$DK/D\pi$ scattering and an exotic virtual bound state from lattice QCD

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Based on arXiv:2403.10498 in collaboration with Christopher Thomas, David Wilson

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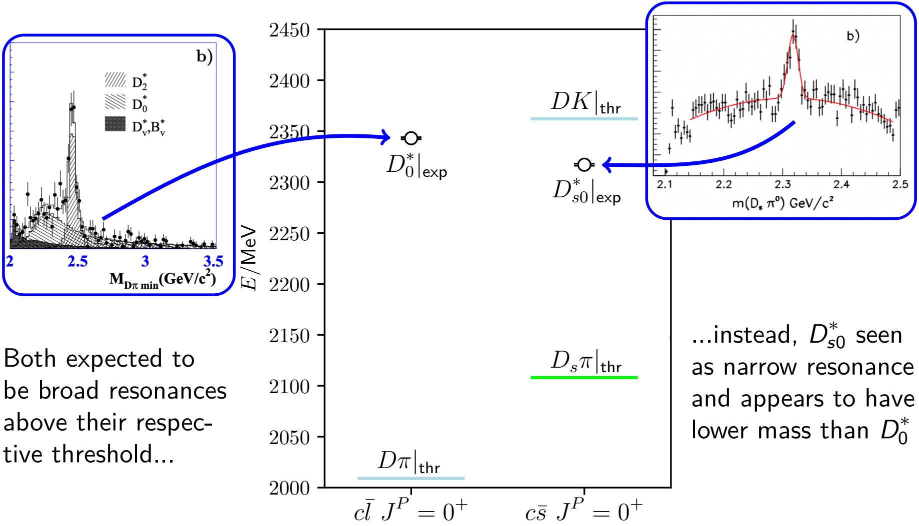
Open-charm $J^P = 0^+$ sector



Both expected to be broad resonances above their respective threshold...

...instead, D^*_{s0} seen as narrow resonance and appears to have lower mass than D^*_0

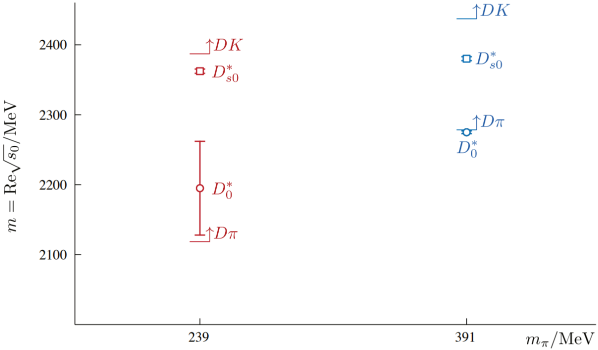
Previous work

Many previous studies of $D\pi$ and DK $S-$wave scattering

- D. Mohler *et al.* 1308.3175
- C.B. Lang *et al.* 1403.8103
- G. S. Bali *et al.* 1706.01247
- C. Alexandrou *et al.* 1911.08435
- A. M. Torres *et al.* 1412.1706
- L. Liu *et al.* 1208.4535
- X.-Y. Guo *et al.* 1801.10122
- Z.-H. Guo *et al.* 1811.05585
- B. Huang *et al.* 2205.02619
- M, F.M. Lutz *et al.* 2209.10601
- F. Gil-Domínguez *et al.* 2306.01848
- D. Mohler *et al.* 1208.4059
- H.Yan *et al.* 2404.13479

Previous work

- Observed single poles in elastic $D\pi$ and DK amplitudes [G.K.C. Cheung *et al.* 2008.06432, G. Moir *et al.* 1607.07093, L. Gayer *et al.* 2102.04973]

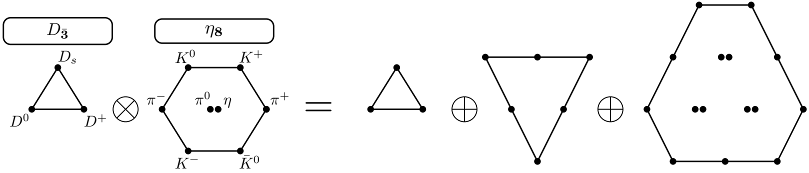


- m_π dependence? - Recent $D\pi$ study near physical m_π [H. Yan *et al.* 2404.13479]
- Potential exotic poles? - Virtual bound state in $D\bar{K}|_{I=0}$ scattering
- Second pole in $D\pi$ scattering at higher energies?

Flavour symmetric point

- $m_u = m_d = m_s \Rightarrow$ Flavour SU(3) global symmetry

$$\bar{\mathbf{3}} \otimes \mathbf{8} = \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}$$



- $\{D\pi, DK, Ds\bar{K}, ...\} \in D_{\bar{\mathbf{3}}}\eta_{\mathbf{8}}$ scattering
- Flavour $\bar{\mathbf{3}} \sim c\bar{q}$
- Flavour $\mathbf{6}$, $\overline{\mathbf{15}} \sim c\bar{q}q\bar{q}$
- $D_{\bar{\mathbf{3}}}\eta_{\mathbf{8}}$ $S-$wave $\rightarrow J^P = 0^+$

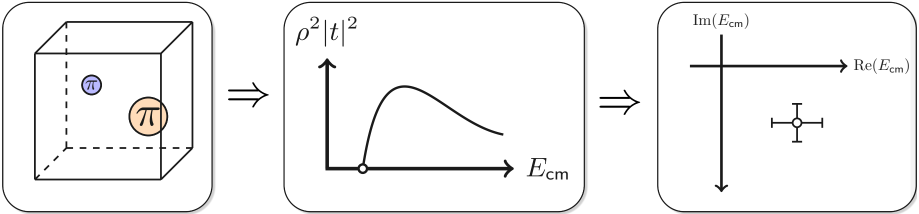
Lattice details

- Anisotropic lattices, $a_s/a_t \approx 3.5$
- 3 dynamical (light) + 1 quenched (charm) Wilson Clover fermions
- $m_u = m_d = m_s \approx m_s^{phy} \Rightarrow m_\pi \approx 700$ MeV

$(L/a_s)^3 \times (T/a_t)$	N_{cfgs}	N_{tsrcs}	N_{vecs}
$12^3 \times 96$	219	4	48
$14^3 \times 128$	397	4	64
$16^3 \times 128$	533	4	64
$18^3 \times 128$	358	4	96
$20^3 \times 128$	503	4	128
$24^3 \times 128$	607	4	160

- Volumes $L/a_s = \{16, 20, 24\}$ used in Lüscher analysis
- Single lattice spacing $a_t^{-1} = 4655$ MeV

Methodology



- Obtain finite-volume spectra from lattice QCD
- Constrain amplitudes via Lüscher quantisation condition [Lüscher, Sharpe, Hansen, Briceño ...]

$$\det\left[\mathbb{1} + i\rho(s)\cdot t(s)\cdot\left(\mathbb{1} + i\overline{\mathcal{M}}(s,L)\right)\right] = 0$$

- Study singularity structure
  - Elastic amplitude has two Riemann sheets
  - Bounds states on physical sheet $\mathrm{Im}[k] > 0$
  - Virtual bounds states and resonances on unphysical sheet $\mathrm{Im}[k] < 0$

Flavour $\bar{\mathbf{3}}$ Sector

Finite-volume spectra

Operator basis

- $\mathcal{O}^\dagger \sim \bar{q}\boldsymbol{\Gamma} c$ for $q \in \{u, d, s\}$
- $\mathcal{O}^\dagger_{\mathbb{M}_1\mathbb{M}_2}(\mathbf{P}) \sim \sum_{\mathbf{p}_1,\mathbf{p}_2} CG\ \Omega^\dagger_{\mathbb{M}_1}(\mathbf{p}_1)\ \Omega^\dagger_{\mathbb{M}_2}(\mathbf{p}_2)$

GEVP

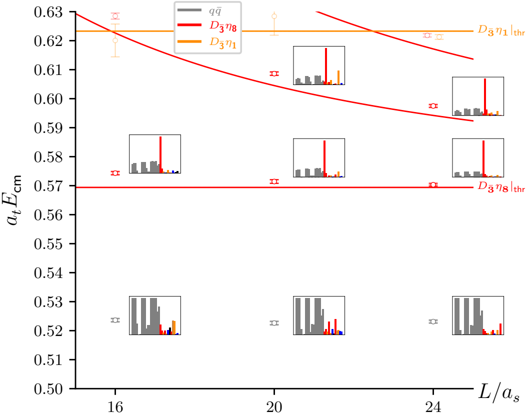
- $C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}^\dagger_j(0)|0\rangle \Rightarrow C_{ij}(t)v^{\mathbf{n}}_j = \lambda^{(\mathbf{n})}(t,t')C_{ij}(t')v^{\mathbf{n}}_j$

$$\lambda^{(\mathbf{n})}(t,t') \sim e^{-E_{\mathbf{n}}(t-t')}$$

- Correlation functions computed in distillation framework [M. Peardon *et al.* 0905.2160]

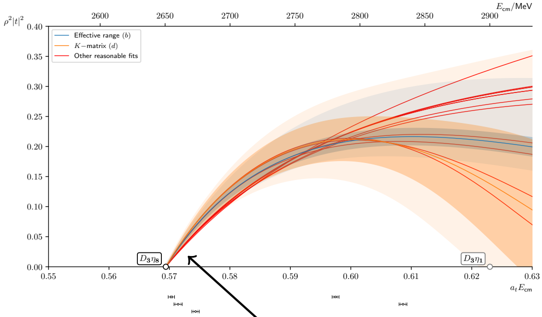
Flavour $\bar{3}$ Sector

- Up to inelastic threshold, $[000]A_1^+$ irrep $\sim J^P = 0^+$

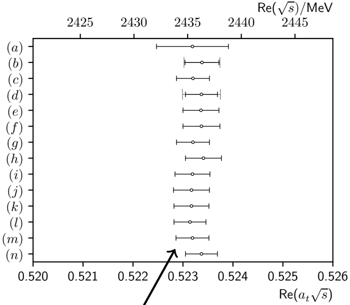


- $\sim \bar{q}c$ dominated state below threshold
- Small upward shifts from non-interacting meson-meson energies

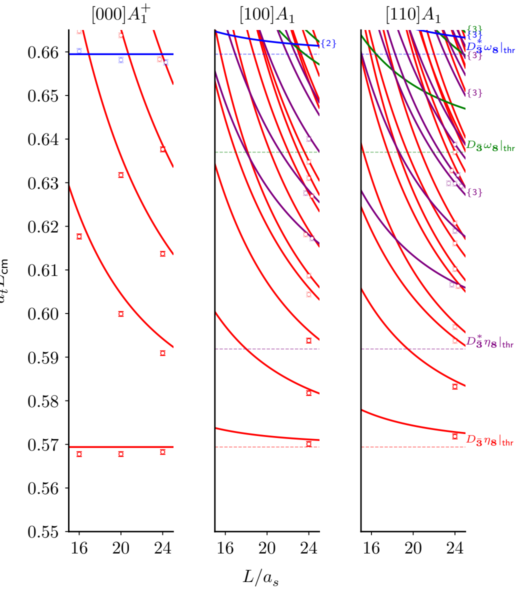
Amplitudes and poles



Rapid growth at threshold



Bound state locations



$\bar{q}\Gamma c \not\ni \mathbf{6}$ ⇒**manifestly flavour exotic**

More constraints needed around threshold...

... include $[100]A_1, [110]A_1$ irreps

$[nmp]\Lambda$	$D_{\bar{\mathbf{3}}}\eta_{\mathbf{8}}(\,^{2S+1}\ell_J)$
$[100]A_1$	$^1S_0$, $^1P_1$, $^1D_2$, $^1F_3$, ...
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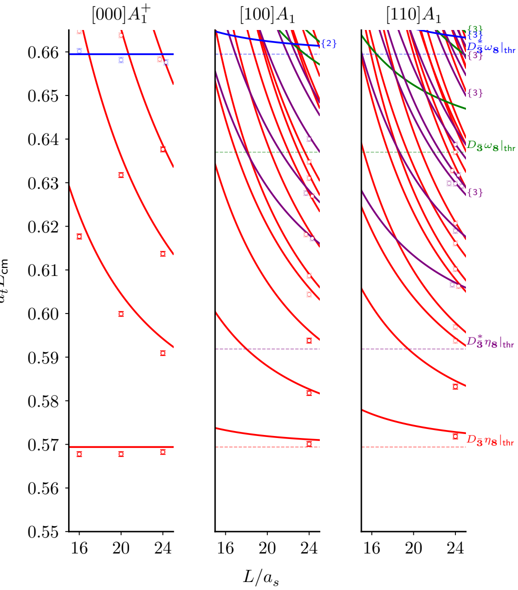

$\bar{q}\Gamma c \not\to$ **6** $\Rightarrow$**manifestly flavour exotic**

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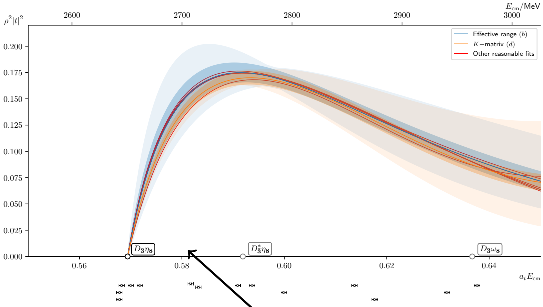
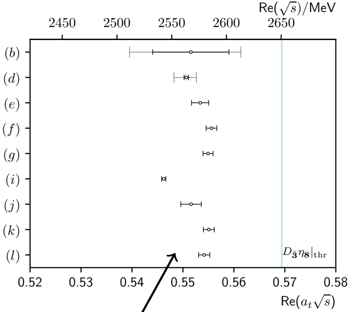


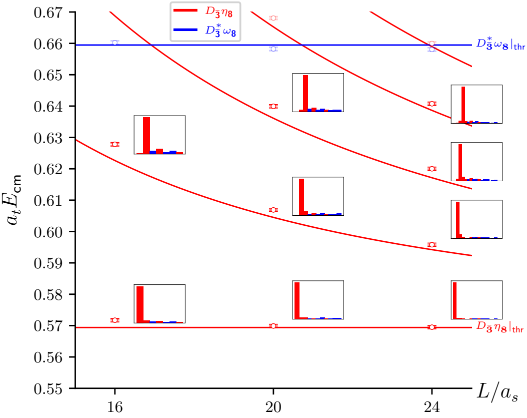


Again, rapid growth at threshold

Unphysical sheet pole locations

- **Virtual bound state** near threshold

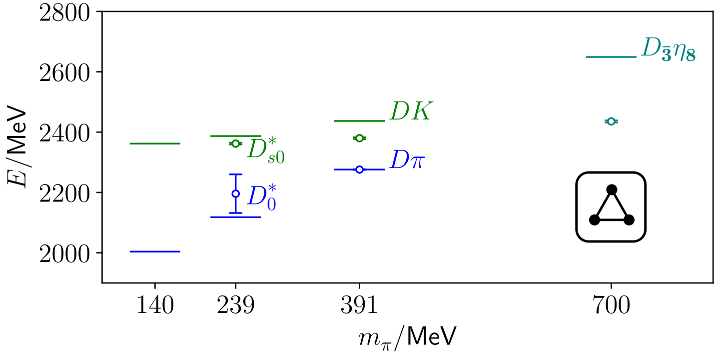
- Again, **manifestly flavour exotic**
- Only $\mathcal{O}^{\dagger}_{\mathbb{M}_1\mathbb{M}_2}(\mathbf{P}) \sim \sum_{\mathbf{p}_1,\mathbf{p}_2} CG\ \Omega^{\dagger}_{\mathbb{M}_1}(\mathbf{p}_1)\ \Omega^{\dagger}_{\mathbb{M}_2}(\mathbf{p}_2)$



- Resultant amplitudes show **weak** interactions

Intepretation

- Flavour $\bar{\mathbf{3}}$ pole appears to be consistent with quark model expectations
- Contains (strange, isospin) $= (0, \frac{1}{2})$ and $(1, 0)$ components
- Breaking flavour $SU(3)$...



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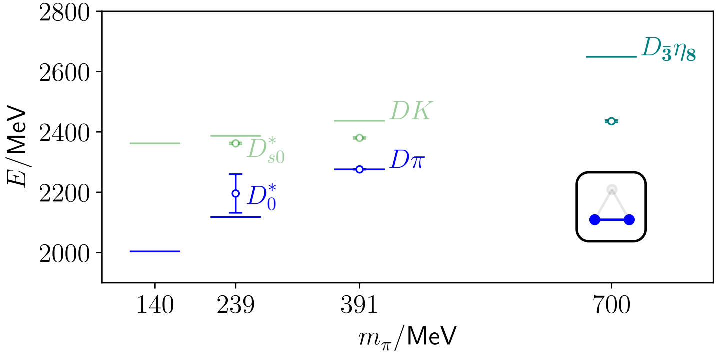
  ... $(0, \frac{1}{2})$ contributes to $D\pi$ scattering



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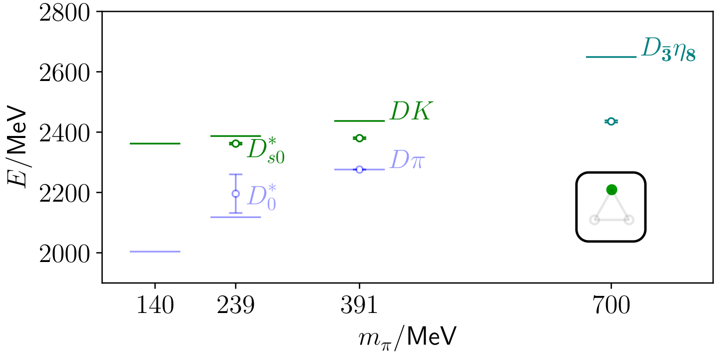
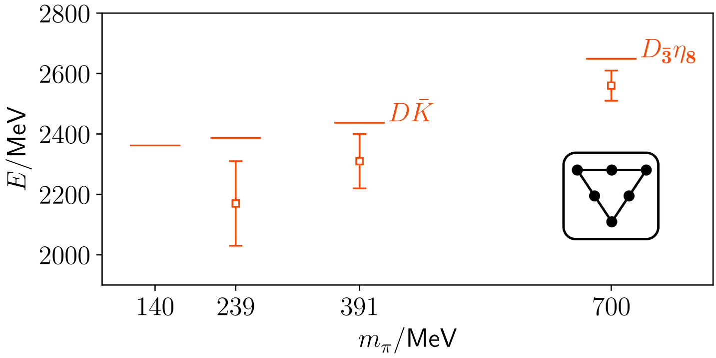
  ... $(0, \frac{1}{2})$ contributes to $D\pi$ scattering
  ... $(1, 0)$ contributes to DK scattering



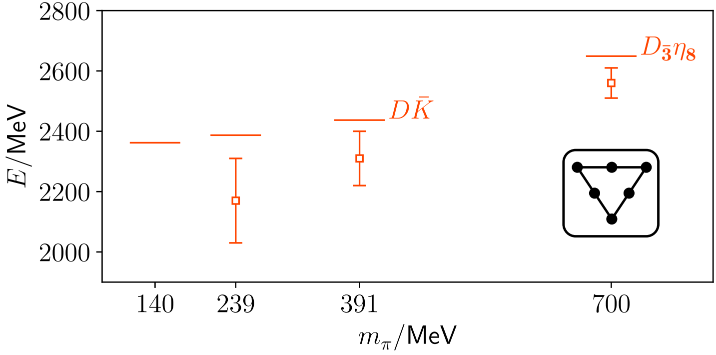
Intepretation

- Flavour exotic **6** virtual bound state in qualitative agreement with UChPT
- (strange, isospin) = $(-1,0)$, $(0,\frac{1}{2})$ and $(1,1)$ components
- Breaking flavour $SU(3)$...



Intepretation

- Flavour exotic **6** virtual bound state in qualitative agreement with UChPT
- (strange, isospin) = $(-1, 0)$, $(0, \frac{1}{2})$ and $(1, 1)$ components
- Breaking flavour $SU(3)$...
  ... $(0, \frac{1}{2})$ could also contribute to $D\pi$ scattering
  $\Rightarrow$ potential two-pole structure!



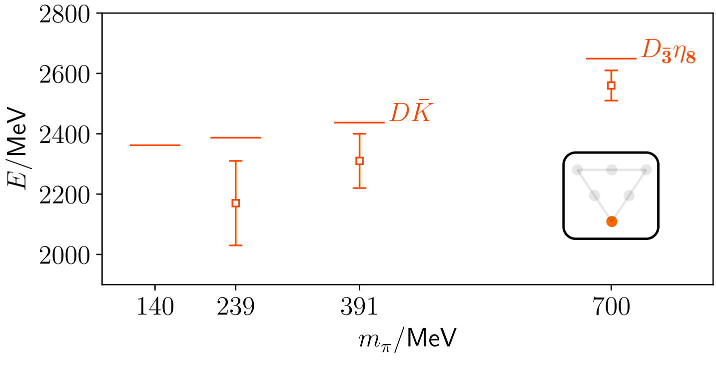
Intepretation

- Flavour exotic **6** virtual bound state in qualitative agreement with UChPT
- (strange, isospin) $= (-1, 0)$, $(0, \frac{1}{2})$ and $(1, 1)$ components
- Breaking flavour $SU(3)$...

  ... $(0, \frac{1}{2})$ could also contribute to $D\pi$ scattering

  $\Rightarrow$ potential two-pole structure!

  ... however, only $(-1, 0)$ component seen in previous lattice studies



Summary and outlook

Summary

- **At $\mathbf{m_\pi} \approx$ 700 MeV**, 2 poles found
  Bound state in $\bar{\mathbf{3}}$ & **virtual** bound state in **6**
- Flavour $\bar{\mathbf{3}}$ corresponds to poles in elastic $D\pi$ and DK scattering
- Flavour **6** pole $\rightarrow$ possible $D\pi$ two-pole structure ...
  ... but need to study its light quark mass dependence

Outlook

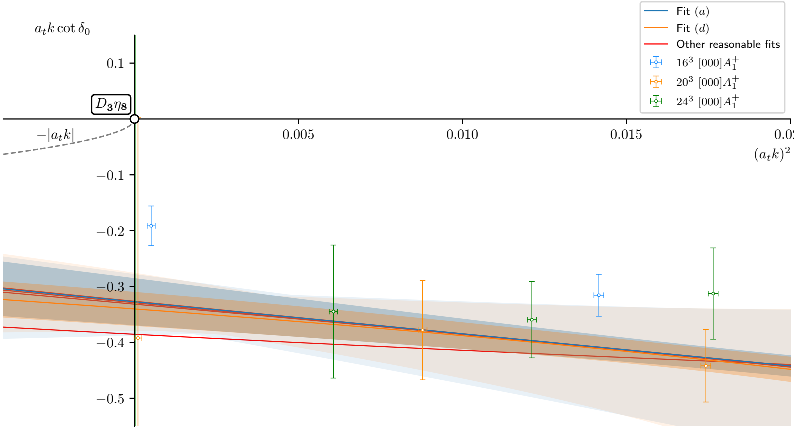
- $DK/D\pi$ scattering at flavour symmetric point with lower m_π
- $D\pi - D\eta - D_s\bar{K}$ scattering at closer to physical m_π
- Analogous studies of open charm $J^P = 1^+$ scattering

Thank you for listening!
Any questions?

Backup slides

Flavour $\overline{\mathbf{15}}$ results

- Fits represented as $k \cot \delta$



- No significant energy dependence and no poles nearby

  $\longrightarrow$ Weak interactions

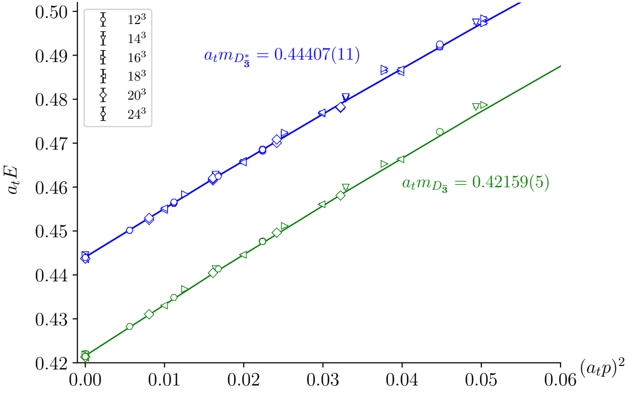
Backup slides

Details of lattice action

- Using anisotropic lattices, $a_s/a_t \approx 3.5$
- 3 dynamical quarks (u,d,s) and 1 quenched (charm) quark
- Gauge sector: tree-level Symanzik-improved anisotropic action
- Fermion sector: tadpole-improved anisotropic Sheikholeslami-Wohlert (Clover) actionwith spatial stout-smearing
- Single lattice spacing $a_t^{-1} = 4655$ MeV

Stable $D_{\bar{\mathbf{3}}}$ and $D^*_{\bar{\mathbf{3}}}$ mesons

- Using $\mathcal{O}^\dagger(\mathbf{P}) \sim \sum_{\mathbf{x}} e^{-i\mathbf{P}\cdot\mathbf{x}} \bar{q}\mathbf{\Gamma} c$ for $q \in \{u, d, s\}$ in GEVP
- $\mathbf{P} = \frac{2\pi}{L}\mathbf{n}$ for $|\mathbf{n}|^2 \leq 2, 3, 4$ on $L/a_s = 12, 14, \{16, 18, 20, 24\}$ volumes
- Fit to relativistic dispersion relation $(a_t E_{\mathbf{n}})^2 = (a_t m)^2 + \frac{1}{\xi^2}\left(\frac{2\pi}{L/a_s}|\mathbf{n}|\right)^2$



Flavour **6** Sector subductions

- Additional data from $[100]A_1, [110]A_1$
  $\rightarrow$ Include contributions from $J^P = 1^-, 2^+, 3^+$

$[000]\Lambda^P$	J^P	Channel	${}^{2S+1}\ell_J$
$[000]A_1^+$	0^+, 4^+, ...	$D_{\bar{3}}\eta_8$	${}^1S_0$
		⋮	⋮
$[000]T_1^-$	1^-, 3^-, 4^-, ...	$D_{\bar{3}}\eta_8$	${}^1P_1$, ${}^1F_3$
		$D^*_{\bar{3}}\eta_8$	${}^3P_1$, ${}^3F_3$, ${}^3F_4$
		⋮	⋮
$[000]E^+$	2^+, 4^+, ...	$D_{\bar{3}}\eta_8$	${}^1D_2$
		$D^*_{\bar{3}}\eta_8$	${}^3D_2$
		⋮	⋮
$[000]T_2^+$	2^+, 3^+, 4^+, ...	$D_{\bar{3}}\eta_8$	${}^1D_2$
		$D^*_{\bar{3}}\eta_8$	${}^3D_2$, ${}^3D_3$
		⋮	⋮

$[nmp]\Lambda$	$\lvert\lambda\rvert^{(\tilde{\eta})}$	Channel	${}^{2S+1}\ell_J$
$[100]A_1$	0^+, 4, ...	$D_{\bar{3}}\eta_8$	${}^1S_0$, ${}^1P_1$, ${}^1D_2$, ${}^1F_3$
		$D^*_{\bar{3}}\eta_8$	${}^3P_1$, ${}^3D_2$, ${}^3F_3$, ${}^3F_4$
		⋮	⋮
$[110]A_1$	0^+, 2, 4, ...	$D_{\bar{3}}\eta_8$	${}^1S_0$, ${}^1P_1$, ${}^1D_2$, ${}^1F_3$
		$D^*_{\bar{3}}\eta_8$	${}^3P_1$, ${}^3D_2$, ${}^3D_3$, ${}^3F_3$, ${}^3P_2$, ${}^3F_2$, ${}^3F_4$
		⋮	⋮

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		$\vdots$	$\vdots$
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		$\vdots$	$\vdots$
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		$\vdots$	$\vdots$

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		$\vdots$	$\vdots$
$[110]A_1$	0^+, 2, $4, \ldots$	$D_{\bar{3}}\eta_8$	$^1S_0$, $^1P_1$, $^1D_2$, $^1F_3$
		$D^*_{\bar{3}}\eta_8$	$^3P_1$, $^3D_2$, $^3D_3$, $^3F_3$, $^3P_2$, $^3F_2$, $^3F_4$
		$\vdots$	$\vdots$

Backup slides

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		⋮	⋮
$[000]E^+$	2^+, $4^+, \ldots$	$D_{\bar{3}}\eta_8$	$^1D_2$
		$D^*_{\bar{3}}\eta_8$	$^3D_2$
		⋮	⋮
$[000]T_2^+$	2^+, 3^+, $4^+, \ldots$	$D_{\bar{3}}\eta_8$	$^1D_2$
		$D^*_{\bar{3}}\eta_8$	$^3D_2$, $^3D_3$
		⋮	⋮

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		$D^*_{\bar{3}}\eta_8$	$^3P_1$, $^3D_2$, $^3F_3$, $^3F_4$
		⋮	⋮
$[110]A_1$	0^+, 2, $4, \ldots$	$D_{\bar{3}}\eta_8$	$^1S_0$, $^1P_1$, $^1D_2$, $^1F_3$
		$D^*_{\bar{3}}\eta_8$	$^3P_1$, $^3D_2$, $^3D_3$, $^3F_3$, $^3P_2$, $^3F_2$, $^3F_4$
		⋮	⋮

Flavour **6** Sector subductions

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		⋮	⋮
$[000]T_1^-$	$1^-,\ 3^-,\ 4^-, \ldots$	$D_{\bar{3}}\eta_8$	$^1P_1,\ ^1F_3$
		$D^*_{\bar{3}}\eta_8$	$^3P_1,\ ^3F_3,\ ^3F_4$
		⋮	⋮
$[000]E^+$	$2^+,\ 4^+, \ldots$	$D_{\bar{3}}\eta_8$	$^1D_2$
		$D^*_{\bar{3}}\eta_8$	$^3D_2$
		⋮	⋮
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		$D^*_{\bar{3}}\eta_8$	$^3D_2,\ ^3D_3$
		⋮	⋮

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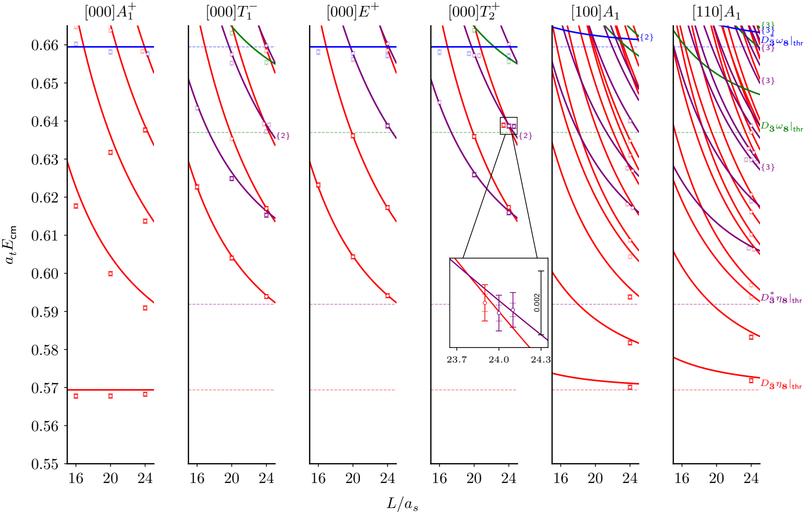
		⋮	⋮
$[110]A_1$	$0^+,\ 2,\ 4, \ldots$	$D_{\bar{3}}\eta_8$	$^1S_0,\ ^1P_1,\ ^1D_2,\ ^1F_3$
		$D^*_{\bar{3}}\eta_8$	$^3P_1,\ ^3D_2,\ ^3D_3,\ ^3F_3,\ ^3P_2,\ ^3F_2,\ ^3F_4$
		⋮	⋮

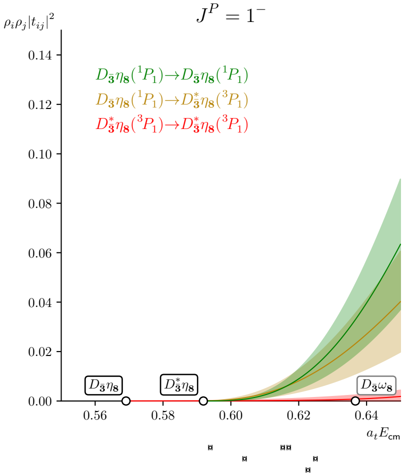
- $P-$wave and $D-$wave suppression at threshold $\sim k^{2\ell}$ (in absence of poles)

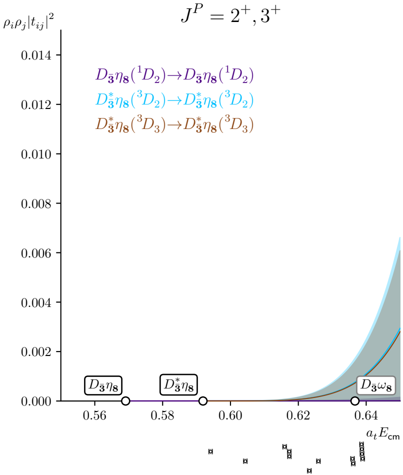
Backup slides

Flavour **6** sector spectra (including [000] T_1^-, [000] E_2^+, [000] T_2^+ irreps)

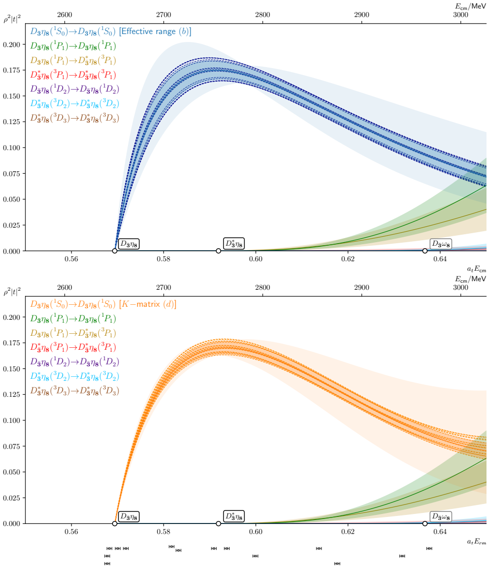


Constrain $J^P = 1^-, 2^+, 3^+$ amplitudes





Backup slides



- Fits with background wave contributions (dashed)
- Fits without background wave contributions (solid)
- Negligible change in fits

Backup slides

Finite-volume spectra to amplitudes

- Parameterise $t-$matrix, $t(s;\alpha)$, with free parameters $\{\alpha\}$
- Solve lüscher quantisation condition

$$\det\left[\mathbb{1} + i\rho(s)\cdot t(s;\alpha)\cdot\left(\mathbb{1} + i\overline{\mathcal{M}}(s,L)\right)\right] = 0 \;\Rightarrow \{E^{\text{pred}}_{\mathbf{n}}[\alpha]\}$$

- Fit to obtained spectra, $\{E_{\mathbf{n}}\}$, minimise

$$\chi^2 = (E - E^{\text{pred}})^{\mathsf{T}}\cdot C^{-1}_{\text{cov}}\cdot(E - E^{\text{pred}})$$

  keeping only 'reasonable fits', $\chi^2/N_{\text{dof}} < 2$
- Diverse set of parameterisations

Backup slides

List of parameterisations

- $k \cot \delta = \frac{1}{a} + \frac{1}{2} r k^2 + P_2 k^4 + \mathcal{O}(k^6)$
- $K-$matrix parameterisations
  $t^{-1}(E_{\mathrm{cm}})_{ij} = \frac{1}{(2k_i)^{\ell_i}} K^{-1}(E_{\mathrm{cm}})_{ij} \frac{1}{(2k_j)^{\ell_j}} + I(E_{\mathrm{cm}})_{ij}$
  where $\mathrm{Im}[I(E_{\mathrm{cm}})_{ij}] = -\rho(E_{\mathrm{cm}})\delta_{ij}$

  Two choices:

  - Chew-Mandelstam prescription
  - Phase-space prescription, $I(E_{\mathrm{cm}}) = -i\rho(E_{\mathrm{cm}})$