Navdeep Singh Dhindsa **02/08/2024**

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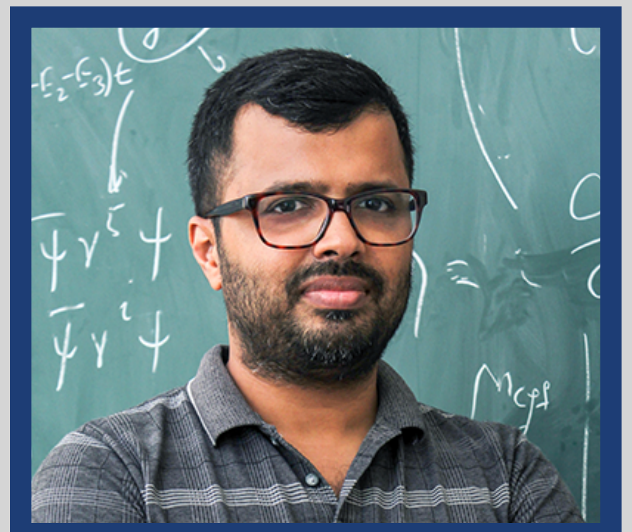
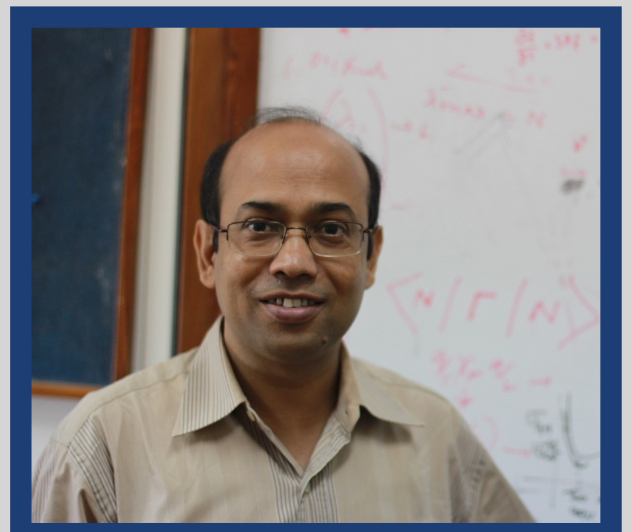
Exploring Single-Flavor Dibaryons: A lattice perspective

Funding resources

Work in collaboration with

M. Padmanath (IMSc Chennai) and Nilmani Mathur (TIFR Mumbai)



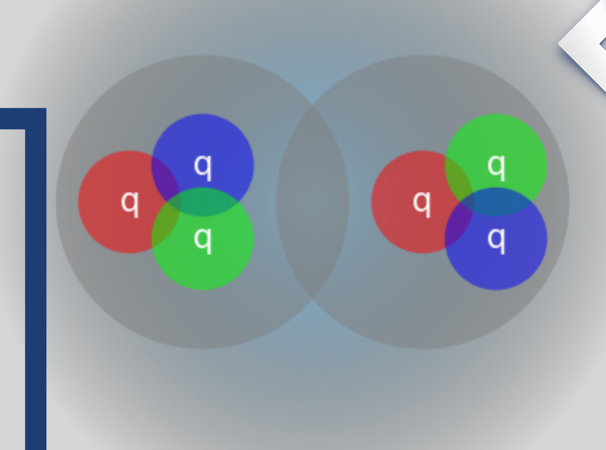


Dibaryons - Status

Hexaquark - Any object with Baryon number 2

Composed of six valence quarks

Can be molecular or compact



- Deuteron only stable dibaryon
- 1950's - Many predictions of various dibaryon states but failed experimental checks
- Experimental evidence of existence of d*

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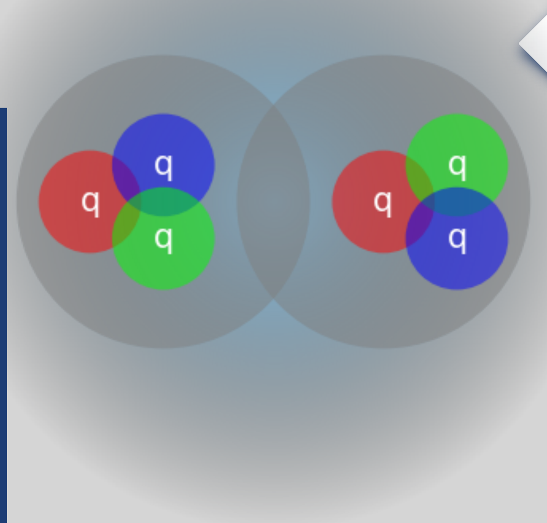
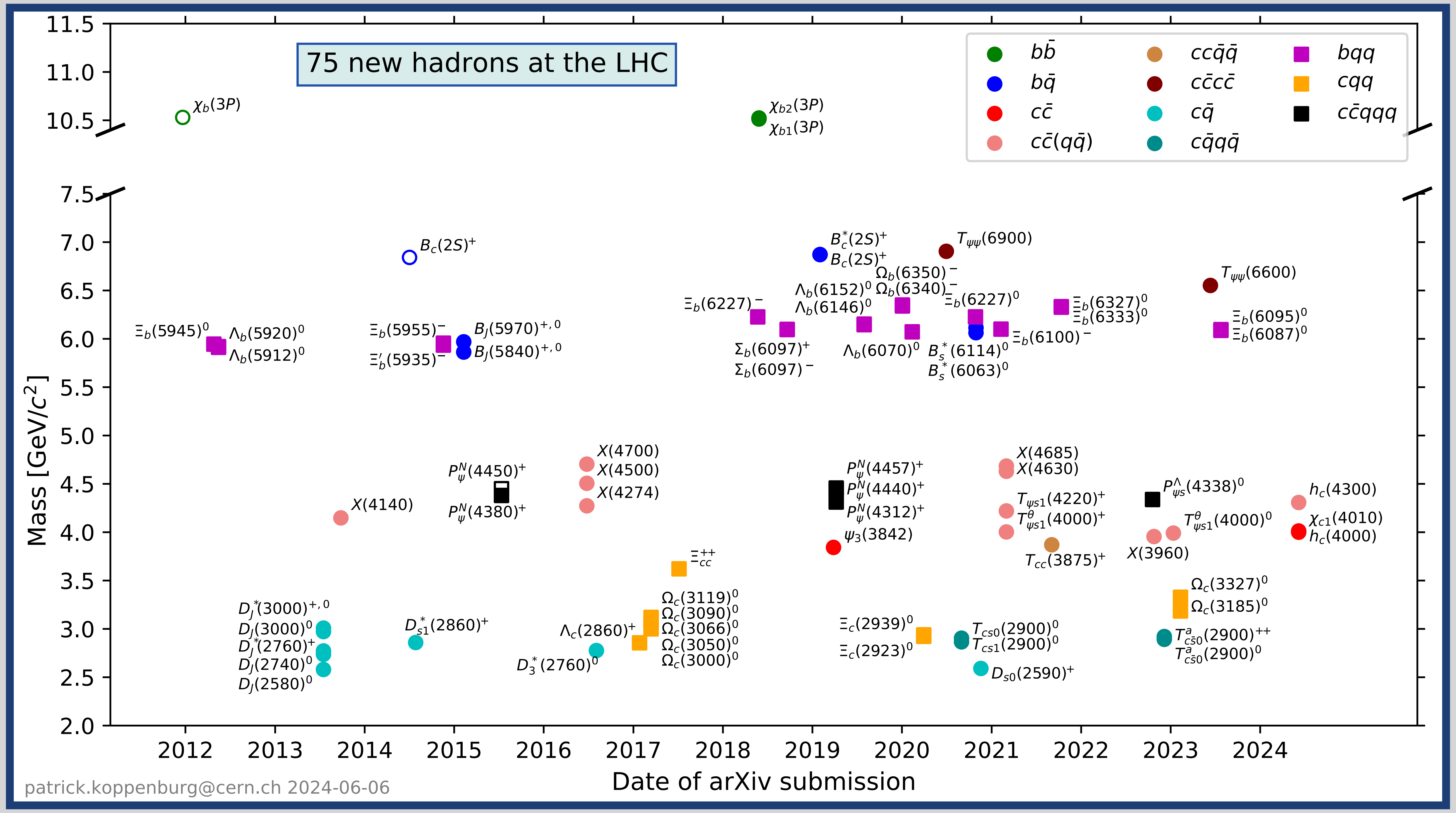


- Deuteron only stable dibaryon
- 1950's - Many predictions of various dibaryon states but failed experimental checks
- Experimental evidence of existence of d*
- Recent renewal in interest due to discoveries of complex quark systems



Based on theory of strong interactions, we cannot rule out more dibaryons in nature.

Review by Clement PPNP (2016)

Dibaryons - Status

Hyperon-Nucleon experimental results indicates:

- ΛN attractive, though less than NN, no strange deuteron.
- ΣN interaction even weaker than ΛN.

Dineutron? Diproton?

NN scattering experiments indicates absence of bound state

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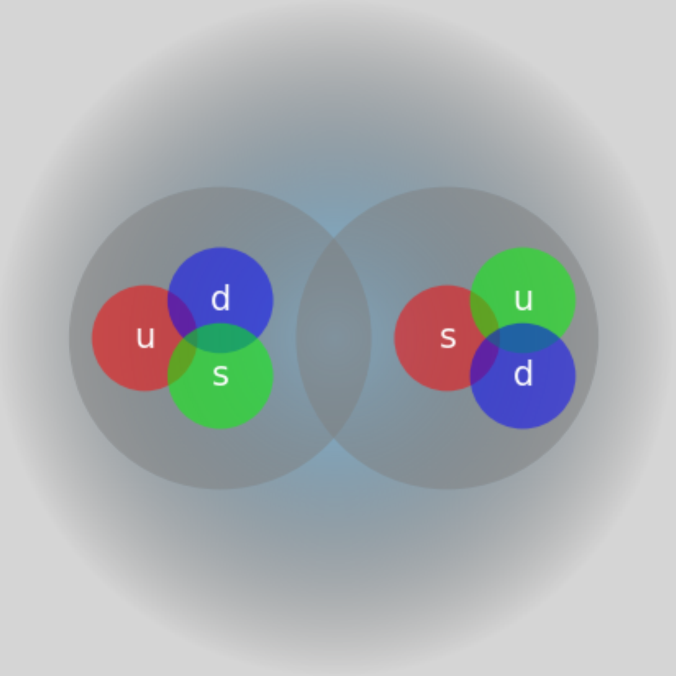

Hyperon-Hyperon:

- $\Lambda\Lambda$ experiments does not rule out bound system.
- Jaffe prediction of dihyperons

Jaffe
PRL 138 (1977) 195

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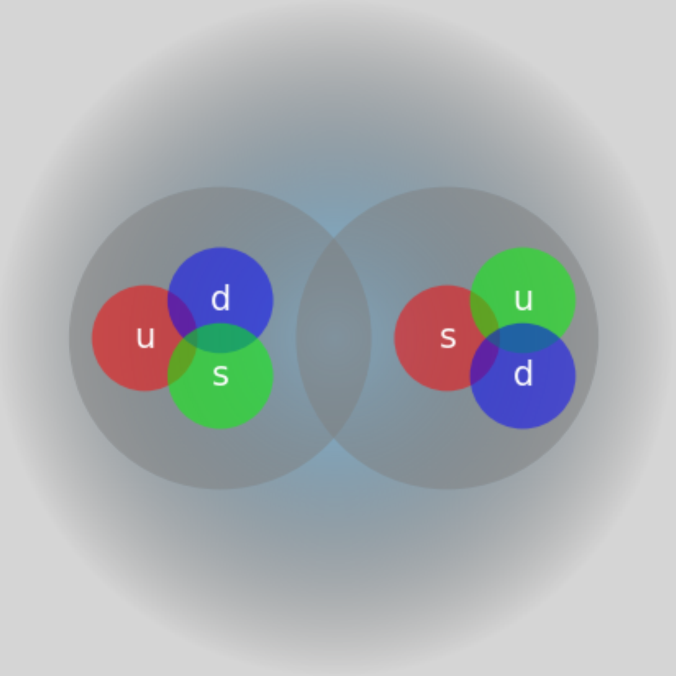

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PRL 138 (1977) 195

✱Dihyperon other calculations vary from very deep bound (even more than Jaffe's prediction) to unbound.

✱ Lattice QCD - gives bound result (8 MeV) - Large pion mass used.

Beane et al.
PRL 106(2011) 162001

Inoue et al.
PRL 106(2011) 162002

Dibaryons - Status

H dibaryon

* NAGARA event - constraint on binding energy
* Dedicated experiments for H dibaryon indicates existence unlikely but its existence not ruled out yet.

Next talk by Jeremy Green

Dibaryons - Status

Dyson and Xuong
PRL 13 (1964) 815

1964 - Prediction of possible bound states

They predicted mass of D_{03} (close to d* which was found later)

H dibaryon

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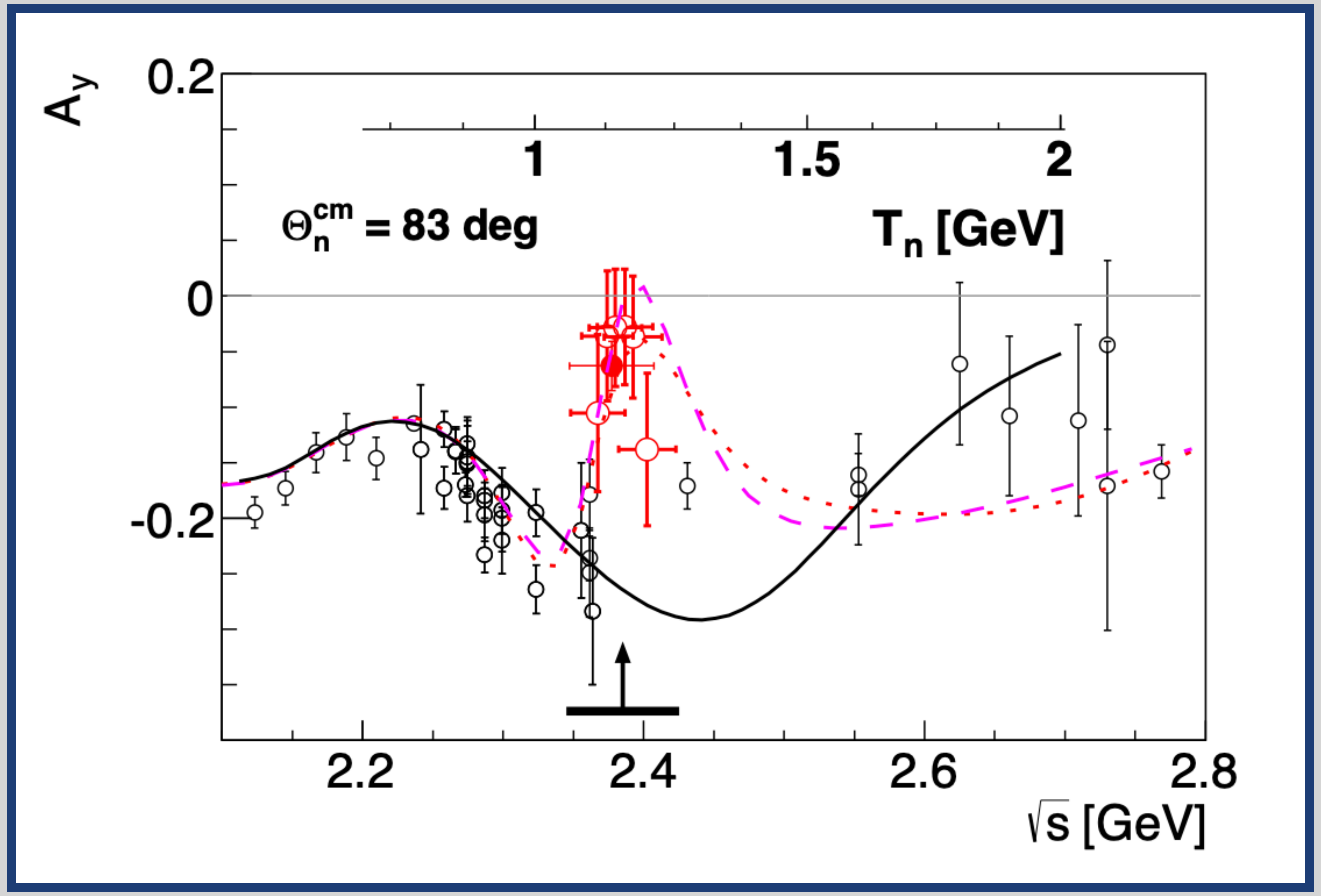

WASA @ COSY collaboration
SAID Data Analysis Center
PRL 112 (2014) 202301

Dibaryons - Lattice

Larger Pion Mass

Deuteron, Dineutron - Bound or Unbound ??

Tension between results

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Tension between results

Signal (m_N hadron)

$\propto e^{-m_N t}$

Noise

$\propto e^{-\frac{3}{2}m_\pi t}$

Error in propagator correlation function dominated by pions because of virtue of lower energy states

Signal to Noise ratio exponentially degrades for $m_q \to 0$

Lepage, TASI (1989)

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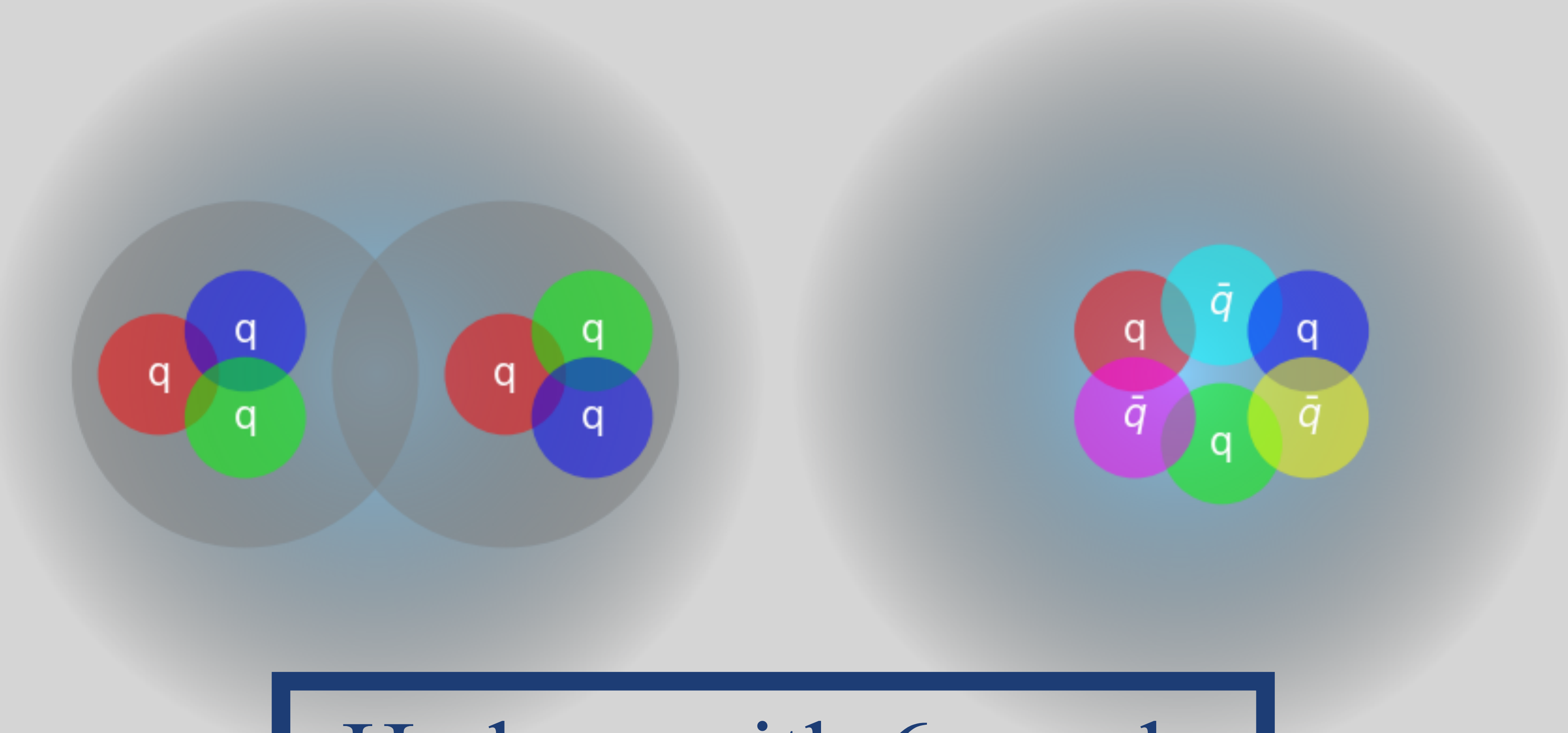
Lepage, TASI (1989)

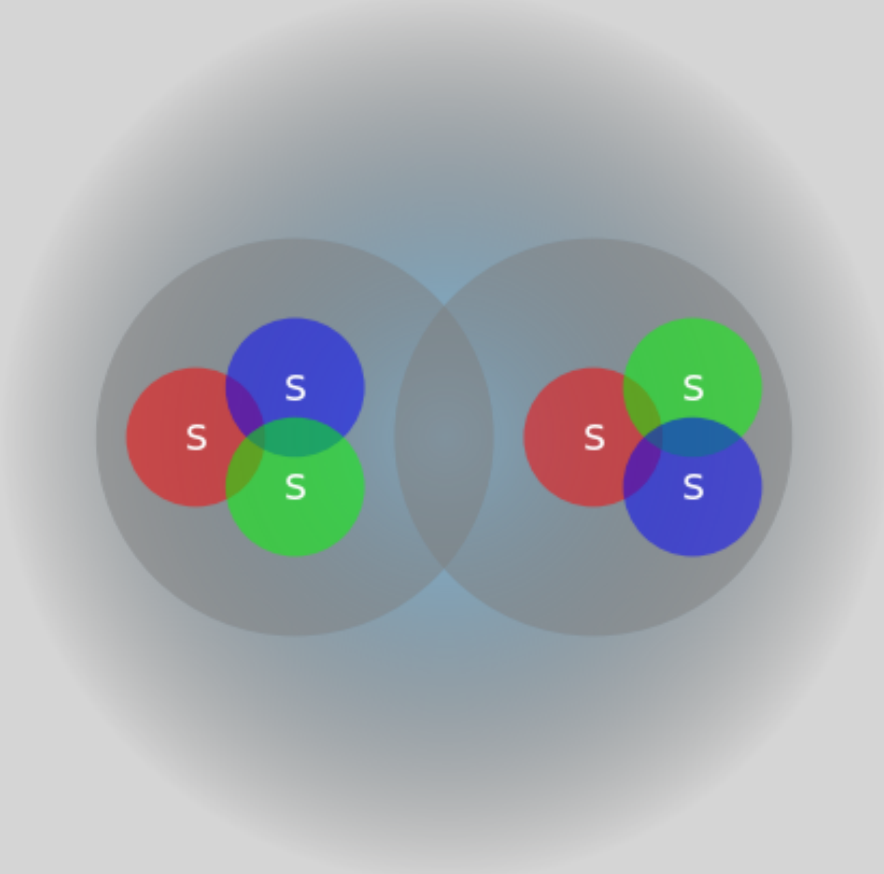
- Multiple ensembles for continuum limit.
- Even more computing power in contractions for exotic hadrons.
- $m_\pi L \geq 4$, to constraint finite volume effects.
- Finer lattices for lesser discretisation errors.

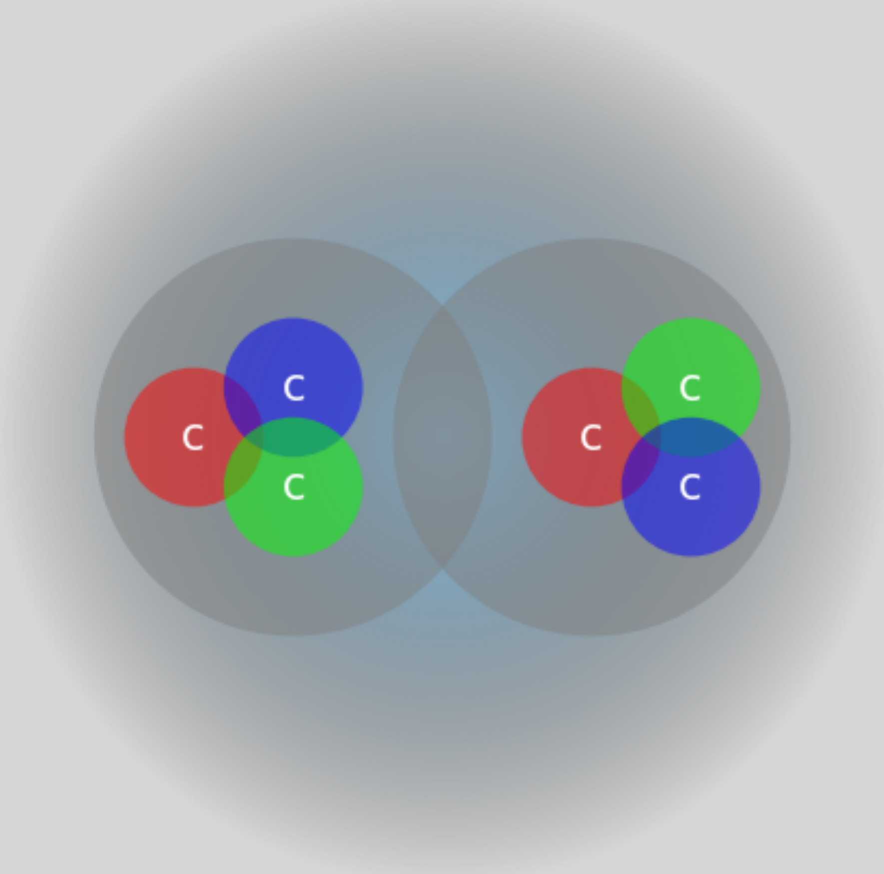
Hexaquark - Dibaryon

Extensive studies of deuteron like heavy dibaryons using Lattice QCD

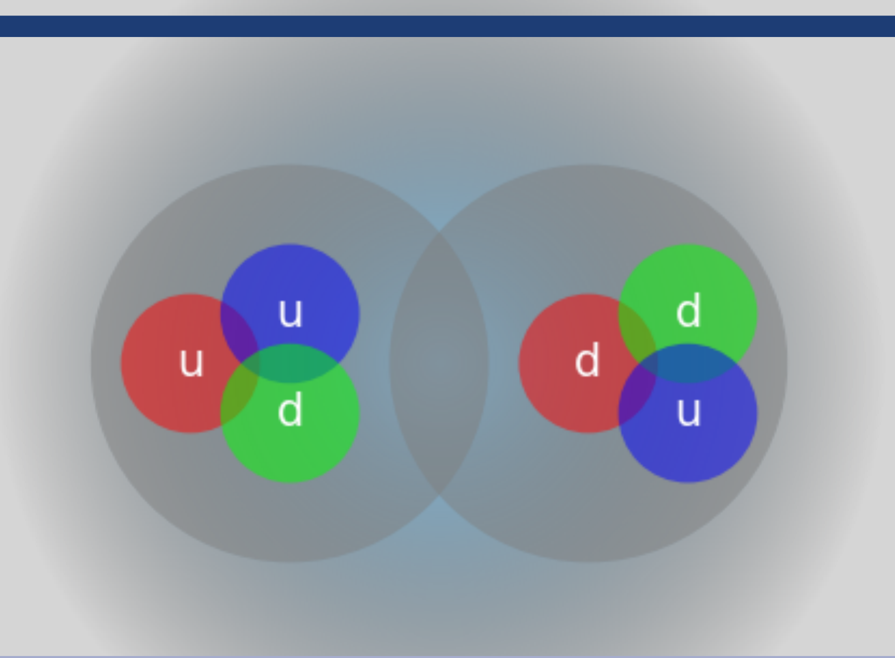


Hadron with 6 quarks





Deuteron



We work with single flavored dibaryons composed of strange and charm quarks named as $\mathscr{D}_{6s}$ and $\mathscr{D}_{6c}$ respectively

Lattice Setup

Bazavov et al., PRD 87 (2013) 5, 054505

- Overlap action on background of Highly Improved Staggered Quark (HISQ) gauge configurations.
- Strange and charm masses set at physical values.
- Up and down set as degenerate masses heavier than physical values.

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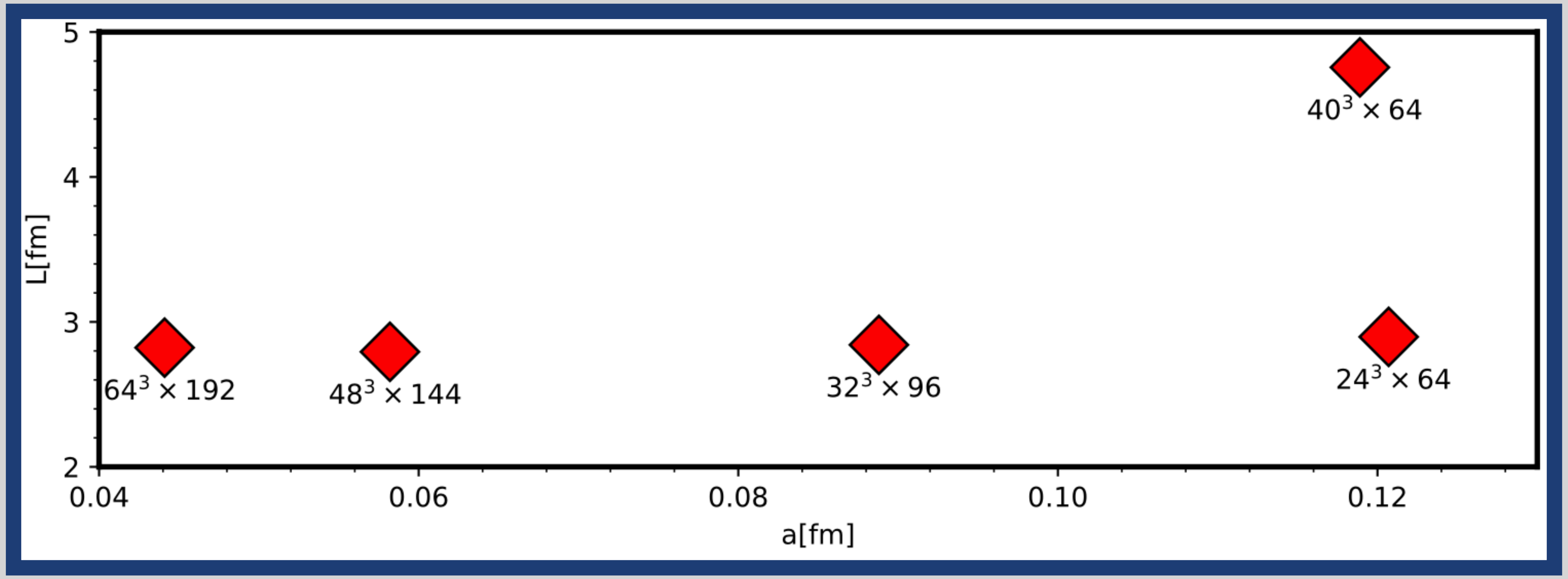
- Overlap action on background of Highly Improved Staggered Quark (HISQ) gauge configurations.
- Strange and charm masses set at physical values.
- Up and down set as degenerate masses heavier than physical values.
- Finest lattice used $a \approx 0.044\ fm$ with Volume as $64^3 \times 192$



Masses from Lattice

- Euclidean two point correlator as: $C_{ji}(t_f - t_i) = \langle 0 | O_j(t_f)\bar{O}_i(t_i) | 0 \rangle = \sum_n \frac{Z_i^{n*} Z_j^n}{2m_n} e^{-m_n(t_f - t_i)}$

- $O_j(t_f)$ and $\bar{O}_i(t_i)$ are the desired interpolating operators and $Z_j^n = \langle 0 | O_j | n \rangle$

- Effective mass $= \log \left[\frac{C(t)}{C(t+1)} \right]$

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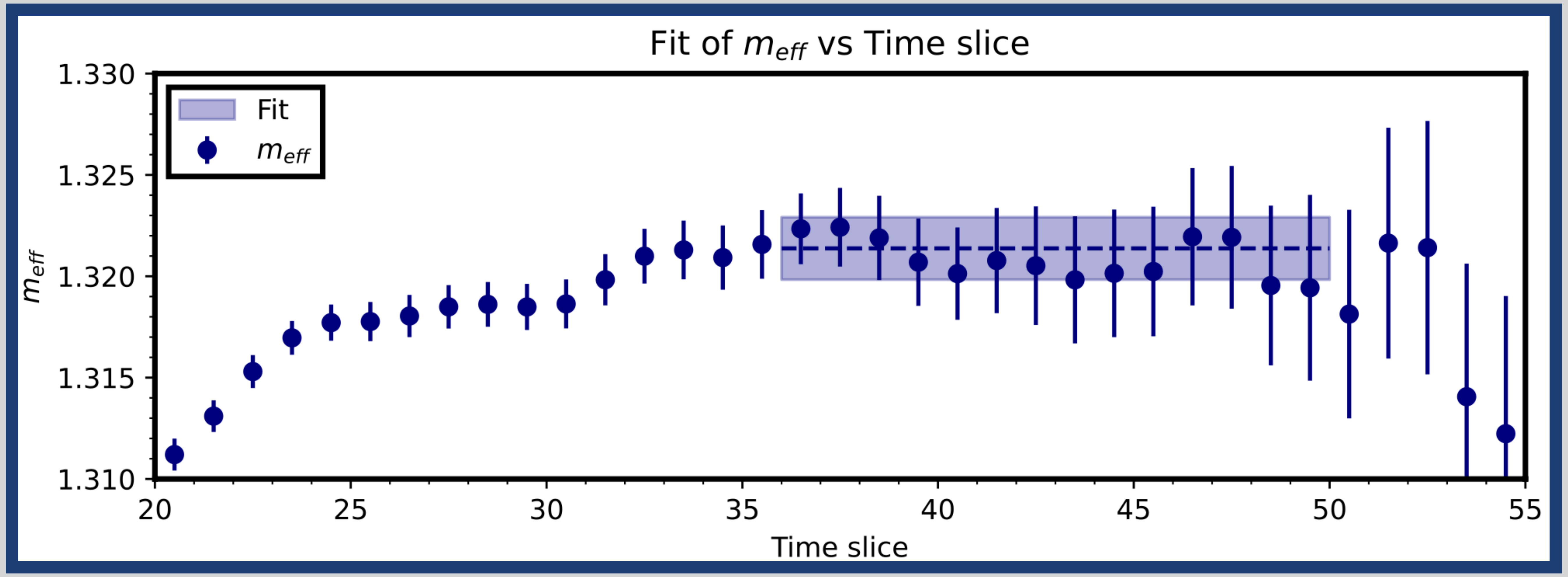

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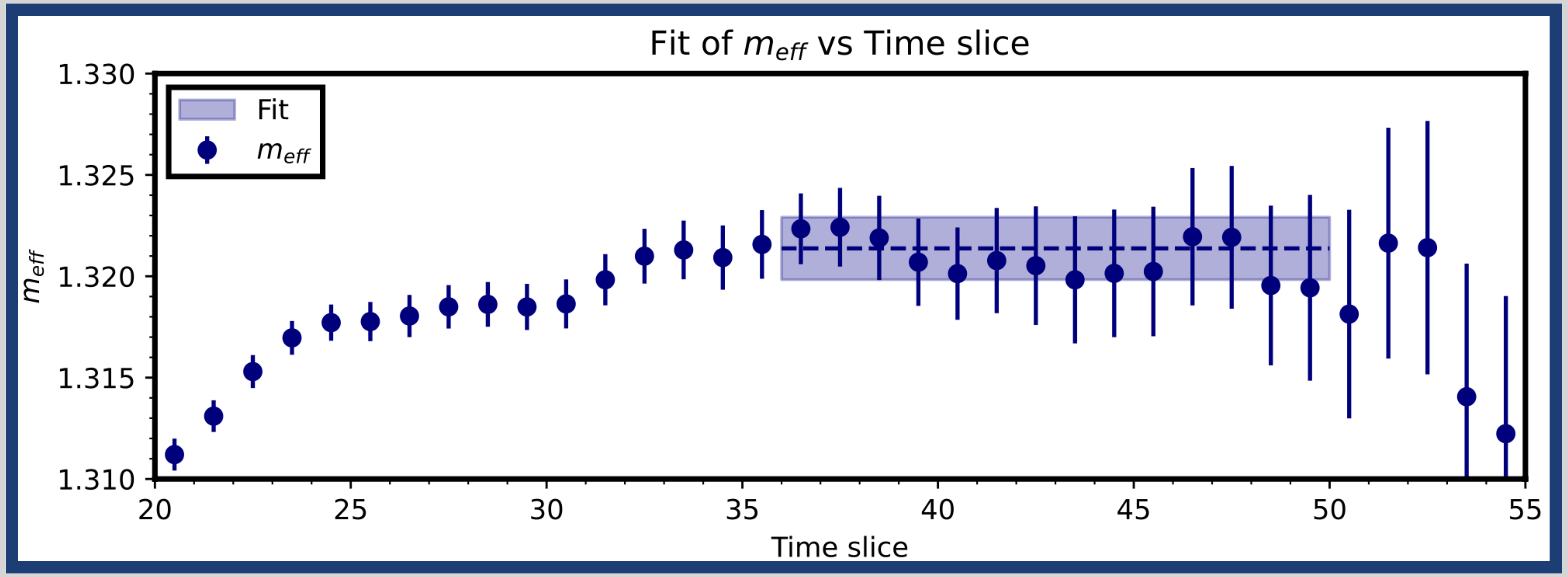

Wall source to point sink.

Cross checked results with boxsink.

Dibaryon Operators

$$\mathcal{O} = \epsilon_{abc} q^a_{\mu_1} q^b_{\mu_2} q^c_{\mu_3}$$

$$\mathcal{O}_d = \mathcal{O}_1 \, . \, CG \, . \, \mathcal{O}_2$$

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- Total wave function anti-symmetric under exchange of baryons.
- Single flavor baryons (symmetric).
- Assume only s wave interactions (symmetric) in dibaryon system.
- Color singlet baryons (symmetric)
- Hence Spin must be anti-symmetric which is in case of even spin (Spin 0 and 2)

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- Use reduction coefficients to project continuum based operators to suitable octahedral group.
- $S = 0$ continuum spin subduces to one dimensional A_1^+ irrep.
- $S = 2$ continuum spin subduces to two dimensional E^+ and three dimensional T_2^+ irrep.

Dibaryon Operators

$$\mathcal{O}^{[0]}_{d,A_1,1} = \frac{1}{2}\left({}^aH_{3/2}\ {}^bH_{-3/2} -{}^a H_{1/2}\ {}^bH_{-1/2} +{}^a H_{-1/2}\ {}^bH_{1/2} -{}^a H_{-3/2}\ {}^bH_{3/2}\right)$$

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For Spin 0, dibaryon operator corresponding to one dimensional A_1^+ irrep.

For Spin 2, 5 such operators corresponding to E^+ and T_2^+ irrep.

Dibaryon Operators

a,b - different embeddings

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S_z	Operator	State
3/2	${}^1H_{3/2}$	111
1/2	${}^1H_{1/2}$	112+121+211
-1/2	${}^1H_{-1/2}$	122+212+221
-3/2	${}^1H_{-3/2}$	222

Non Relativistic Embedding

S_z	Operator	State
3/2	${}^2H_{3/2}$	133+313+331
1/2	${}^2H_{1/2}$	233+323+332+134+341+413+143+431+314
-1/2	${}^2H_{-1/2}$	144+414+441+234+342+423+243+432+324
-3/2	${}^2H_{-3/2}$	244+424+442

Relativistic Embedding

Operator Contraction

Two baryons at source and two at sink

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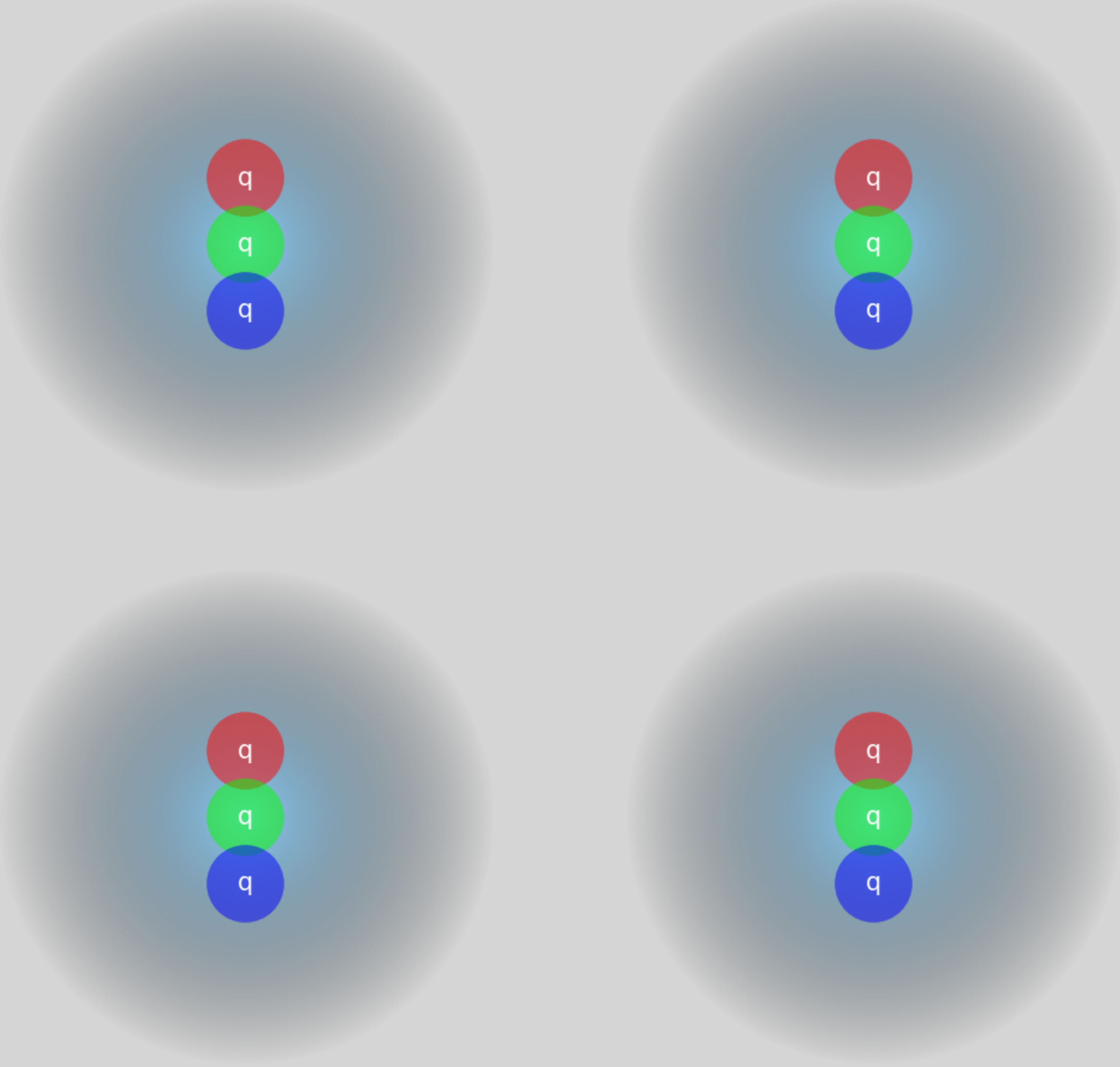

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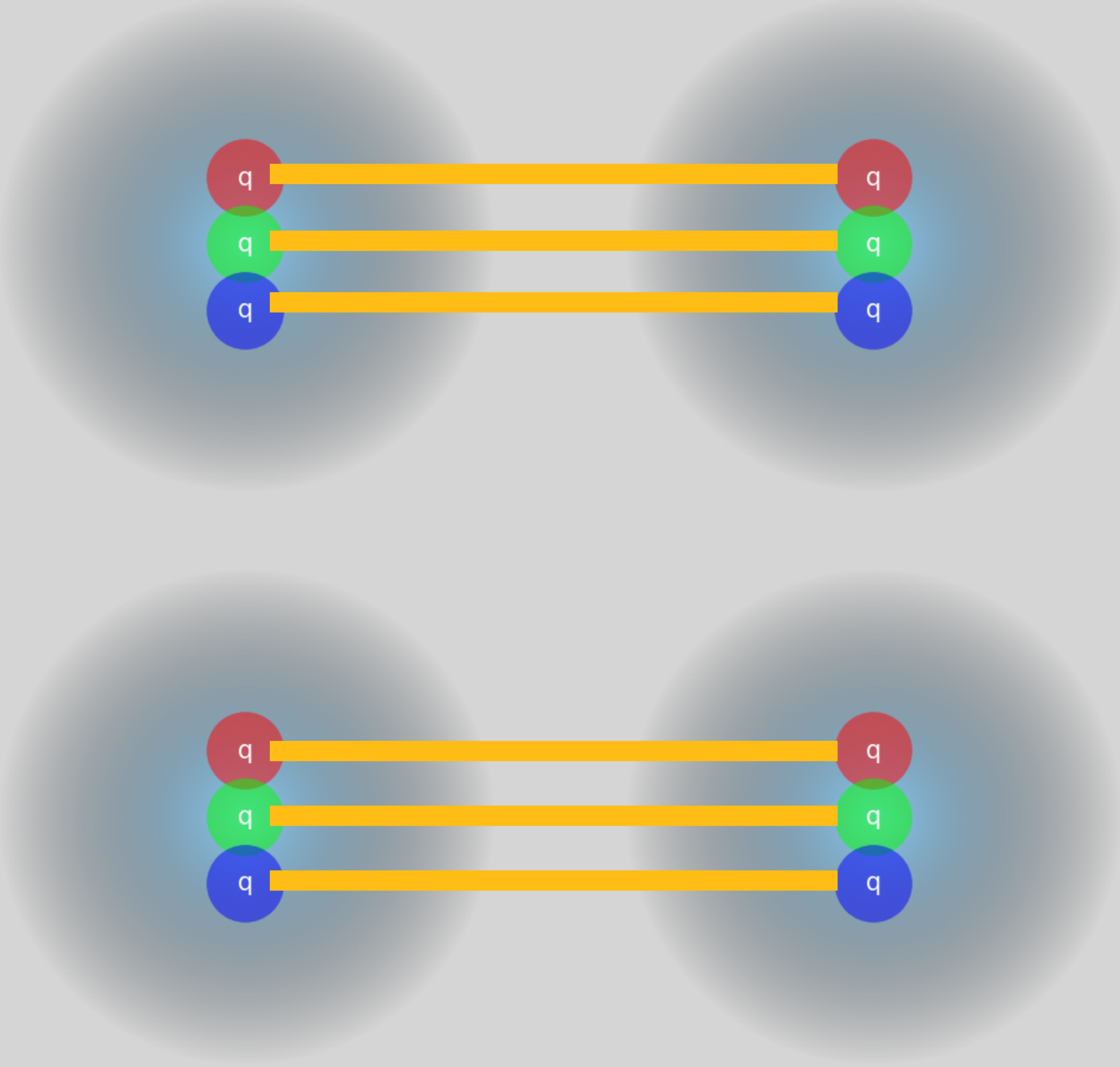

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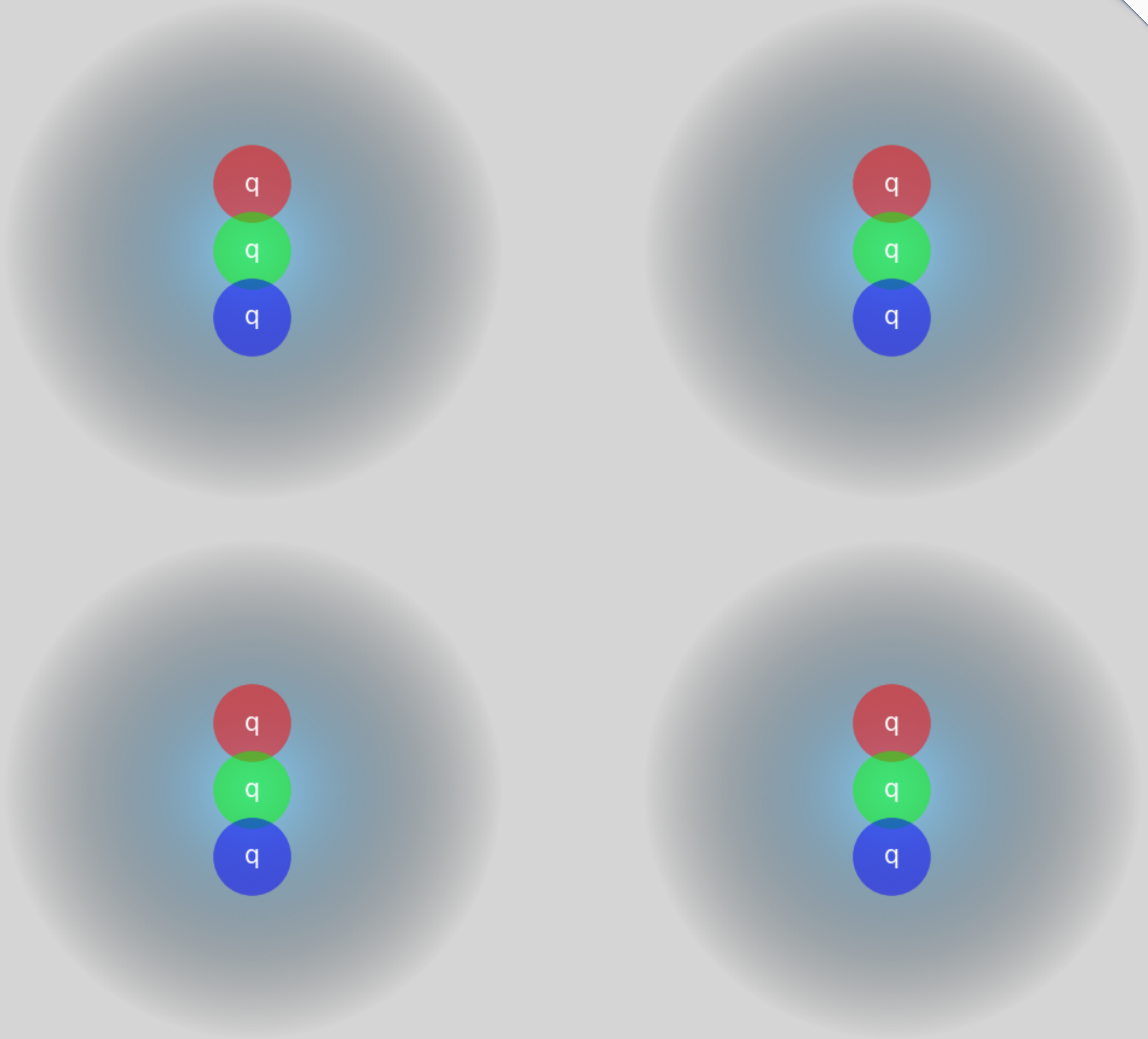

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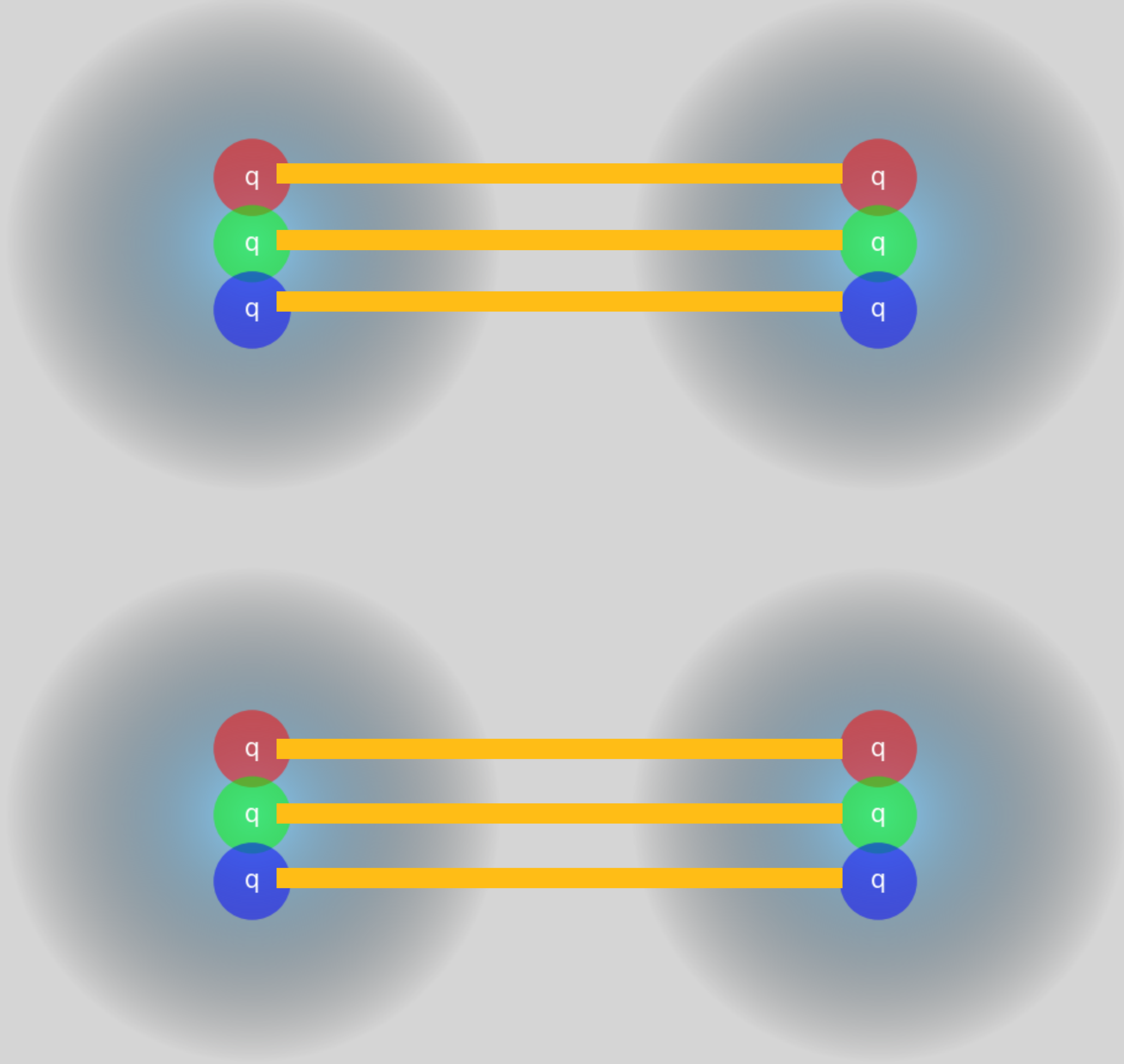

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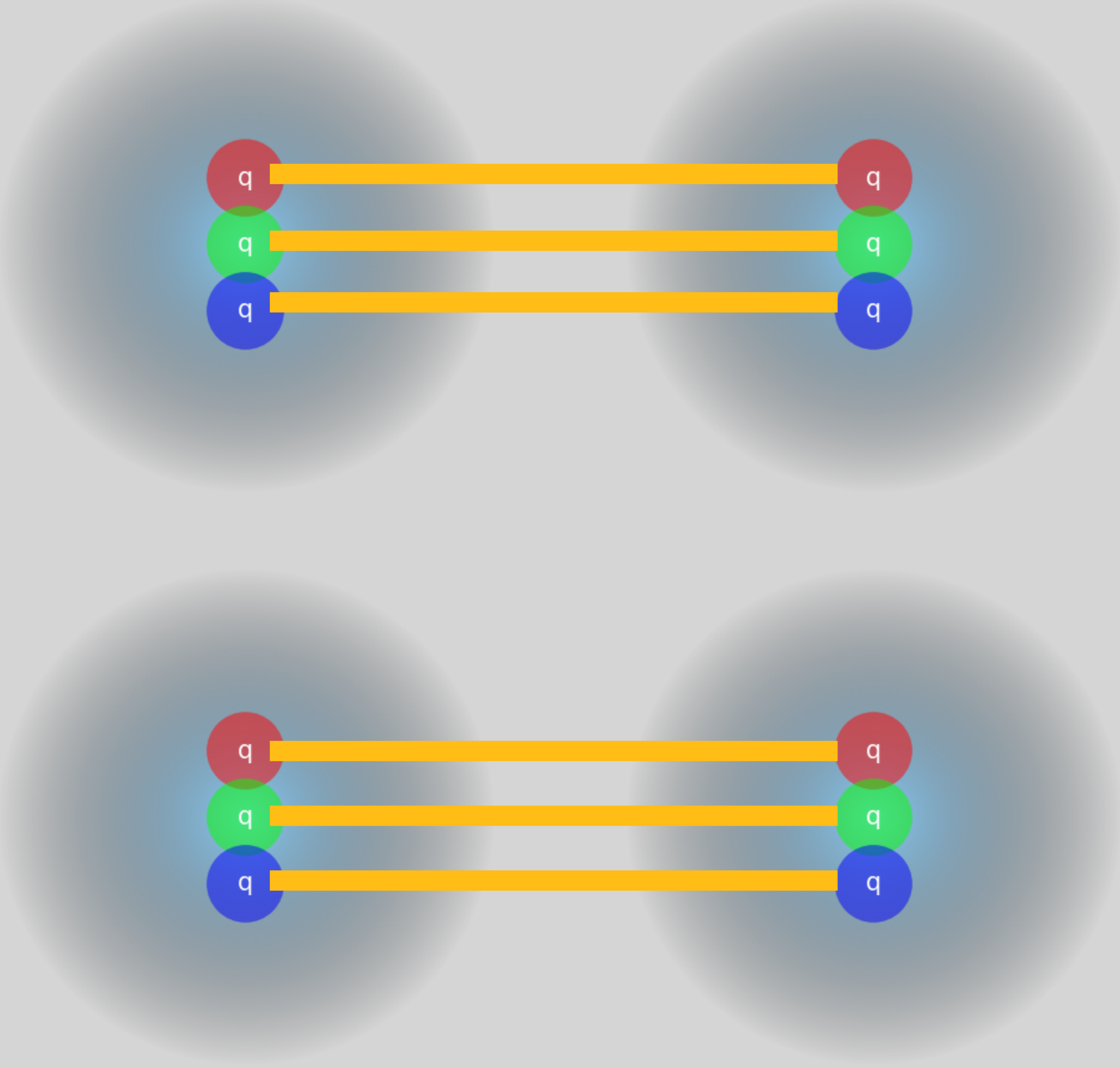

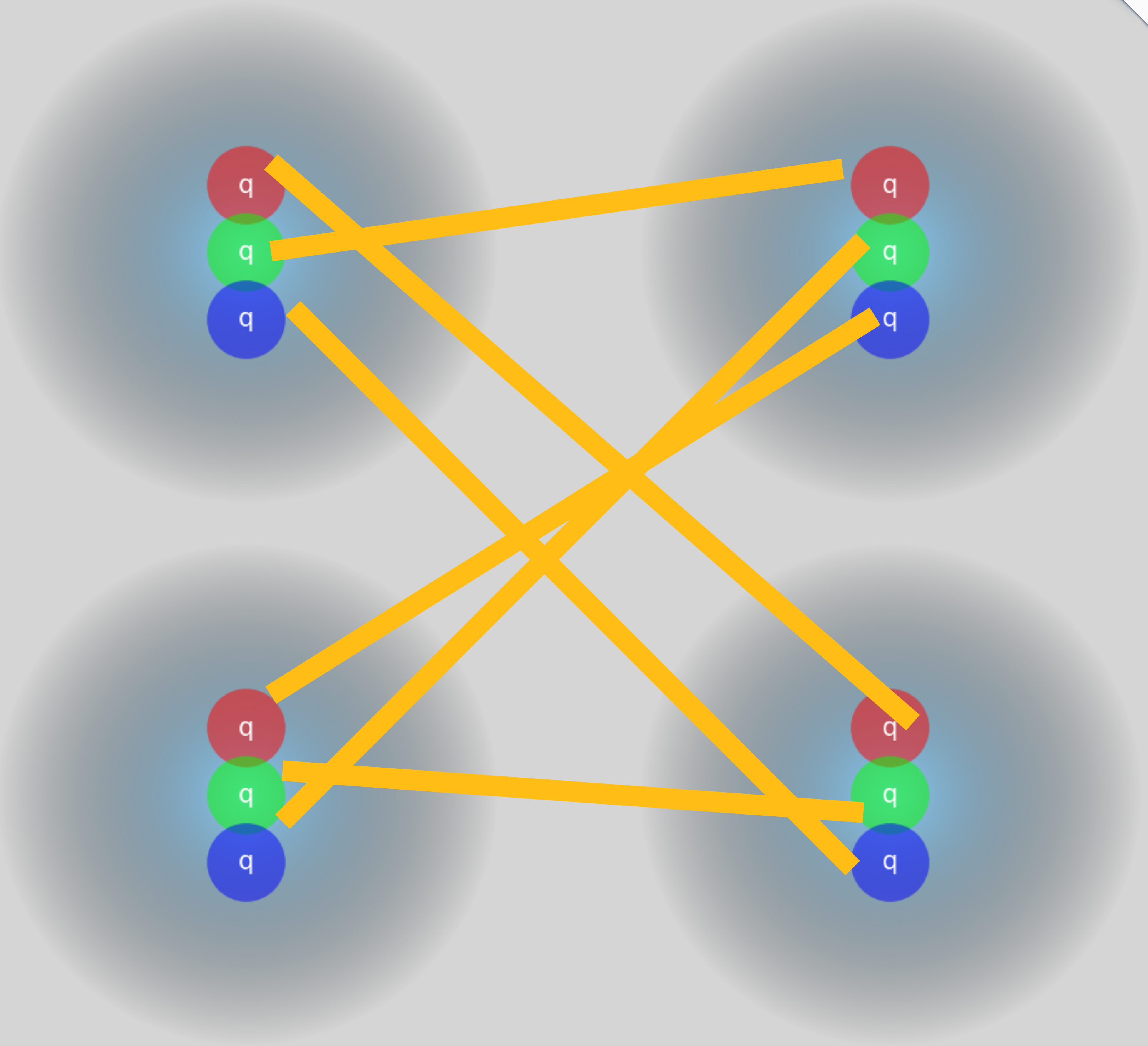

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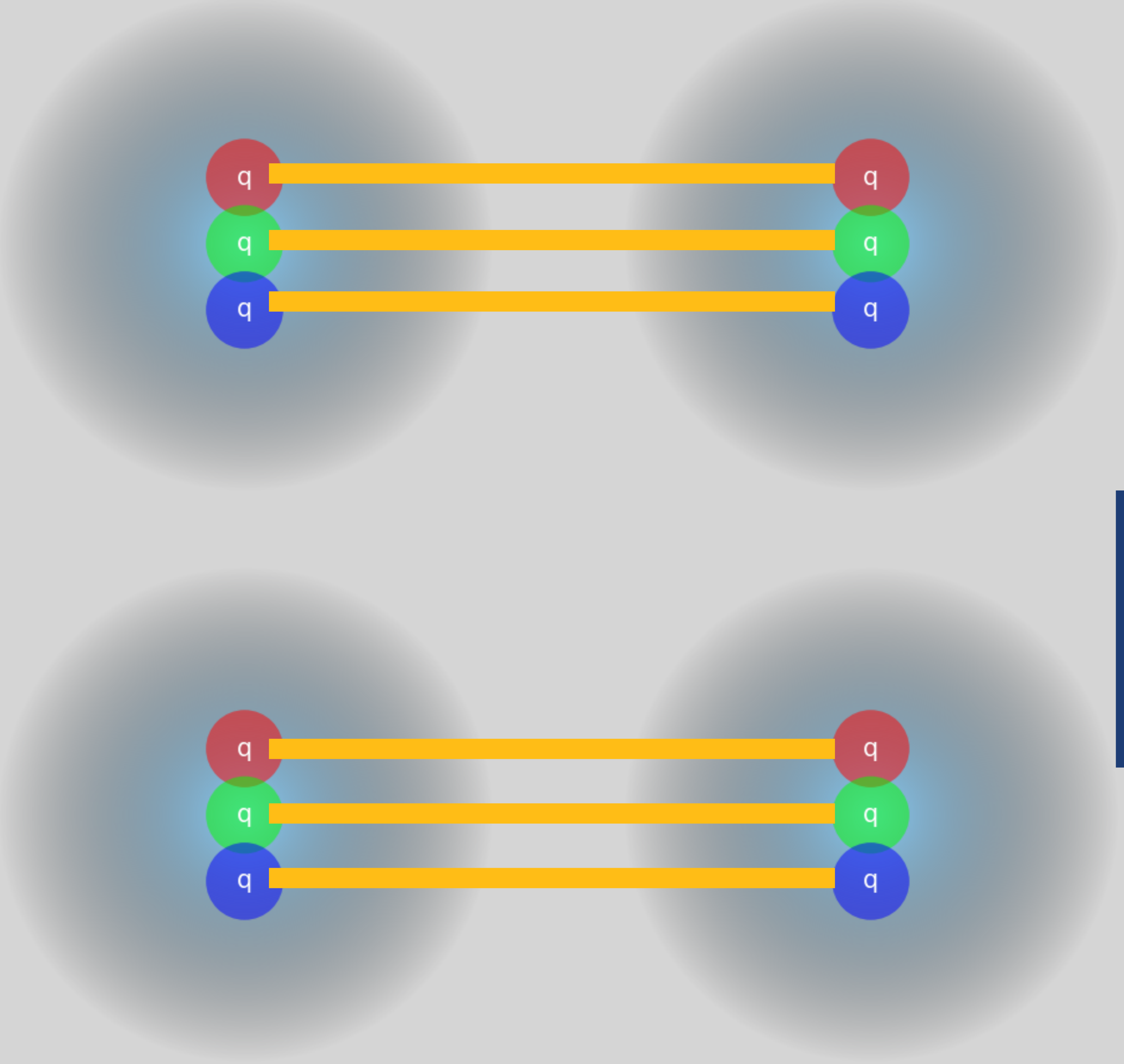
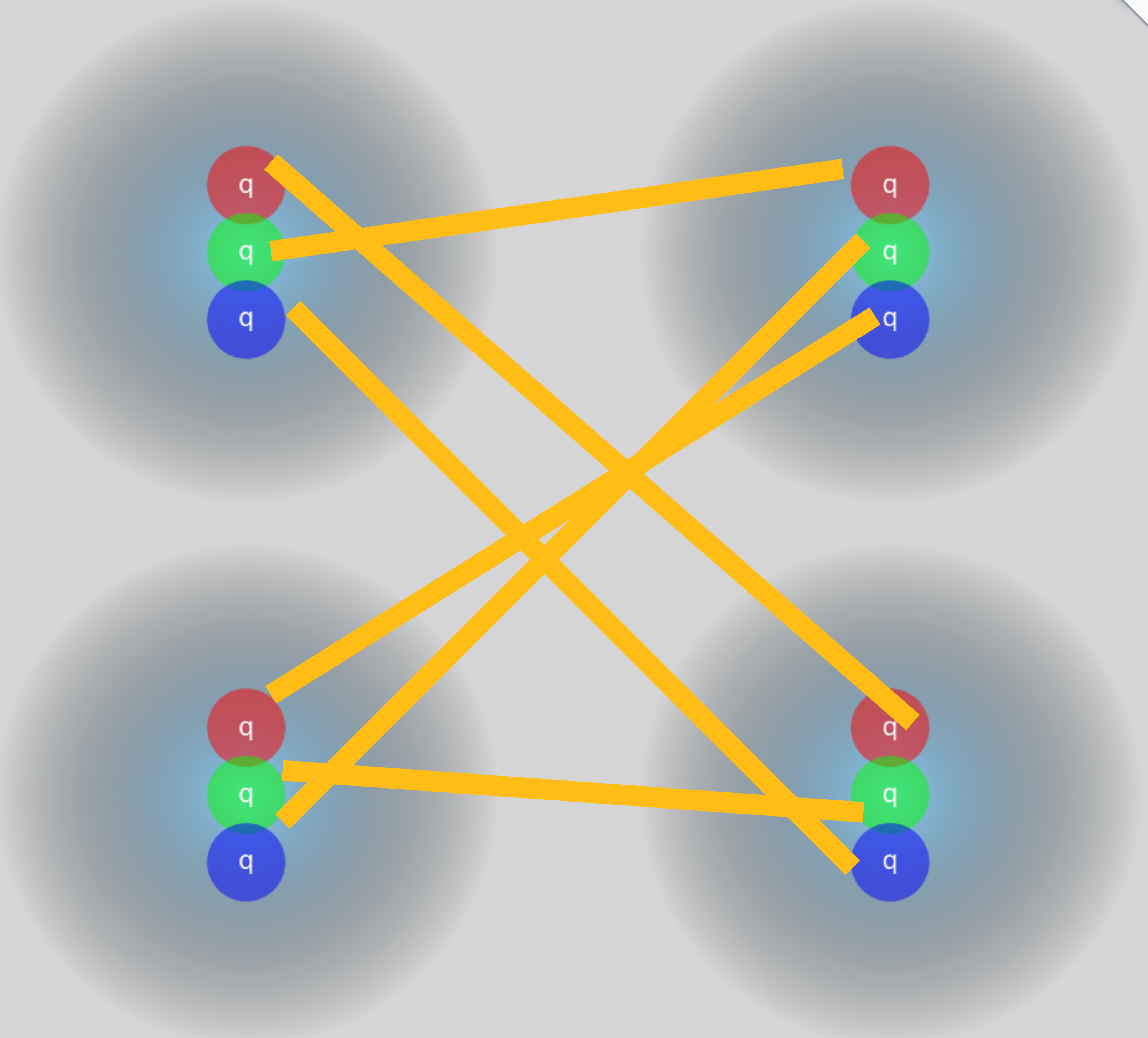


6! = 720 contraction possibilities



720 contractions can happen in 16 different ways depending on different embeddings

Operator Contraction

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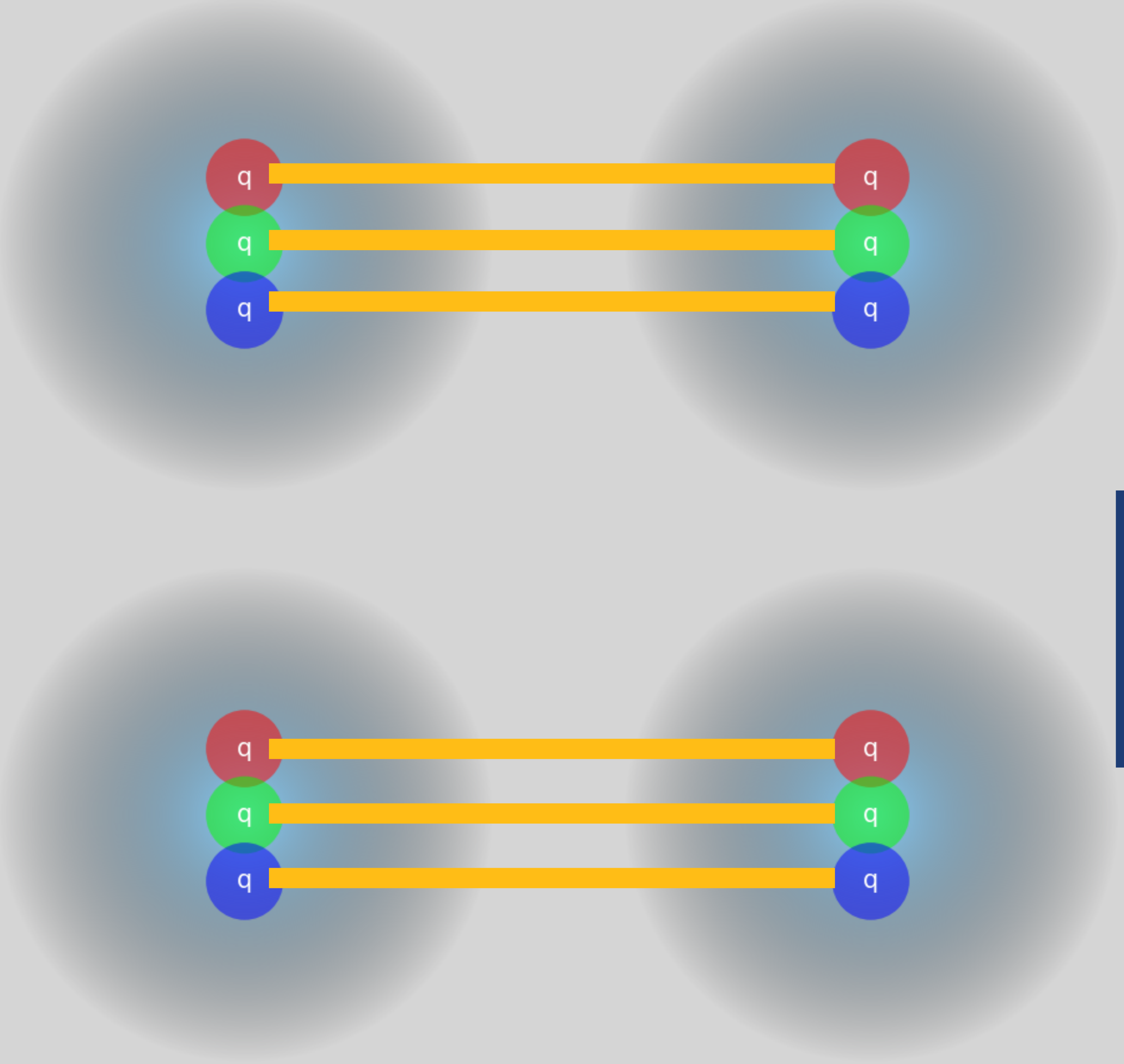

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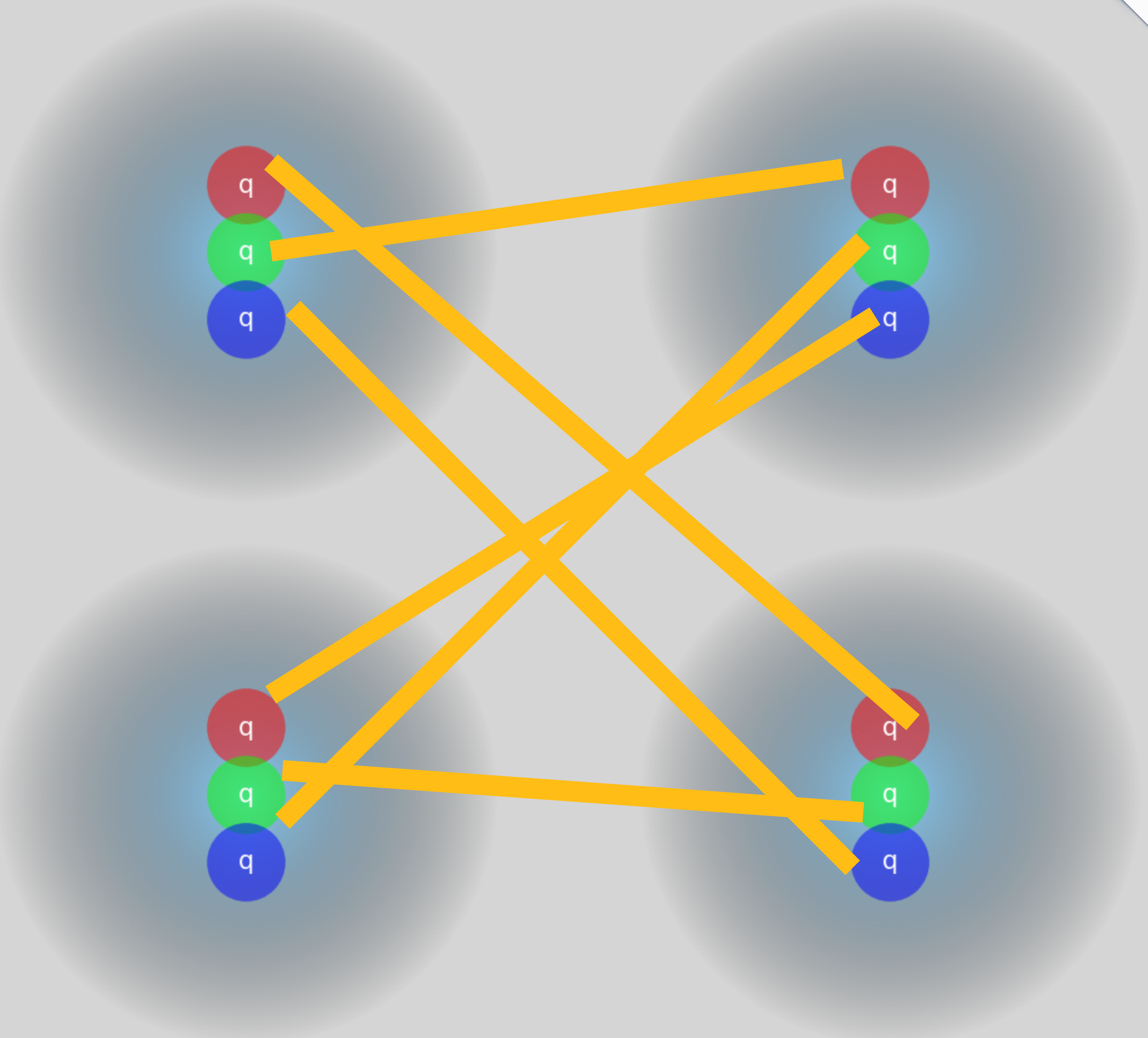
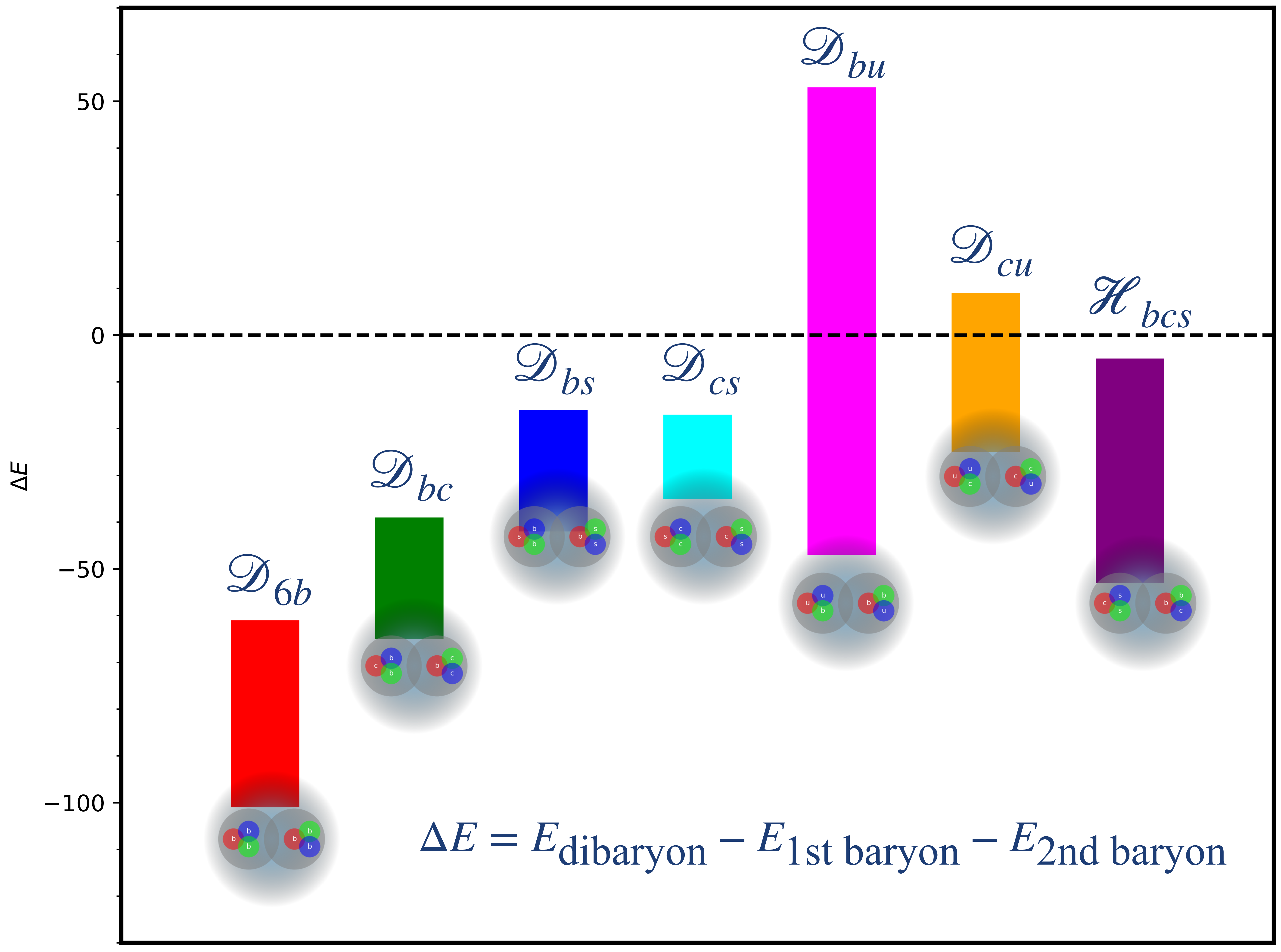


720 contractions can happen in 16 different ways depending on different embeddings

N-N-N-N	N-N-N-R	N-N-R-N	N-N-R-R
N-R-N-N	N-R-N-R	N-R-R-N	N-R-R-R
R-N-N-N	R-N-N-R	R-N-R-N	R-N-R-R
R-R-N-N	R-R-N-R	R-R-R-N	R-R-R-R

Some of these are degenerate and some of these do not contribute at all for different Spin cases

Dibaryons results from Lattice



$\mathcal{D}_{6b}$, $S = 0$

Mathur, Padmanath, Chakraborty
PRL, 130, 111901 (2023)

$\mathcal{D}_{bc}$, $\mathcal{D}_{bs}$, $\mathcal{D}_{cs}$, $\mathcal{D}_{bu}$, $\mathcal{D}_{cu}$

$S = 1$

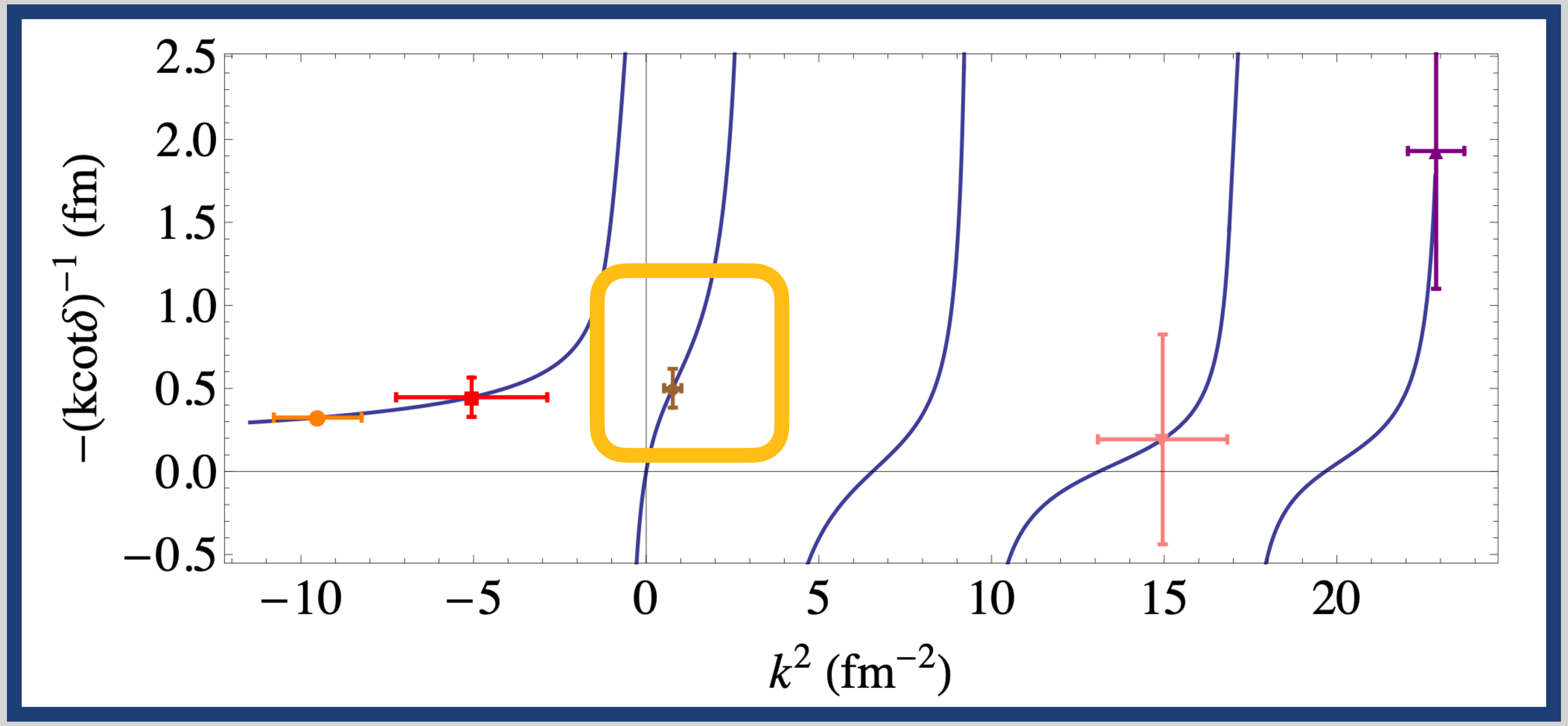
Junnarkar, Mathur PRL, 123, 162003 (2019)

$\mathcal{H}_{bcs}$, $S = 0$

Junnarkar, Mathur PRD, 106, 054511 (2022)

Where does $\mathcal{D}_{6c}$, $\mathcal{D}_{6s}$ stand ??

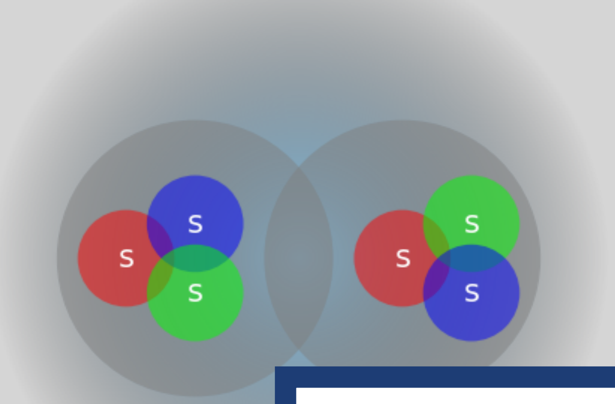
$\mathcal{D}_{6s}$ Existing Results

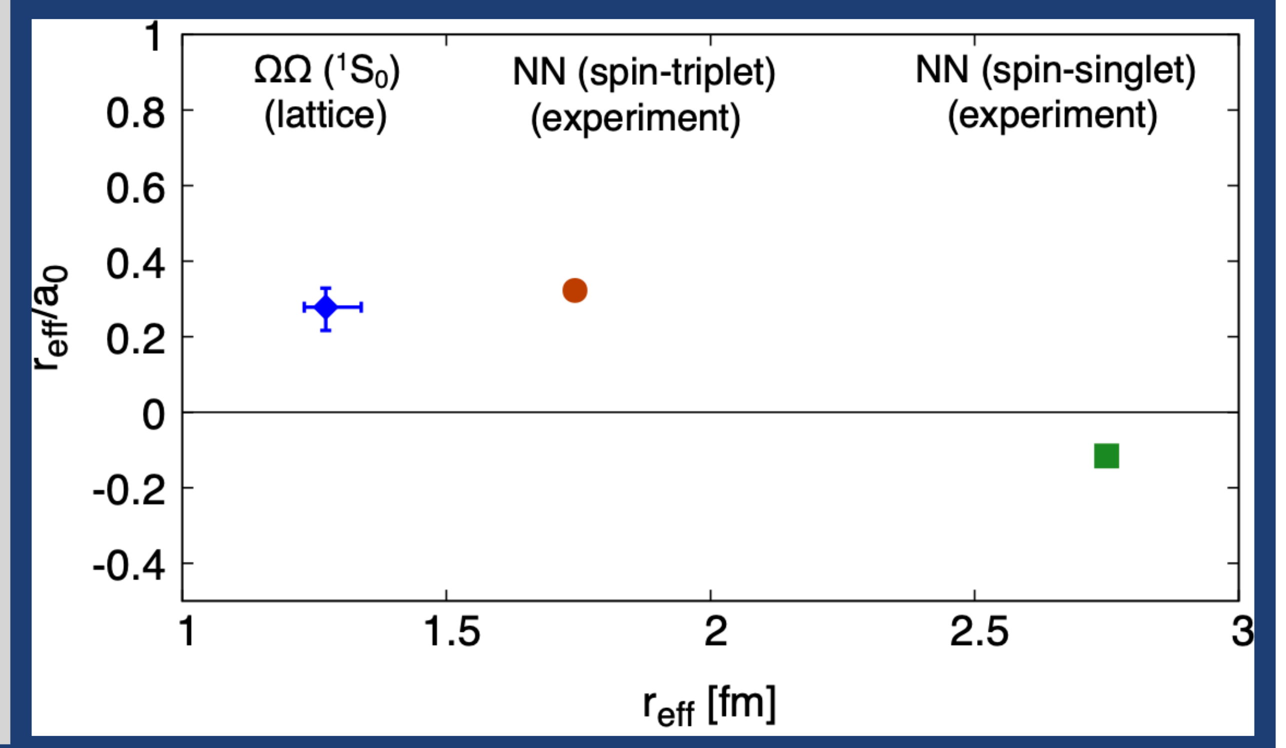


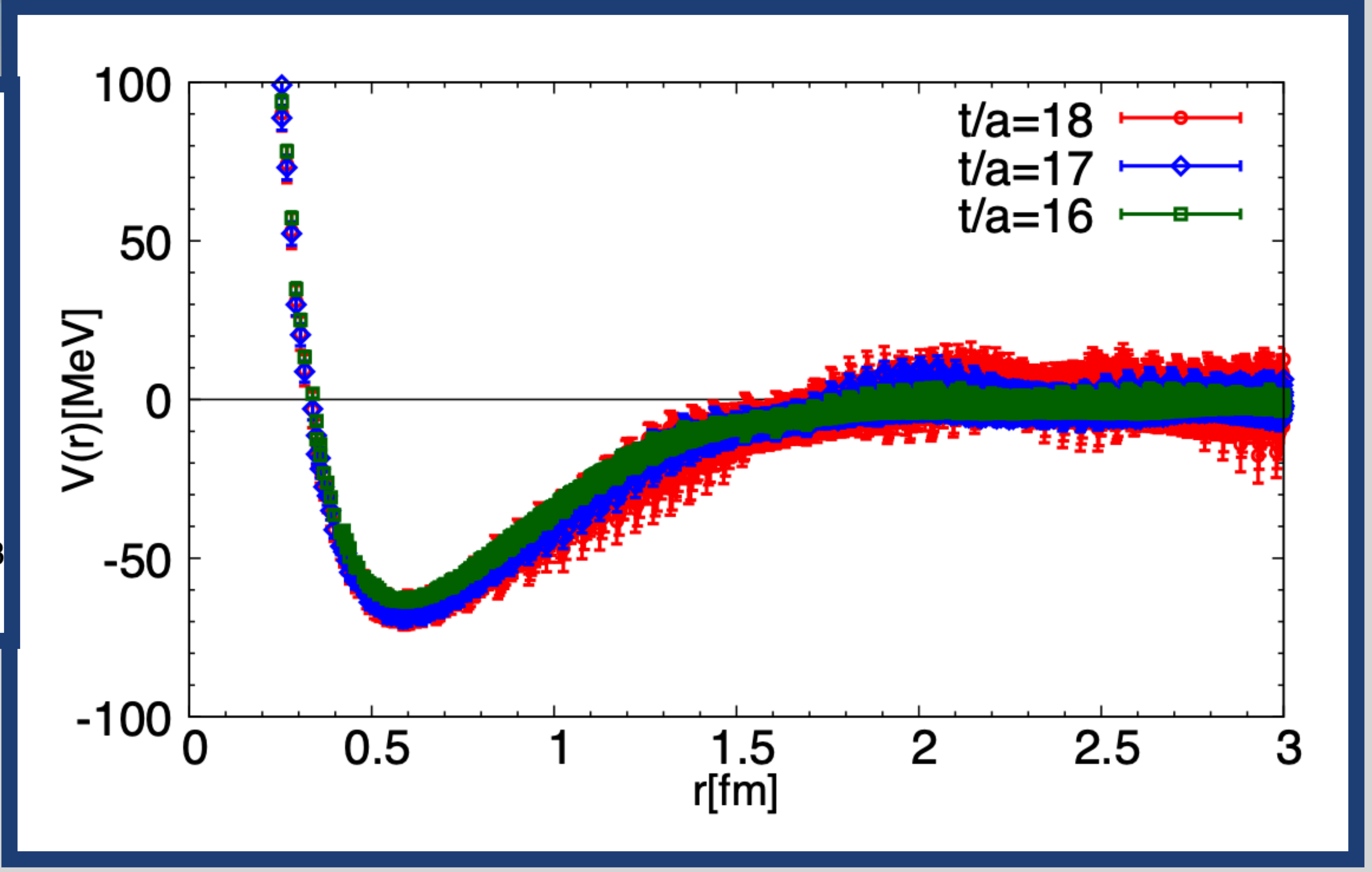
- Weakly repulsive in Spin 0 H^+H^+ irrep, No bound state
- Attractive in Spin 1,2 $G_1^+H^+$ but only single volume used.

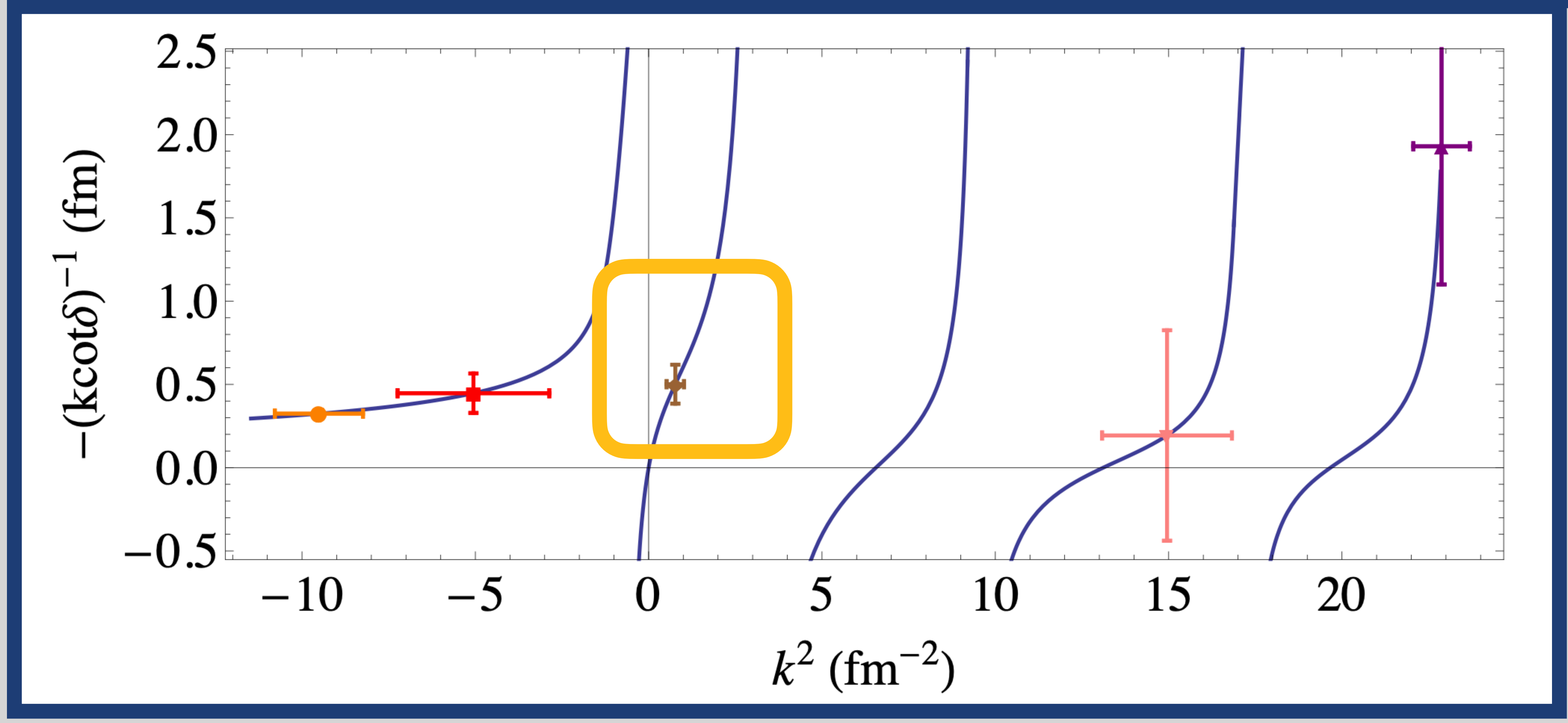
Buchoff, Luu, Wasem
PRD 85, 094511 (2012)

$\mathscr{D}_{6s}$ Existing Results









- Weakly attractive in Spin 0, hence bound state

HALQCD
PRL 120, 212001 (2018)

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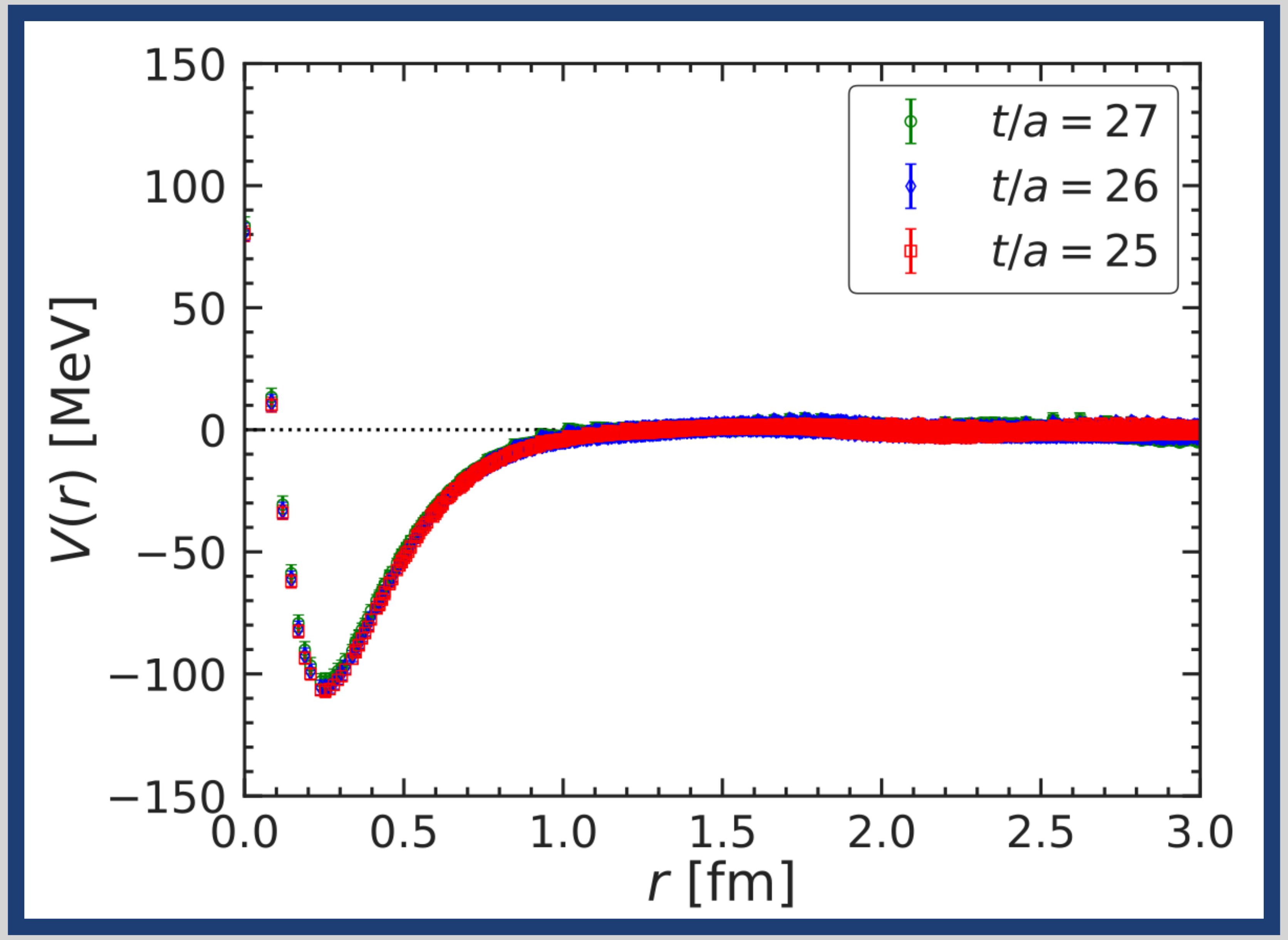
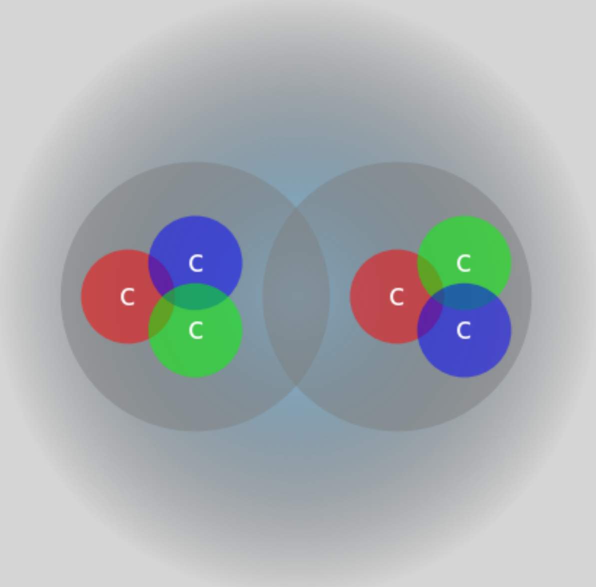
Buchoff, Luu, Wasem
PRD 85, 094511 (2012)

$\mathscr{D}_{6c}$ Existing Results

HALQCD
PRL 127, 072003 (2021)



$\mathcal{D}_{6c}$ Existing Results



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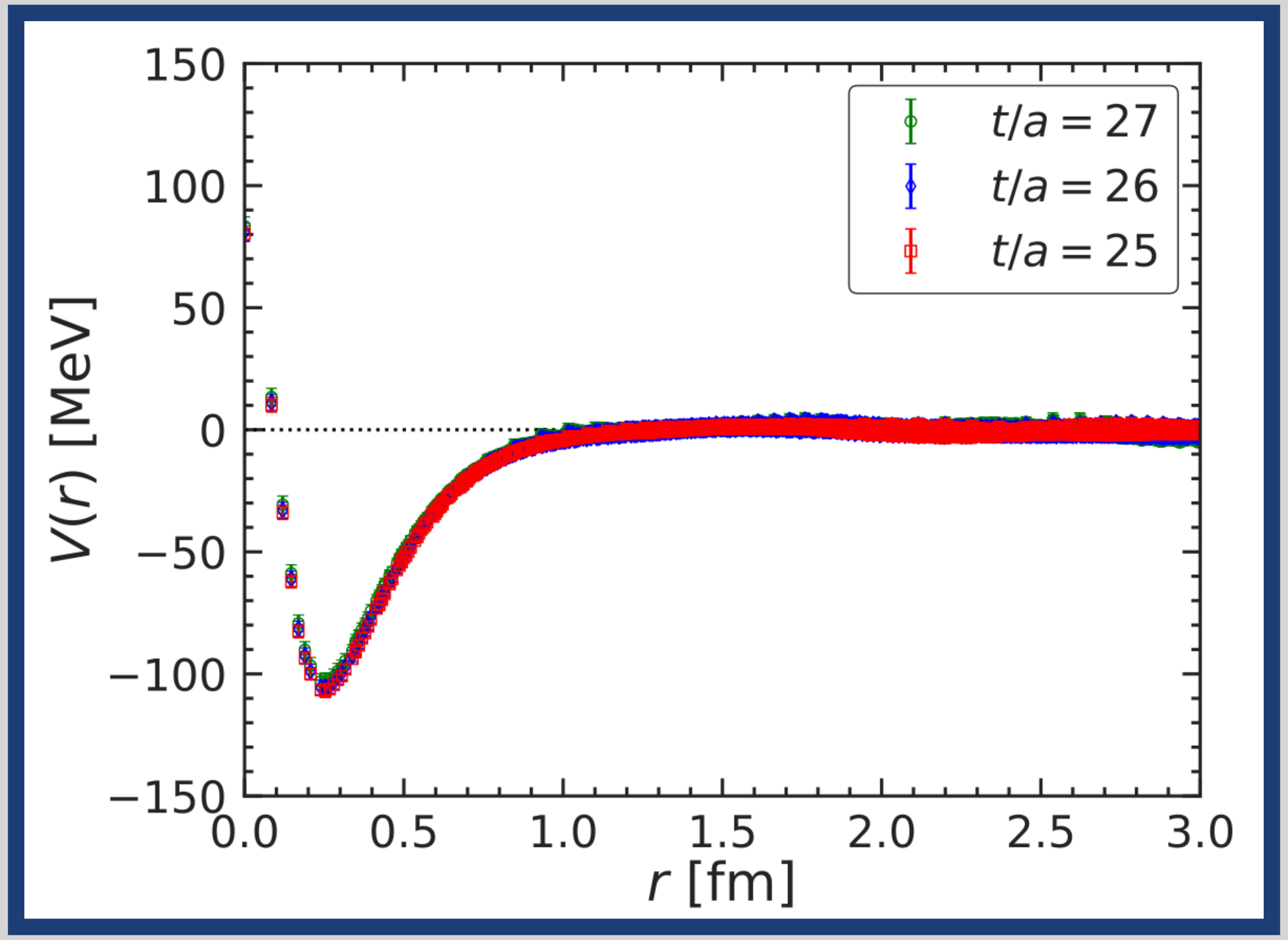
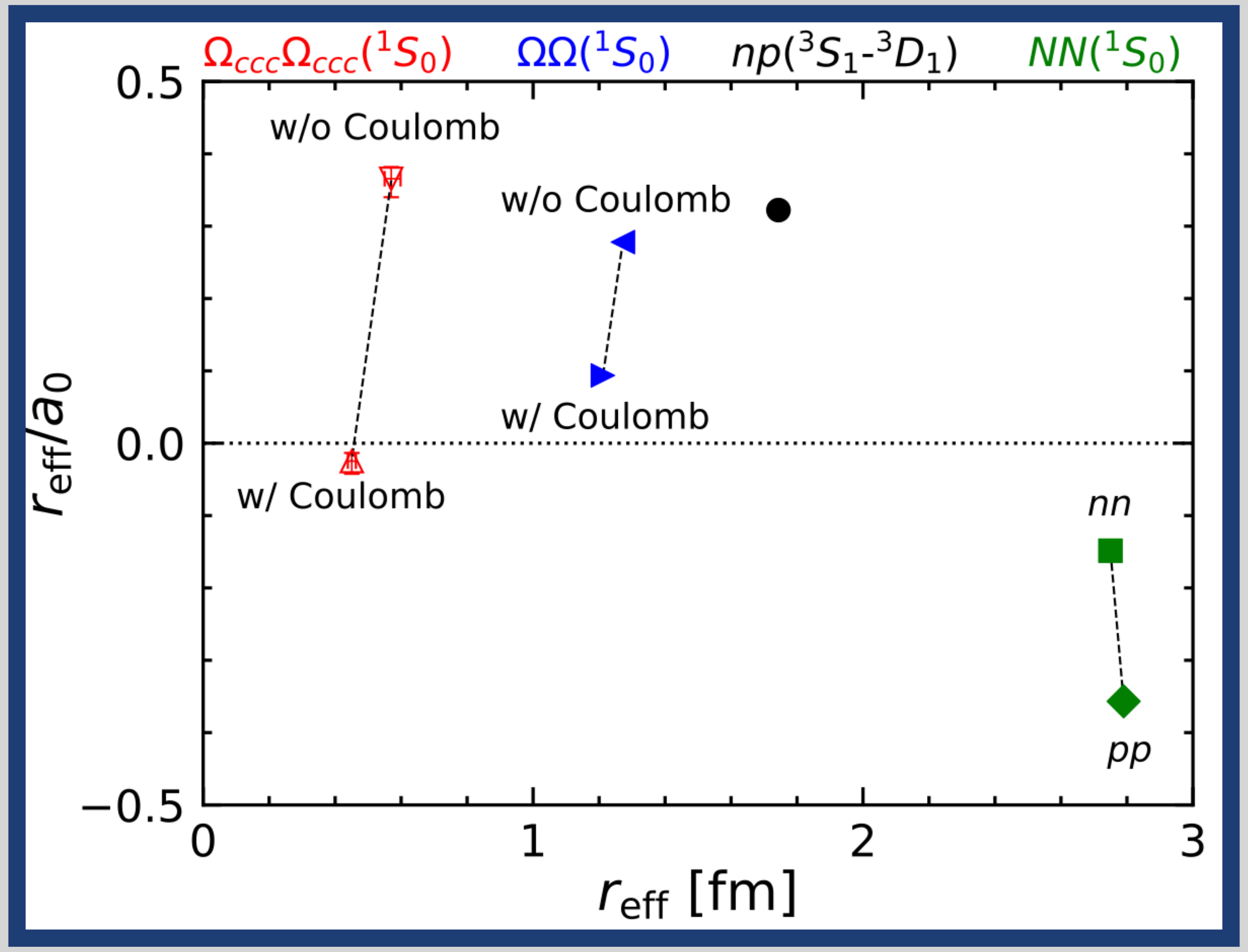
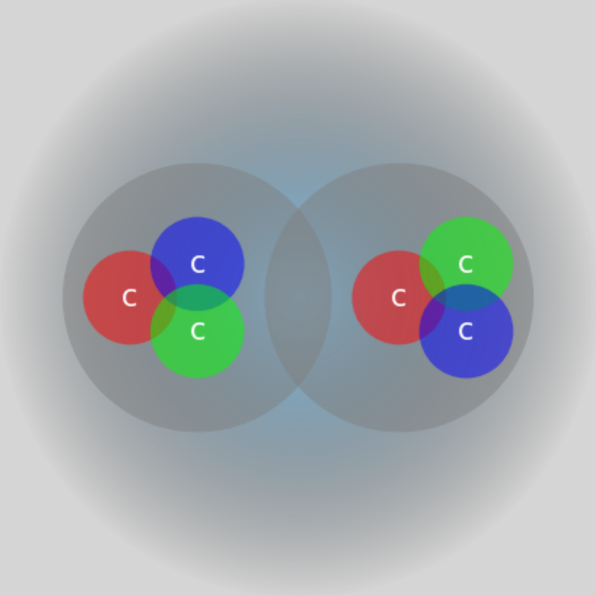
PRL 127, 072003 (2021)

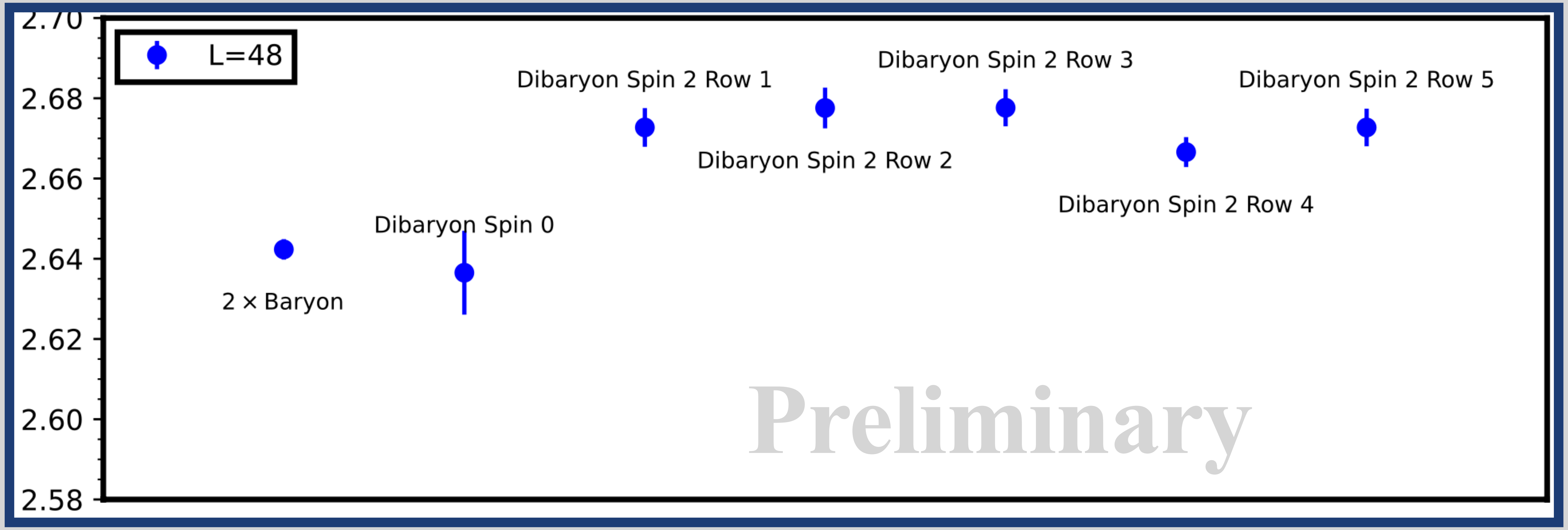




- Simulation with physical charm mass and near physical light quark mass.
- Dibaryon existence without Coulomb interaction.
- Near unitary region with Coulomb interaction (scattering length less than corresponding strange dibaryon calculation).

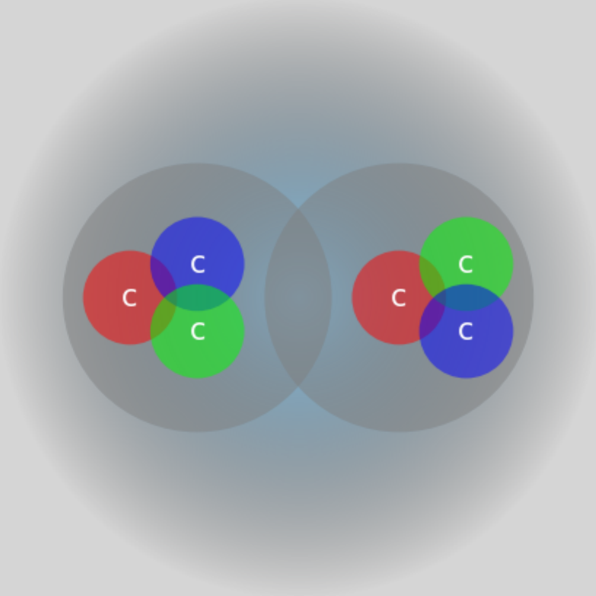
$\mathscr{D}_{6c}$ Results

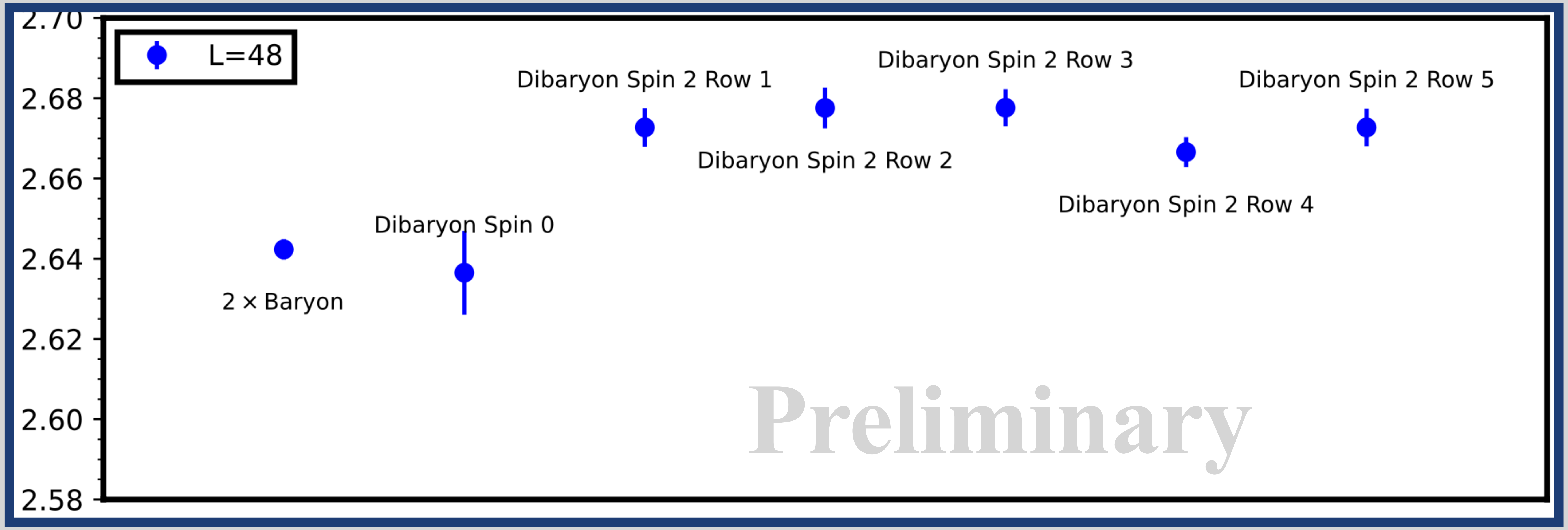




- Two lattice volumes, 4 lattice spacings, this plot with L = 48
- Spin 2 - repulsive interactions, Spin 0 dibaryon energy same as twice of baryon (within error)

$\mathcal{D}_{6c}$ Results

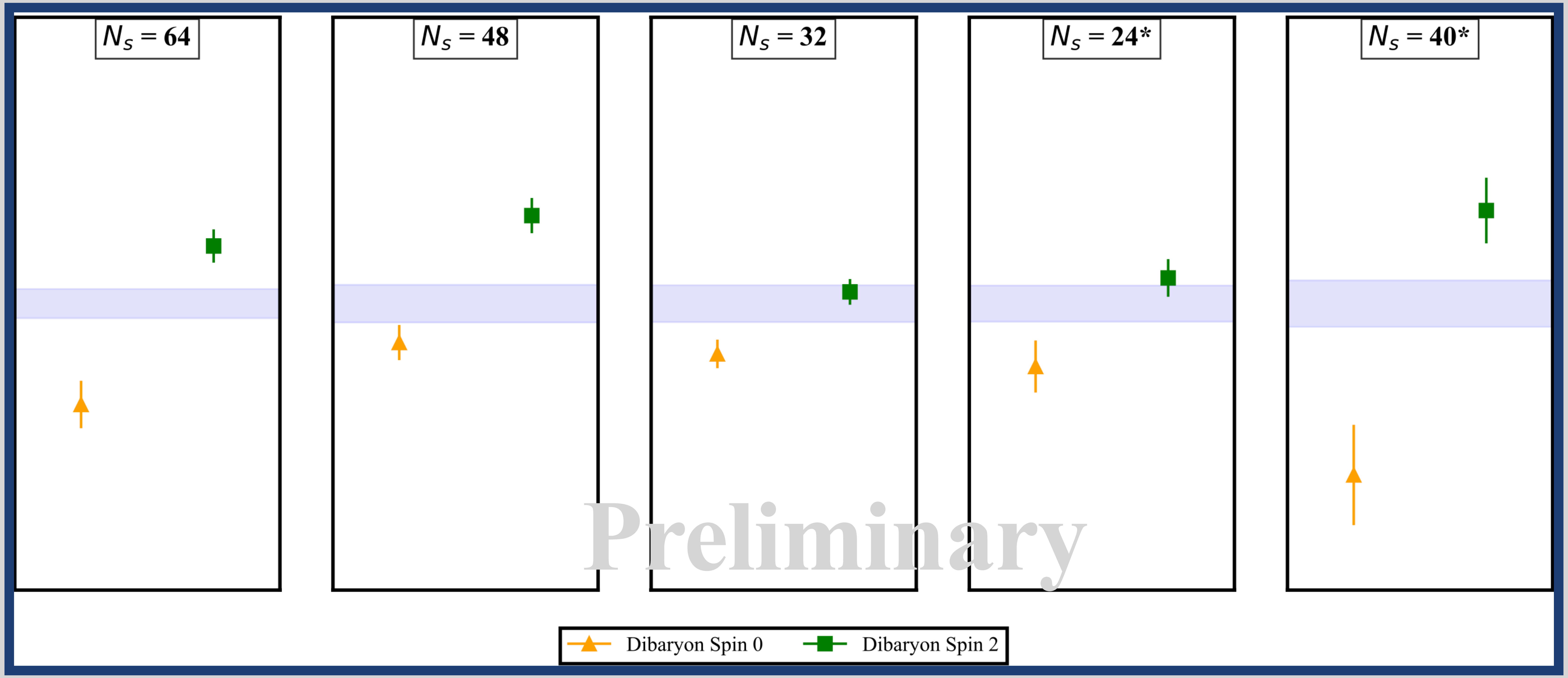




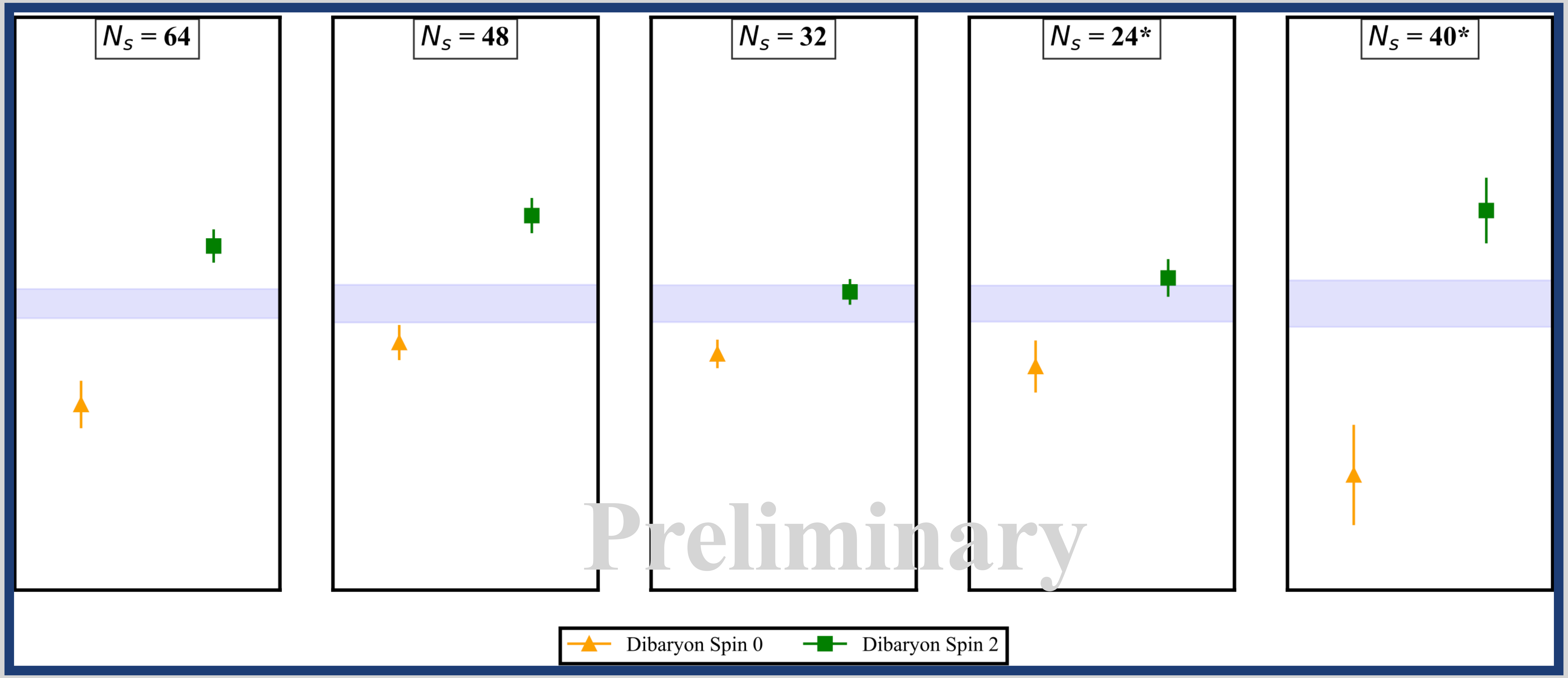
- Bound state, if exist, is shallow
- More probe using amplitude analysis

- Two lattice volumes, 4 lattice spacings, this plot with L = 48
- Spin 2 - repulsive interactions, Spin 0 dibaryon energy same as twice of baryon (within error)

$\mathcal{D}_{6c}$ Results

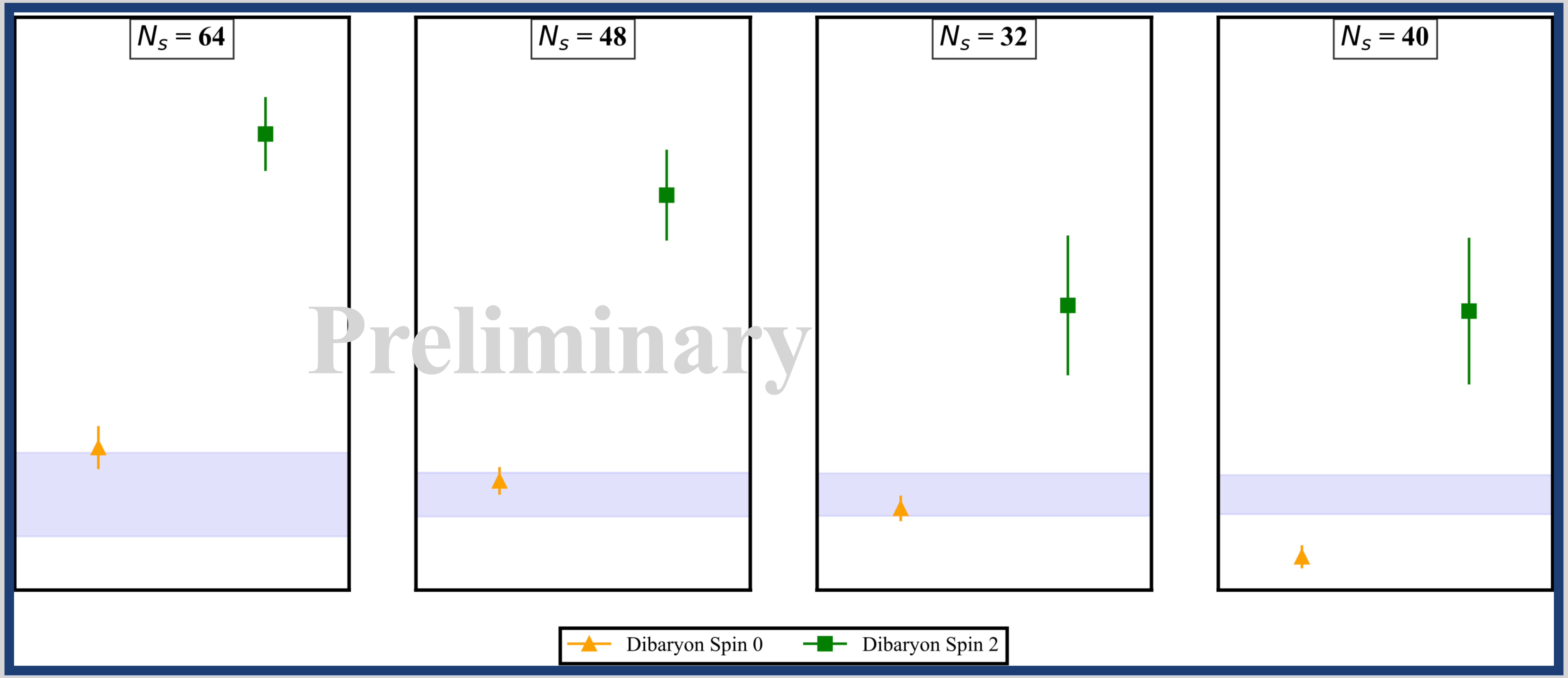


$\mathcal{D}_{6c}$ Results

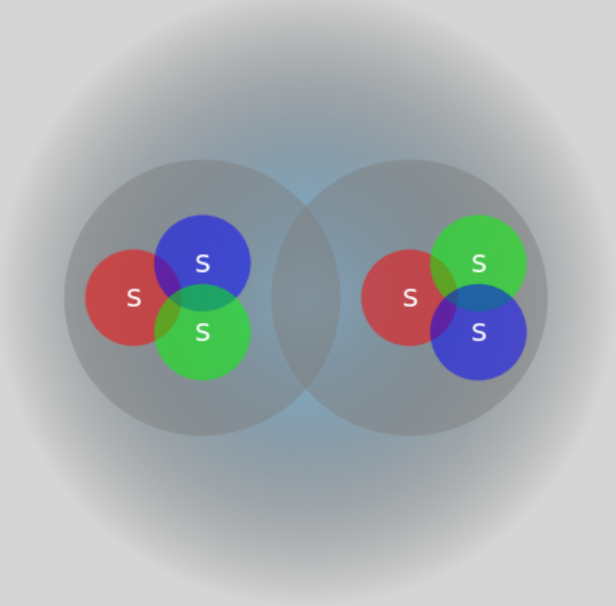


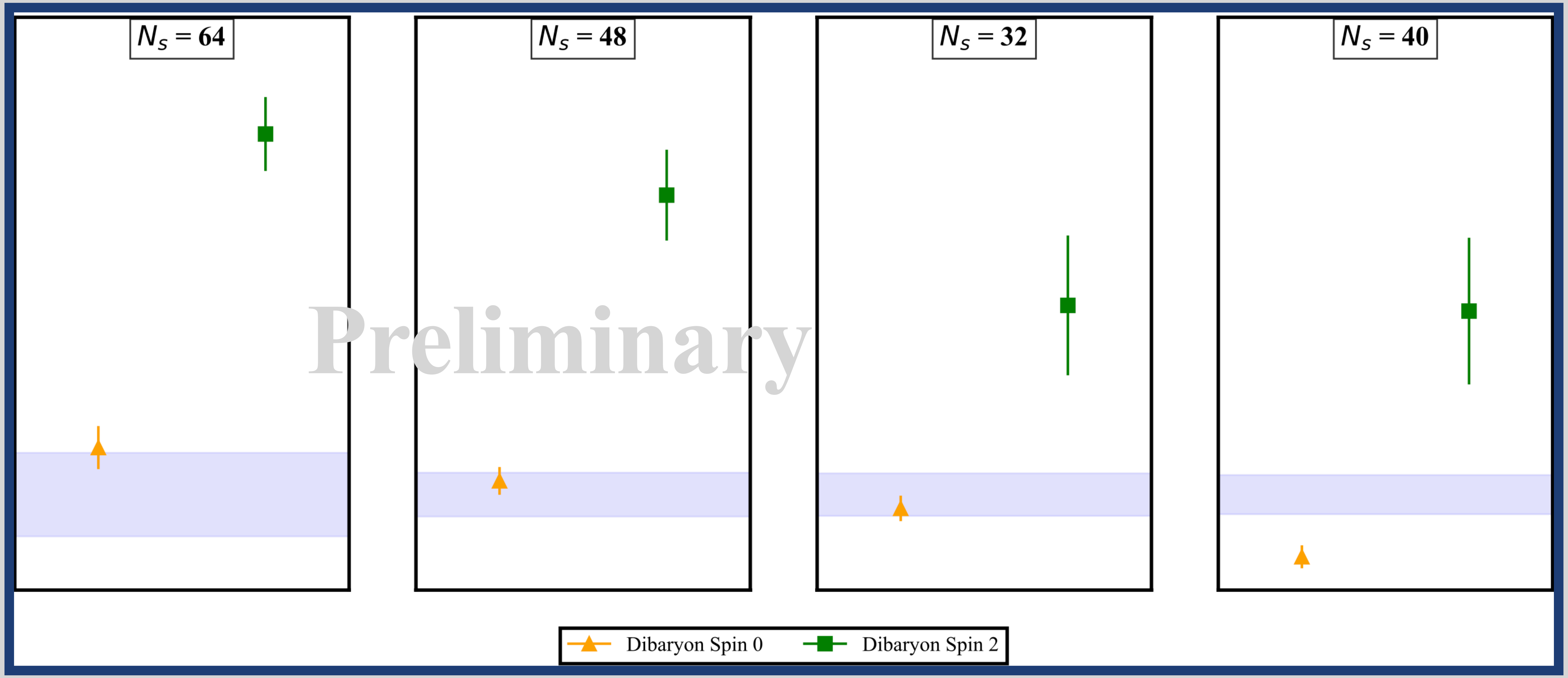
- Bound state, if exist, is shallow
- More probe using amplitude analysis

$\mathcal{D}_{6s}$ Results



$\mathscr{D}_{6s}$ Results





- No bound state
- More probe using amplitude analysis

Summary

★ We observe a positive shift in the S=2 channel, indicating a repulsive interaction and inability to host any bound state for both strange and charm systems.

★ In the charm sector, for spin zero, there is a slight tendency towards negative shifts, although these shifts have smaller magnitudes.

★ In the strange sector, for spin zero, the results generally suggest a non-interacting scenario, with weak interactions and potentially no bound states.

Summary

★ We observe a positive shift in the S=2 channel, indicating a repulsive interaction and inability to host any bound state for both strange and charm systems.

★ In the charm sector, for spin zero, there is a slight tendency towards negative shifts, although these shifts have smaller magnitudes.

★ In the strange sector, for spin zero, the results generally suggest a non-interacting scenario, with weak interactions and potentially no bound states.

★ A more precise conclusion can only be drawn with larger statistics and a comprehensive finite-volume amplitude study.

★ Lattice estimation of d*(2380)…

THANK YOU