Lambda(1405) in the flavor SU(3) limit using a separable potential in the HAL QCD method

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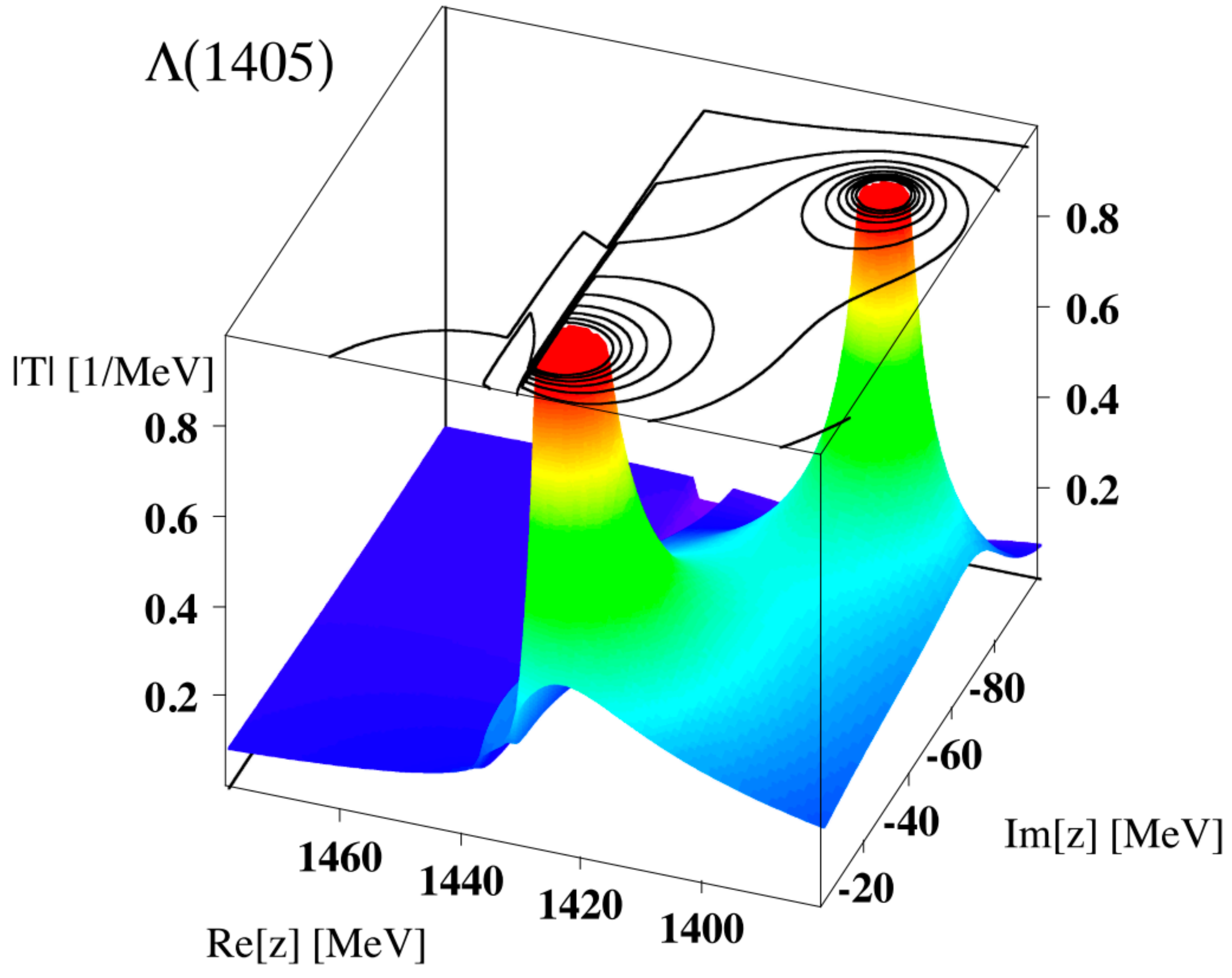
$\Lambda(1405)$ from lattice QCD

- $\Lambda(1405)$: not a simple Λ baryon (exotic hadron)

- one pole? **two poles?**

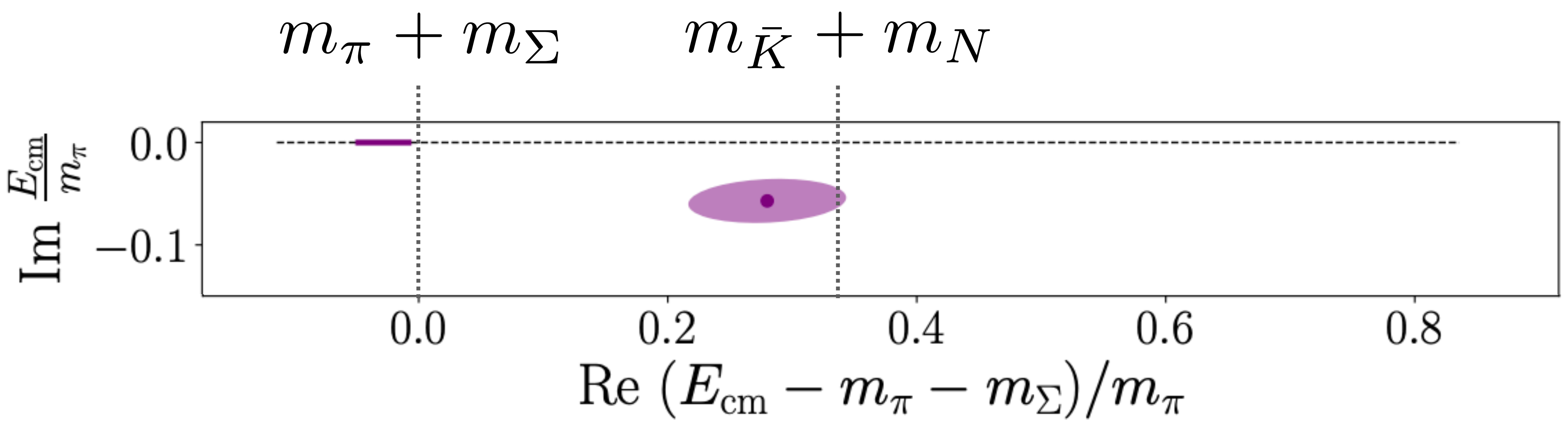
  - chiral unitary model [Oller and Meissner, 2001] [Jido, Oller, Oset, Ramos, Meissner, 2003]



[Hyodo and Jido 2012]

  - lattice QCD using finite-volume method at $m_\pi \approx 200$ MeV [Bulava et al. (BaSc Collab.), 2024]

  → virtual state below $\pi\Sigma$ + resonance below $\bar{K}N$



[Bulava et al. (BaSc Collab.), 2024]

- our work: $\Lambda(1405)$ **from HAL QCD approach**

HAL QCD method

[Ishii, Aoki, Hatsuda 2007]
[Ishii et al. 2011]

- R-correlator:

$$R(\mathbf{r},t) = \frac{\langle O_1(\mathbf{r},t)O_2(\mathbf{0},t)\bar{J}(0)\rangle}{\langle O_1(t)\bar{O}_1(0)\rangle\langle O_2(t)\bar{O}_2(0)\rangle} \approx \sum_n C_{\bar{J},n}\underline{\Psi^{W_n}(\mathbf{r})}e^{-(W_n-m_1+m_2)t}$$

Nambu-Bethe-Salpeter (NBS) wave function

- time-dependent HAL QCD method

(μ: reduced mass)

$$\int d^3r'\ U(\mathbf{r},\mathbf{r}')R(\mathbf{r}',t) \simeq \left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right)R(\mathbf{r},t)$$

$$\approx V(r)\delta^{(3)}(\mathbf{r}-\mathbf{r}')$$

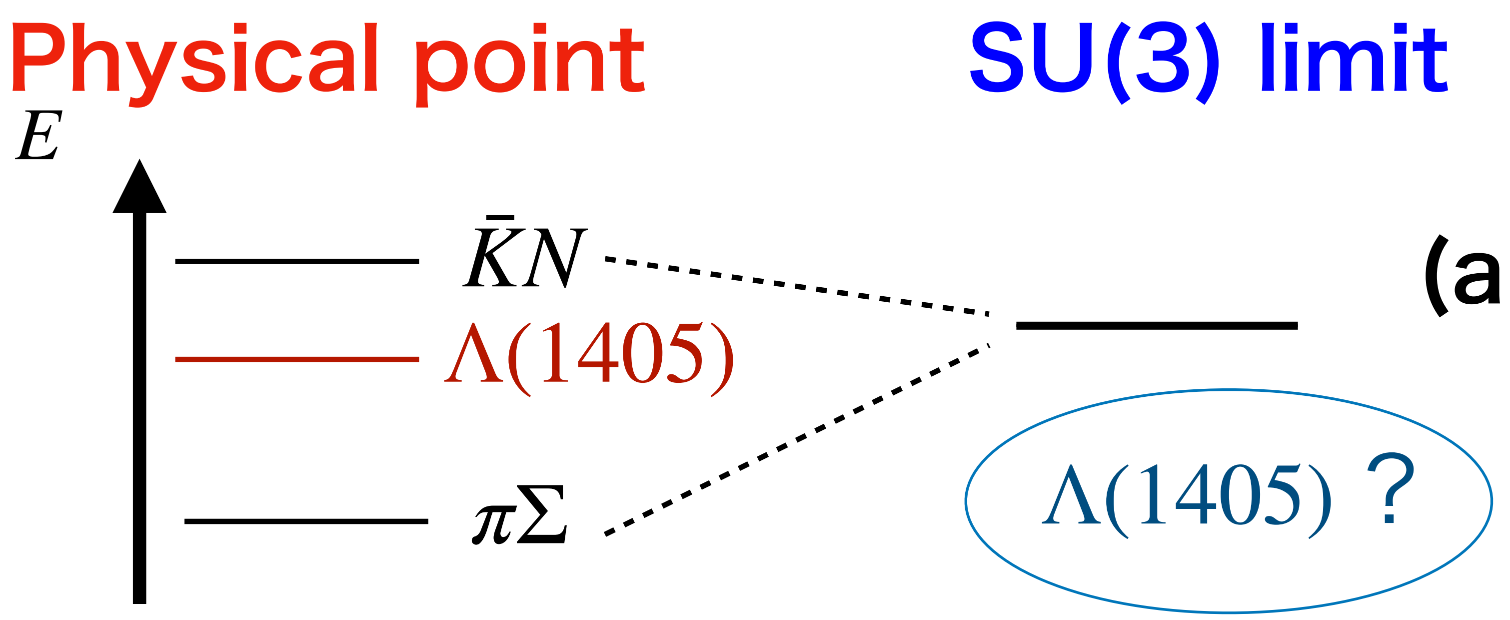
(local (leading-order) approximation)

$$\rightarrow V(\mathbf{r}) \simeq \frac{1}{R(\mathbf{r},t)}\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right)R(\mathbf{r},t)$$

- **Talks of HAL QCD studies**
  - Ξ_{cc}-Ξ_{cc} (T. Doi, Jul 29)
  - Λ_c-N (L. Zhang, Jul 31)
  - J/ψ-N, η_c-N (Y. Lyu, Jul 31)
  - Left-hand cut (S. Aoki, Aug 1)
  - Neural network (L. Wang, Aug 2)

Setups

- study $\Lambda(1405)$ in **flavor SU(3) limit**



(almost) single-channel analysis

- channels: $\underset{\text{meson}}{\underline{8}} \otimes \underset{\text{baryon}}{\underline{8}} = 27 \oplus 10 \oplus 10^* \oplus \underline{8_s \oplus 8_a \oplus 1}$
    - chiral unitary model: one pole in **singlet channel** & the other in **octet channel**

[Jido, Oller, Oset, Ramos, Meissner, 2003]
[Guo, Kamiya, Mai, Meissner, 2023]

- **neglect coupling between 8_s and 8_a** in this work

$$\begin{pmatrix} V_{8_s8_s}(r) & V_{8_s8_a}(r) \\ V_{8_a8_s}(r) & V_{8_a8_a}(r) \end{pmatrix} \approx \begin{pmatrix} V_{8_s8_s}(r) & 0 \\ 0 & V_{8_a8_a}(r) \end{pmatrix}$$

cf. chiral perturbation theory with Weinberg-Tomozawa term:

- **no coupling between 8_s and 8_a**
- same interactions for 8_s and 8_a

Lattice setups

- $a \approx 0.12$ fm, 32^4 lattices, $m_M = 459.4(1.7)_{\rm stat}\ \mathrm{MeV}$

$$m_B = 1166.1(4.1)_{\rm stat}\ \mathrm{MeV}$$

- R-correlators $(\mathrm{rep} = 1, 8_s, 8_a)$

$$R^{(\mathrm{rep})}(\mathbf{r}, t) = \frac{\langle M(\mathbf{x}+\mathbf{r}, t) B(\mathbf{x}, t) \bar{\Lambda}^{(X)}(0)\rangle}{\langle M(t)\bar{M}(0)\rangle\langle B(t)\bar{B}(0)\rangle}$$

$\Lambda^{(X)}(t) \sim \sum_{\mathbf{z}} u(\mathbf{z}, t) d(\mathbf{z}, t) s(\mathbf{z}, t)$: 3-quark type

($X = 8$ for $\mathrm{rep} = 8_s, 8_a$
$X = 1$ for $\mathrm{rep} = 1$)

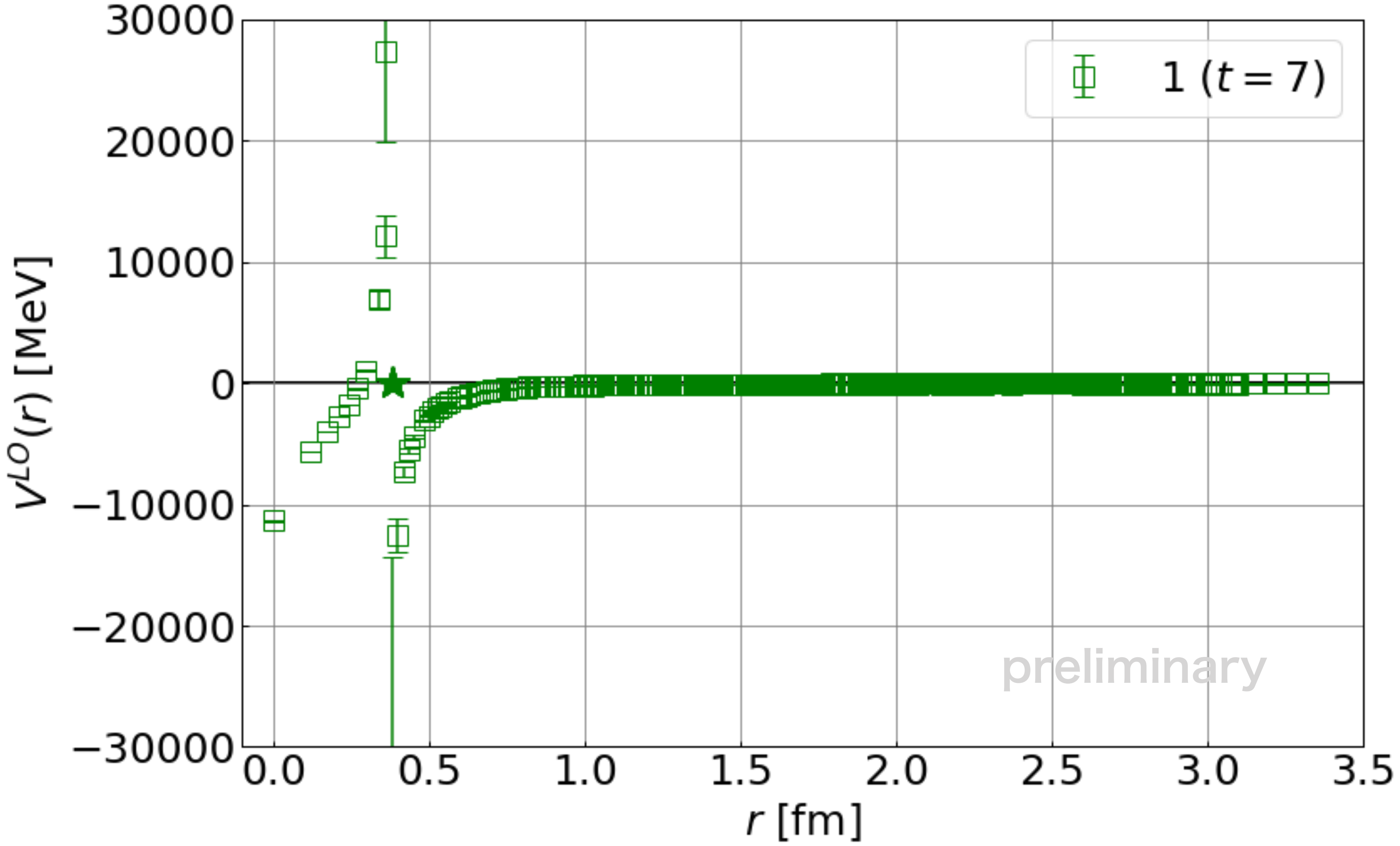
- calculation technique: same as in [KM, Aoki, PoS **LATTICE2023**, 063 (2024)]

- (at least) **one bound state in each channel** from $\langle \Lambda^{(8)}(t)\bar{\Lambda}^{(8)}(0)\rangle$ and $\langle \Lambda^{(1)}(t)\bar{\Lambda}^{(1)}(0)\rangle$

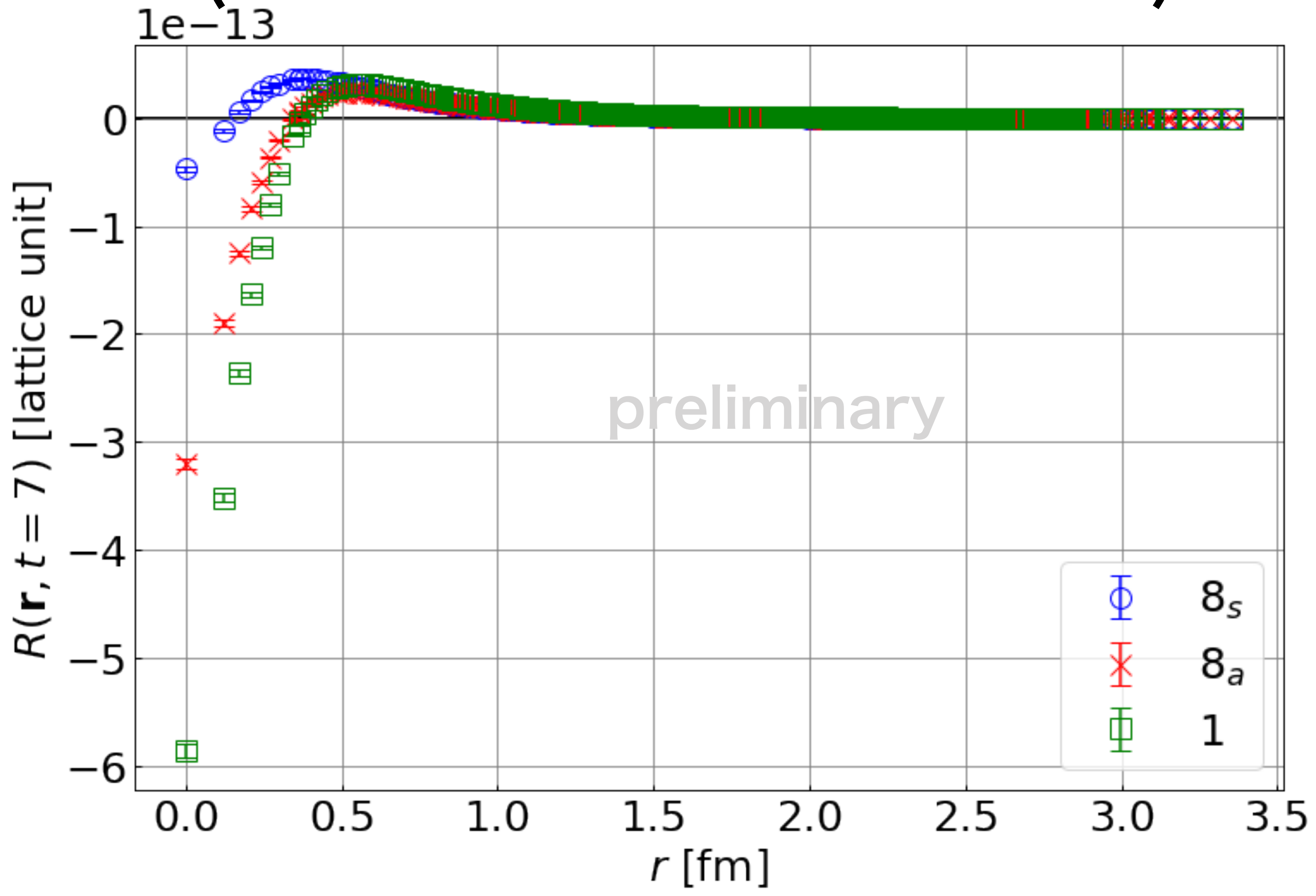
Local potentials

$$V(\mathbf{r}) \simeq \frac{1}{R(\mathbf{r},t)}\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right)R(\mathbf{r},t)$$

- local potential in singlet channel $V_1(r)$



- R-correlators $R(\mathbf{r},t)$ (NBS wave functions)



- **singular behavior in all channels** because of R-correlators crossing zero

no problematic in principle,
but difficult to obtain reliable results

- alternative approach: **separable potential** $U(\mathbf{r},\mathbf{r}') \approx \eta v(\mathbf{r}) v(\mathbf{r}')$ $(\eta = \pm 1)$

Separable potentials in the HAL QCD method

- time-dependent equation

$$\left(R(\mathbf{r},t) = \frac{\langle O_1(\mathbf{r},t)O_2(\mathbf{0},t)\bar{J}(0)\rangle}{\langle O_1(t)\bar{O}_1(0)\rangle\langle O_2(t)\bar{O}_2(0)\rangle} \right)$$

$$\int d^3r'\ U(\mathbf{r},\mathbf{r}')R(\mathbf{r}',t) \simeq \left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right)R(\mathbf{r},t)$$

$$\approx \eta v(\mathbf{r})v(\mathbf{r}'), \quad (\eta = \pm 1)$$

(separable potential approximation)

$$\rightarrow \quad \eta v(\mathbf{r}) \underline{\int d^3r'\ v(\mathbf{r}')R(\mathbf{r}',t)} \approx \left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right)R(\mathbf{r},t)$$

constant (indep. of $\mathbf{r}$)

→ **no singular behavior for** $v(\mathbf{r})$

☑ checked validity of separable potential approx. in KN system

How to extract separable potentials

- time-dependent (TD) equation for separable potential: $(\eta = \pm 1)$

$$\eta v(\mathbf{r}) \underbrace{\int d^3r' \; v(\mathbf{r}')R(\mathbf{r}',t)}_{= A[R,v]\text{: constant (indep. of } \mathbf{r})} = \underbrace{\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right) R(\mathbf{r},t)}_{= \mathcal{D}R(\mathbf{r},t)}$$

$$\times \int d^3r \; R(\mathbf{r},t)$$

$$\eta (A[R,v])^2 = \int d^3r \; \underbrace{R(\mathbf{r},t)\mathcal{D}R(\mathbf{r},t)}_{\text{real}}$$

$$\eta = \mathrm{sgn}[\eta(A[R,v])^2] = \mathrm{sgn}\left[\int d^3r \; R(\mathbf{r},t)\mathcal{D}R(\mathbf{r},t)\right]$$

$$A[R,v] = \sqrt{|\eta(A[R,v])^2|} = \sqrt{\left|\int d^3r \; R(\mathbf{r},t)\mathcal{D}R(\mathbf{r},t)\right|}$$

$$v(\mathbf{r}) = \frac{\mathcal{D}R(\mathbf{r},t)}{\eta A[R,v]}$$

Setups for separable potentials

- neglect coupling between 8_s and 8_a

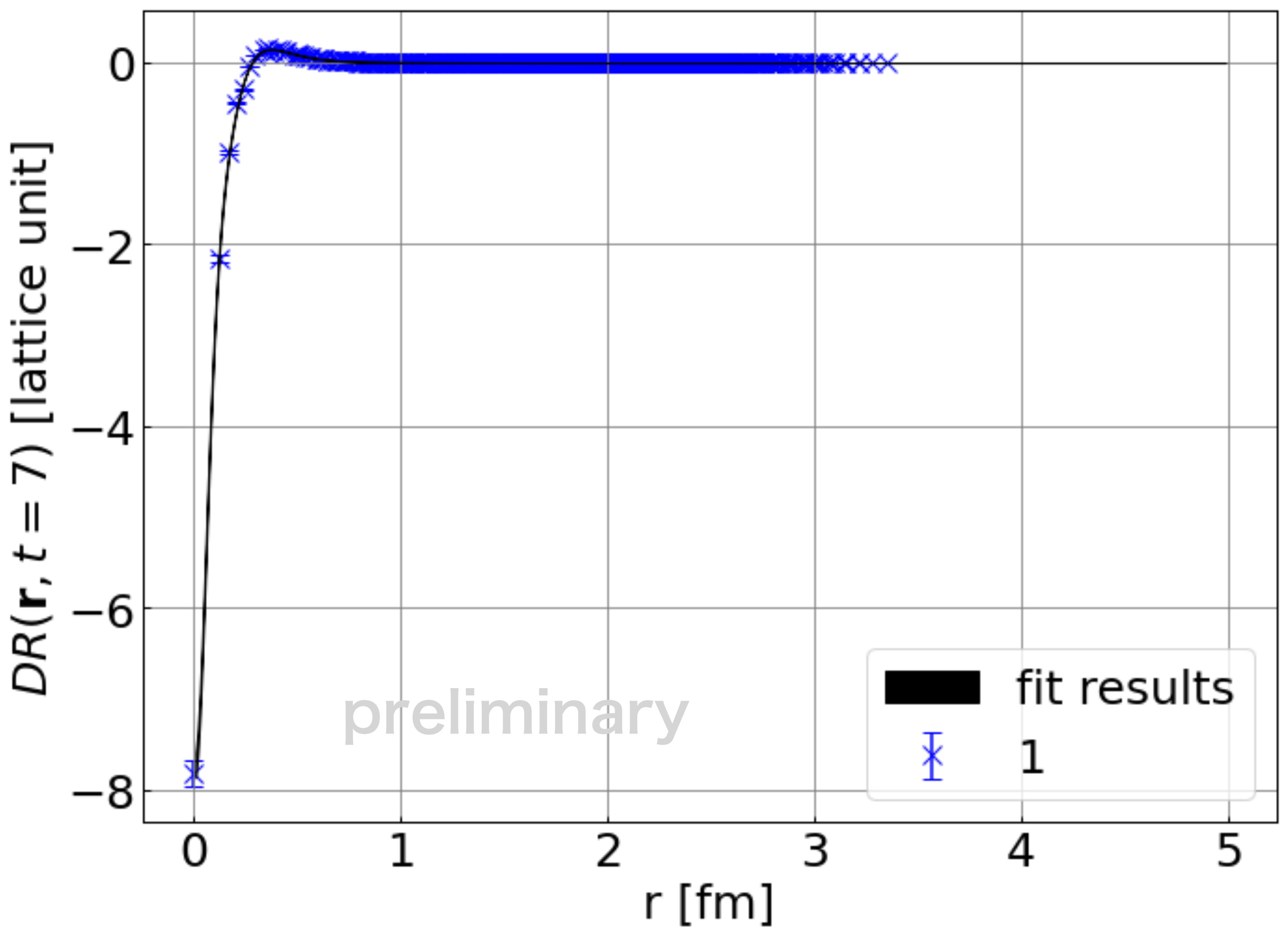
$$U_1(\mathbf{r},\mathbf{r}') \approx \eta_1 v_1(\mathbf{r}) v_1(\mathbf{r}')$$

$$\begin{pmatrix} U_{8_s8_s}(\mathbf{r},\mathbf{r}') & U_{8_s8_a}(\mathbf{r},\mathbf{r}') \\ U_{8_a8_s}(\mathbf{r},\mathbf{r}') & U_{8_a8_a}(\mathbf{r},\mathbf{r}') \end{pmatrix} \approx \begin{pmatrix} \eta_{8_s} v_{8_s}(\mathbf{r}) v_{8_s}(\mathbf{r}') & 0 \\ 0 & \eta_{8_a} v_{8_a}(\mathbf{r}) v_{8_a}(\mathbf{r}') \end{pmatrix}$$

- fitting for $\mathcal{D}R(\mathbf{r},t)$ using multi-Gaussians to obtain potentials in continuum

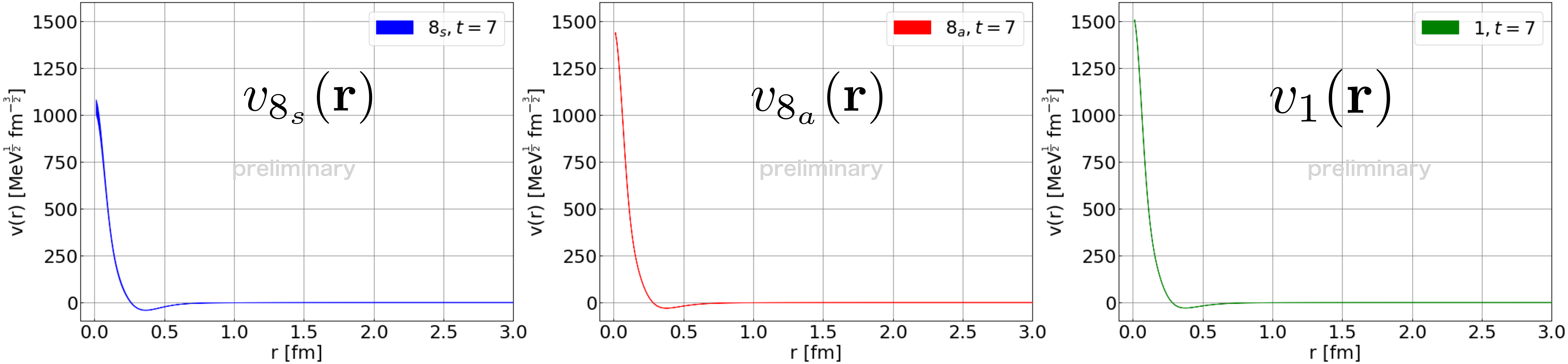
$$\left(\mathcal{D}R(\mathbf{r},t) = \left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2} \right) R(\mathbf{r},t) \right)$$

$\mathcal{D}R(\mathbf{r},t)$ in singlet channel



Results of separable potentials

- Results of $v(\mathbf{r})$, η



	8_s channel	8_a channel	1 channel
η	-1	-1	-1

- $\eta = -1$ for all three channels ➡ attractive interactions
- magnitude of $v(\mathbf{r})$ in short distance is larger for singlet channel

Binding energies

- solve Schrödinger equation in the Gaussian expansion method with separable potentials [Hiyama, Kino, Kamimura, 2003]

- systematic error includes
  - timeslice dependence
  - finite-volume effects

- our results (preliminary)

	8_s channel	8_a channel	1 channel
$E_{\rm bind}$ [MeV]	$59.9(5.3)_{\rm stat}(^{+5.6}_{-7.7})_{\rm syst}$	$52.6(3.8)_{\rm stat}(^{+2.1}_{-5.4})_{\rm syst}$	$69.1(6.2)_{\rm stat}(^{+7.7}_{-12.4})_{\rm syst}$

- c.f. estimates from $\langle \Lambda^{(X)}(t)\bar{\Lambda}^{(X)}(0)\rangle$ $(X = 1, 8)$:

	$8_s(8_a)$ channel	1 channel
$E_{\rm bind}$ [MeV]	$23.1(28.0)_{\rm stat}$	$78.0(12.3)_{\rm stat}$

- consistent with the results from $\langle \Lambda^{(X)}(t)\bar{\Lambda}^{(X)}(0)\rangle$ within (large) errors
- $E^{8_s}_{\rm bind}, E^{8_a}_{\rm bind} < E^{1}_{\rm bind}$ is satisfied ⟵ same as chiral unitary model

Summary

- we study $\Lambda(1405)$ in flavor SU(3) limit from the **meson-baryon scatterings in lattice QCD** using the HAL QCD method

- R-correlator has a zero point, which leads to singular behavior of the potential

- to avoid such behavior, we employ a **separable potential** instead of the usual local approximation for the HAL QCD method

- our results of the potentials show **attractive interactions** and produce consistent binding energies within (large) errors

- first time application of the HAL QCD method with separable potentials

Future work

- more precise consistency check by reducing errors of the estimates from $\langle \Lambda^{(X)}(t)\bar{\Lambda}^{(X)}(0)\rangle$ $(X=1,8)$
  - variational method using $\langle (MB)(t)\bar{\Lambda}(0)\rangle$ and $\langle (MB)(t)(\bar{M}B)(0)\rangle$ additionally
- **coupled-channel analysis** for 8_s and 8_a channel with separable potentials
- studies with more realistic setups
  - **(2+1)-flavor** simulation ⬅ coupled-channel analysis is required
- **more complicated separable form** in the HAL QCD potential
  - a sum of separable terms $U(\mathbf{r},\mathbf{r}') \approx \sum_i \eta^{(i)} v^{(i)}(\mathbf{r}) v^{(i)}(\mathbf{r}')$

Backups

Setups in detail ($\Lambda(1405)$ in SU(3) limit)

- use 3-quark source operator in the R-correlator

(cf. $m_M = 368$ MeV, $m_B = 1151$ MeV in chiral unitary model)

$$R^{(\mathrm{rep})}(\mathbf{r},t) = \frac{\langle (M(\mathbf{r},t)B(\mathbf{0},t))_{(\mathrm{rep})}\underline{\bar{\Lambda}(0)}\rangle}{\langle M(t)M^\dagger(0)\rangle\langle B(t)B^\dagger(0)\rangle} \sim \sum_{\mathbf{z}} \bar{u}(\mathbf{z})\bar{d}(\mathbf{z})\bar{s}(\mathbf{z})$$

: 3-quark type (octet, singlet)

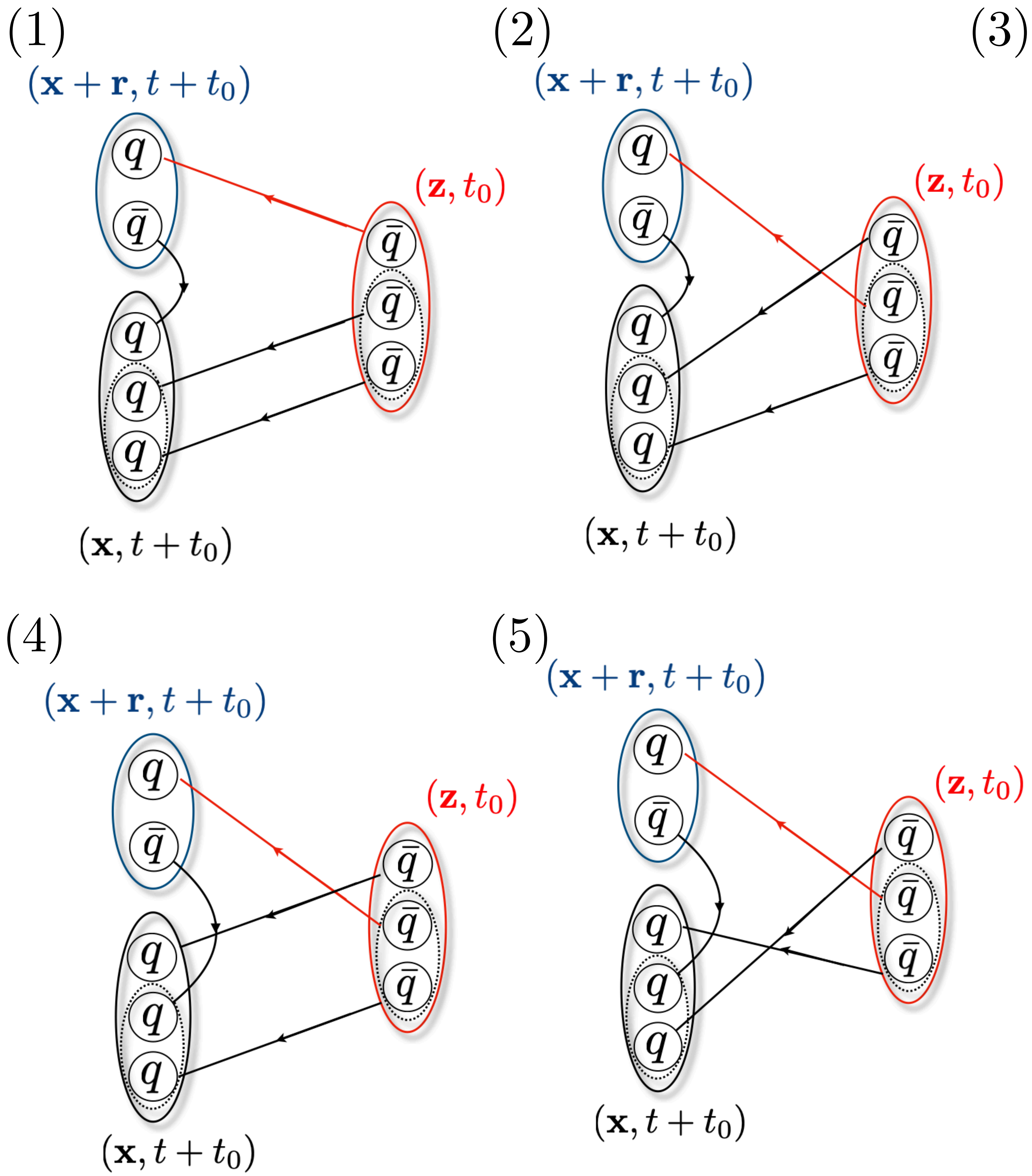
$$(\mathrm{rep} = (1, 8_a, 8_s))$$

$$(MB_\alpha)^{S=-1,I=0}_{8_1} = \frac{\sqrt{10}}{10}(K\Xi_\alpha)^{I=0} - \frac{\sqrt{10}}{10}(\bar{K}N_\alpha)^{I=0} - \frac{\sqrt{15}}{5}(\pi\Sigma_\alpha)^{I=0} - \frac{\sqrt{5}}{5}\eta^8\Lambda^8_\alpha$$

$$(MB_\alpha)^{S=-1,I=0}_{8_2} = \frac{\sqrt{2}}{2}(K\Xi_\alpha)^{I=0} + \frac{\sqrt{2}}{2}(\bar{K}N_\alpha)^{I=0}$$

$$(MB_\alpha)^{S=-1,I=0}_{1} = \frac{1}{2}(K\Xi_\alpha)^{I=0} - \frac{1}{2}(\bar{K}N_\alpha)^{I=0} + \frac{\sqrt{6}}{4}(\pi\Sigma_\alpha)^{I=0} - \frac{\sqrt{2}}{4}\eta^8\Lambda^8_\alpha$$

Quark contractions



$$F_\alpha^{8_1}(\mathbf{r},t) = -\sqrt{10}[(1)+(2)+(3)-(4)-2(5)]$$

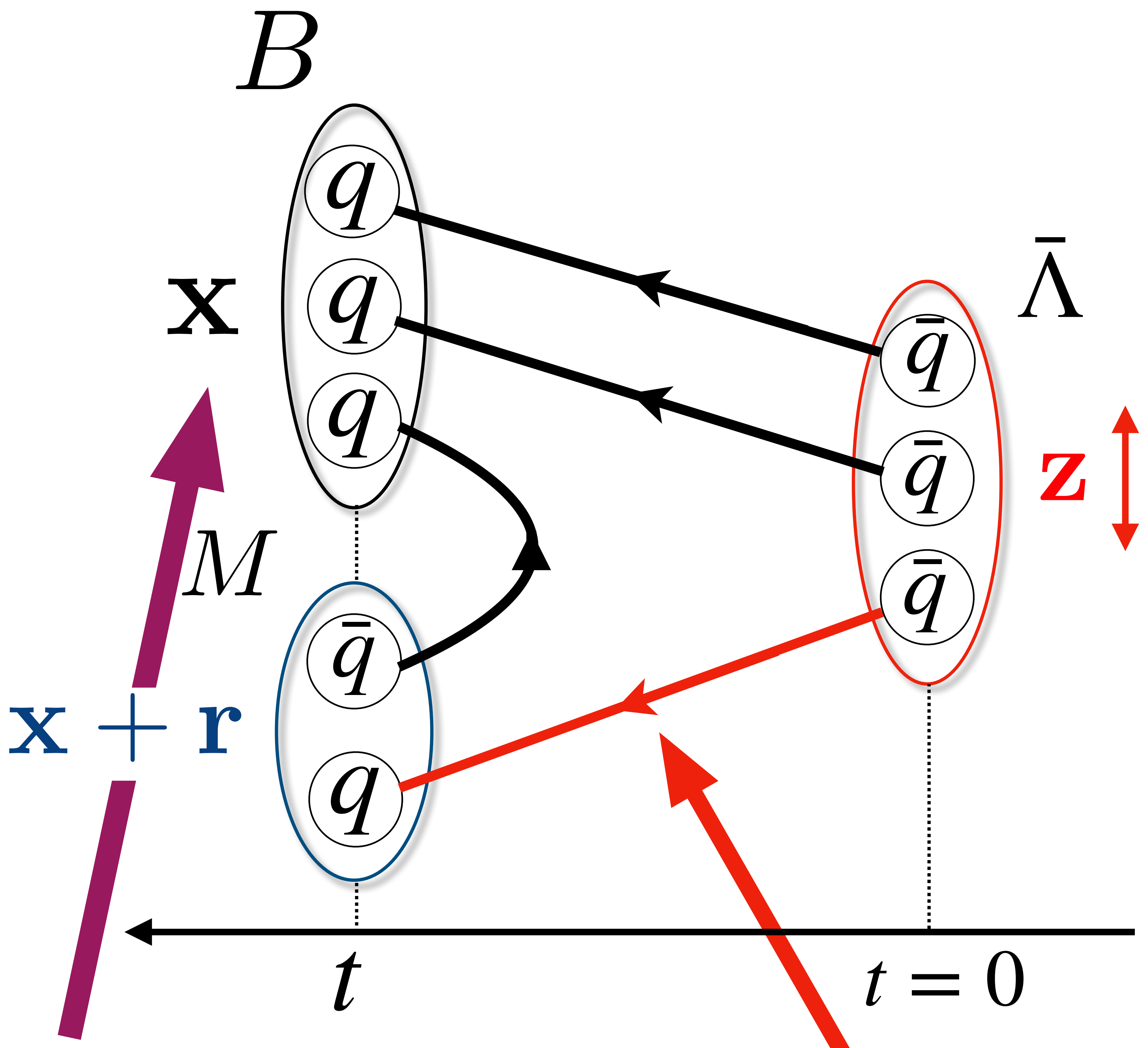
$$F_\alpha^{8_2}(\mathbf{r},t) = -\frac{\sqrt{6}}{2}[(1)+(2)+(3)+(4)]$$

$$F_\alpha^{1}(\mathbf{r},t) = -\frac{4}{\sqrt{3}}[(1)-2(2)+(3)-(4)+(5)]$$

Details of calculation

- R-correlators $(\mathrm{rep} = 1, 8_s, 8_a)$

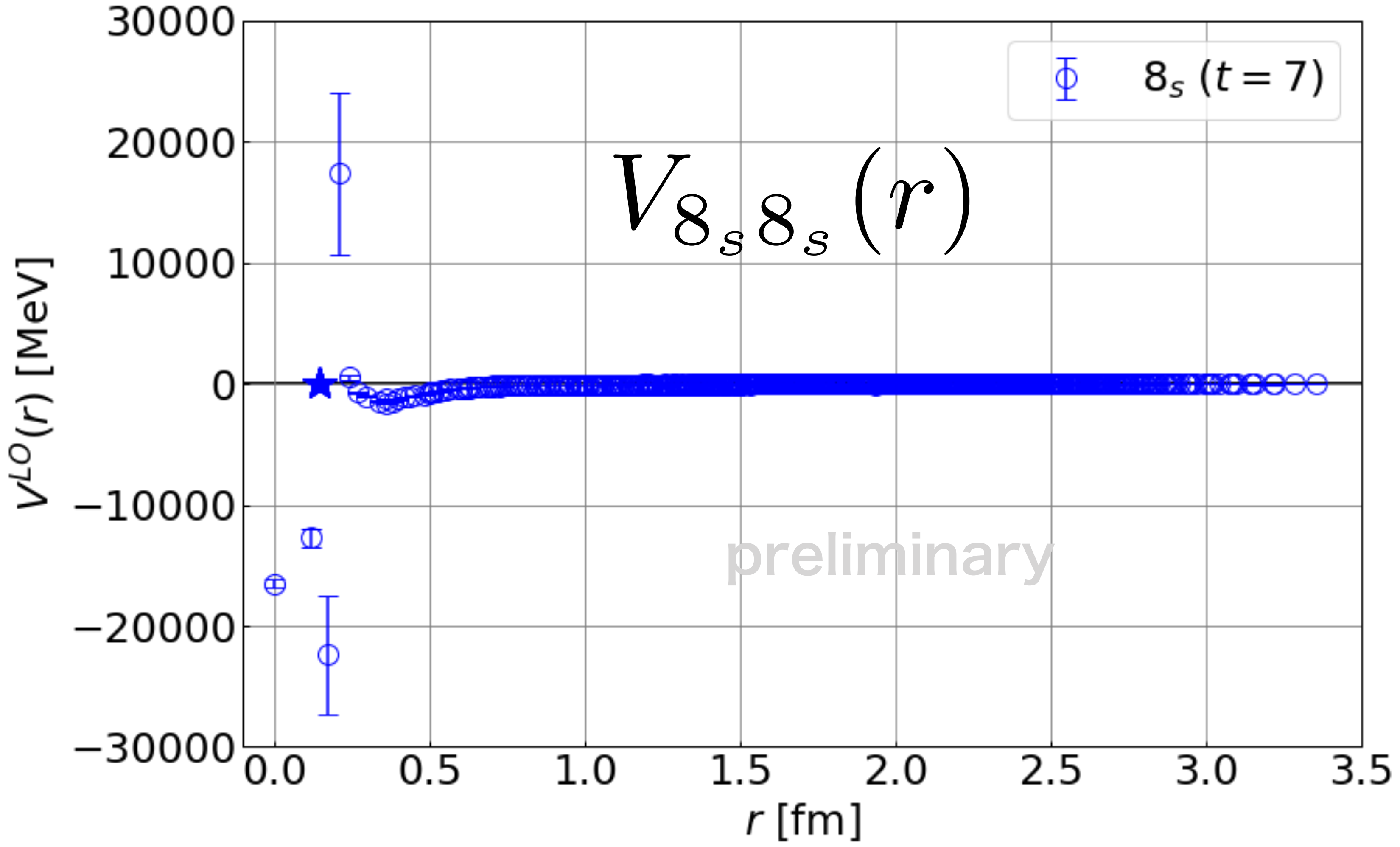
$$R^{(\mathrm{rep})}(\mathbf{r}, t) = \frac{\langle M(\mathbf{x}+\mathbf{r}, t) B(\mathbf{x}, t) \bar{\Lambda}^{(X)}(0) \rangle}{\langle M(t) \bar{M}(0) \rangle \langle B(t) \bar{B}(0) \rangle}$$

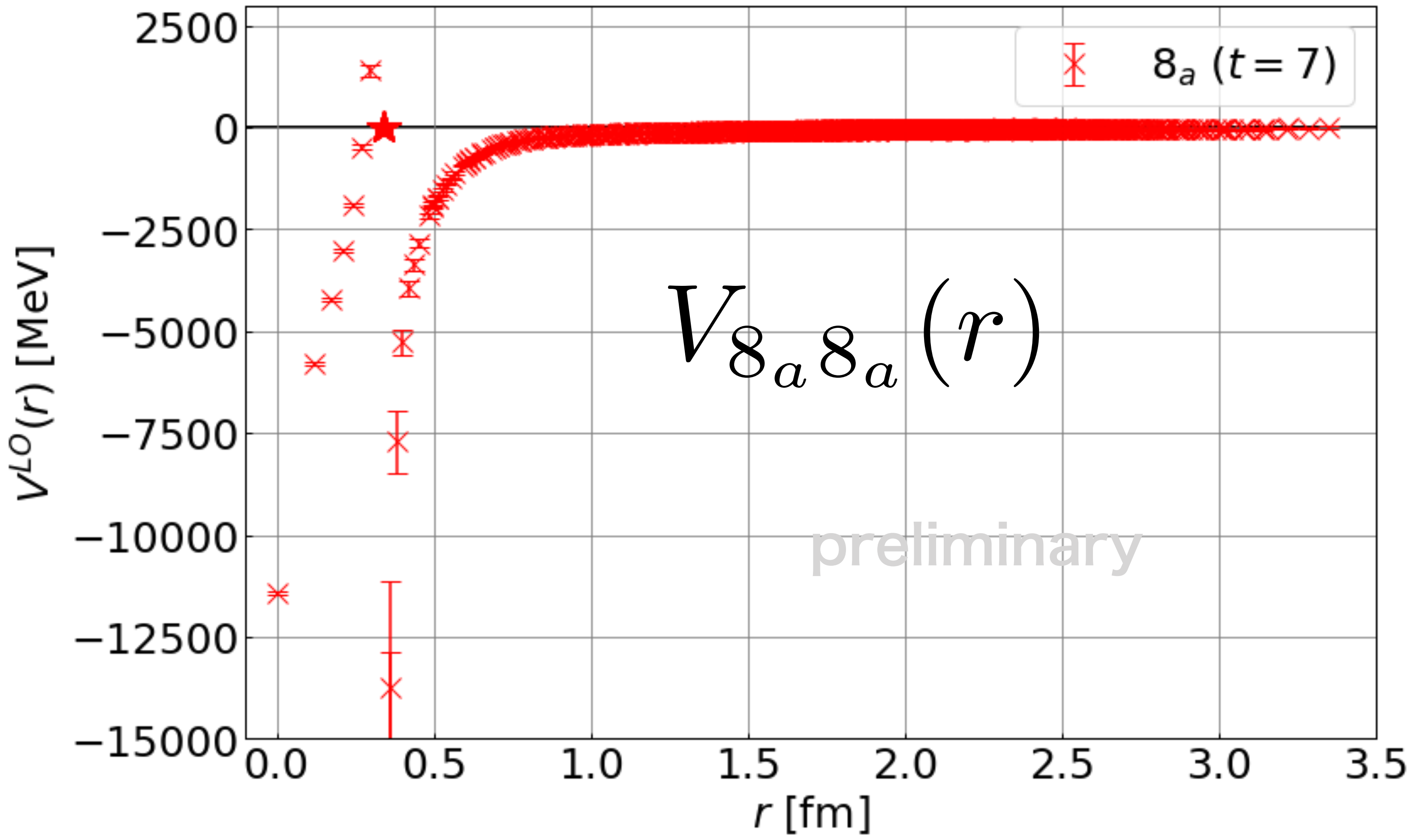


move $\mathbf{x}$ to increase statistics (CAA + TSM)
[Bali, Collins, Schäfer 2010]
[Blum, Izubuchi, Shintani 2013]

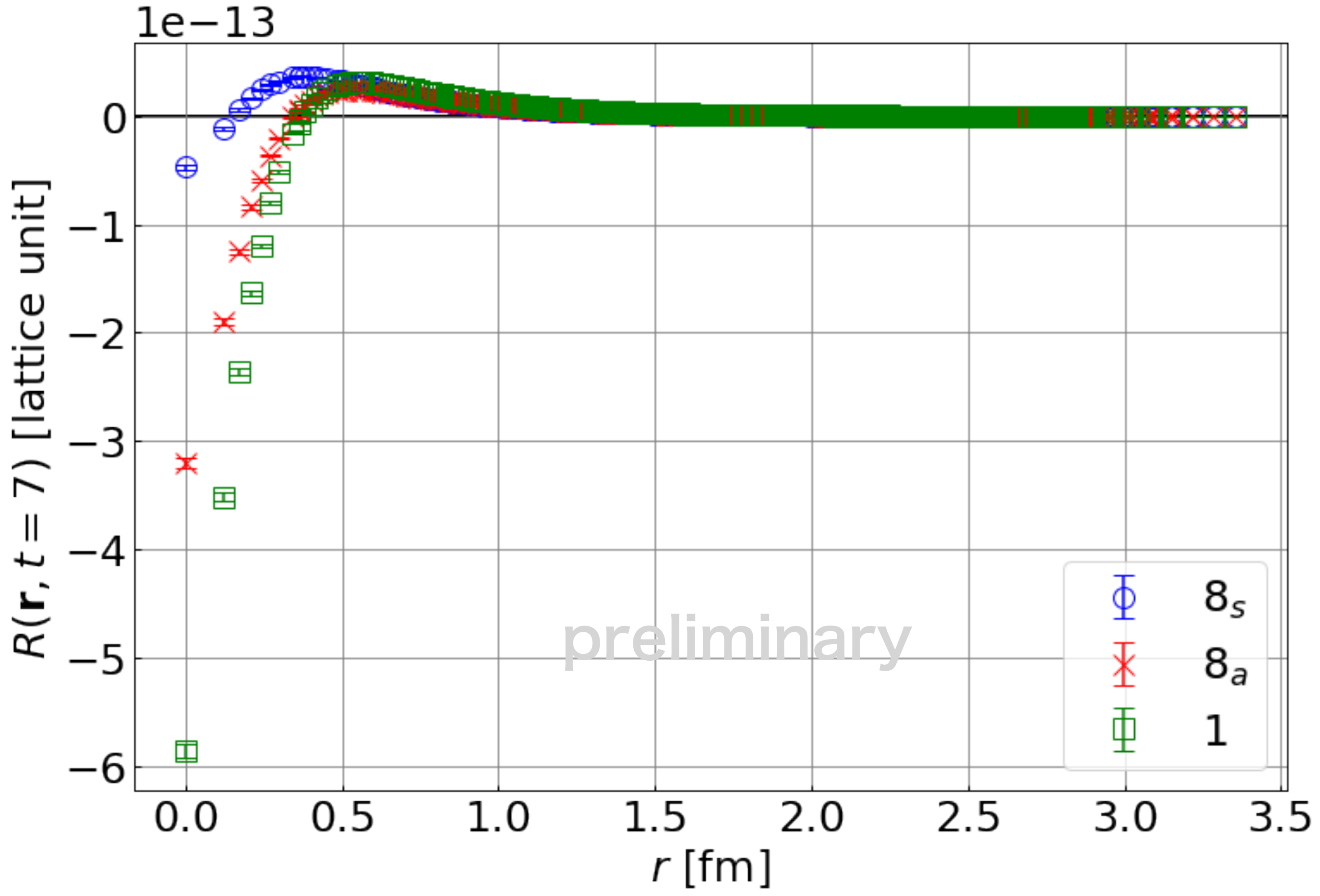
all-to-all-propagator calculation using stochastic method

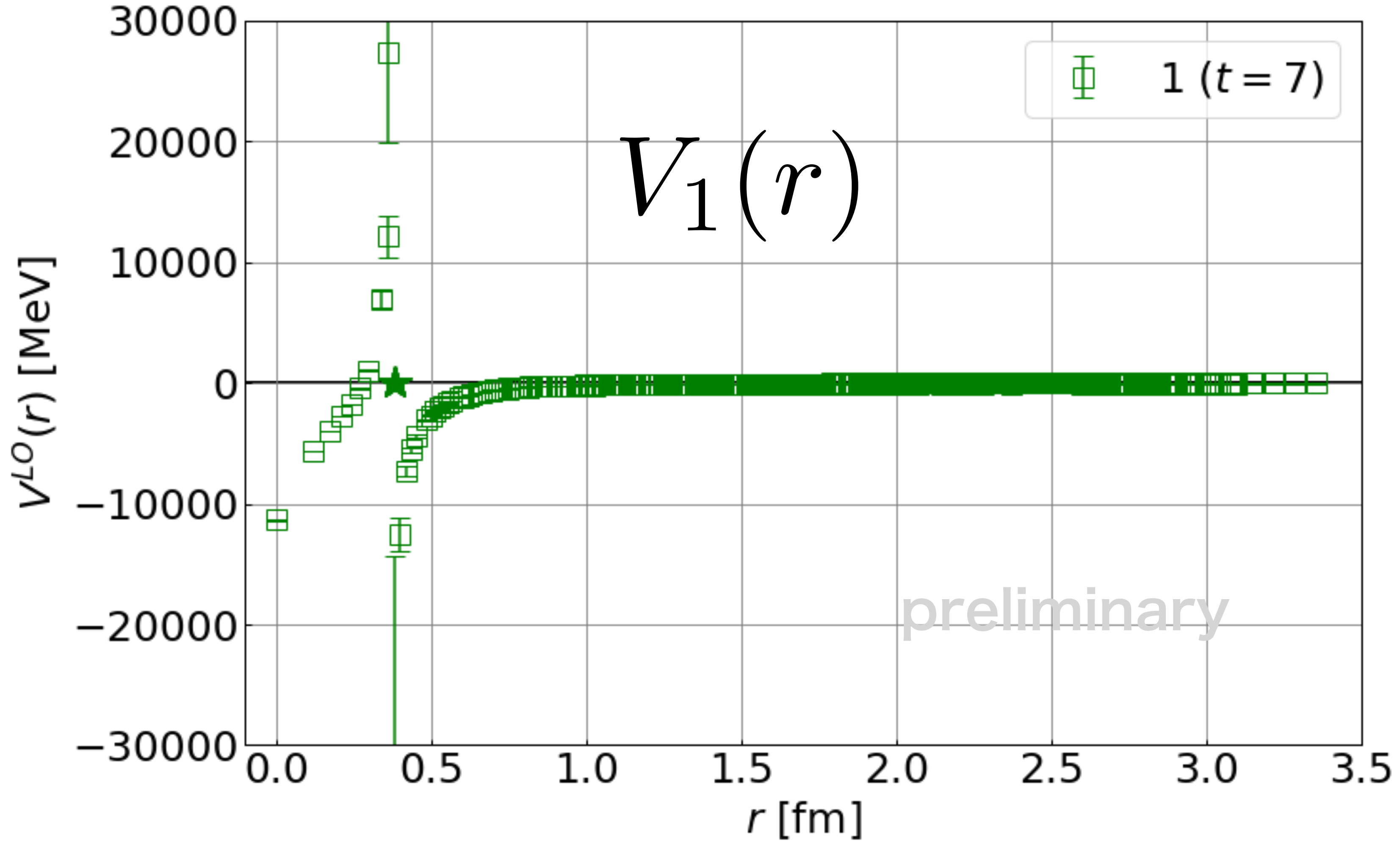
Local potentials in all channels





R-correlators $R(\mathbf{r}, t)$





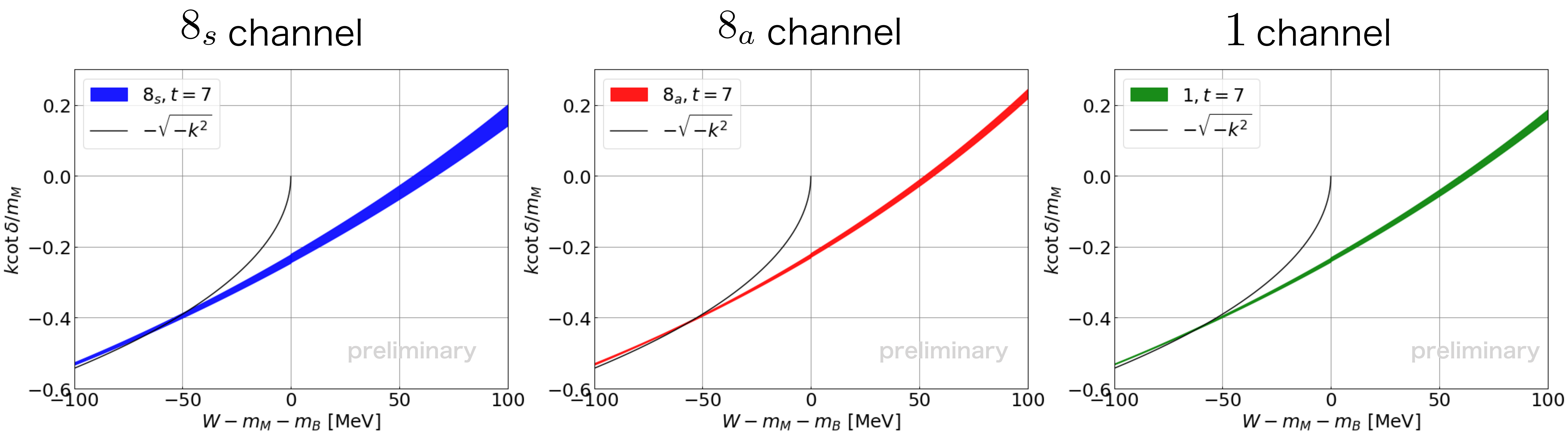
$$V(\mathbf{r}) \simeq \frac{1}{R(\mathbf{r},t)}\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right)R(\mathbf{r},t)$$

- singular behavior because of the R-correlators crossing zero

Phase shifts

- solve Lippmann-Schwinger equation with the sapearable potentials

- results of $k \cot \delta(k)$



- typical behavior when systems have attractive interactions

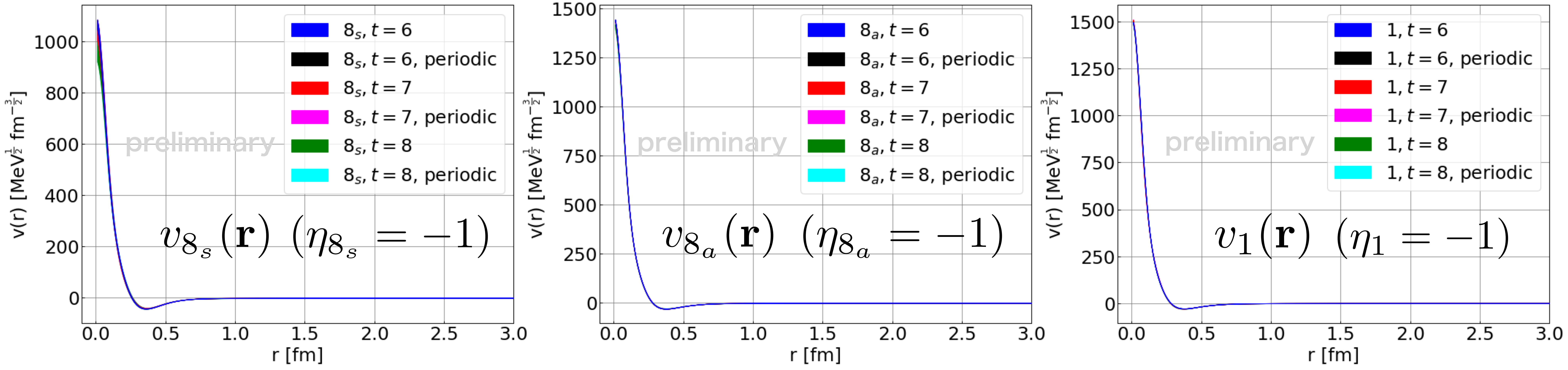
- one bound state appears

Separable potential in each case

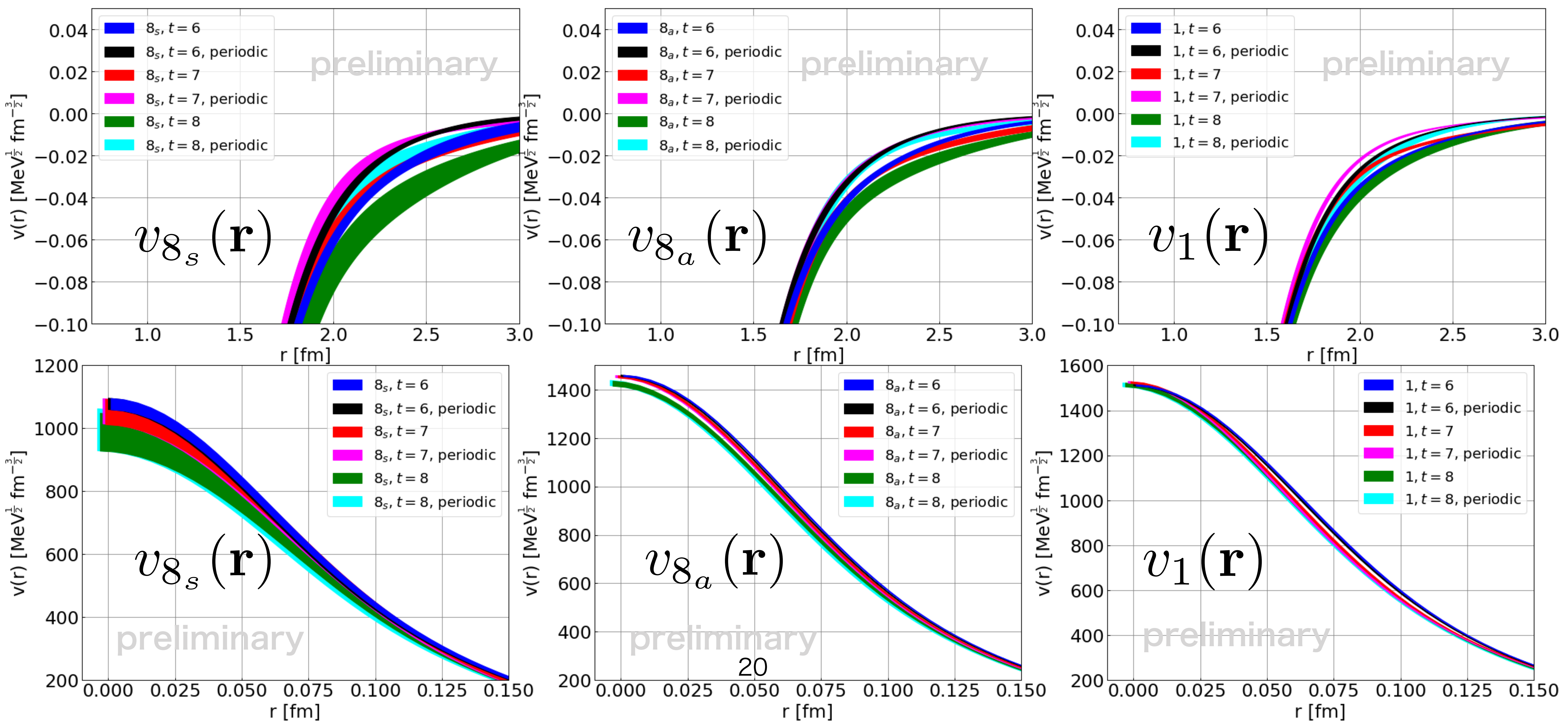
- "periodic": using periodic funtion as a fit function

$$f_p(\mathbf{r}) = f(\mathbf{r}) + \sum_{\mathbf{n}} f(\mathbf{r} + L\mathbf{n}), \quad \mathbf{n} \in \{(0,0,\pm1),(0,\pm1,0),(\pm1,0,0)\}$$

- plot of $v(\mathbf{r})$



- enlarged plot



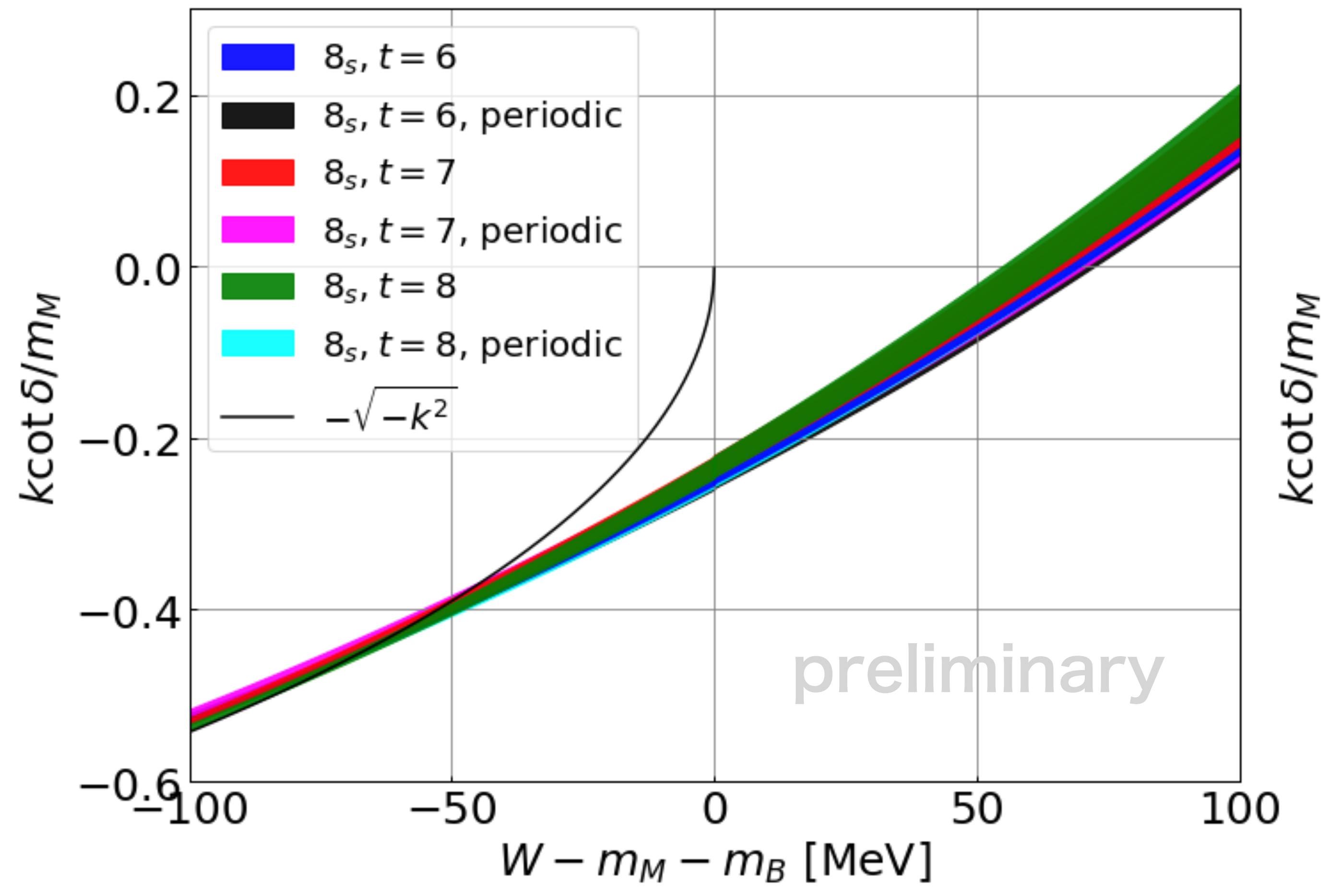
Observables in each case

- "periodic": using periodic funtion as a fit function

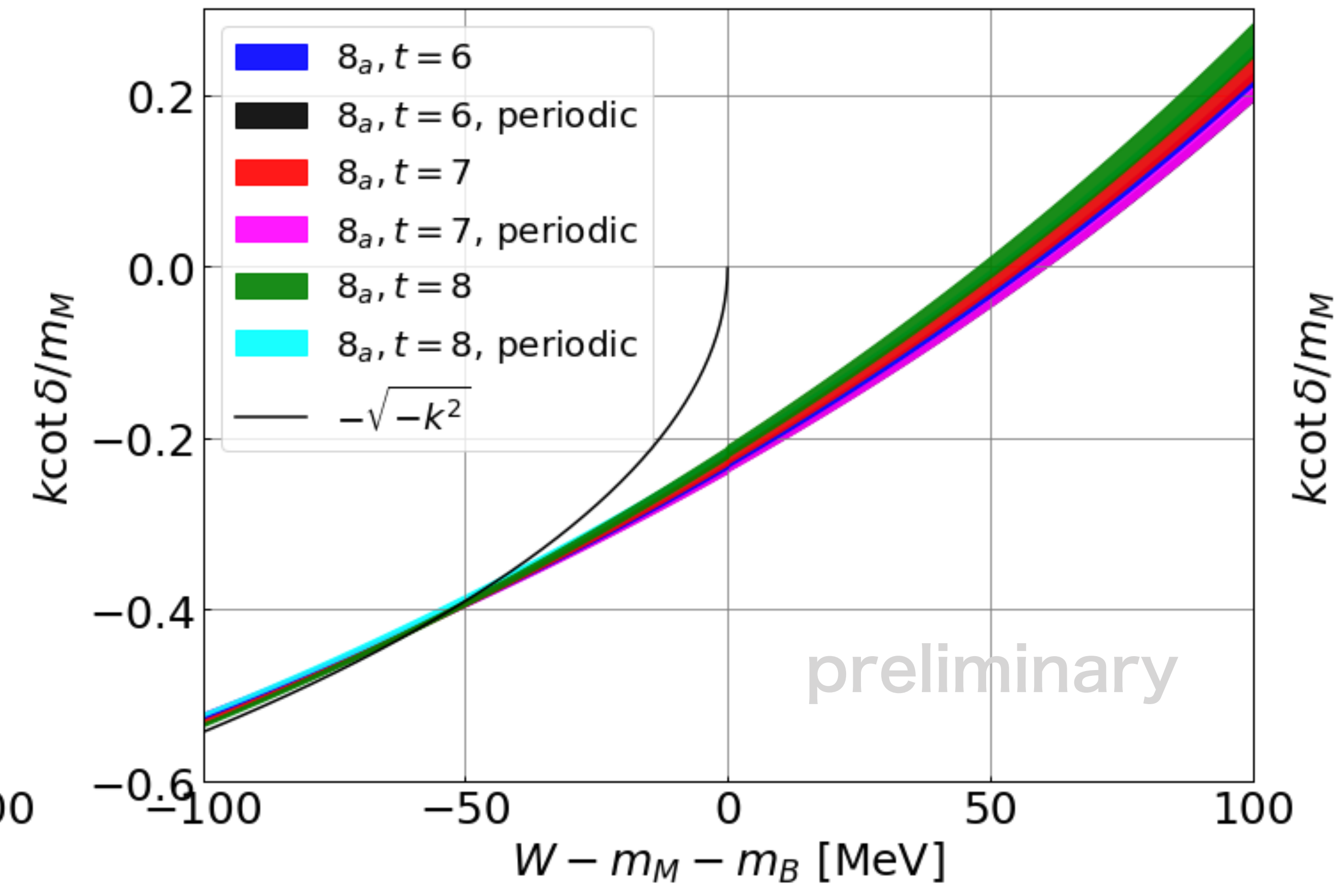
$$f_p(\mathbf{r}) = f(\mathbf{r}) + \sum_{\mathbf{n}} f(\mathbf{r} + L\mathbf{n}), \quad \mathbf{n} \in \{(0,0,\pm1),(0,\pm1,0),(\pm1,0,0)\}$$

- $k \cot \delta(k)$

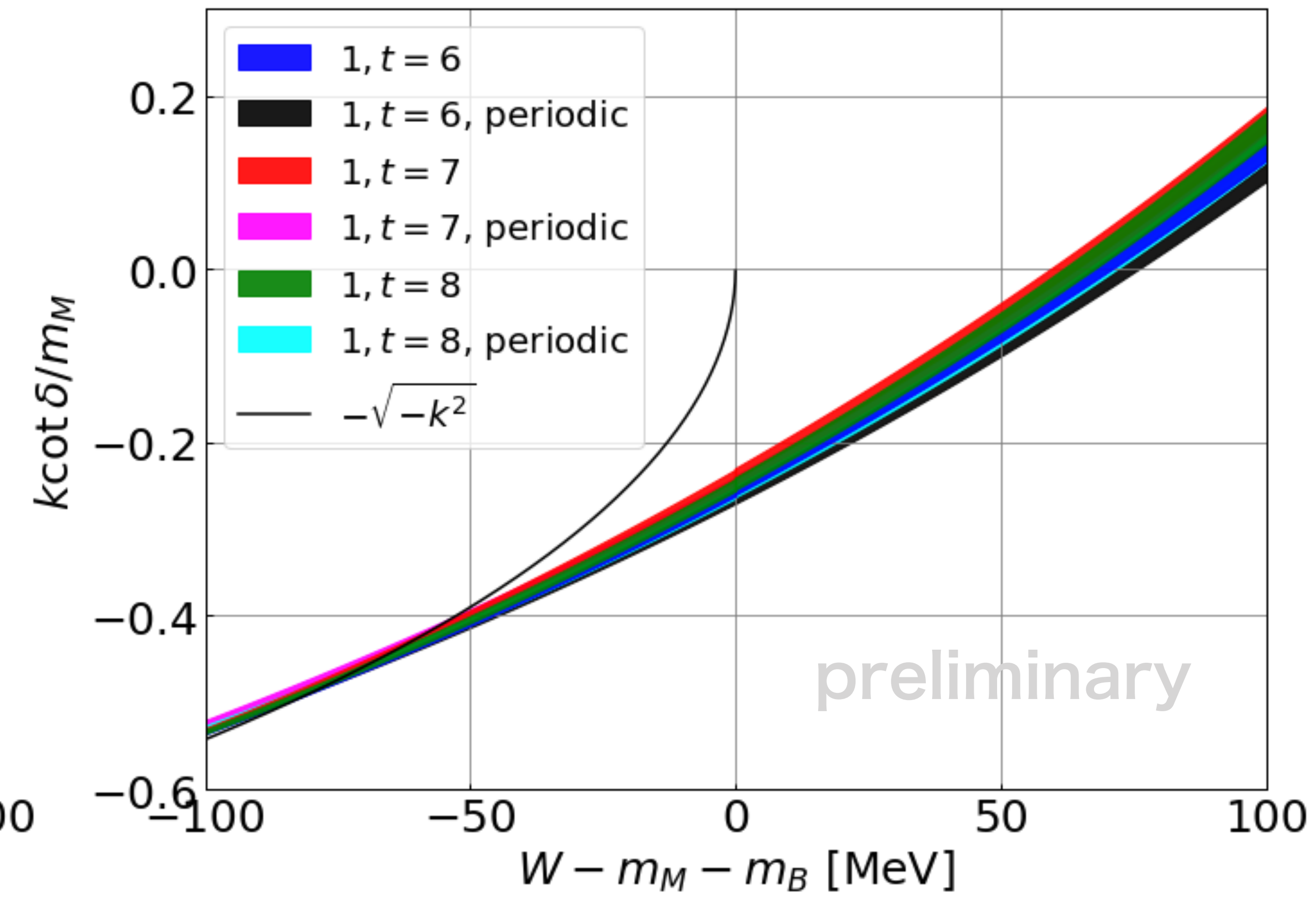
8_s channel



8_a channel



1 channel



- binding energies (MeV) (preliminary)

	8_s	8_a	1
t=6	61.4(5.3)	54.7(1.5)	76.8(3.4)
t=7	54.4(9.7)	53.7(3.6)	58.3(4.1)
t=8	65.5(12.5)	52.4(5.6)	69.9(6.7)

periodic	8_s	8_a	1
t=6	59.9(5.3)	53.4(1.7)	75.7(3.3)
t=7	52.2(9.9)	52.6(3.8)	56.7(4.9)
t=8	62.2(9.5)	47.2(6.2)	69.1(6.2)

	singlet	octet	octet
from 2pt	78.0(12.3)	23.1(28.0)	23.1(28.0)

Discussions

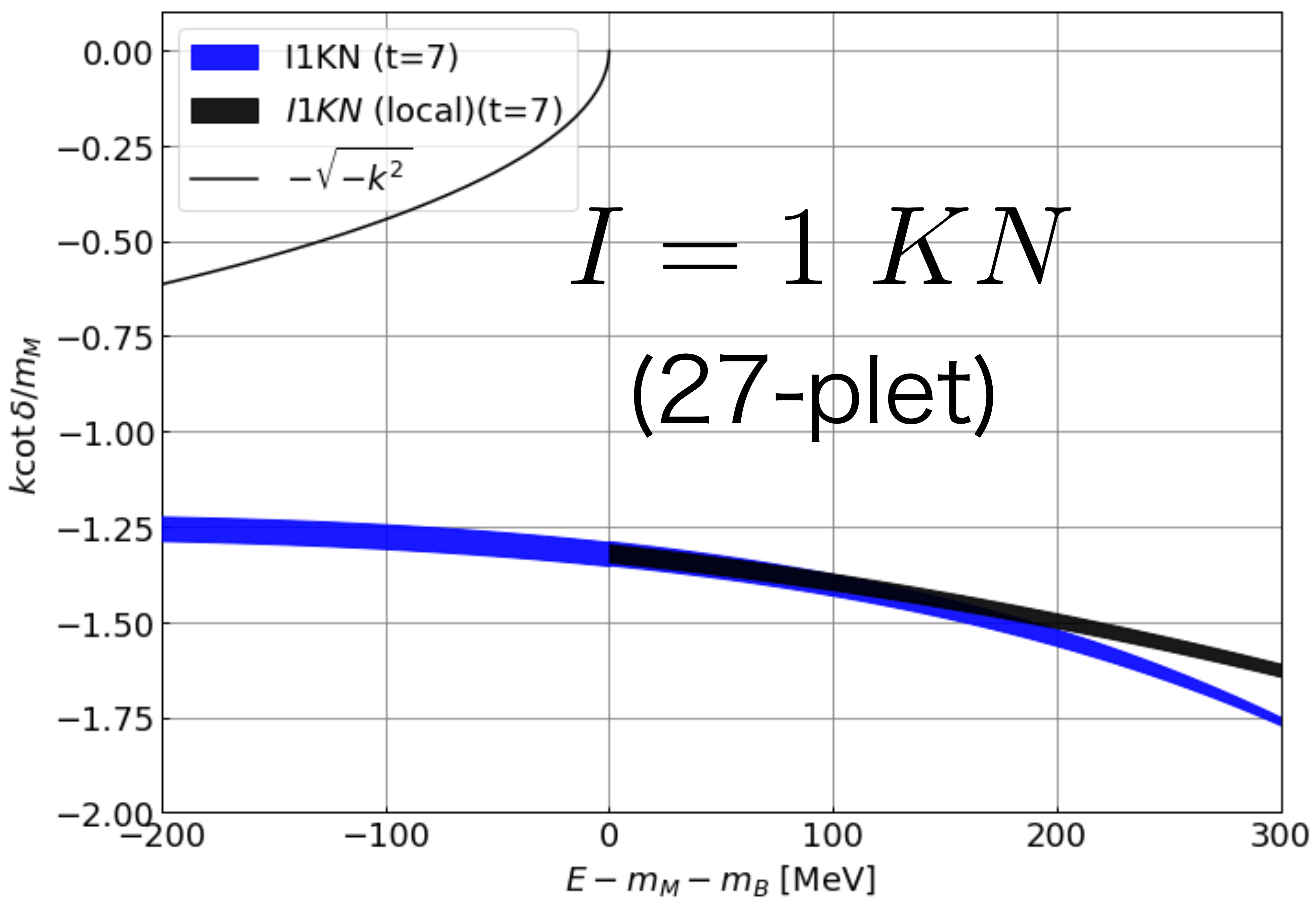
- singular behavior:
  - it appears due to the zeros of $R(\mathbf{r}, t)$ (wave functions)
  - such behavior does not happen in the usual QM
    ➡ singular behavior: effects beyond QM (effects from QFT)

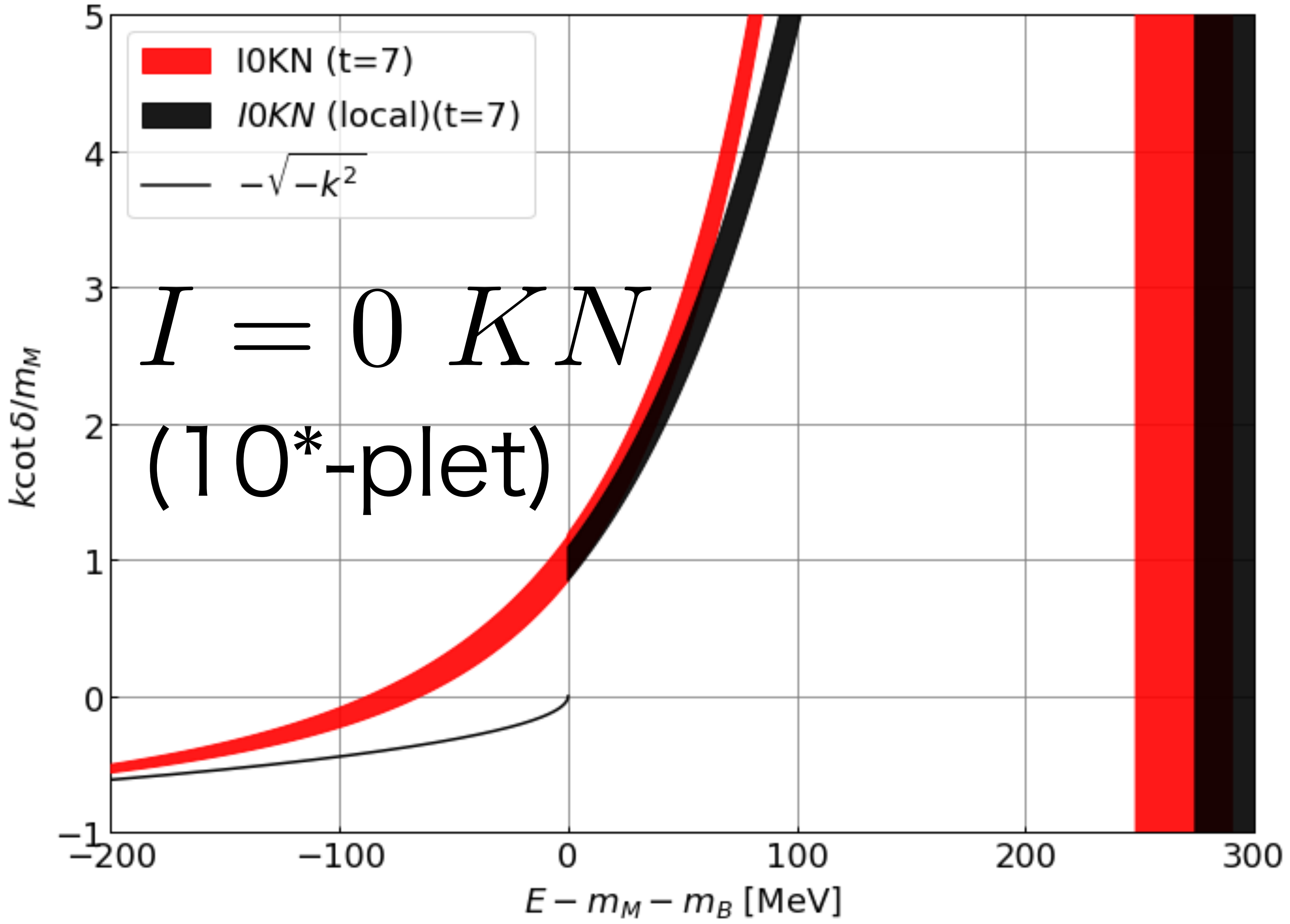
    non-locality of the HAL QCD potential
  - HAL QCD method with separable potentials allow us to avoid singular behavior
- systematic error of binding energy (BE):
  - mainly comes from timeslice dependence of the potentials in short distance ($r \lesssim 0.15$ fm)

  ➡ BE is sensitive to short distance behavior of separable potentials

Reliability check of separable potential

- compare phase shifts calculated using **local** and **separable potentials**
- systems: $I=1$ and $I=0$ KN in SU(3) limit at $m_M \approx 672$ MeV, $m_B \approx 1490$ MeV
- $k\cot\delta$





- same phase shifts up to ~100 MeV for $I=1$ KN while ~10 MeV for $I=0$ KN