

Universality of the continuum limit for the H dibaryon

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University of Liverpool, UK

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- ▶ Quark mass dependence of deuteron binding.
- ▶ Hyperon interactions \rightarrow neutron stars?
- ▶ Nuclear structure.

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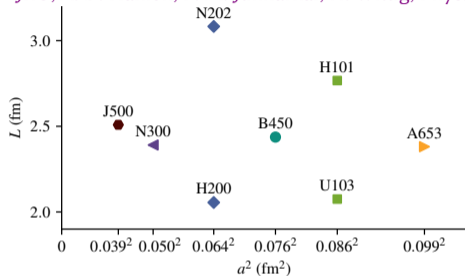
Need to control all systematics:

- ▶ Identification of spectrum.
- ▶ Finite-volume effects.
- ▶ Physical quark masses.
- ▶ **Discretization effects.**

Previous result

Weakly bound H dibaryon from $SU(3)$ -flavor-symmetric QCD

JRG, A. D. Hanlon, P. M. Junnarkar, H. Wittig, Phys. Rev. Lett. **127**, 242003 (2021)



Eight $N_f = 3$ clover ensembles from CLS.

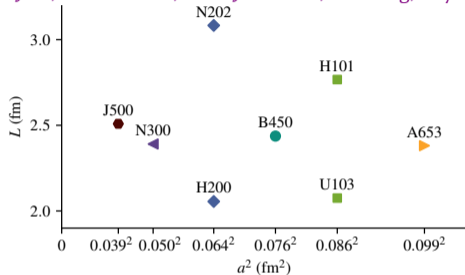
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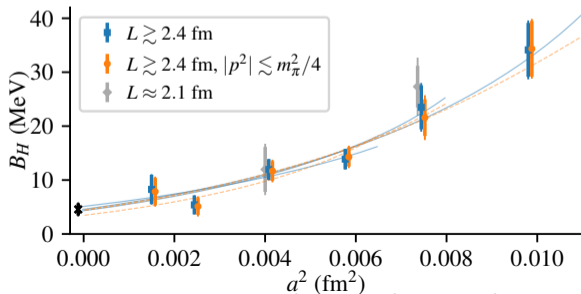
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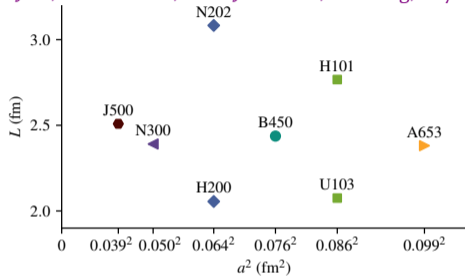
In continuum: $B_H = 4.56(1.13)(0.63)$ MeV.



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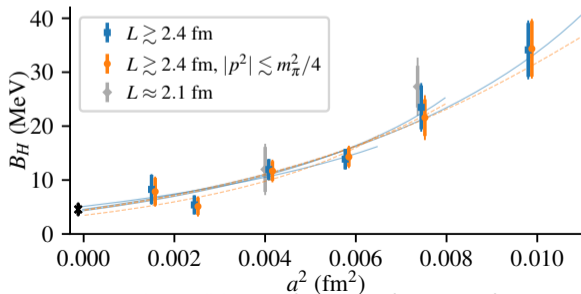
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See also:

Takashi Inoue, EFB25, last summer

Robert Perry, Wednesday 11:35



With $O(a)$ improved action, corrections start at a^2 :

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} + a^2 \sum_i O_i + O(a^3).$$

Dimension-six operators O_i are gluonic, $\bar{q}q$, or $(\bar{q}q)^2$:

- ▶ Some break $O(4)$ rotational symmetry \rightarrow modified dispersion relations.
- ▶ Some break chiral symmetry.

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We see percent-level effects on baryon-baryon energies
but $O(100\%)$ effects on scattering observables such as the scattering length.

Can we understand what is causing these large effects? Study using different actions.

1. Methods
2. Actions and ensembles
3. Spectrum
4. Binding energy and scattering parameters
5. Summary

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Solve GEVP and extract energy differences $\Delta E = E_{BB} - E_B(\mathbf{p}_1) - E_B(\mathbf{p}_2)$ from correlator ratios.

Reconstruct using continuum dispersion relation $E_{BB}^{\text{recon}} = \Delta E + \sqrt{m_B^2 + \mathbf{p}_1^2} + \sqrt{m_B^2 + \mathbf{p}_2^2}$.

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Use Lüscher quantization condition for S wave, fit to spectrum with model

$$p \cot \delta_0(p^2) = \sum_i c_i p^{2i}, \quad c_i = c_{i0} + c_{i1} a^2.$$

Bound state from pole condition $p \cot \delta_0(p^2) = -\sqrt{-p^2}$.

OpenLat: $N_f = 3$ exponentiated clover

$$4 + m_0 + c_{\text{SW}} \frac{i}{4} \sigma_{\mu\nu} \hat{F}_{\mu\nu} \rightarrow (4 + m_0) \exp \left(\frac{c_{\text{SW}}}{4 + m_0} \frac{i}{4} \sigma_{\mu\nu} \hat{F}_{\mu\nu} \right)$$

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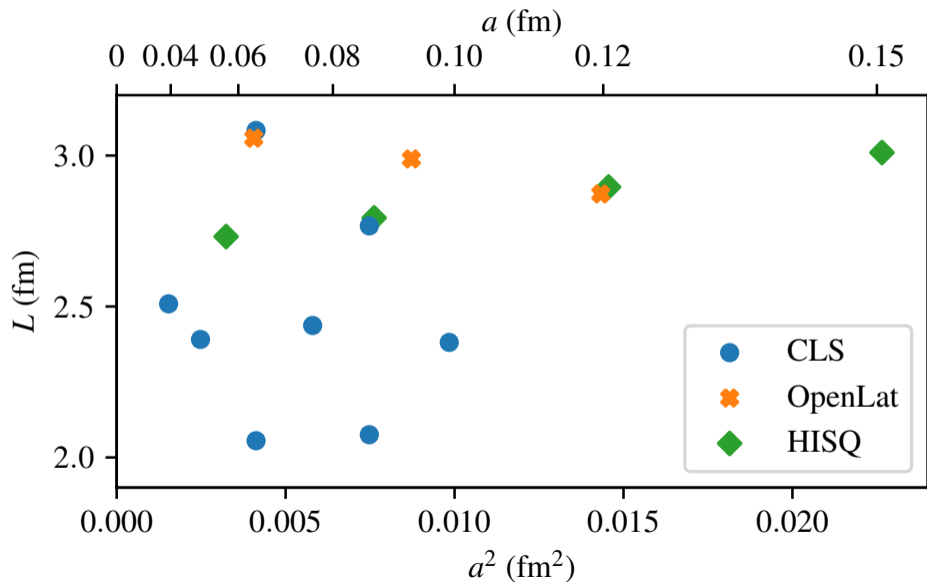
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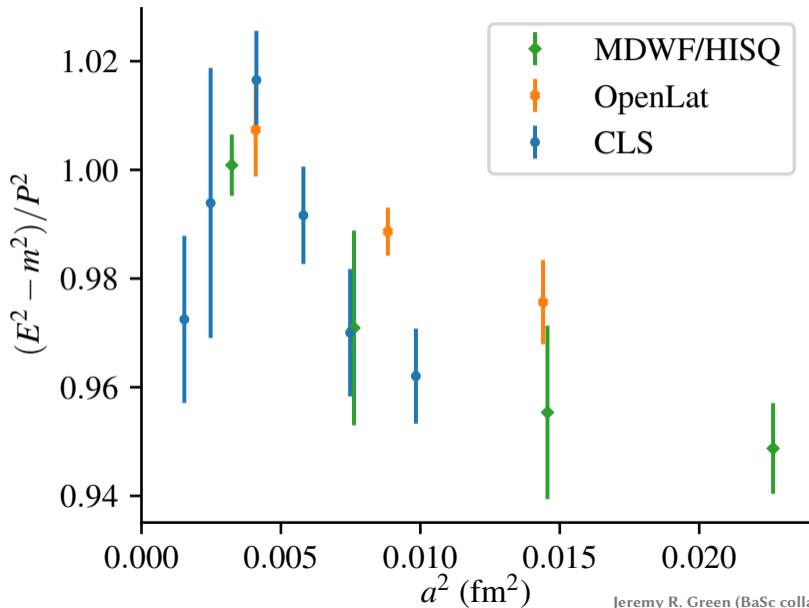
MDWF/HISQ: tadpole-improved one-loop improved gauge action,

$N_f = 3 + 1$ rooted improved staggered sea fermions coupled to smeared links incl. 3-link term,
Möbius domain wall valence fermions coupled to flowed ($t/a^2 = 1$) gauge field.

Ensembles



Octet baryon dispersion relation



Average over lowest few nonzero momenta.

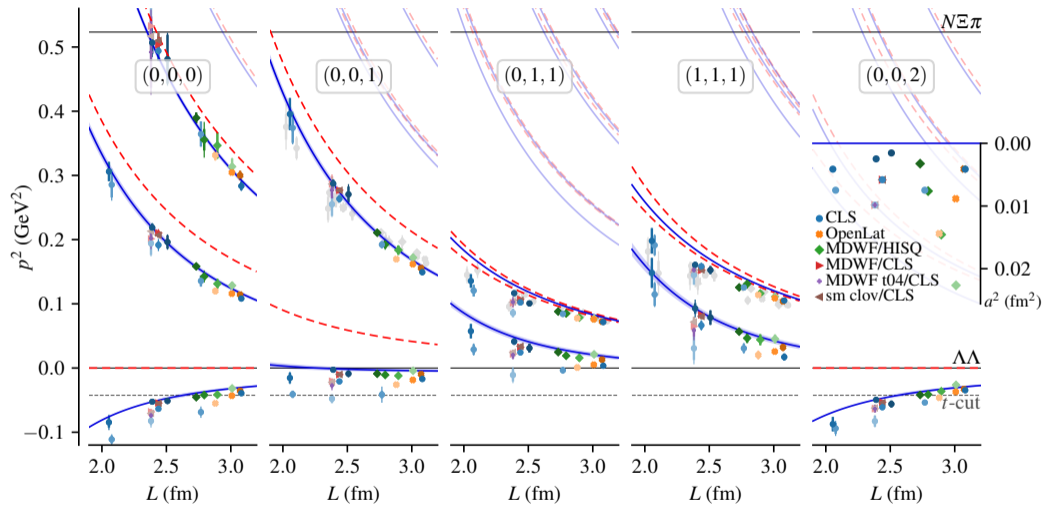
Step-by-step change between CLS and MDWF/HISQ:

- ▶ Clover valence on flowed ($t/a^2 = 1$) CLS.
- ▶ MDWF on flowed ($t/a^2 = 1$) CLS.

Try less smearing:

- ▶ MDWF on flowed ($t/a^2 = 0.4$) CLS.

Spectrum

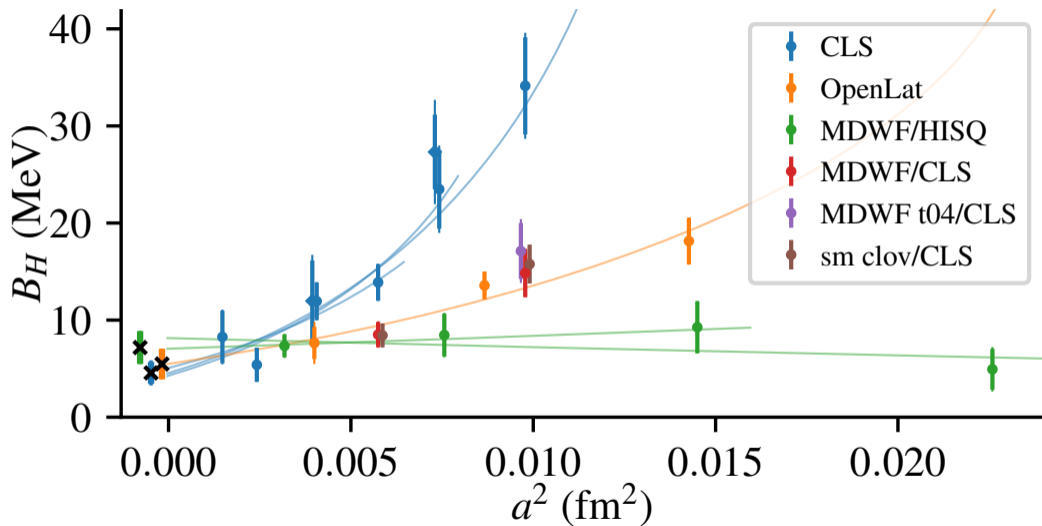


Points: lattice levels.

Red dashed: noninteracting levels.

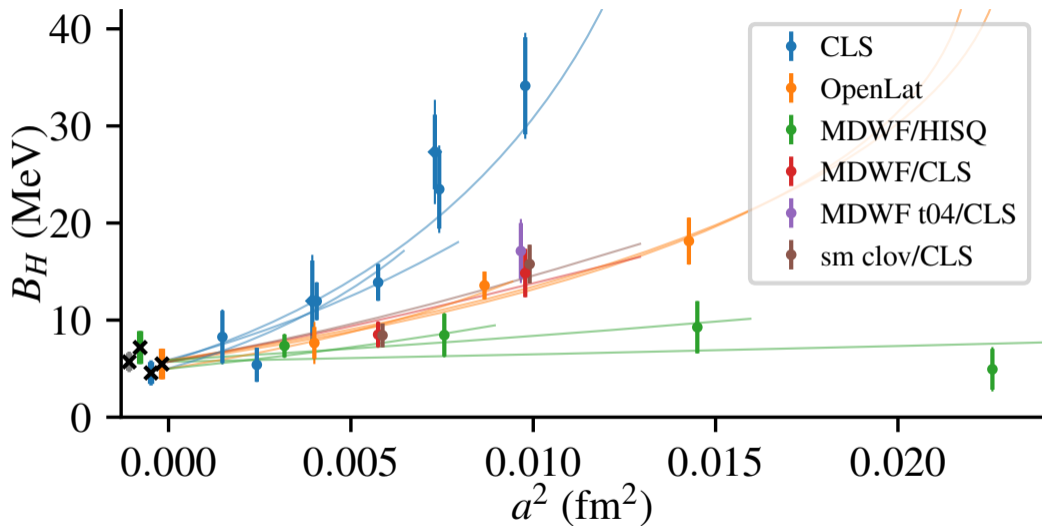
Blue: continuum (published).

Binding energy of H dibaryon



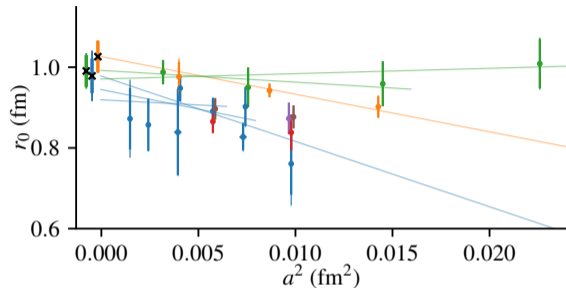
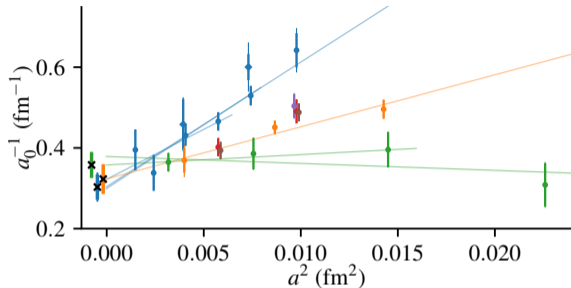
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Good fit quality for each action separately or all combined.

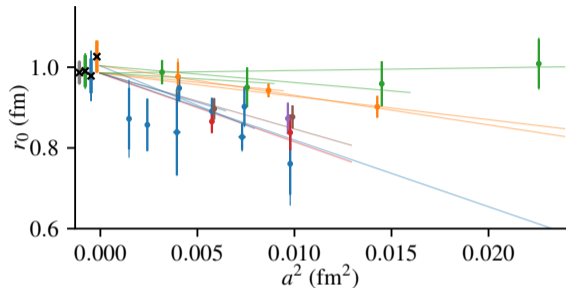
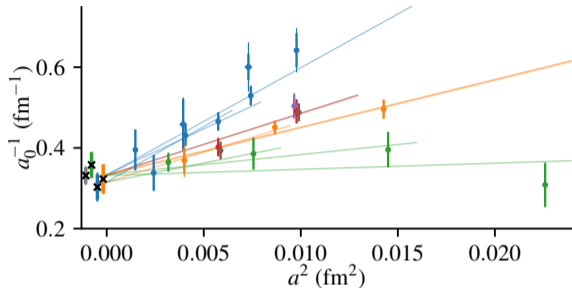
Scattering parameters



$$p \cot \delta_0(p) = -\frac{1}{a_0} + \frac{1}{2}r_0p^2 + O(p^4)$$

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Summary

- ▶ $B_H \approx 6$ MeV at $m_\pi = m_K \approx 415$ MeV obtained from three independent continuum extrapolations. Charm sea has negligible effect.
- ▶ Size of discretization effects varies significantly among different actions.
- ▶ Exponentiated clover term yields smaller lattice artifacts.
- ▶ Coupling valence fermions to smeared links moves data closer to continuum value.
- ▶ No significant difference between MDWF and clover valence using same smeared links.
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Controlling discretization effects is important!

MDWF on HISQ: tuning

