Pole trajectories of the $\Lambda(1380)$ and $\Lambda(1405)$ resonances from the combination of lattice and experimental data

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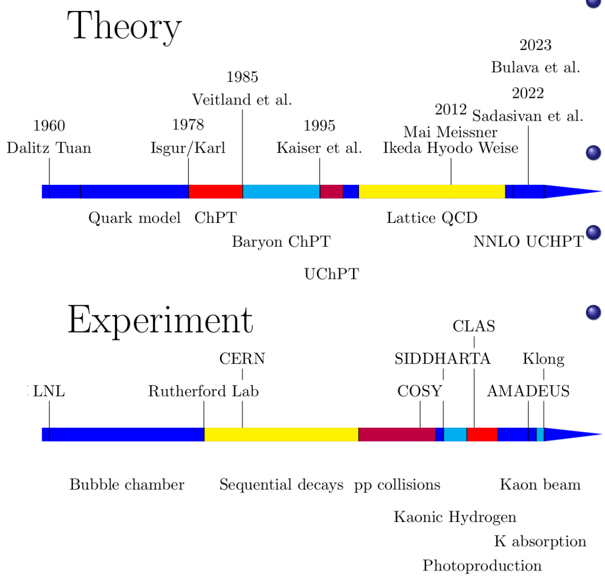
August 2, 2024

Resonant meson-baryon systems $\Lambda(1405)$



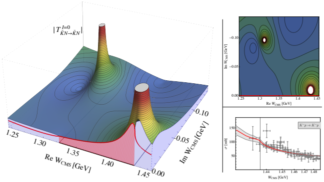
- Current status: Two pole structure using Chiral Unitary approach
- Lower broad pole around $\pi\Sigma$
- Higher narrow pole around $\bar{K}N$
- Check the combined effects of experimental and lattice data on the Chiral Unitary models. (Lu et al. (PRL.2023))

Transition amplitude

Plan:

- Chiral Perturbation Theory dictates the form of the interaction at low energies
- Constraints from *S* matrix (Unitarity/Analyticity/Crossing)
- Unitary scattering amplitude from the Bethe-Salpeter equation
- Fit free parameters to experimental/lattice data
- Extract complex pole positions for complex energies

Goal:



- The pole positions are independent of the particular reaction. They are universal property of the resonance.

From lattice QCD to infinite-volume quantities

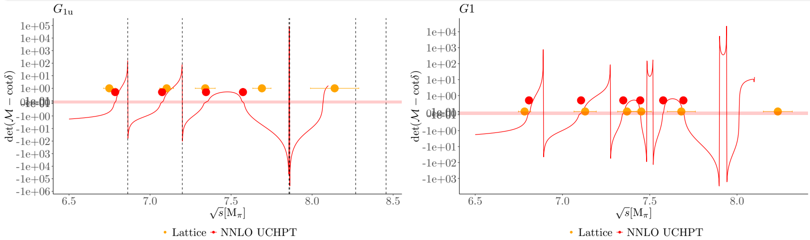
$I = 0$ fixed, we need 4 channels, two-body quantization condition

Lüscher formula in coupled channels $(\pi\Sigma, \bar{K}N, \eta N, K\Xi)$

$$q^2_{\mathrm{cmf}} = \sqrt{\frac{(s-(m_{2a}+m_{1a})^2)\cdot(s-(m_{2a}-m_{1a})^2)}{(4\cdot s)}}, s = E^2_{\mathrm{cmf}}, u^2_a = \frac{L^2 q^2_{\mathrm{cmf}}}{(2\pi)^2}$$

$$QC = \mathscr{M} - \cot\delta; \mathscr{M} = diag(u_a R^a_{00}); a \in (\pi\Sigma, \bar{K}N, \eta N, K\Xi); R_{lm} = \frac{1}{\gamma\pi^{3/2} u_a^{l+1}} \mathscr{Z}_{\ell m}(s_a, \gamma, u^2_a)$$

We only consider the lowest partial-wave s-wave.



Chiral Unitary models: Potential model

Degrees of freedom: Meson and Baryon octet

- To relate observables we compute Höhler partial wave amplitudes $f_{0+}(E_2)$
- Scattering length: $f_{0+}^{MB}(m_M + m_B) = a_{MB}$
- $\mathscr{S} = \{K^- p, \bar{K}^0 n, \pi^0 \Lambda, \pi^0 \Sigma^0, \pi^+ \Sigma^-, \pi^- \Sigma^+, \eta \Lambda, \eta \Sigma^0, K^+ \Xi^-, K^0 \Xi^0\}$

Partial wave amplitude

$$T(E_2) = 8\pi E_2 f_{0+} = -V(E_2)\frac{1}{1 - G(E_2)V(E_2)}$$

$$V_{ij}^{\mathrm{WT}}(\sqrt{s}) = -\frac{C_{ij}^{\mathrm{WT}}}{8F_i F_j}\mathcal{N}_i \mathcal{N}_j (2\sqrt{s} - m_i - m_j)$$

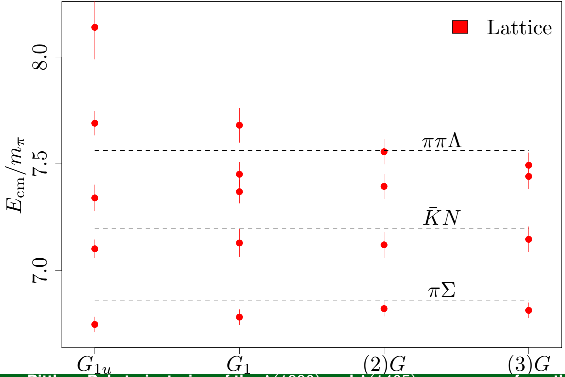
$$V_{ij}^{\mathrm{NLO}}(\sqrt{s}) = \frac{\mathcal{N}_i \mathcal{N}_j}{F_i F_j}\left(C_{ij}^{\mathrm{NLO1}} - 2C_{ij}^{\mathrm{NLO2}}\left(E_i E_j + \frac{q_i^2 q_j^2}{3\mathcal{N}_i \mathcal{N}_j}\right)\right).$$

We project to $I = 0$ $(\pi\Sigma, \bar{K}N, \eta N, K\Xi)$

Lattice input

Bulava et al. (PRL 2024)

- $D200$ CLS ensemble: $N_f = 2+1$ non perturbatively improved Wilson
- $m_\pi \sim 200\mathrm{MeV},\ m_\pi L \sim 4.18$



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- We only consider levels below the lowest 3-particle threshold $\pi\pi\Lambda$

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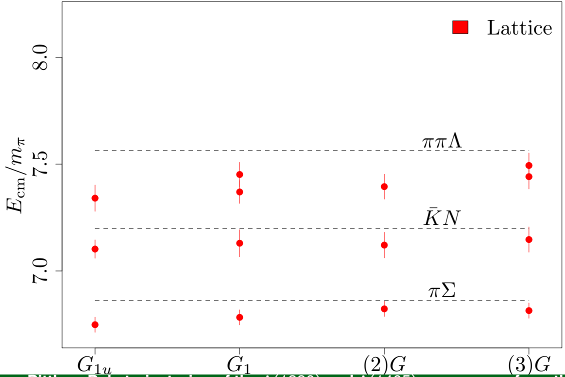

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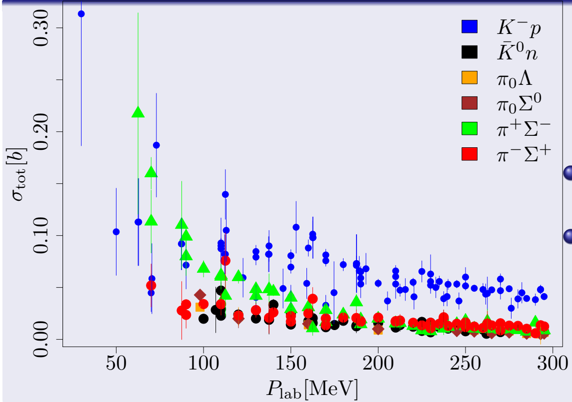
Experimental data

Cross sections: $K^-p \to \{K^-p, \bar{K}^0 n, \pi_0\Lambda, \pi_0\Sigma^0, \pi^+\Sigma^-, \pi^-\Sigma^+\}$



$$\sigma_i = \frac{2\pi}{q_i}|q_i f_{0\ 0i}^{+}|^2$$

- Data are noisy
- Number of data points are not uniform

Amadeus,SIDDHARTA

- $|f_{0+}^{\pi^-\Lambda\to K^-n}|$ at $\sqrt{s} = 1400$MeV.
- Energy shift and width of kaonic hydrogen

Unitary models

Parameters

- Propapapagator: Subtraction constants:

$$a_{\pi\Sigma}, a_{\bar{K}n}, a_{\eta\Lambda}, a_{K\Xi}$$

- Low Energy Constants(LEC.) :

$$b_0, b_D, b_F, d_1, d_2, d_3, d_4$$

Model 1 $V \equiv V_{WT}$

- Fitting parameters $(a_{\pi\Sigma}, a_{\bar{K}n}, a_{\eta\Lambda})$

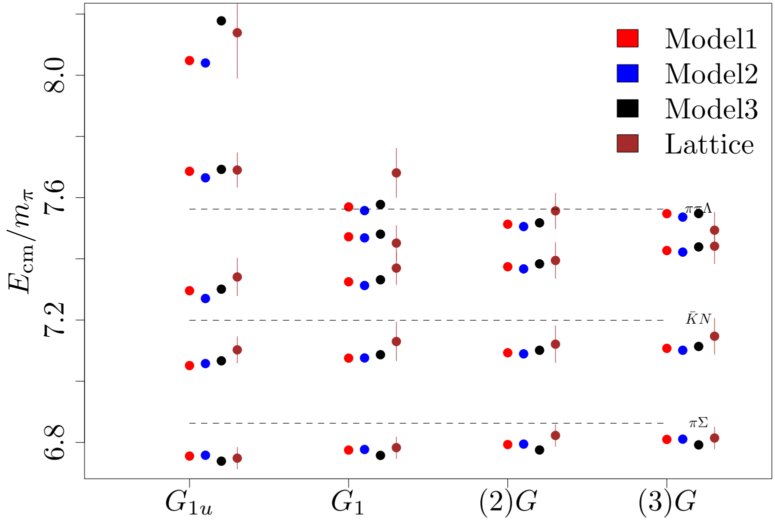
Model 2: $V \equiv V_{WT} + V_{\text{born}}$

- Fitting parameters $a_{\pi\Sigma}, a_{\bar{K}n}, a_{\eta\Lambda}$,

Model 3 $V \equiv V_{WT} + V_{\text{born}} + V_{\text{NLO}}$

- Fitting parameters $a_{\pi\Sigma}, a_{\bar{K}n}, a_{\eta\Lambda}, b_0, b_D, b_F, d_1, d_2, d_3, d_4$

Fit to BaSC results



Including additional data

Baryon masses

$$M_N = m_0 - 2(b_0 + 2b_F)M_\pi^2 - 4(b_0 + b_D - b_F)M_K^2$$
$$M_\Lambda = m_0 - 2/3(3b_0 - 2b_D)M_\pi^2 - 4/3(3b_0 + 4b_D)M_K^2$$
$$M_\Sigma = m_0 - 2(b_0 + 2b_D)M_\pi^2 - 4(b_0)M_K^2$$
$$M_\Xi = m_0 - 2(b_0 - 2b_F)M_\pi^2 - 4(b_0 + b_D + b_F)M_K^2$$

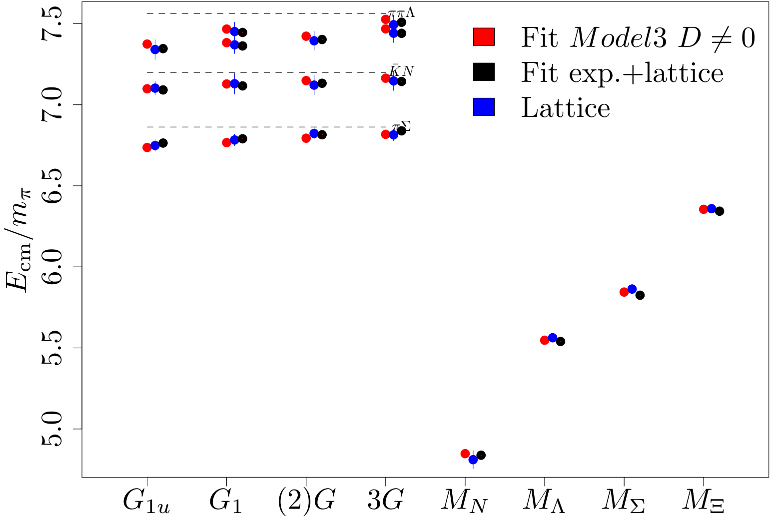
Fitting mass differences:

$$M_N - M_N(\text{phys.}) = m_0 - 2(b_0 + 2b_F)(M_\pi^2 - M_\pi^2(\text{phys.})) - 4(b_0 + b_D - b_F)(M_K^2 - M_K^2(\text{phys.}))$$

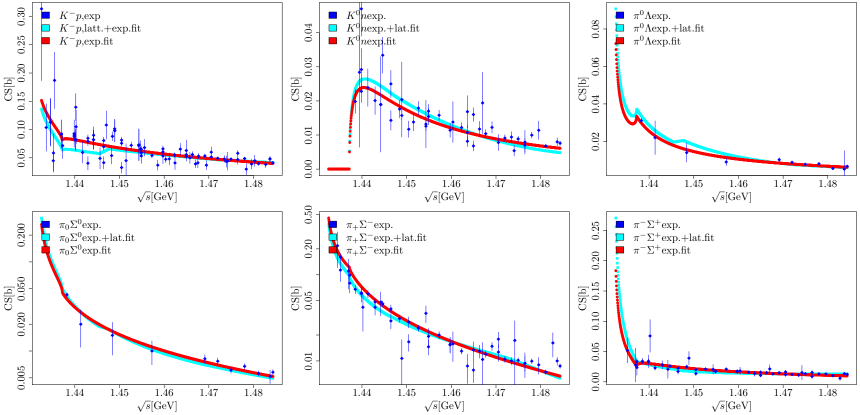
Fitting weighted χ^2

$$\chi^2_{\text{dof}} = \frac{\sum_a N_a}{A((\sum_a N_a) - n)} \sum_{a=1}^{A} \frac{\chi_a^2}{N_a} \quad \text{with} \quad \chi_a^2 = \sum_{i=1}^{N_a} \left(\frac{f_i^a(\vec{\aleph}) - \hat{f}_i^a}{\Delta \hat{f}_i^a} \right)^2$$

Fit results I.



Fit results II.



Summary, Outlook

Summary

- We have fitted lattice and experimental data together to constrain the parameters of chiral unitary models

Work in Progress

- Extraction of the resonance poles are ongoing
- Goal: determine the pole trajectory towards the physical point

Thank you for your attention

Support is acknowledged from the project EXCELLENCE/0421/0195 “Nice quarks,” cofinanced by the European Regional Development Fund and the Republic of Cyprus through the Research and Innovation Foundation