Extraction of S and P wave DD* scattering phase shift using twisted boundary conditions.

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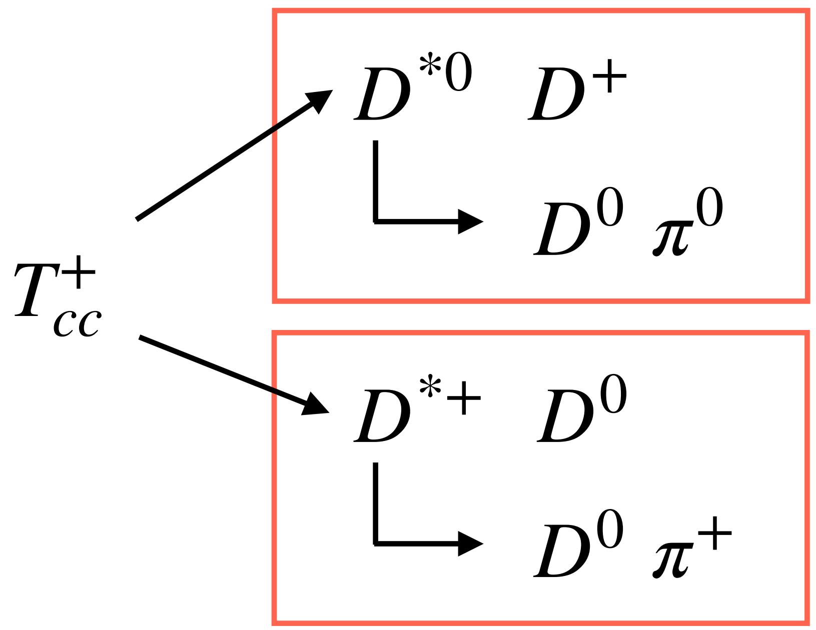
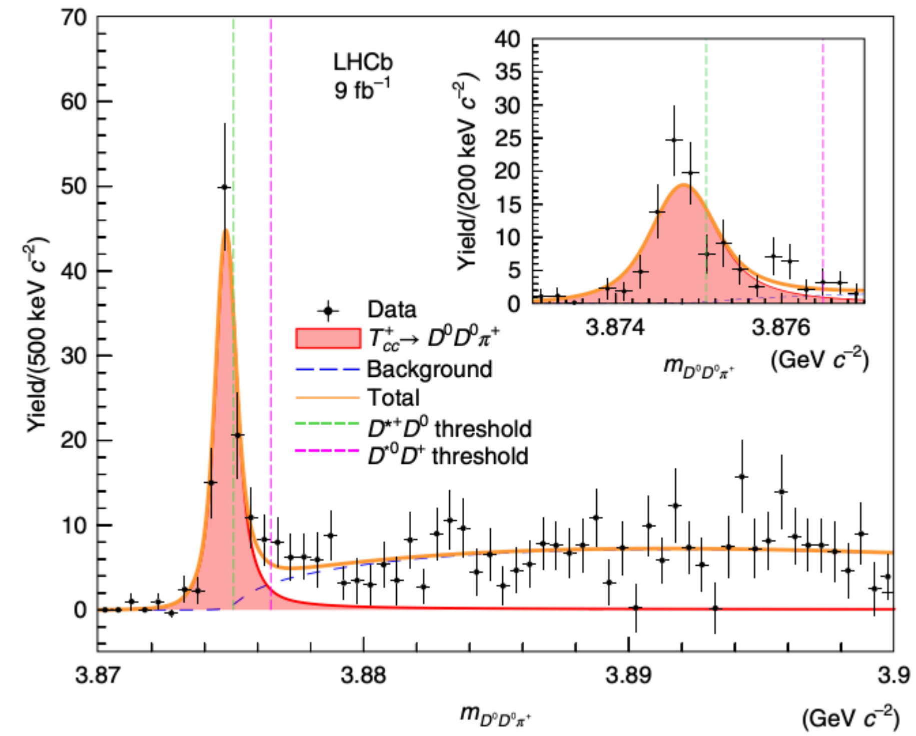
Motivation for investigating T_{cc}^{+}

- Experimental observation in a decay



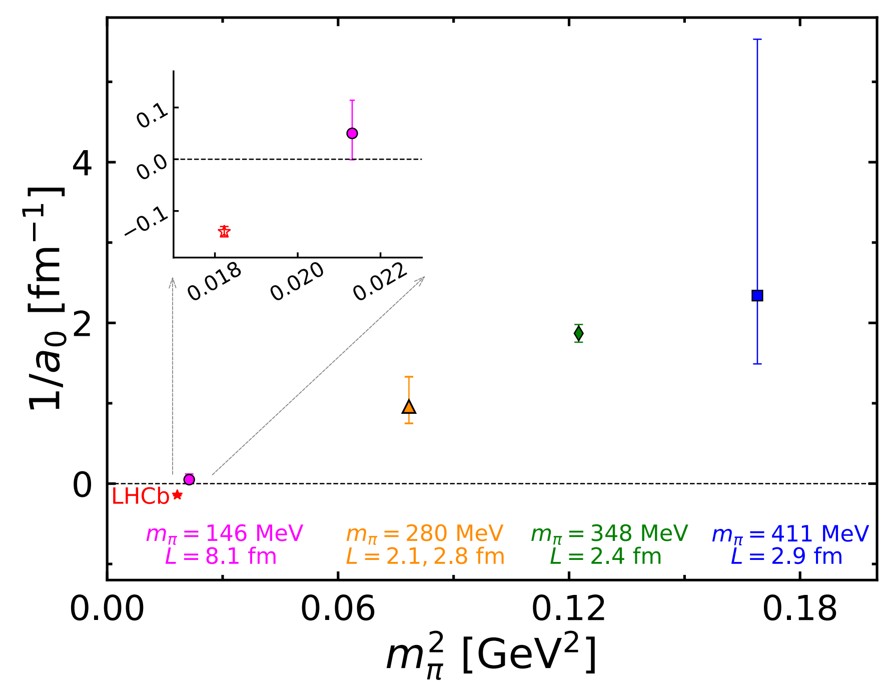
with
$cc\bar{u}\bar{d}$
$I\,(J^P) = 0\,(1^+)$.

LHCb collab. Nat. Commun. 13, 3351 (2022).



- Some lattice QCD calculations imply the existence of T_{cc}^{+} as the bound state of D and D^*.

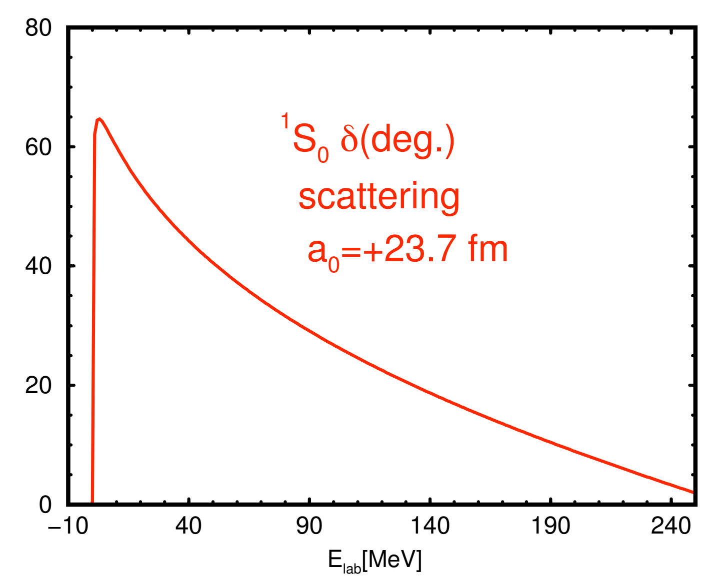
HALQCD, Phys. Rev. Lett. 131, 161901 (2023)
S. Collins et al, Phys. Rev. D 109, 094509 (2024)



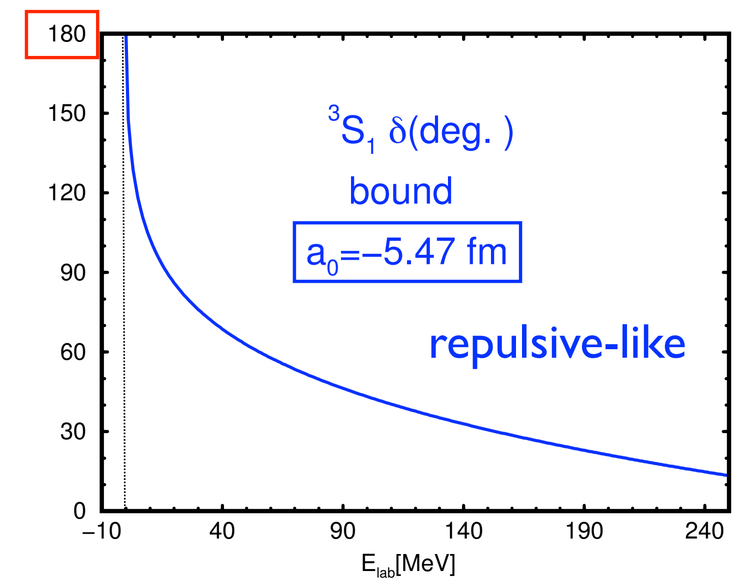
Our Aim

Nucleon-Nucleon scattering

- The behavior of **scattering phase shift** at **low energy** plays a significant role as a signal of a bound state (Levinson's theorem).



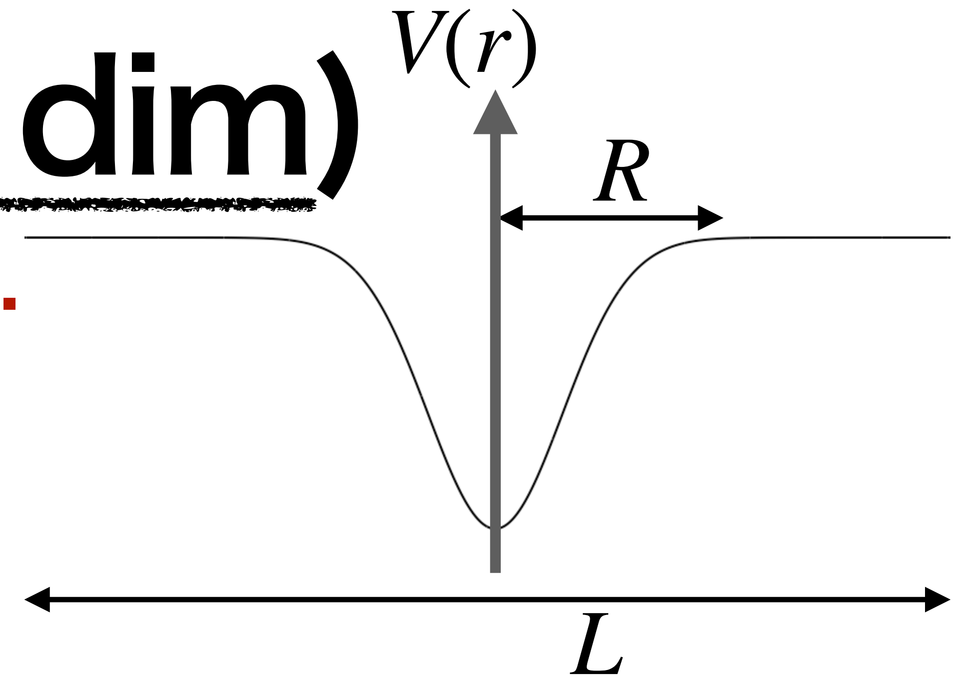
No bound state



Deuteron

- We calculate scattering phase shifts by **Lüscher's method**.
- We access the detailed low energy information by employing the **twisted boundary conditions**.

Lüscher's formula(1dim)



Under the twisted boundary condition.

$$\left(\frac{-1}{m}\frac{\partial^2}{\partial x^2} + V(x)\right)\psi(x) = E\psi(x)$$

Solutions in $|x| > R$

$$\psi(x) = e^{-ik|x|} + e^{2i\delta(k)}e^{ik|x|}, \qquad E = \frac{k^2}{m}$$

Twisted BC : $e^{i\theta}\psi(x - \frac{L}{2}) = \psi(x + \frac{L}{2})$

Quantization cond.

$$e^{2i\delta(k)}e^{ikL} = e^{i\theta} \longleftrightarrow \delta(k) + kL = 2\pi n + \theta \ (n \in \mathbb{Z})$$

$\delta(k)$ in $0 < k < 2\pi/L$ is calculable by changing θ

without using other large volume gauge configurations.

Lüscher's formula(3dim)

Under the twisted boundary condition (up to P wave).

$$\begin{vmatrix} \cot\delta_0(k) - M^{\vec{\theta}}_{SS}(q) & M^{\vec{\theta}}_{SP}(q) \\ M^{\vec{\theta}}_{SP}(q) & \cot\delta_1(k) - M^{\vec{\theta}}_{PP}(q) \end{vmatrix} = 0$$

$$e^{i\theta_j}\psi(x - \frac{L}{2}\vec{e}_j) = \psi(x + \frac{L}{2}\vec{e}_j)$$

$$\vec{\theta} = (\theta_x, \theta_y, \theta_z)$$

$M^{\vec{\theta}}_{ab}(q)$: combination of generalized zeta functions, depends on $\vec{\theta}$.

S and P wave contributions can be highly mixed with each other because trivial irrep A_{1g} (A_g) is contained in both decomposition.

$\vec{\theta}$ Symmetry	$(0,0,0)$ O_h	$(0,0,\theta)$ C_{4v}	$(\theta,\theta,0)$ C_{2v}	(θ,θ,θ) C_{3v}	$(0,0,\pi)$ D_{4v}	$(\pi,\pi,0)$ D_{2v}	(π,π,π) D_{3v}
S	A_{1g}	A_1	A_1	A_1	A_{1g}	A_g	A_{1g}
P	T_{1u}	$A_1 \oplus E$	$A_1 \oplus B_1 \oplus B_2$	$A_1 \oplus E$	$A_{2g} \oplus E_u$	$E_{1u} \oplus B_{2u} \oplus B_{3u}$	$A_{2u} \oplus E_u$

Strategy to obtain $\delta_0(k)$, $\delta_1(k)$

S. Ozaki, S. Sasaki, Phys. Rev. D87 (2013) 014506

$$\vec{\theta} = (0,0,0),\ (\pi,0,0),\ (\pi,\pi,0),\ (\pi,\pi,\pi)$$

Step.1

$$\cot\delta_0(k) = \frac{Z_{00}(1;q^2)}{\pi^{3/2}q}$$

Effective range expansion

$$k^{2l+1}\cot\delta_l(k) = \frac{1}{a_l} + \frac{r_l}{2}k^2 + \mathcal{O}(k^4)$$

Input by fit

Using $\vec{\theta} = (\theta,\theta,\theta)$

Step.2

$$\cot\delta_1(k) = M_{PP}^{(111)}(q) + \frac{|M_{SP}^{(111)}(q)|^2}{\cot\delta_0(k) - M_{SS}^{(111)}(q)}$$

Input by fit

Using $\vec{\theta} = (0,0,\theta)$

Step.3

$$\cot\delta_0(k) = M_{PP}^{(001)}(q) + \frac{|M_{SP}^{(001)}(q)|^2}{\cot\delta_1(k) - M_{SS}^{(001)}(q)}$$

Simulation setup

- 2+1 flavor PACS-CS gauge configuration with $L^3 \times T = 32^3 \times 64,\ La = 2.9$ fm. Phys. Rev. D 79, 034503 (2009).

m_π [MeV]	N_{conf}
295	800
410	450

- **Non perturbatively improved clover** action for **up** and **down** quarks.

- **Relativistic heavy quark (RHQ)** action for **charm** and **bottom** quarks.

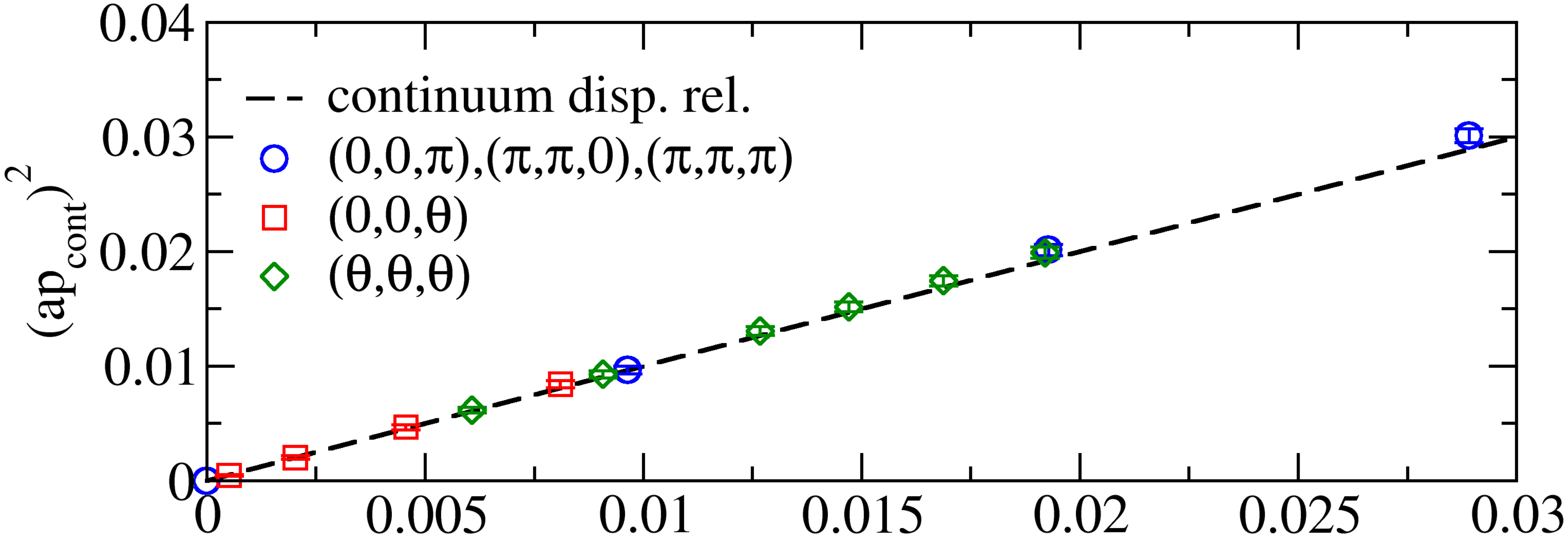
- We also calculate BB^* case for comparison.

- Using operators

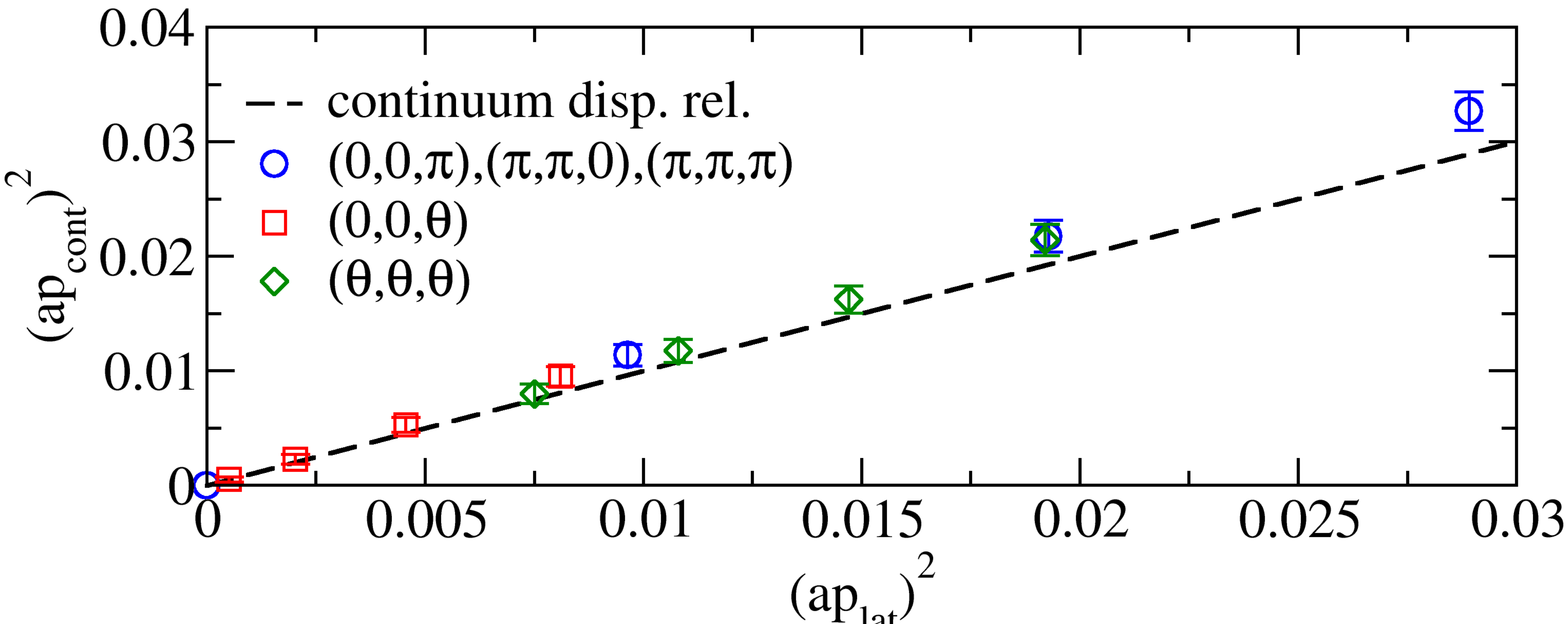
$$O_j = \frac{(\bar{u}\gamma_5 Q)\cdot(\bar{d}\gamma_j Q) - (\bar{d}\gamma_5 Q)\cdot(\bar{u}\gamma_j Q)}{\sqrt{2}} \quad (Q = c,\ b)$$

Check of dispersion relations

For D meson



For B meson

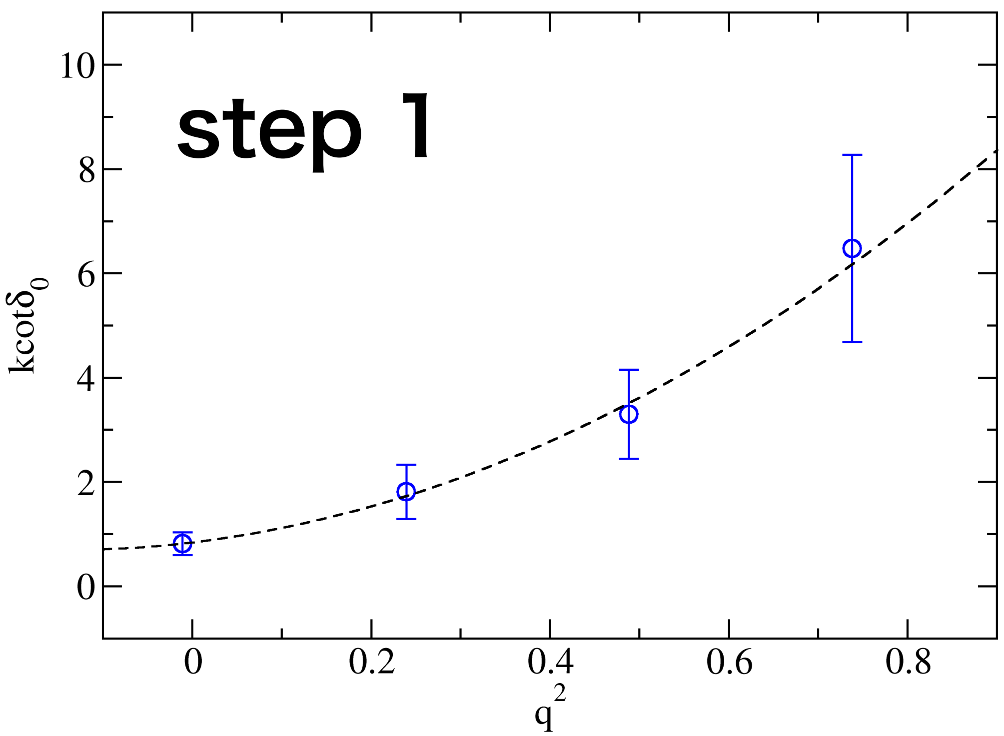


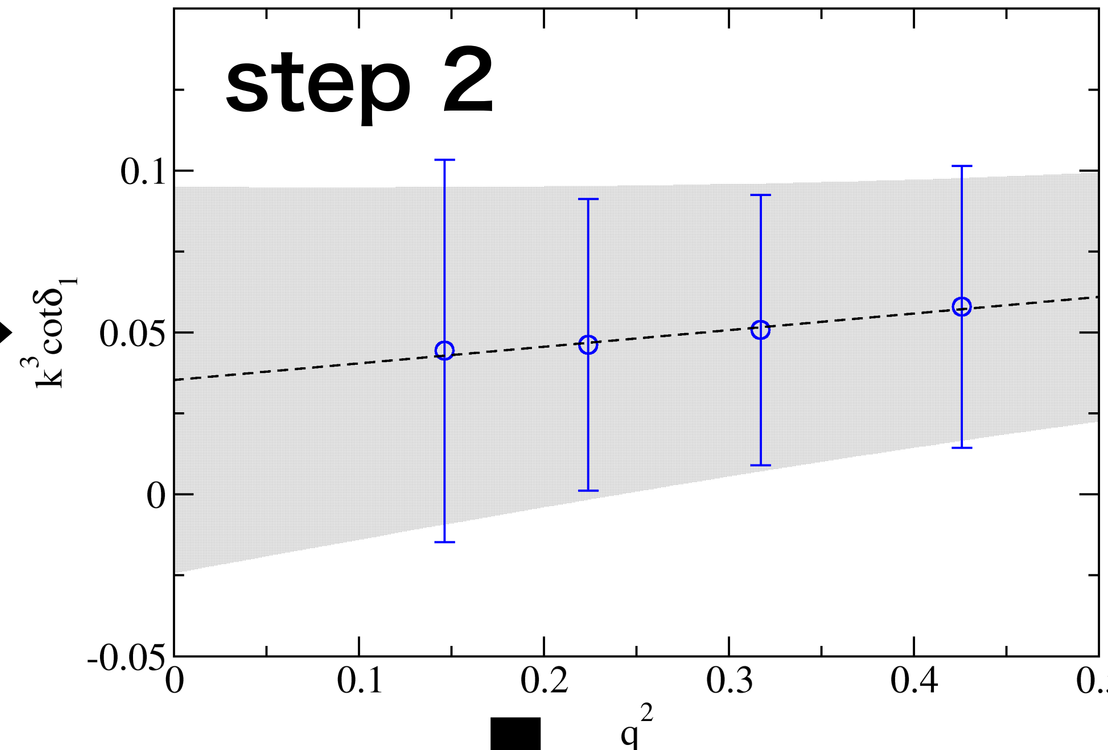
$$p_{\rm cont}^2 = E^2 - m^2$$

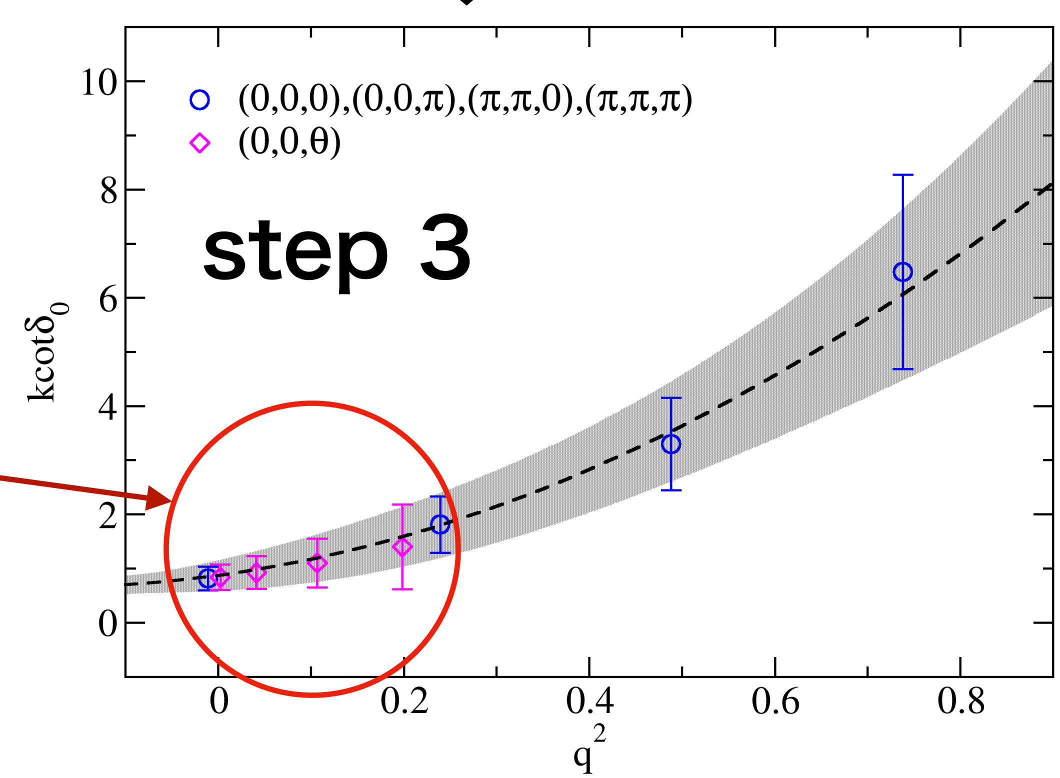
$$p_{\rm lat}^2 = \left(\frac{\vec{\theta}}{L}\right)^2$$

Calculation according to the strategy

For DD^* system with $m_\pi = 410$ MeV



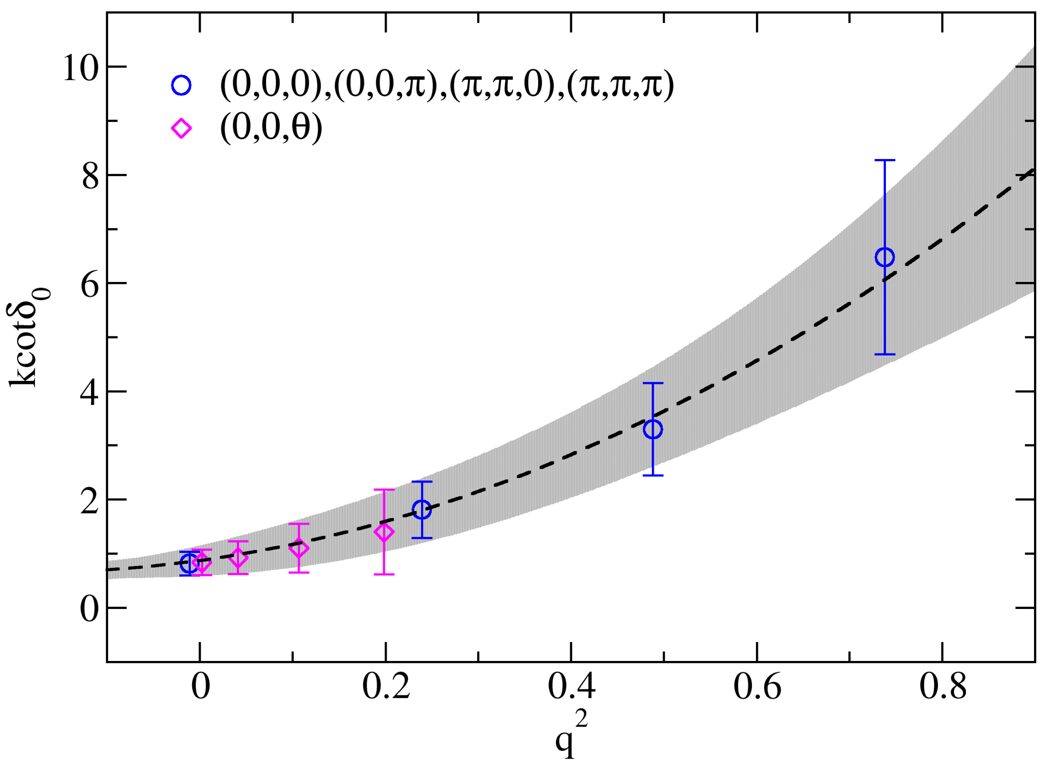




Horizontal axis: $q^2 = \left(\dfrac{Lk}{2\pi}\right)^2$

The low energy behavior can be obtained through P wave calculations.

Result of $k \cot \delta_0(k)$

<table>
<tr><th>m_π</th><th>Charm</th><th>Bottom</th></tr>
<tr><td>410 MeV</td><td>


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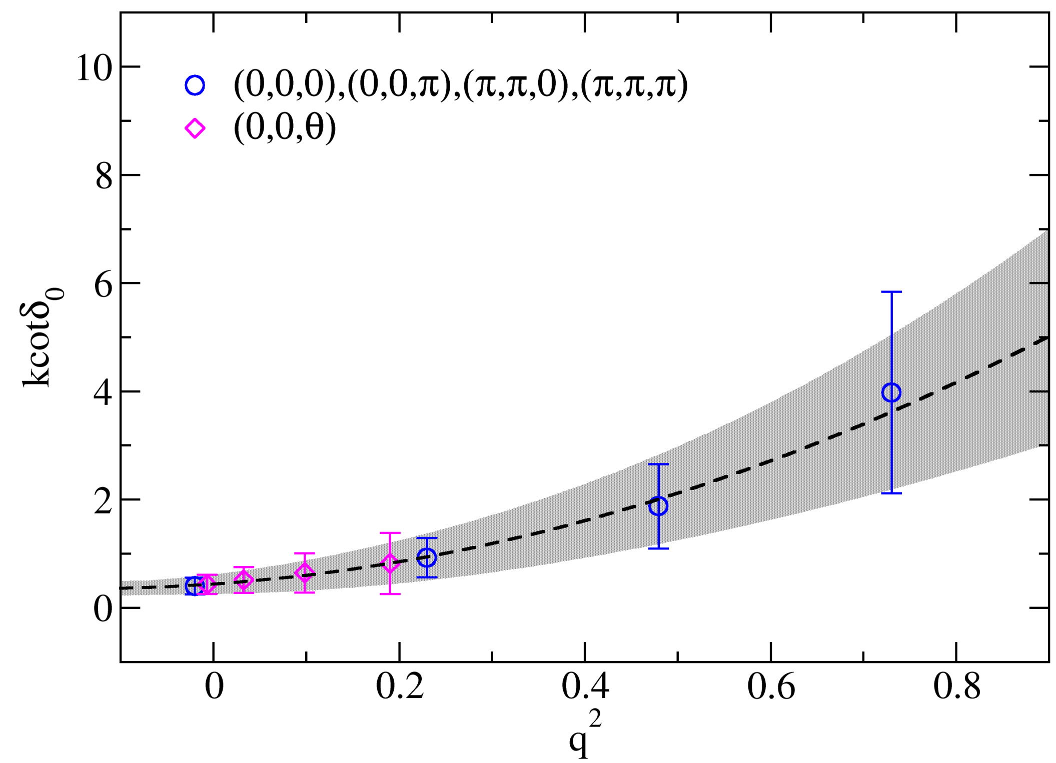

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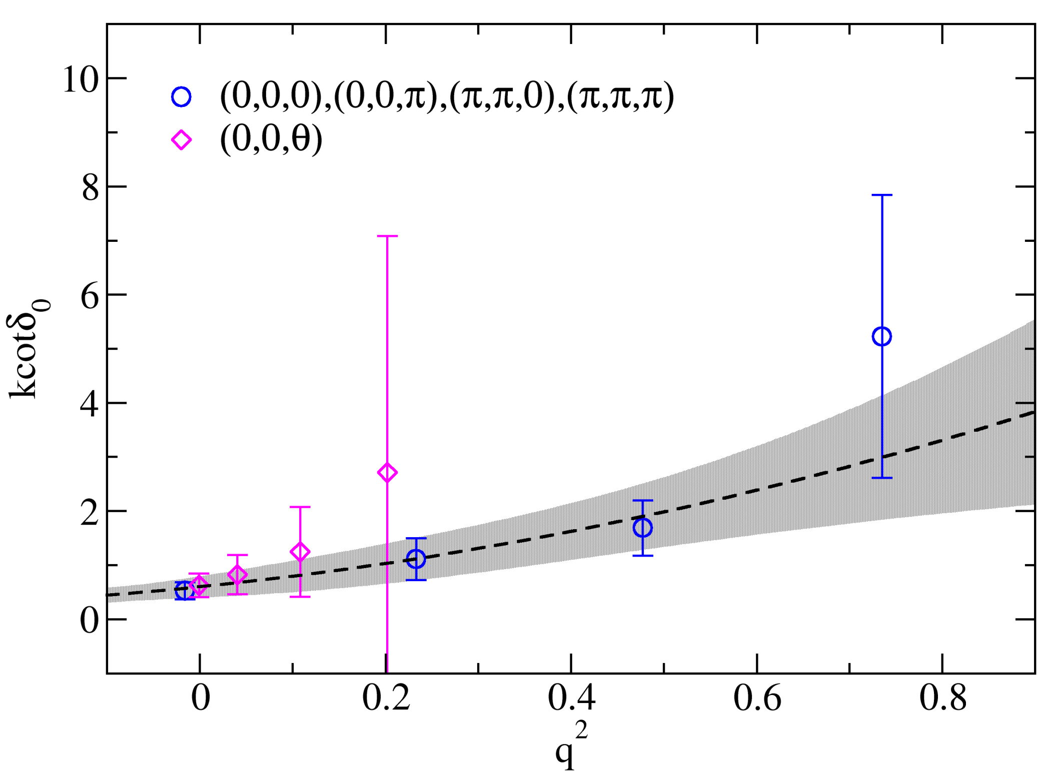
<tr><td>295 MeV</td><td>


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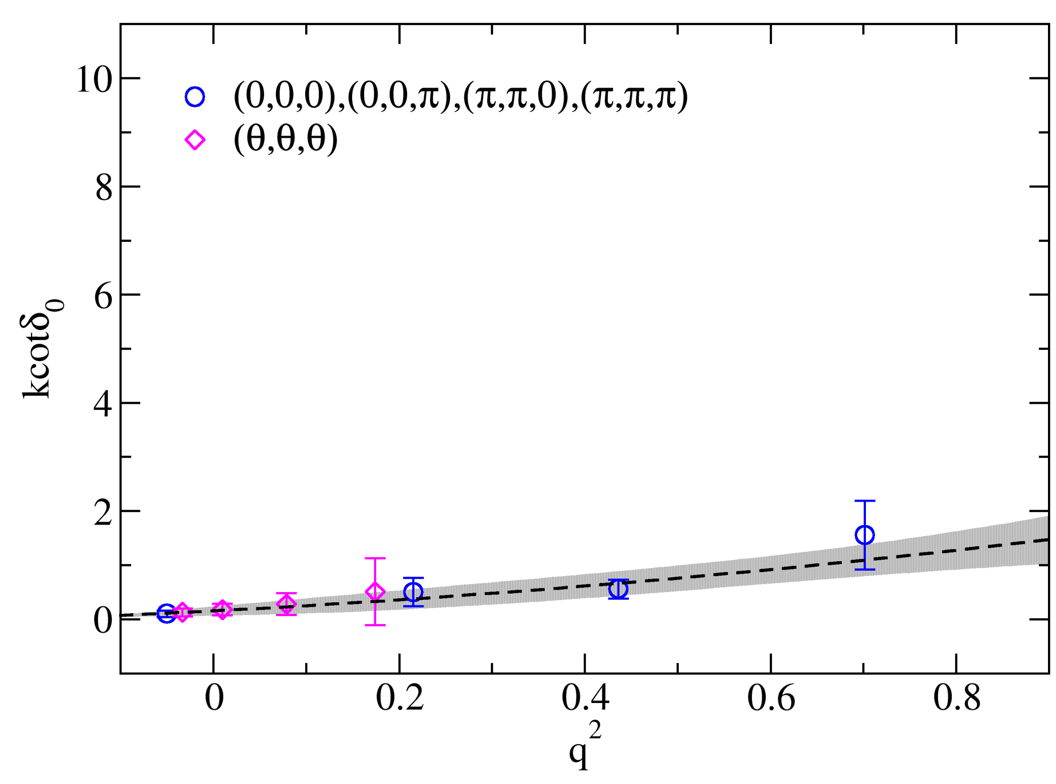

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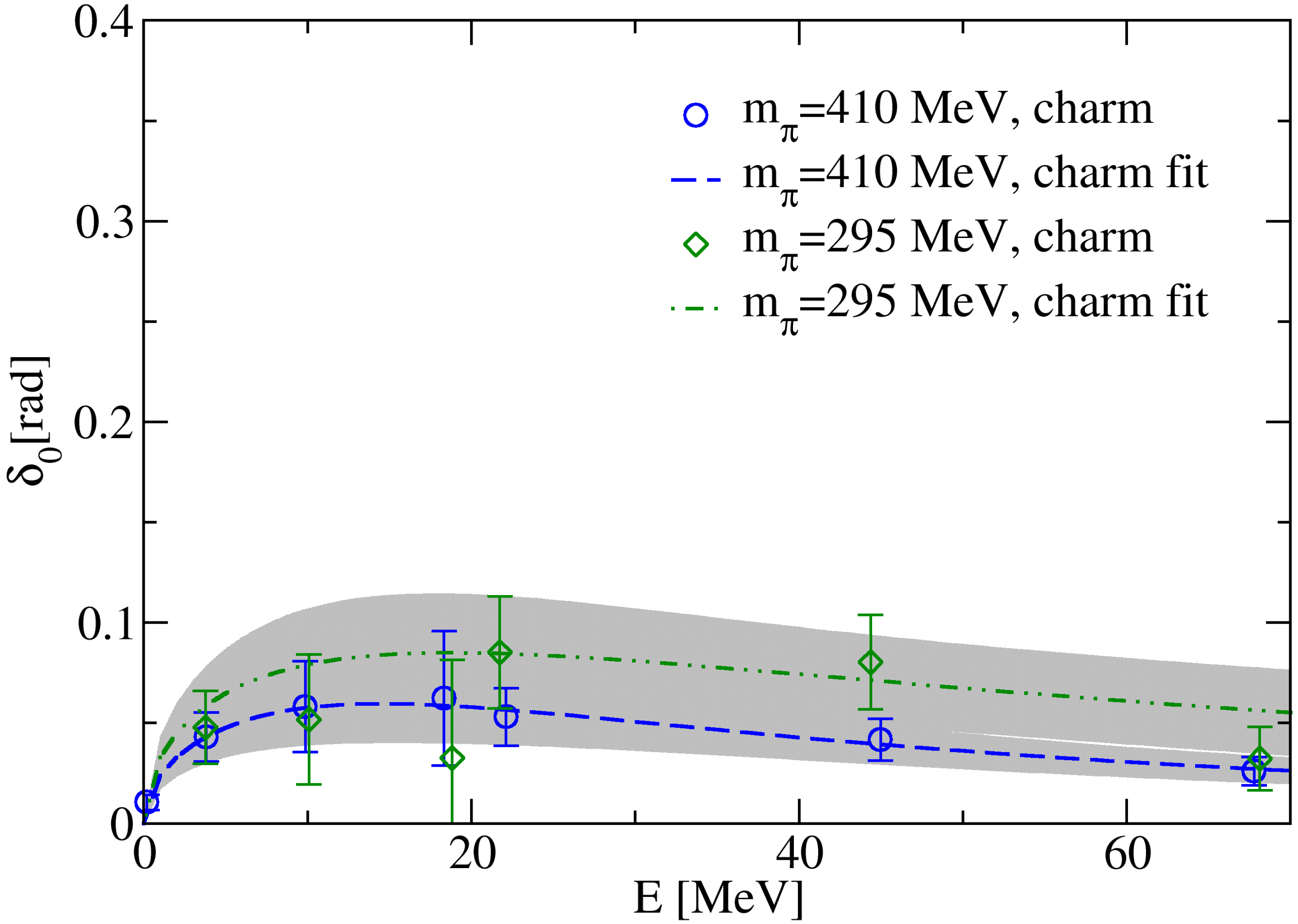
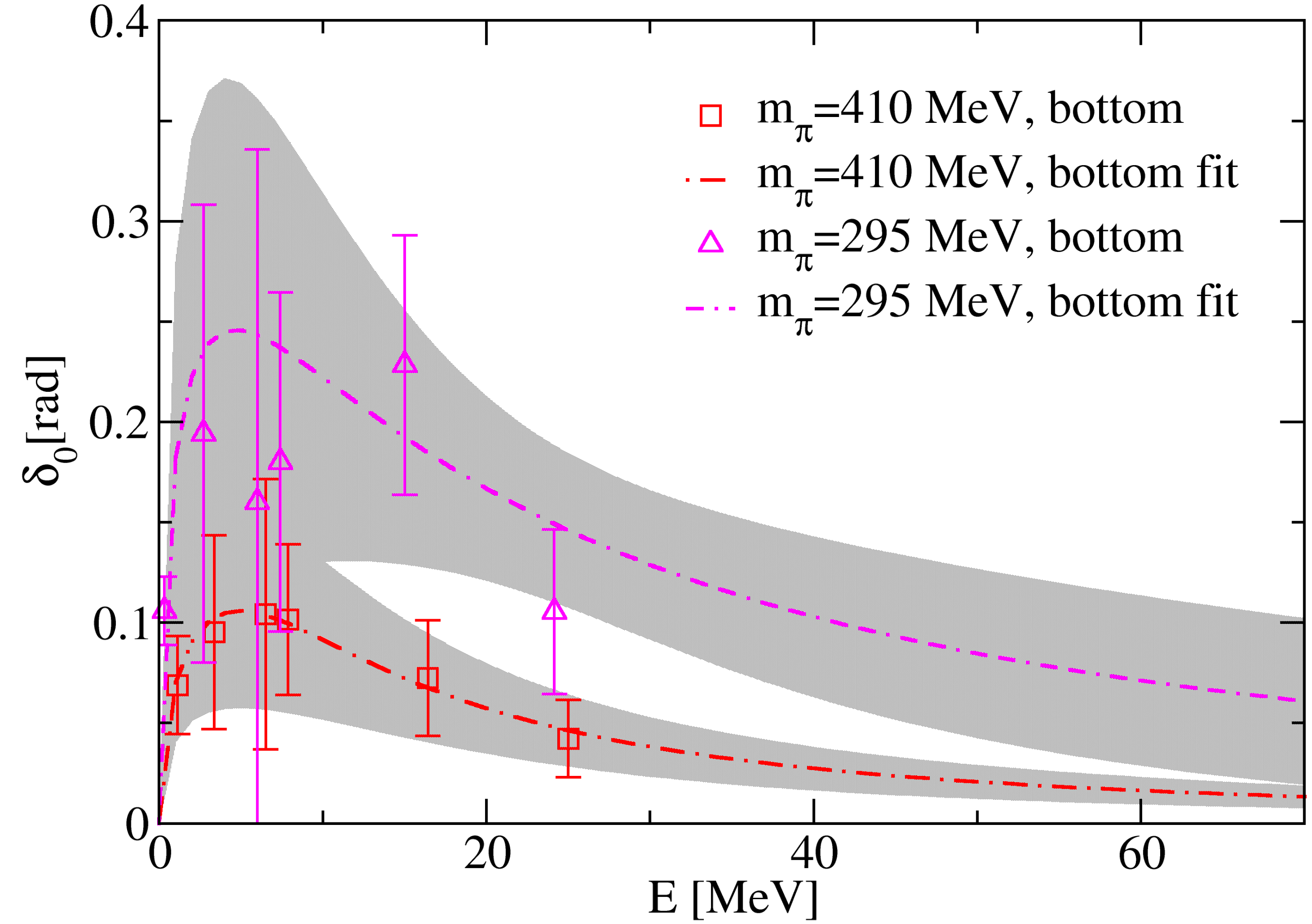
S wave scattering phase shifts

For *DD**



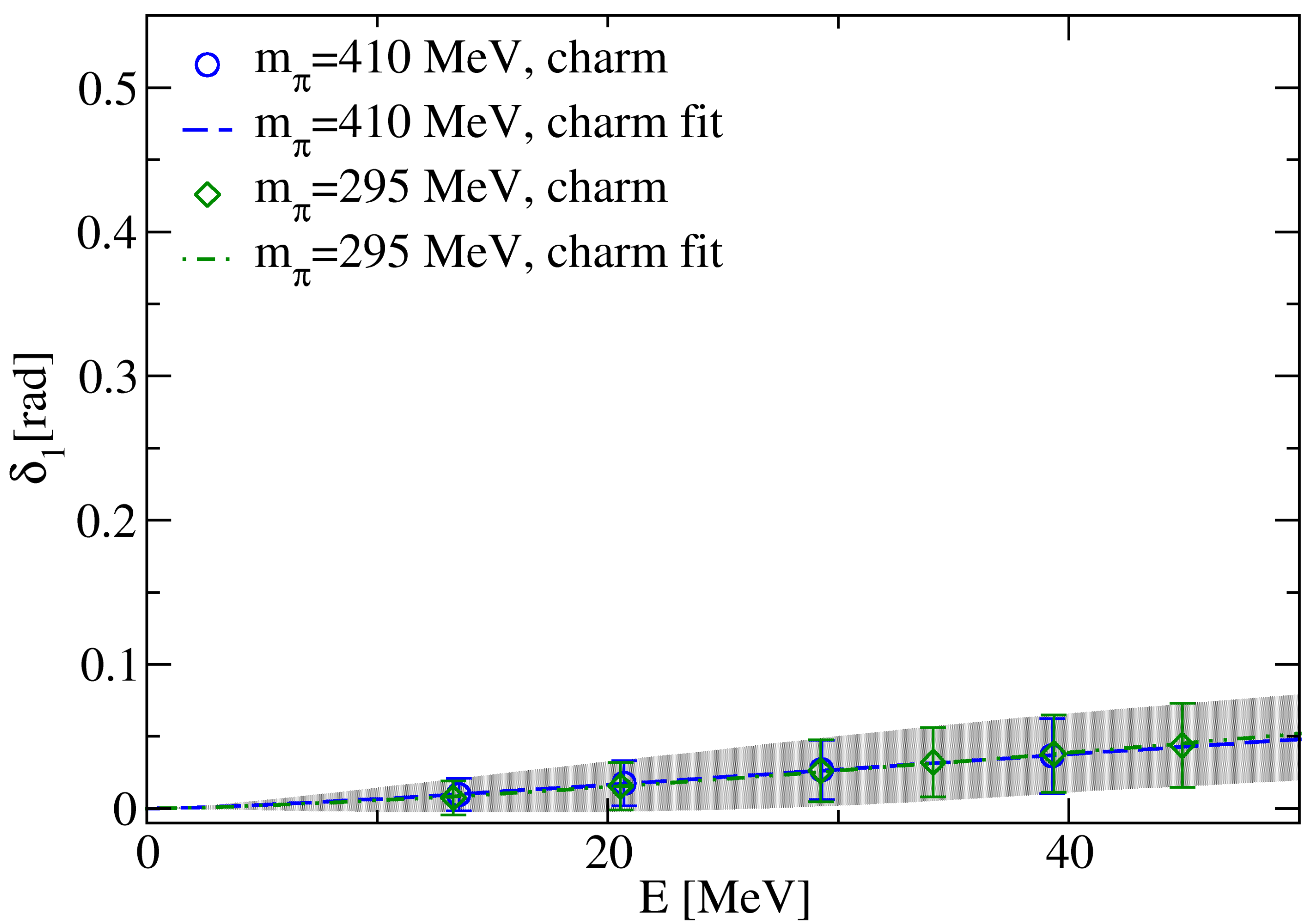
For *BB**



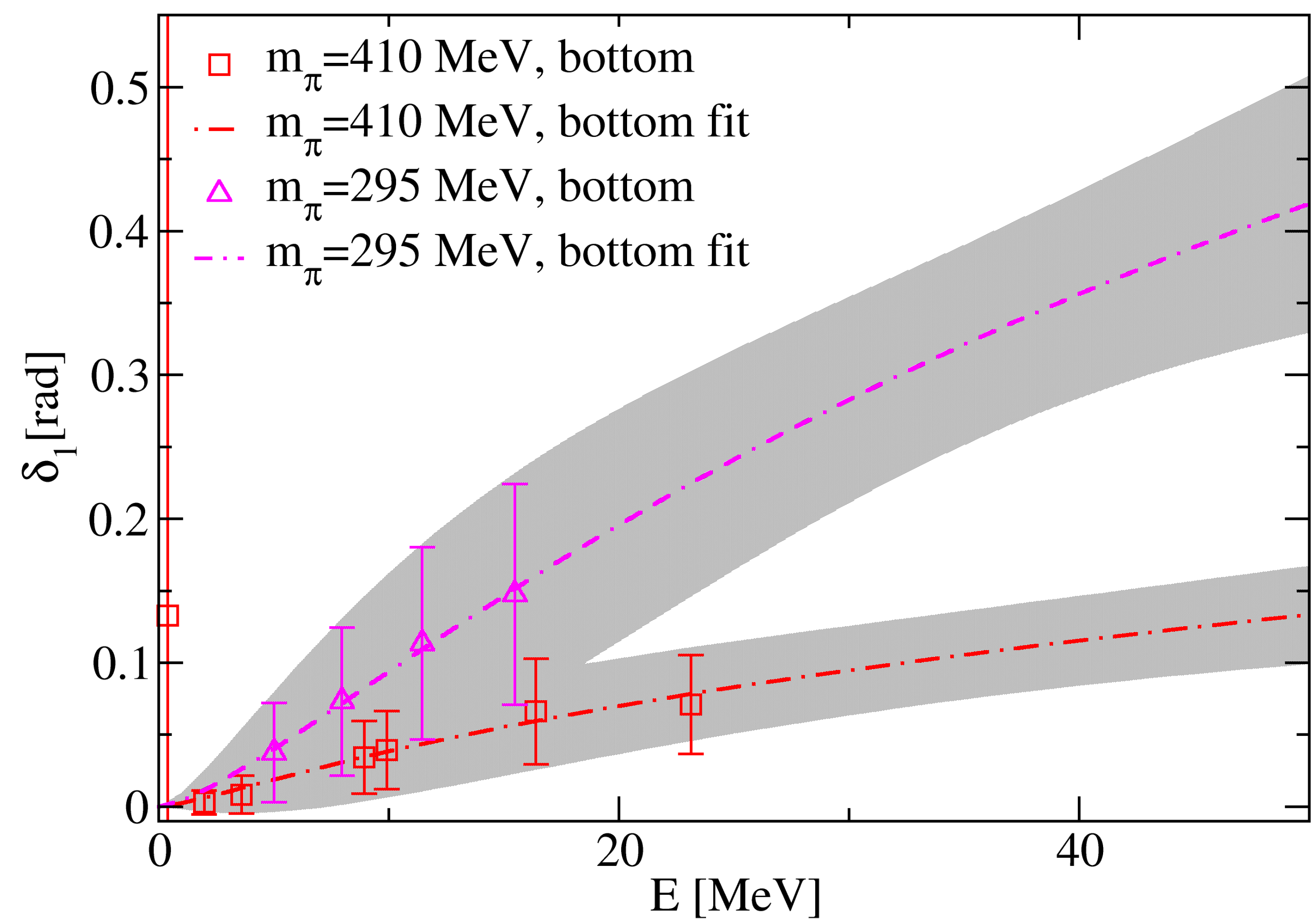
Scattering phase shifts get close to the behavior of a bound state as m_π gets smaller and heavy quark mass get heavier.

P wave scattering phase shifts

For DD^*



For BB^*



- P wave effects are measured and we separated.
- δ_1 seems to be independent of m_π for DD^* case while it depends on m_π for BB^*.

Summary

- **S** and **P** wave scattering phase shift are extracted through Lüscher's method under twisted BCs.
  - We properly separated the S and P wave effects in Lüscher's formula.
  - $QQ\bar{u}\bar{d}$ system seems to get close to **a bound state** as $m_\pi \to m_{\rm phys}$ and heavy quark mass gets heavier.

Prospects

- More statistics for $m_\pi = 295\ \mathrm{MeV}$, and lighter m_π.
- A bound state in $QQ\bar{u}\bar{d}$ system?