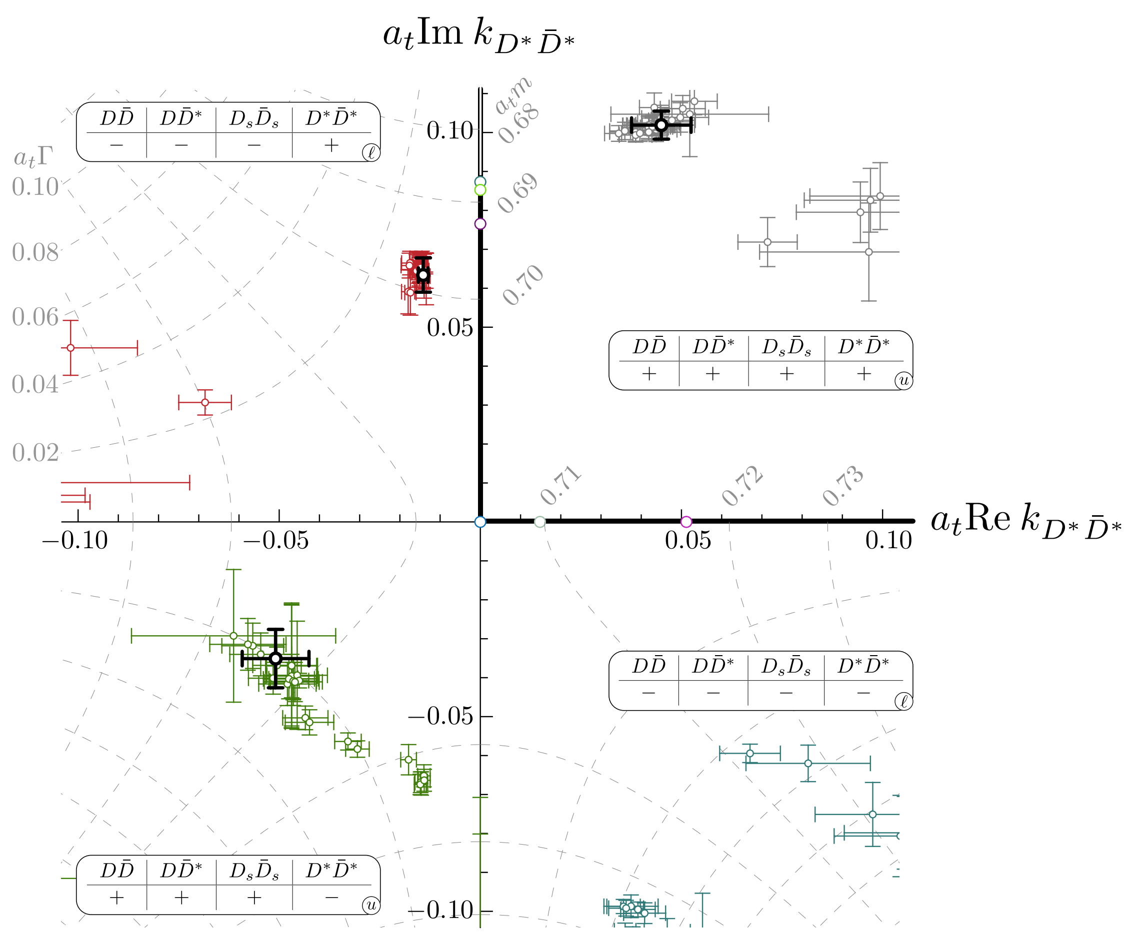


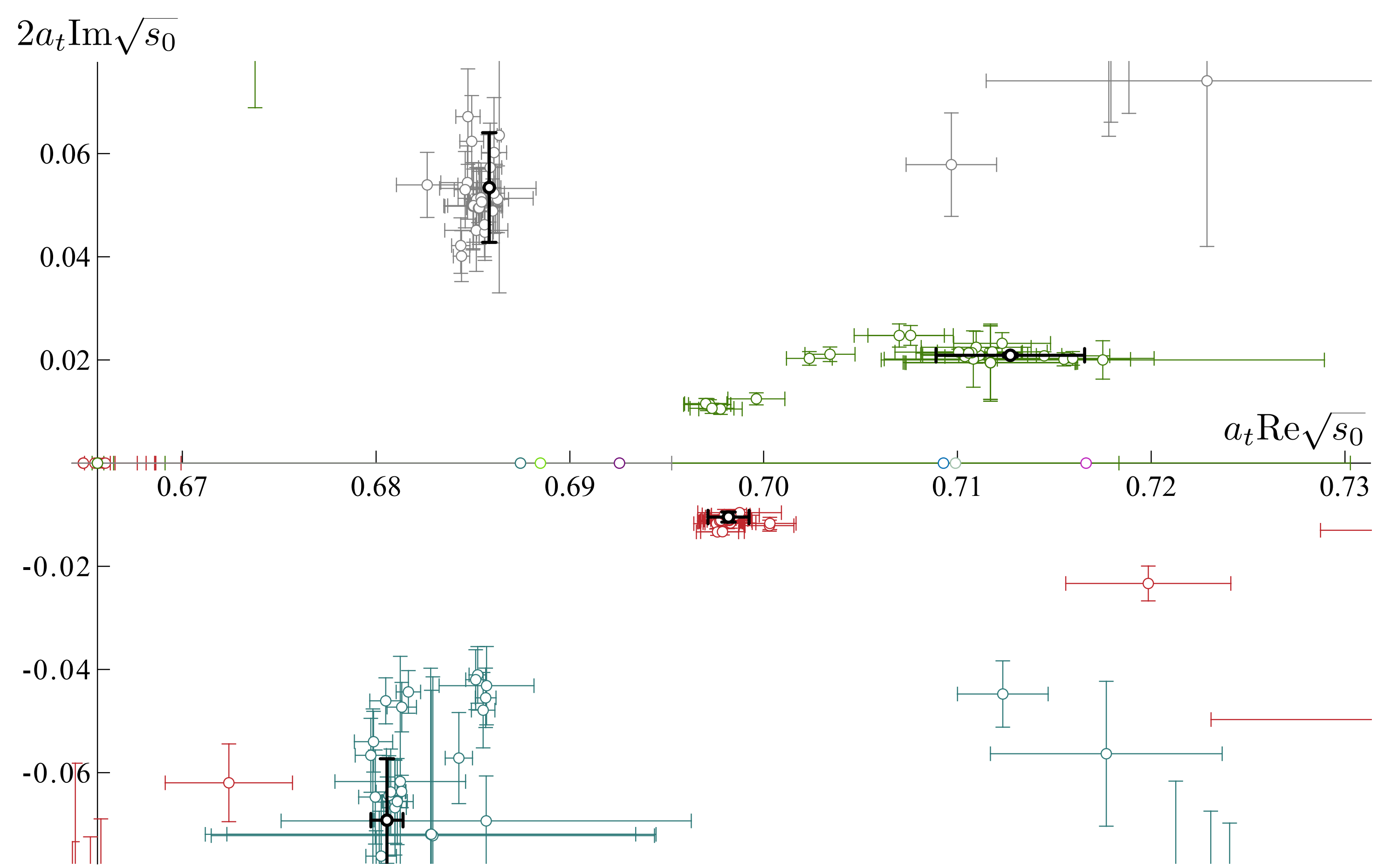
sign Im k_i

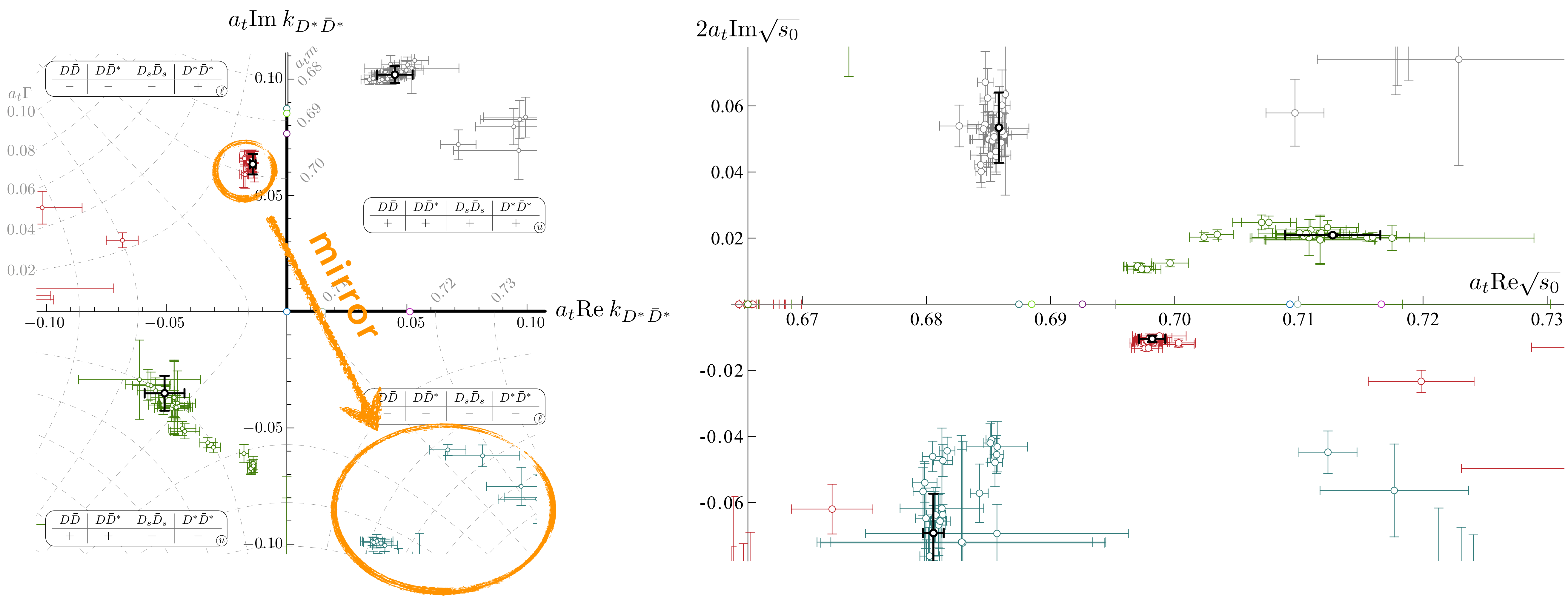
$D\bar{D}^*$	$\psi\omega$	$D^*\bar{D}^*$	$D_s\bar{D}_s^*$	$\psi\phi$
−	−	−	−	−



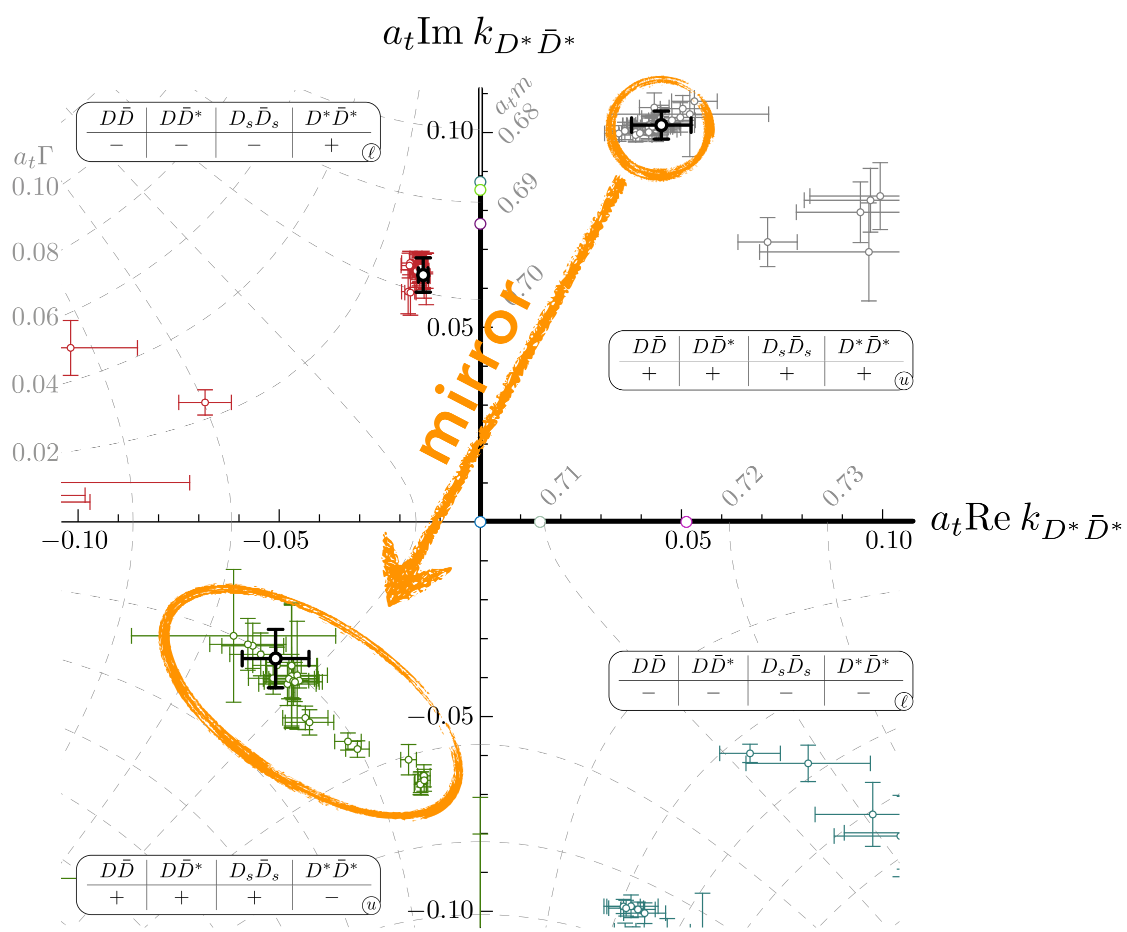


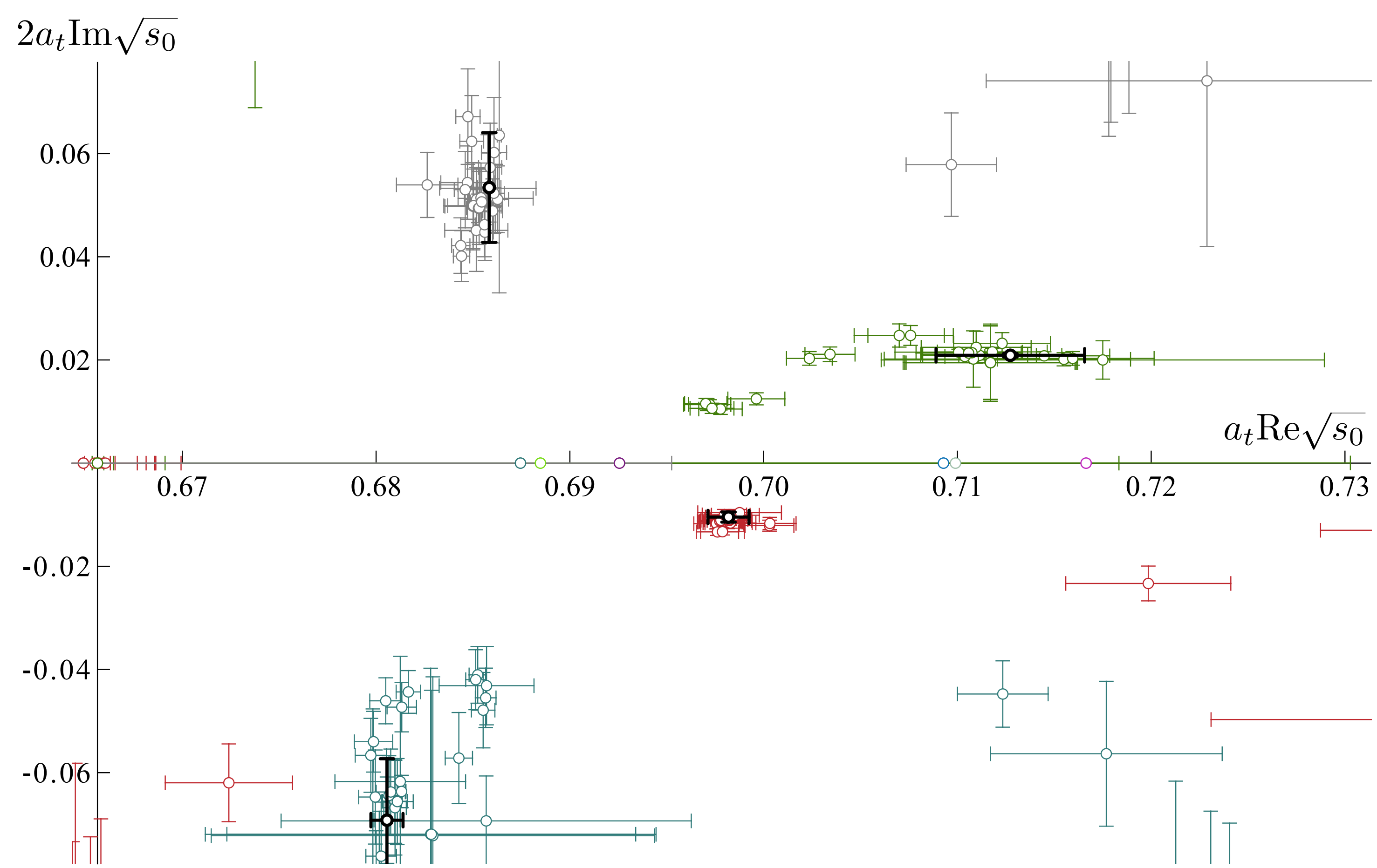

$a_t \mathrm{Im}\, k_{D^*\bar{D}^*}$
$a_t \mathrm{Re}\, k_{D^*\bar{D}^*}$
$a_t\Gamma$
$a_t m$
$D\bar{D}$ | $D\bar{D}^*$ | $D_s\bar{D}_s$ | $D^*\bar{D}^*$
− − − + ℓ
+ + + + u
− − − − ℓ
+ + + − u
0.68
0.69
0.70
0.71
0.72
0.73


$2a_t \mathrm{Im}\sqrt{s_0}$
$a_t \mathrm{Re}\sqrt{s_0}$



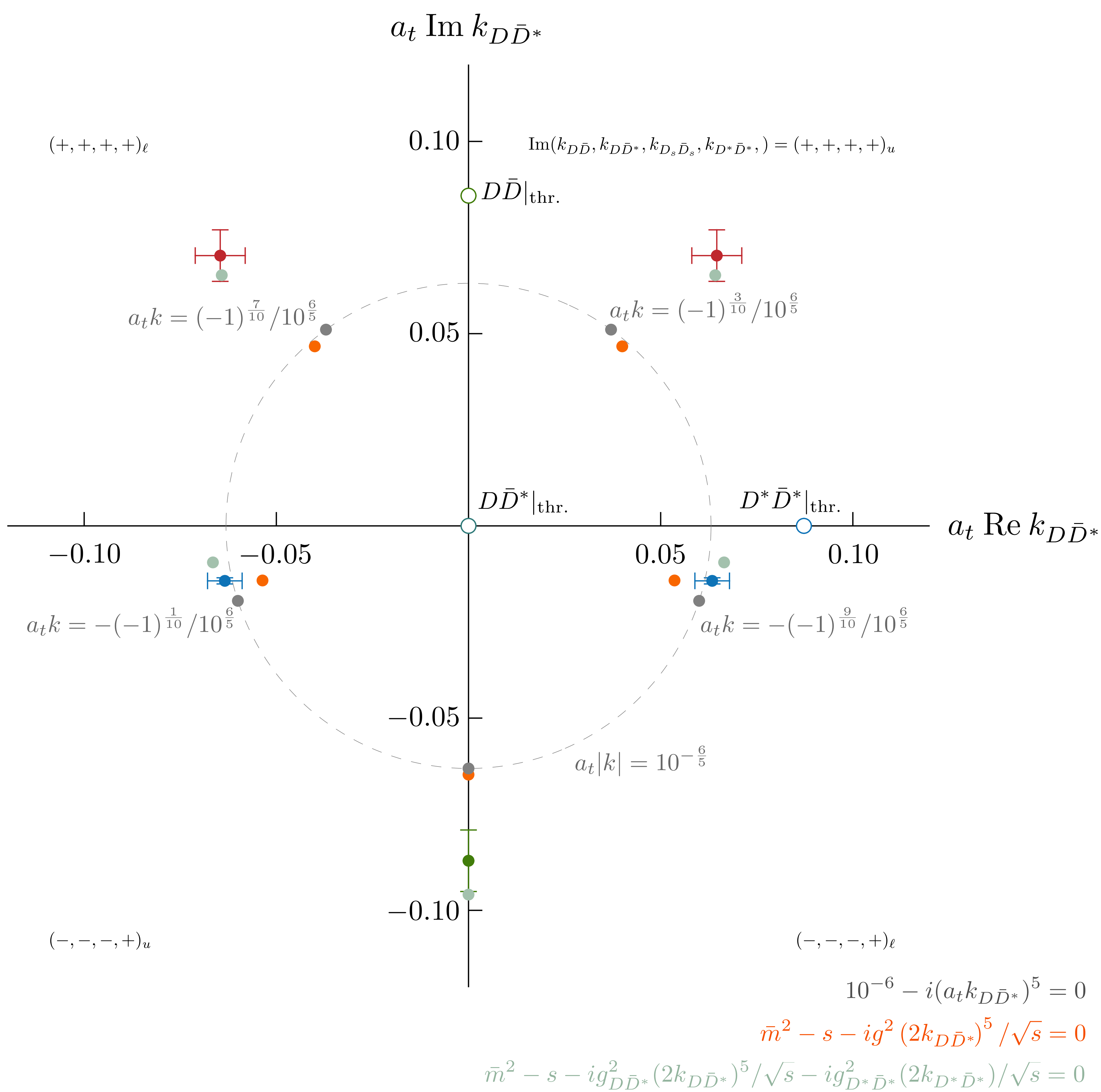
mirror pole - similar to a Flatté



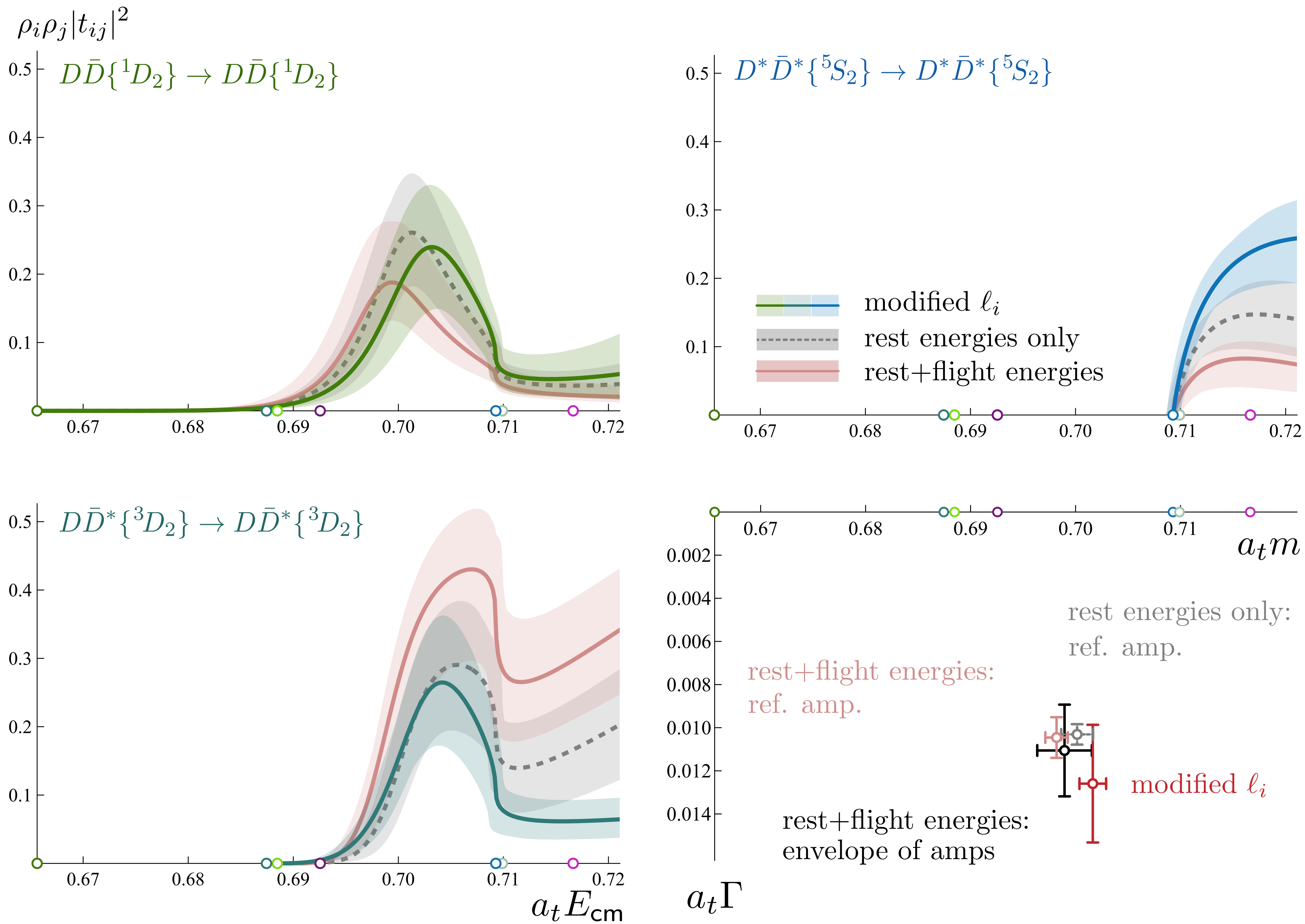


"green" pole is a mirror of the physical sheet pole

physical sheet pole arises because of the large g_{DD^*}



a_t Im $k_{D\bar{D}^*}$
a_t Re $k_{D\bar{D}^*}$
0.10
0.05
−0.05
−0.10
$(+,+,+,+)_\ell$
$\mathrm{Im}(k_{D\bar{D}}, k_{D\bar{D}^*}, k_{D_s\bar{D}_s}, k_{D^*\bar{D}^*},) = (+,+,+,+)_u$
$D\bar{D}|_{\mathrm{thr.}}$
$D\bar{D}^*|_{\mathrm{thr.}}$
$D^*\bar{D}^*|_{\mathrm{thr.}}$
$a_t k = (-1)^{\frac{7}{10}}/10^{\frac{6}{5}}$
$a_t k = (-1)^{\frac{3}{10}}/10^{\frac{6}{5}}$
$a_t k = -(-1)^{\frac{1}{10}}/10^{\frac{6}{5}}$
$a_t k = -(-1)^{\frac{9}{10}}/10^{\frac{6}{5}}$
$a_t|k| = 10^{-\frac{6}{5}}$
$(-,-,-,+)_u$
$(-,-,-,+)_\ell$
$10^{-6} - i(a_t k_{D\bar{D}^*})^5 = 0$
$\bar{m}^2 - s - ig^2\,(2k_{D\bar{D}^*})^5/\sqrt{s} = 0$
$\bar{m}^2 - s - ig^2_{D\bar{D}^*}(2k_{D\bar{D}^*})^5/\sqrt{s} - ig^2_{D^*\bar{D}^*}(2k_{D^*\bar{D}^*})/\sqrt{s} = 0$

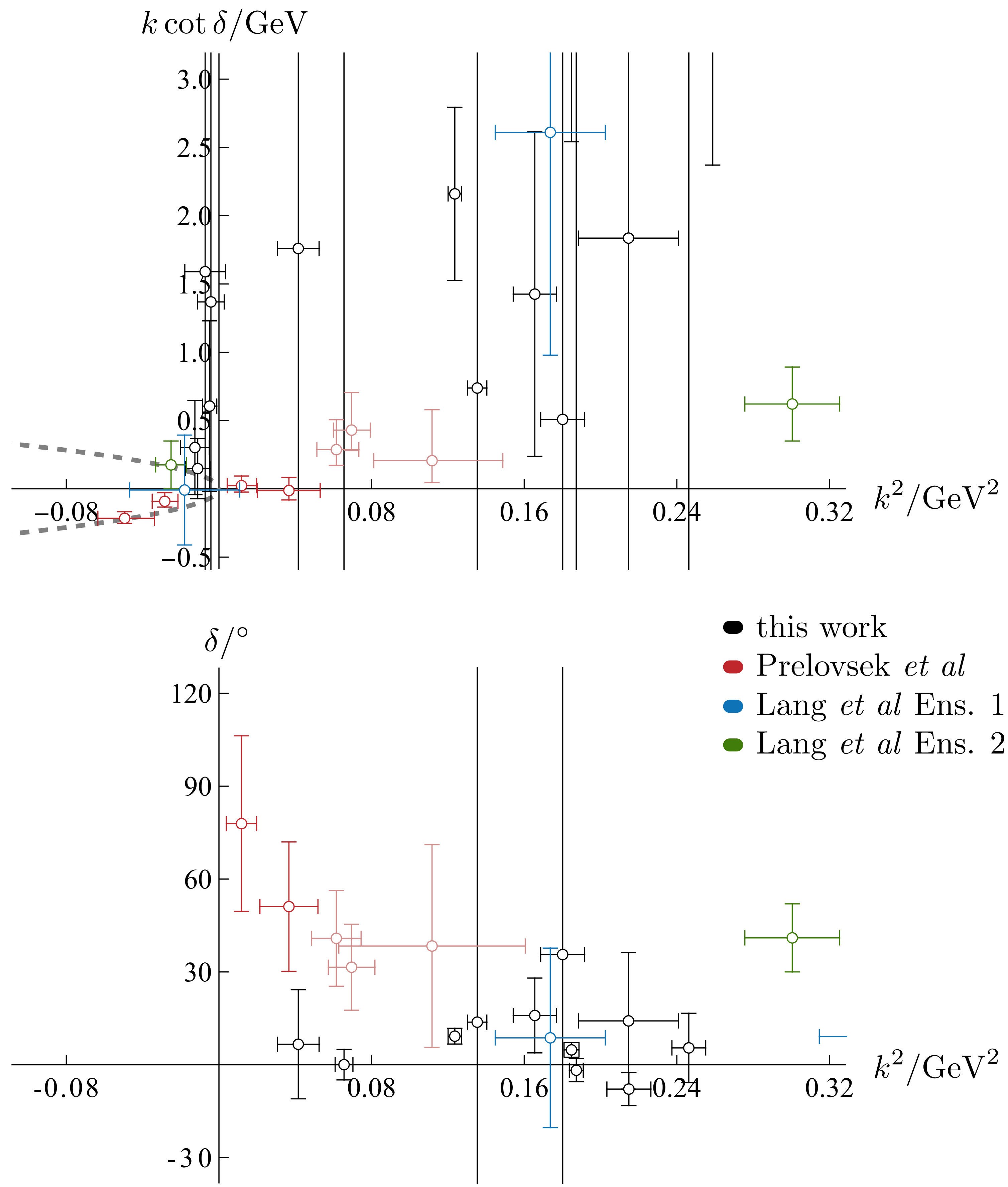


- different physical sheet pole
- no obvious nearby (+,+,+,-) sheet pole (there are some with a_t E >0.74)

Results from Prelovsek, Padmanath et al, suggest effects at DDbar and DsDsbar thresholds

- pion mass ~ 280 MeV
- light quark heavier than physical, strange quark lighter than physical

hard to justify such a large change due to the light quark mass (no one-pion-exchange term)

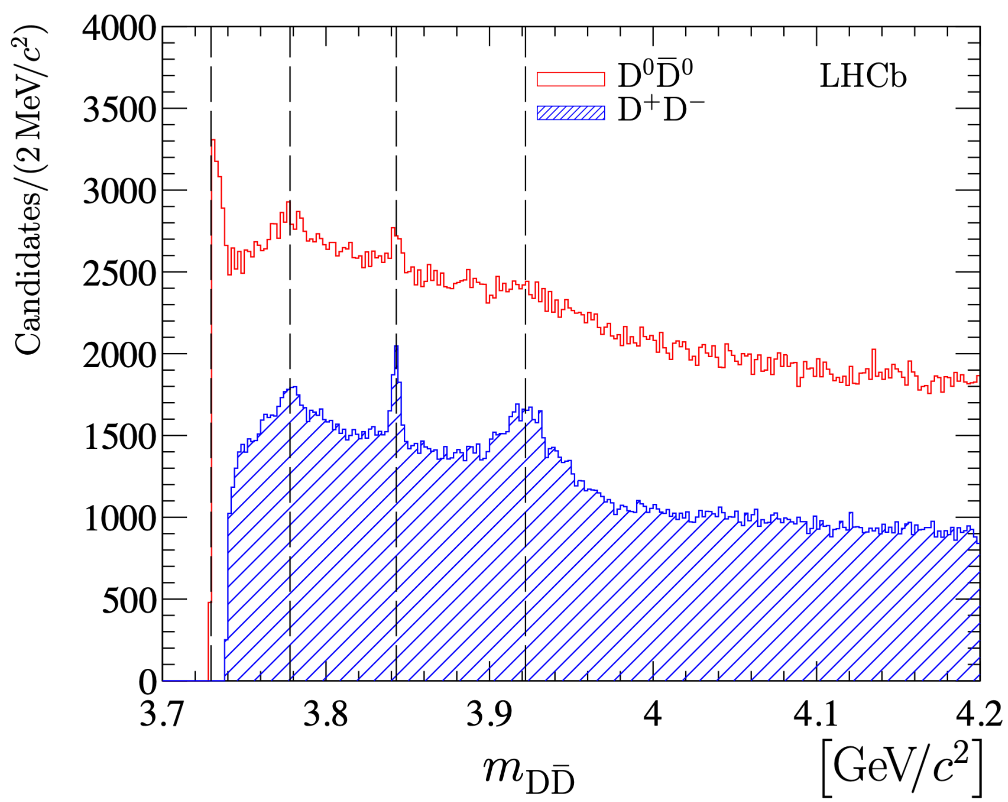


Many models with meson-meson components find strong effects in S-wave DDbar

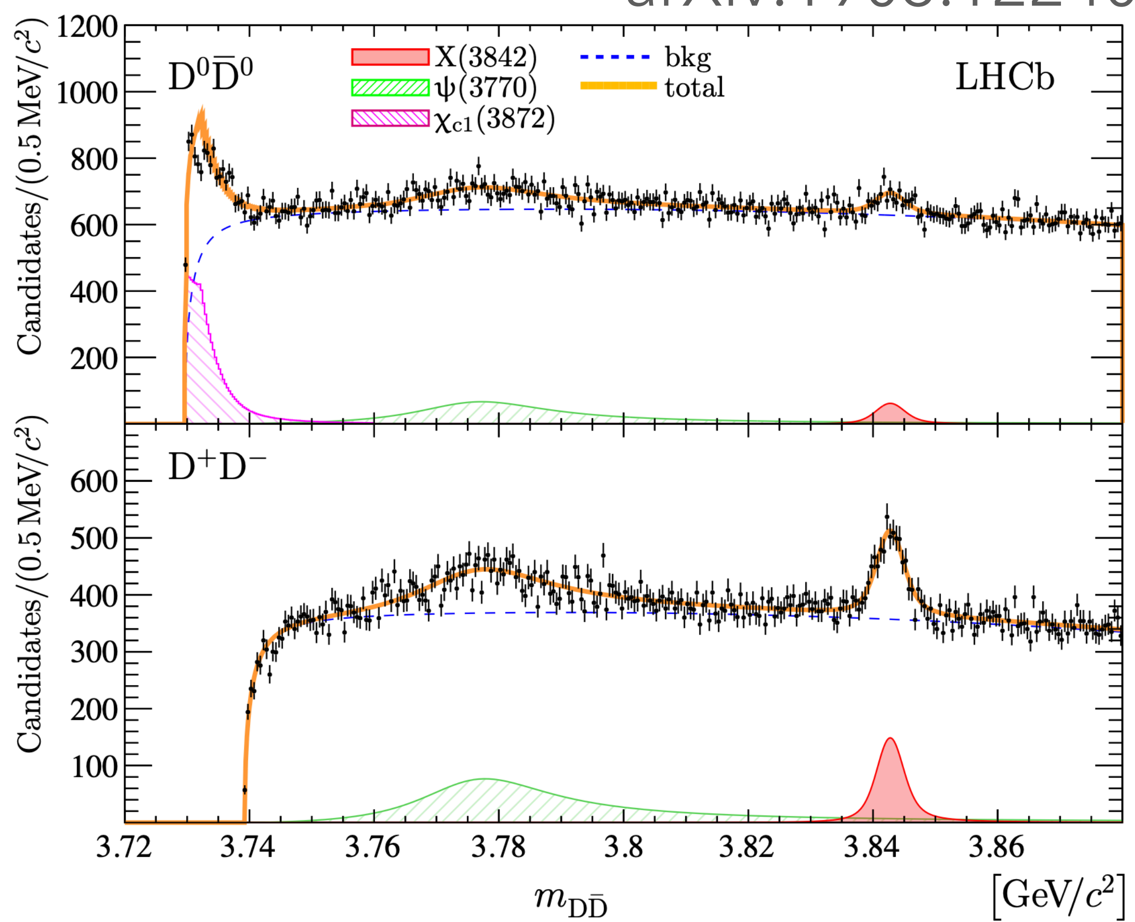
Several suggestions of a near-threshold state in DDbar scattering

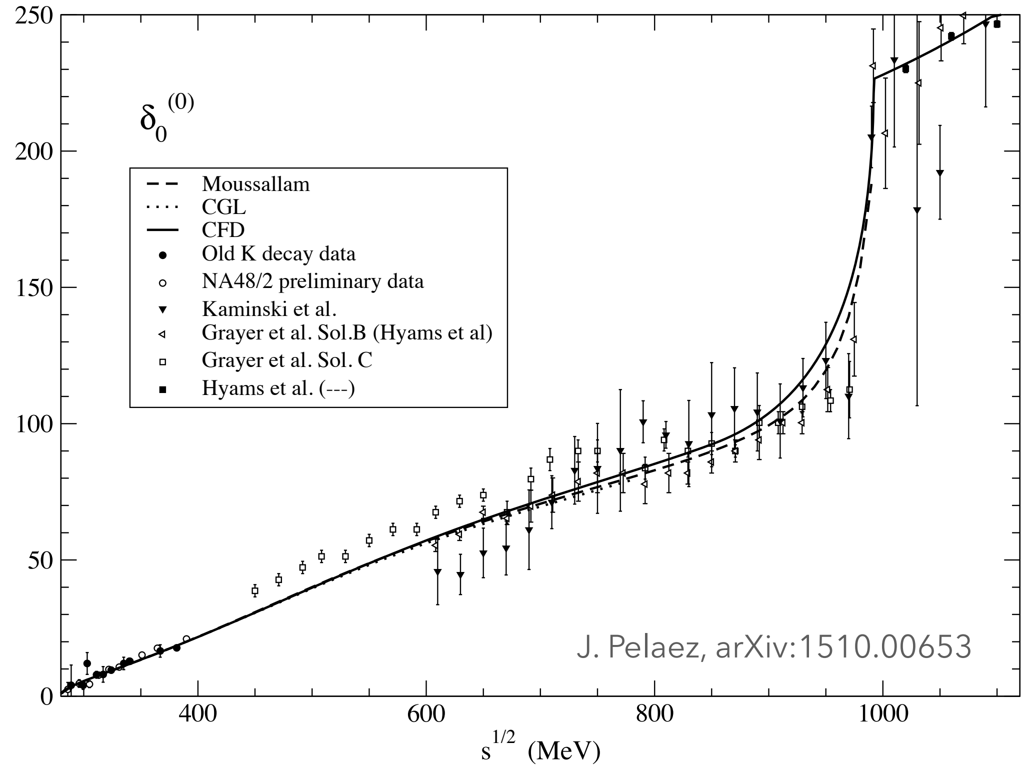
- γγ to DDbar (BaBar, Belle)
- near threshold structure partly due to Born/t-channel photon exchange
- see e.g. Guo & Meißner 2012, Wang et al 2021, Deineka et al 2022

Recent LHCb analyses find a peak at DDbar threshold but attribute this to "feed-down" from X(3872) decays

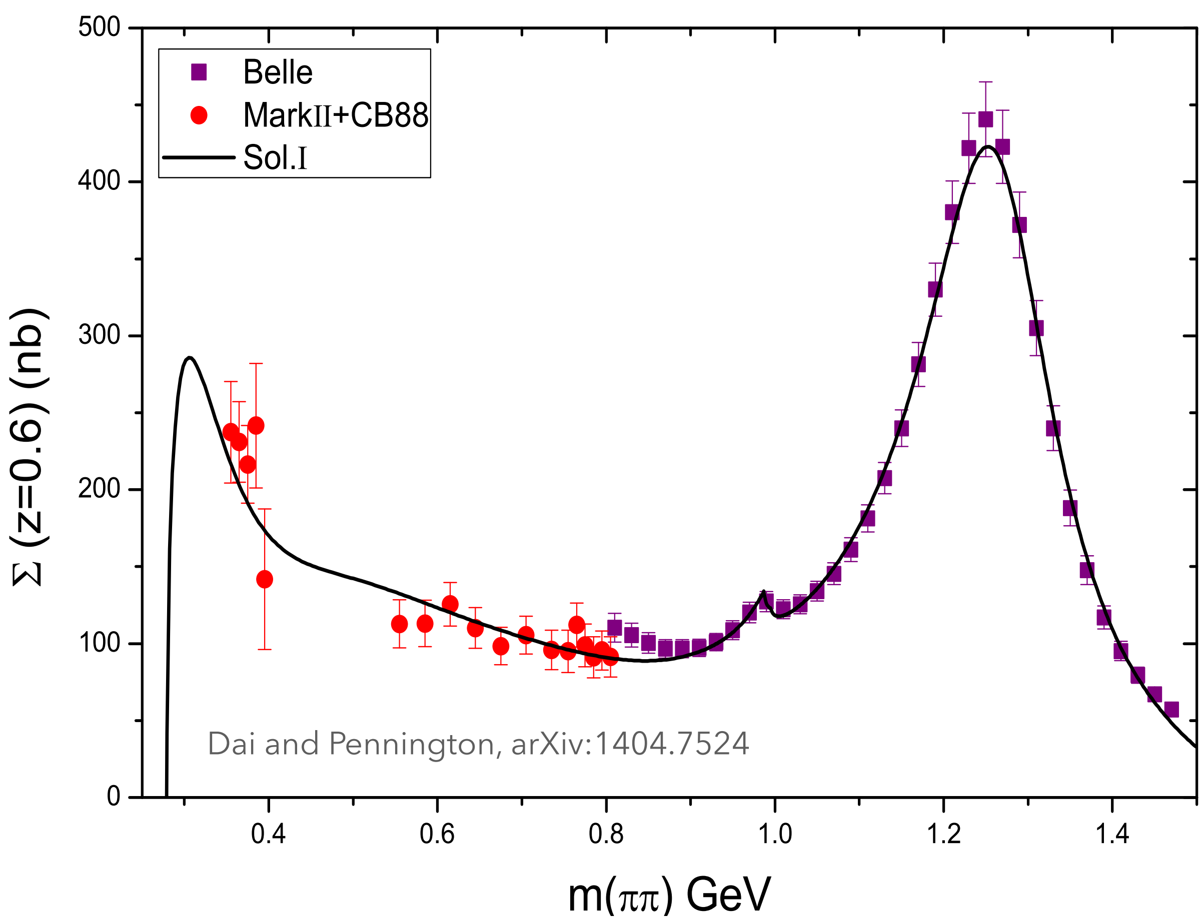


arXiv:1903.12240

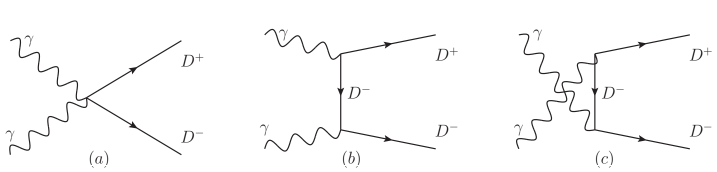




$$\pi\pi \to \pi\pi \quad (S - \text{wave})$$



$$\gamma\gamma \to \pi\pi$$



extra structure at threshold, not linked to a resonance or bound state