Hans Böckler Stiftung

Spectral analysis for Nπ and N$\pi\pi$ states in both parity sectors using distillation with domain wall fermions

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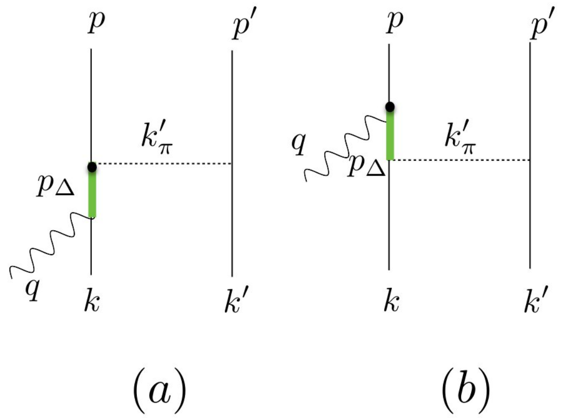
Introduction

Physical Motivation

- fundamental understanding of nucleon spectrum is relevant for e.g. nucleon-neutrino interactions
- $N \rightarrow N\pi, N\pi\pi, \Delta$ transitions are relevant for the resonance regime of nucleon-neutrino interactions
- $N\pi$ state are contributing to excited state systematics for nucleon axial-vector/vector currents *L. Barca et.al., Phys.Rev.D 107 (2023) 5*



from D. Simons, N. Steinberg, et al. arXiv:2210.02455

Computational Motivation

- access to a large database of distillation data (created for the g-2 project of the RBC/UKQCD collaborations)
- the creation of distillation data is expensive
- once computed and stored: using distillation data is cheap
- domain-wall fermions offer better chiral properties

Ensemble – Overview

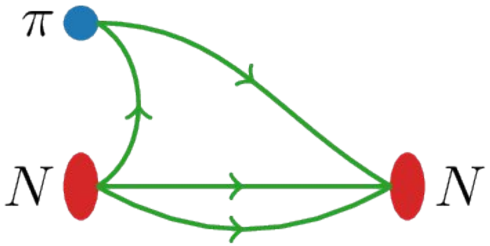
Ens-Id	$L^3 \times T \times L_s$	m_π/MeV	m_K/MeV	a^{-1}/GeV	N_{conf}	N_c	$m_\pi L$
4	$24^3 \times 48 \times 24$	274.8(2.5)	530.1(3.1)	1.7312(28)	94	60	3.8
D	$32^3 \times 64 \times 24$	274.8(2.5)	530.1(3.1)	1.7312(28)	60	60	5.1
9	$32^3 \times 64 \times 12$	278.9(4.9)	531.2(4.9)	2.3549(49)	60	60	3.8
L	$64^3 \times 128 \times 24$	278.9(4.9)	531.2(4.9)	2.3549(49)	20	60	7.6
1	$32^3 \times 64 \times 24$	208.1(1.1)	514.0(1.8)	1.7312(28)	34	60	3.8
3	$32^3 \times 64 \times 24$	211.3(2.3)	603.8(6.1)	1.7312(28)	34	60	3.8
C	$64^3 \times 128 \times 24$	139.32(30)	499.44(88)	1.7312(28)	25	120	5.2

physical point information from:

- 1 and 3 → m_K
- 4,D,9 and L → a^{-1}
- all ensembles → m_π
- all ensembles → volume

- all **2+1 DWF + Iwasaki ensembles** are generated by the RBC/UKQCD collaborations
- the lattice spacings are taken from 48I and 64I
- values of m_π and m_K are only measured for one of the pairs
- masses for ensemble C are taken from 48I

Distillation



Distillation for baryons requires the following building blocks:

Basis

first N_d eigenvectors $V^n(x)$ of the 3-dim Laplace operator (smeared links)

$$L(\boldsymbol{x},\boldsymbol{y}) = -\delta_{\boldsymbol{x},\boldsymbol{y}} + \frac{1}{6a^2}\sum_i U_i(\boldsymbol{x})\delta_{\boldsymbol{x},\boldsymbol{y}-a\hat{\boldsymbol{i}}} + U_i^\dagger(\boldsymbol{x}-a\hat{\boldsymbol{i}})\delta_{\boldsymbol{x},\boldsymbol{y}+a\hat{\boldsymbol{i}}}$$

Modified Elementals

$$\mathcal{E}^{\ell nm}(t_x,\boldsymbol{p}) = \sum_{\boldsymbol{x}} \varepsilon_{abc} V_a^\ell(\boldsymbol{x},t) V_b^n(\boldsymbol{x},t) V_c^m(\boldsymbol{x},t) e^{-i\boldsymbol{p}\cdot\boldsymbol{x}}$$

C. Egerer et al. Physical Review D 99, 034506 (2019)
C. Lang et al., Physical Review D 87, 054502 (2013)

Perambulators

$$\mathcal{G}^{mn}(t_y,t_x) = \sum_{\boldsymbol{y}} V^m(t_y,\boldsymbol{y})^\dagger G^n(t_y,t_x,\boldsymbol{y})$$

where $G^n(t_y,t_x,\boldsymbol{y})$ denotes the propagator with source $V^n(x)$

Momentum insertion

$$\mathcal{P}_{cc'}^{nm}(t,\boldsymbol{p}) = \sum_{\boldsymbol{x}} \left[V_c^m(\boldsymbol{x},t)\right]^\dagger e^{i\boldsymbol{p}\cdot\boldsymbol{x}} V_{c'}^m(\boldsymbol{x},t)$$

Distillation

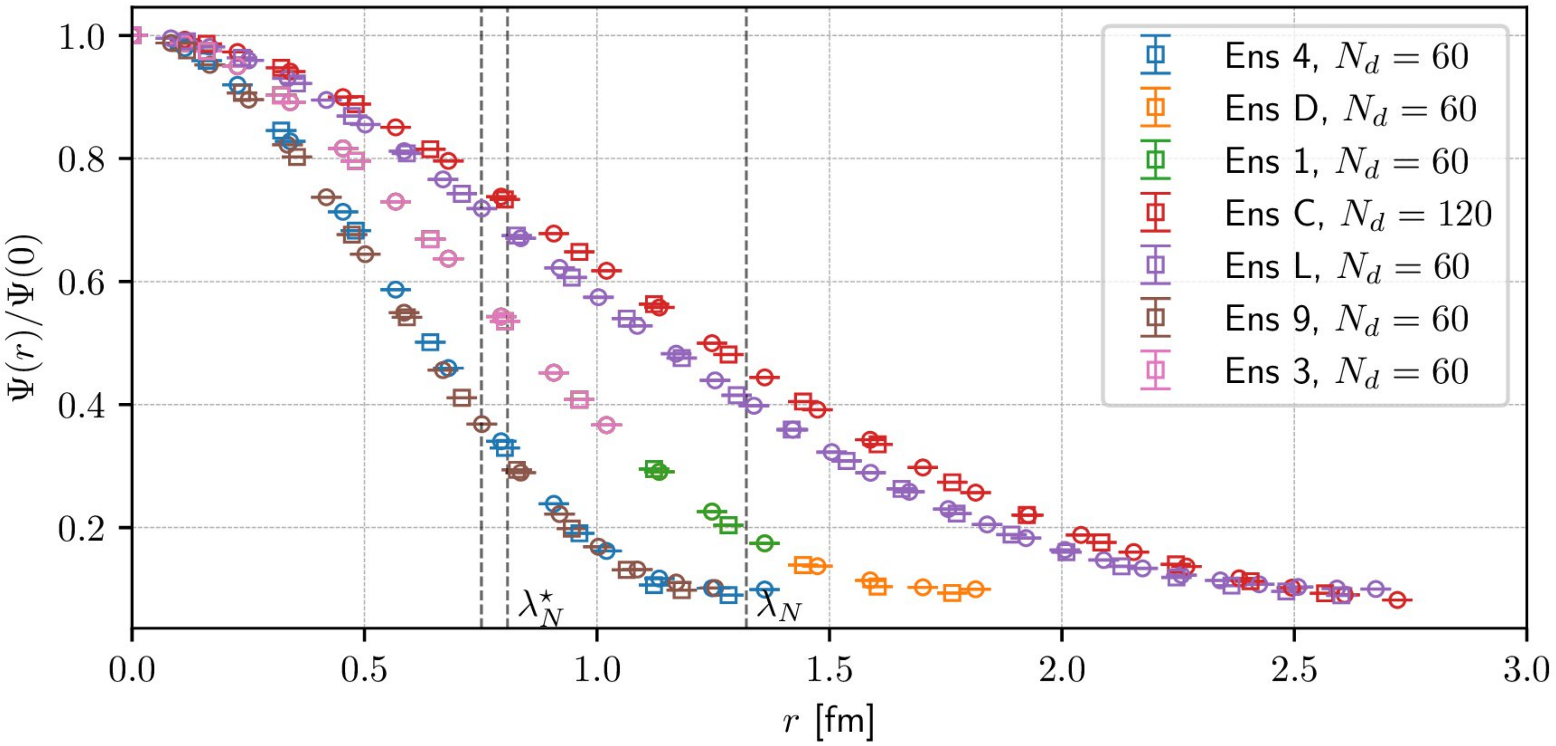
Profile

$$\Psi(\boldsymbol{r}) = \sum_{\boldsymbol{x},t} \sqrt{\mathrm{tr}_c\left[\square(\boldsymbol{x},\boldsymbol{x}+\boldsymbol{r},t)\square(\boldsymbol{x}+\boldsymbol{r},\boldsymbol{x},t)\right]}$$

with the distillation operator

$$\square(\boldsymbol{x},\boldsymbol{y},t)_{cc'} = \sum_{k=1}^{N_d} V_c^k(\boldsymbol{x},t)\left[V_{c'}^k(\boldsymbol{y},t)\right]^\dagger$$

M. Peardon et al., Physical Review D 80, 054506 (2009)

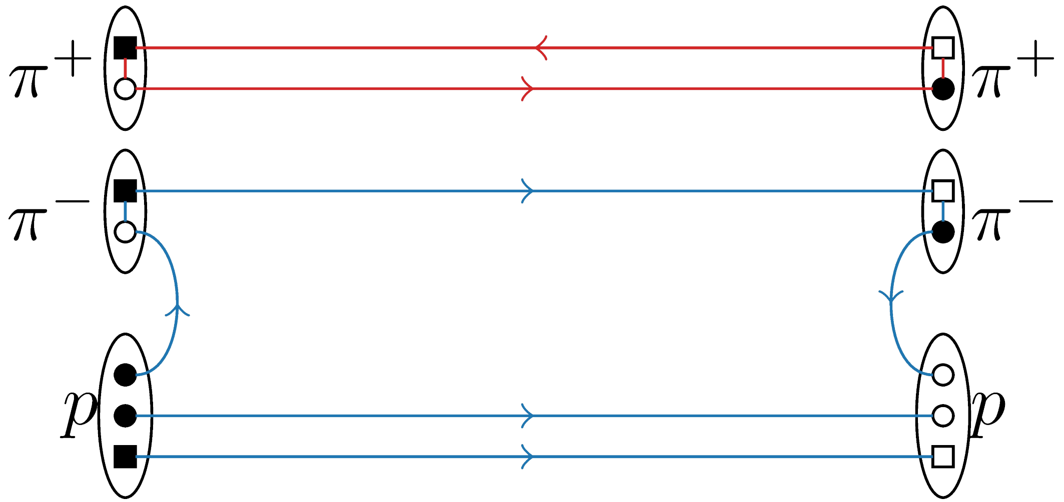


Remarks

- Profile Ψ is a measure for the **smearing** due to the distillation operator
- narrow profile → larger overlap with high mode states and more statistics
- λ_N and $\lambda_N^\star$ denote the approx. Compton wavelengths of the Nucleon, N(1535) and N(1650)

Automatic Wick-Contractor

- $p\,\pi^+\,\pi^- \rightarrow p\,\pi^+\,\pi^-$ has 144 diagrams → need for automation of contractions
- general process: $N + \sum_i \pi_i \rightarrow N + \sum_j \pi_j$ has the following properties:
  - ✓ there are three (sequential) propagator connecting the baryonic fermions
  - ✓ the remaining fermions are part of a loop over pions



Automatic Wick-Contractor

1. anticommute fermionic fields to a predefined order
2. contract all fermions using Wick's theorem
3. find all (sequential) propagators and loops
4. contract with the corresponding Γ structures
5. translate everything into the usage of distillation objects

- everything can be boiled down to tensor contractions with perambulators, momentum insertions and modified elementals
- for most ensembles: calculations can be done on single nodes

Operator set and GEVP

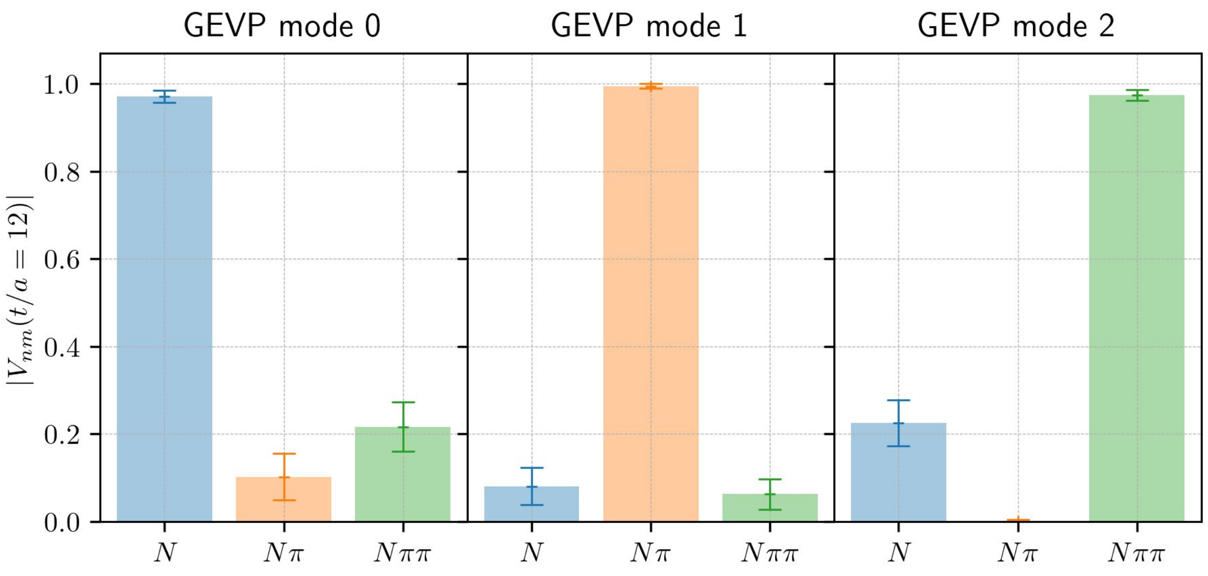
- use of the Generalized Eigenvalue Problem (GEVP)
- operator set of the positive parity channel
  - $\mathcal{O}_N^+(t,\mathbf{0})$
  - $\mathcal{O}_{N\pi}(t) = \mathcal{O}_N^+(t,\boldsymbol{p})\mathcal{O}_\pi(t,-\boldsymbol{p}) - \mathcal{O}_N^+(t,-\boldsymbol{p})\mathcal{O}_\pi(t,\boldsymbol{p})$ with implicit G_1^+ projection
  - $\mathcal{O}_{N\pi\pi}(t) = \mathcal{O}_N^+(t,\mathbf{0})\mathcal{O}_\pi(t,\mathbf{0})\mathcal{O}_\pi(t,\mathbf{0})$
- operator set of the negative parity channel
  - $\mathcal{O}_N^-(t,\mathbf{0})$
  - $\mathcal{O}_{N\pi^S}(t) = \gamma^5\mathcal{O}_N^+(t,\mathbf{0})\mathcal{O}_\pi(t,\mathbf{0})$
  - $\mathcal{O}_{N\pi^P}(t) = \mathcal{O}_N^+(t,\boldsymbol{p})\mathcal{O}_\pi(t,-\boldsymbol{p}) + \mathcal{O}_N^+(t,-\boldsymbol{p})\mathcal{O}_\pi(t,\boldsymbol{p})$ with implicit G_1^- projection
- we project to the isospin $(I, I_3) = \left(\frac{1}{2}, \frac{1}{2}\right)$
- nucleon: $\mathcal{O}_N^\pm(t,\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\varepsilon^{abc}P^\pm\psi^a(x)\left[u^b(x)^T C\gamma^5 d^c(x)\right]$ with ψ chosen to represent n or p
- pion: $\mathcal{O}_\pi(t,\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\psi(x)\gamma^5\phi(x)$ with ψ and ϕ chosen to represent $\pi^+, \pi^0,$ or π^-

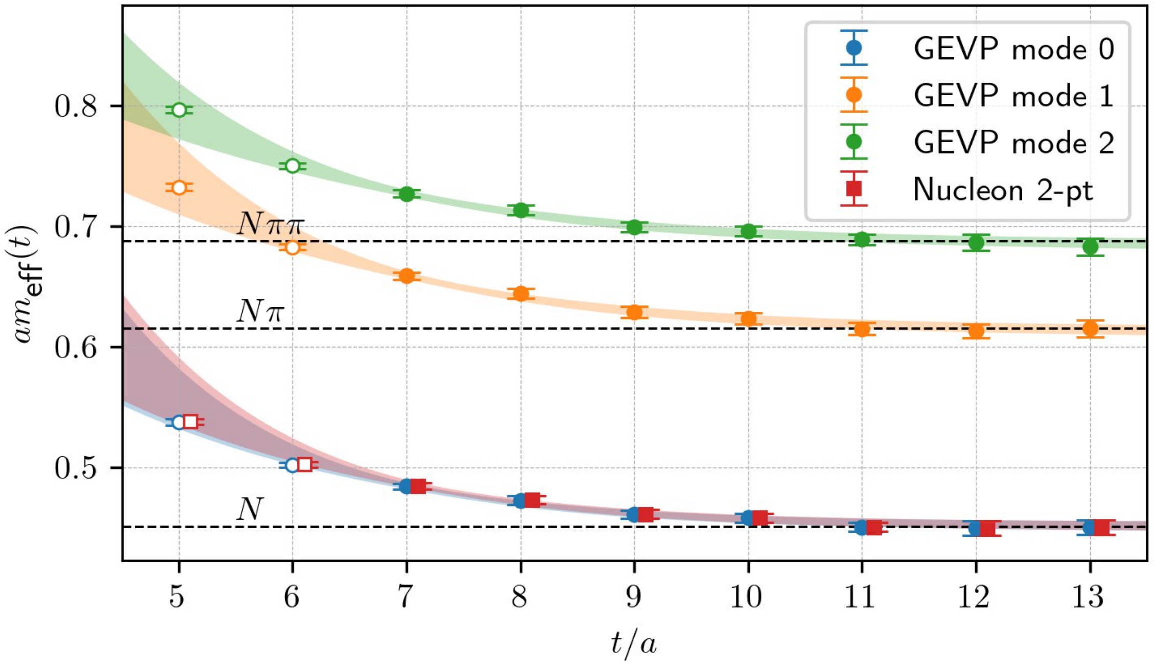
Positive Parity Sector: Example Results (Ensemble L)

- effective energy of mode 1 and 2 converge to energies of non-interacting $N\pi$ and $N\pi\pi$ states
- dominant states align with the energy
- there is only a marginal difference between mode 0 and nucleon 2-point function
- $N\pi$ and $N\pi\pi$ have no significant overlap with nucleon 2-point function

(as expected from χPT *O. Bär, Phys.Rev.D* 92 (2015) 7, 074504; *Phys.Rev.D* 97 (2018) 9, 094507)

$m_\pi L = 7.6$





Positive Parity Sector: Example Results (Ensemble L)

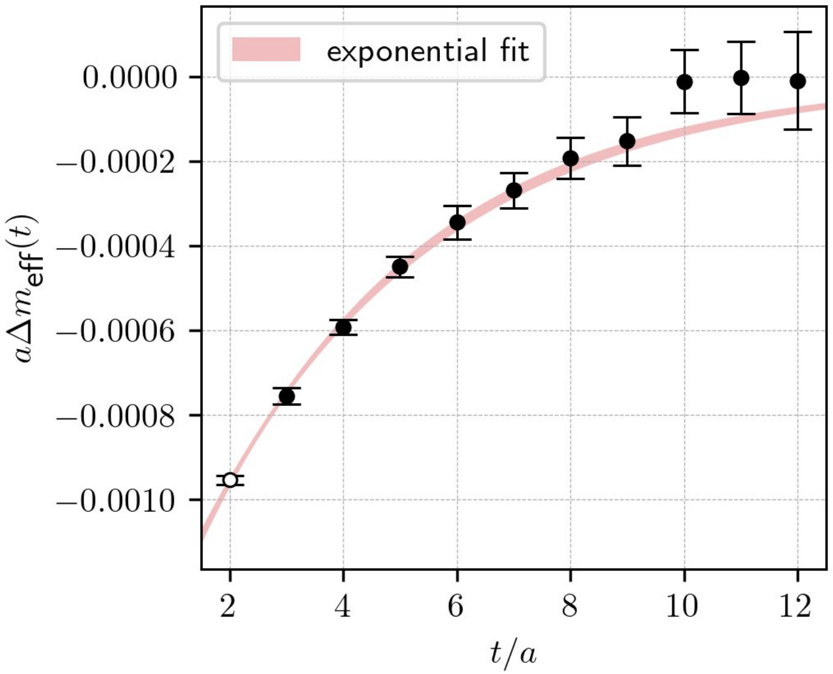
- consider difference between GEVP0 and Nucleon 2-point function

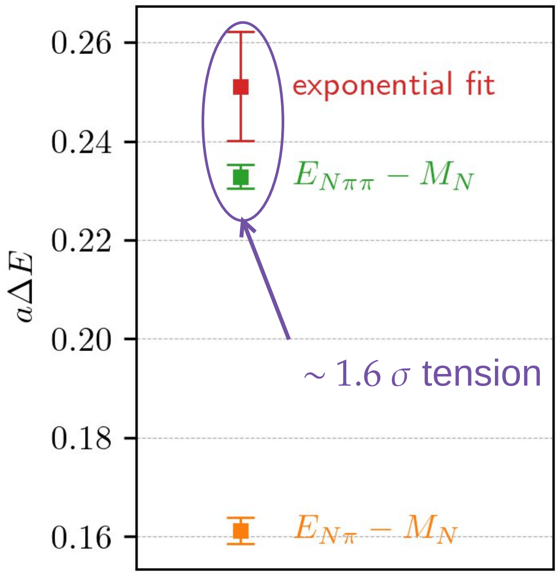
$$a\Delta m_{\text{eff}}(t) = a\left(m_{\text{eff}}^{\text{GEVP0}}(t) - m_{\text{eff}}^{2pt}(t)\right)$$

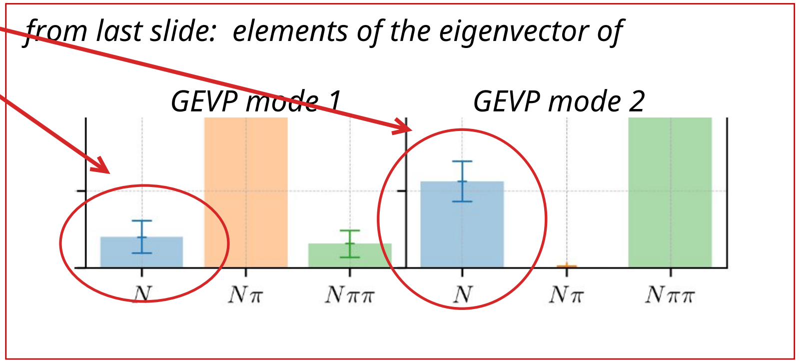
- GEVP0 has no $N\pi$ and $N\pi\pi$ contributions
- Difference shows the contributions in Nucleon 2-point function
- for Ensemble L we get that

$$a\Delta m_{\text{eff}}(t) \approx a_0 e^{-(E_{N\pi\pi}-M_N)t}$$

- in agreement with eigenvectors of GEVP







Finite Volume Correction

finite volume correction using **BχPT:**

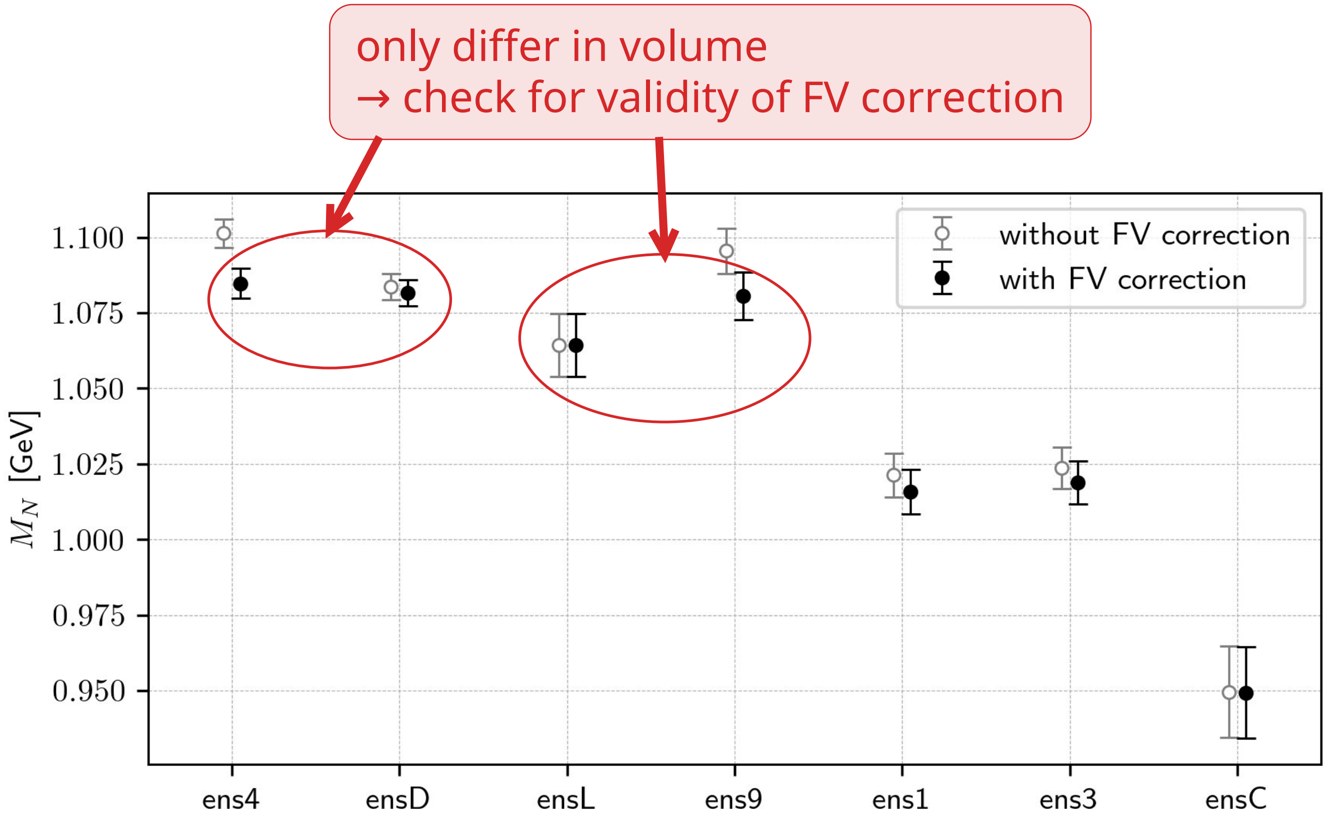
$$m_N(L) - m_N(\infty) = \Delta_a(L) + \Delta_b(L)$$

with

$$\Delta_a(L) = \frac{3g_A^2 m_0 m_\pi^2}{16\pi^2 f_\pi^2} \int_0^\infty \mathrm{d}x \sum_{\boldsymbol{n}\neq 0} K_0\left(L|\boldsymbol{n}|\sqrt{m_0^2 x^2 + m_\pi(1-x)}\right)$$

and

$$\Delta_b(L) = \frac{3m_\pi^4}{4\pi^2 f_\pi^2} \sum_{\boldsymbol{n}\neq 0} \left[(2c_1 - c_3)\frac{K_1(L|\boldsymbol{n}|m_\pi)}{L|\boldsymbol{n}|m_\pi} + c_2 \frac{K_2(L|\boldsymbol{n}|m_\pi}{(L|n|m_\pi)^2}\right]$$



- values for BχPT constants are taken from *G.S. Bali et al. Nuclear Physics B 866, 1 (2013)*

Continuum Extrapolation

- a^2 term (Domain-Wall fermions)
- linear pion models inspired by

A. Walker-Loud, PoS LATTICE2008:005 (2008)

- quadratic pion models inspired by BχPT
- for averaging we use **Akaike information criterion** $\mathrm{AIC} = 2k + \chi^2$

k = Number of fit parameter

Models used:

Tag	Model function $\mathcal{M}(a, m_\pi, m_K)$
$\pi(1)K(0)$	$M_N + c_0 a^2 + c_1(m_\pi - m_\pi^0)$
$\pi(1)K(1)$	$M_N + c_0 a^2 + c_1(m_\pi - m_\pi^0) + c_2(m_K - m_K^0)$
$\pi(1)K(2)$	$M_N + c_0 a^2 + c_1(m_\pi - m_\pi^0) + c_2(m_K^2 - (m_K^0)^2)$
$\pi(2)K(0)$	$M_N + c_0 a^2 + c_1(m_\pi^2 - (m_\pi^0)^2)$
$\pi(2)K(1)$	$M_N + c_0 a^2 + c_1(m_\pi^2 - (m_\pi^0)^2) + c_2(m_K - m_K^0)$
$\pi(2)K(2)$	$M_N + c_0 a^2 + c_1(m_\pi^2 - (m_\pi^0)^2) + c_2(m_K^2 - (m_K^0)^2)$
$\pi(2,3)K(0)$	$M_N + c_0 a^2 + c_1(m_\pi^2 - (m_\pi^0)^2) + c_2(m_\pi^3 - (m_\pi^0)^3)$
$\pi(2,3)K(1)$	$M_N + c_0 a^2 + c_1(m_\pi^2 - (m_\pi^0)^2) + c_2(m_K - m_K^0) + c_3(m_\pi^3 - (m_\pi^0)^3)$
$\pi(2,3)K(2)$	$M_N + c_0 a^2 + c_1(m_\pi^2 - (m_\pi^0)^2) + c_2(m_K^2 - (m_K^0)^2) + c_3(m_\pi^3 - (m_\pi^0)^3)$

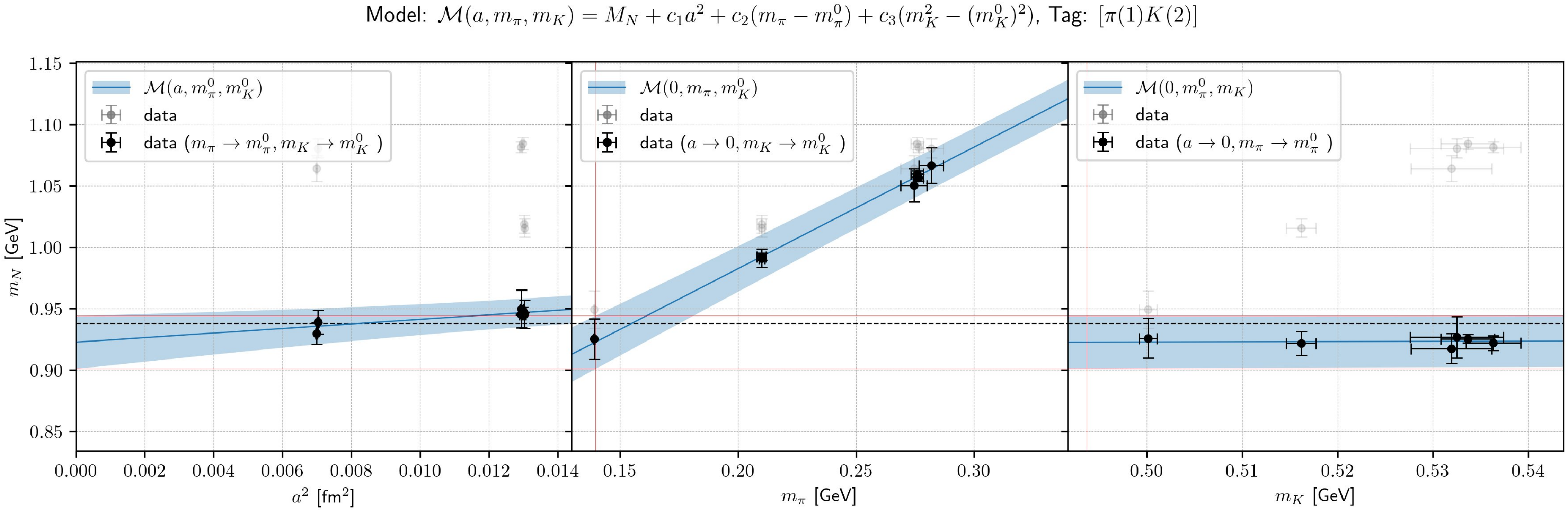
Model averaging for parameter β

$$\bar{\beta} = \sum_{\mathcal{M}} P(\mathcal{M})\beta_{\mathcal{M}}$$

with model probabilities

$$P(\mathcal{M}) = \exp(-\mathrm{AIC}_{\mathcal{M}}) / \sum_{\mathcal{M}'} \exp(-\mathrm{AIC}_{\mathcal{M}'})$$

Continuum Extrapolation – Example Model



Fit results:

Tag	M_N/GeV	c_1/GeV^3	c_2	$c_3 \cdot$ GeV	χ^2/dof	p-value
π(1)K(2)	0.923(22)	0.072(52)	0.991(84)	0.017(98)	0.85/3	0.84

Continuum Extrapolation

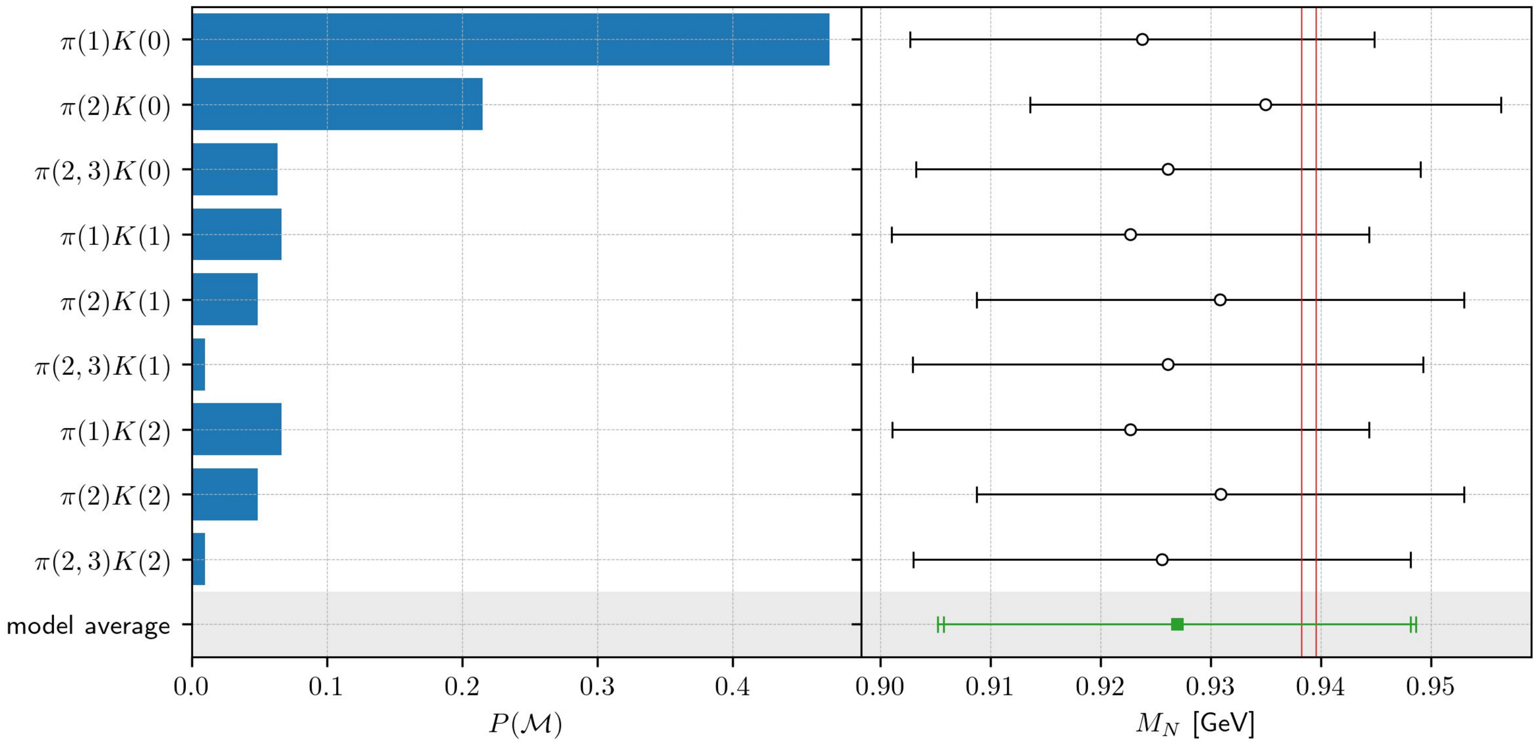
Summary:

- no Kaon dependency
- linear and quadratic pion model fit the data
- m_π^3 term not necessary to fit the data

final **nucleon mass estimate:**

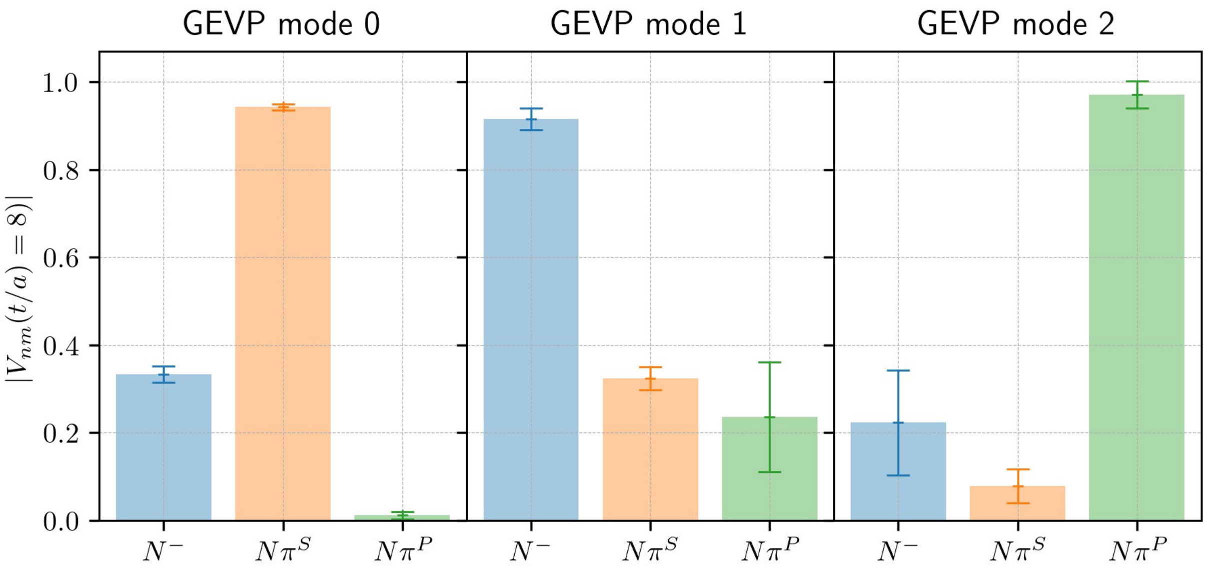
$$M_N \;=\; 0.927\,(21)\,(05)\;\text{GeV}$$

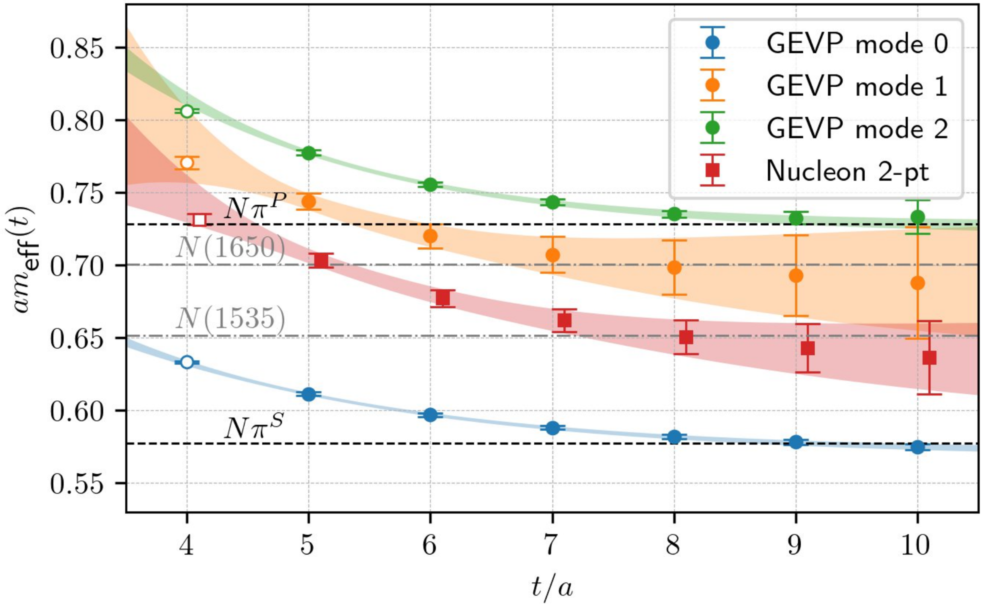
without Isospin-breaking and QED corrections



Negative Parity – Example Result (Ensemble 9)

- disentanglement of the individual states in the GEVP
- inclusion of the negative nucleon 2-point function requires more statistics and narrower profile (Ensemble 9 and 4 yield the best result)
- $N\pi$ states have better signal-to-noise behavior
- mass estimates from GEVP mode 1 and nucleon 2-point function are in agreement with $N(1535)$ and $N(1650)$





Summary and Outlook

Summary

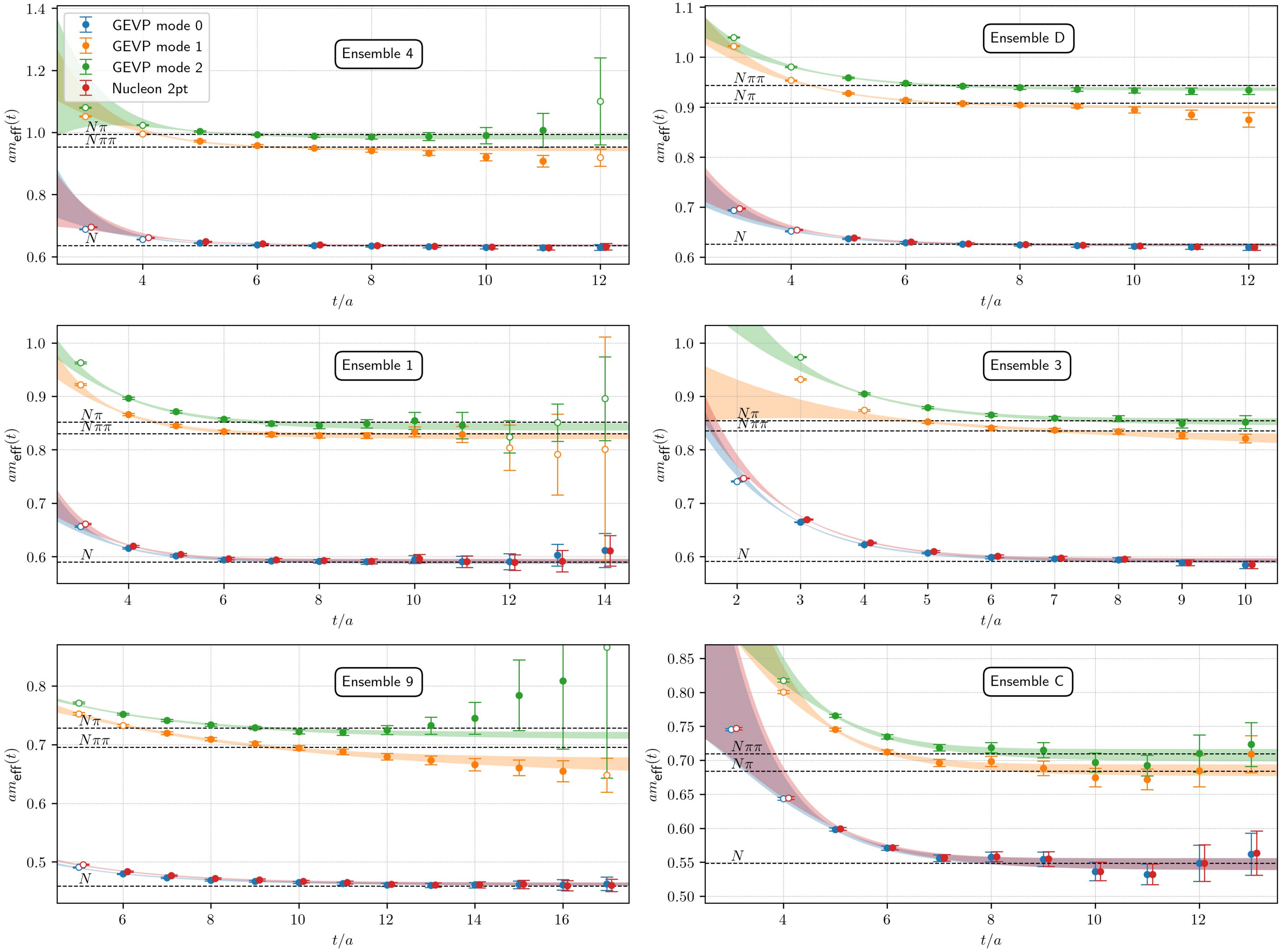
- case study of the effectiveness of distillation for nucleon spectroscopy for (near to) physical pion masses and domain wall fermions
- positive parity channel: reproduction of the χPT result of the negligibility of $N\pi$ and $N\pi\pi$ contributions in the nucleon 2-point function
- negative parity channel: GEVP works, but we need more statistics for more sophisticated analysis
- automated contraction for general nucleon-pion processes in the distillation framework
- continuum extrapolation of the nucleon mass

Outlook

- Repeat the same analysis for other nucleon quantities (e.g. axial-vector current)
- Increase the statistics and number of distillation modes for the negative parity channel
- Include higher momenta in our analysis
- Repeat the analysis for Δ baryons

Backup Slides

Overview GEVP in the positive parity sector



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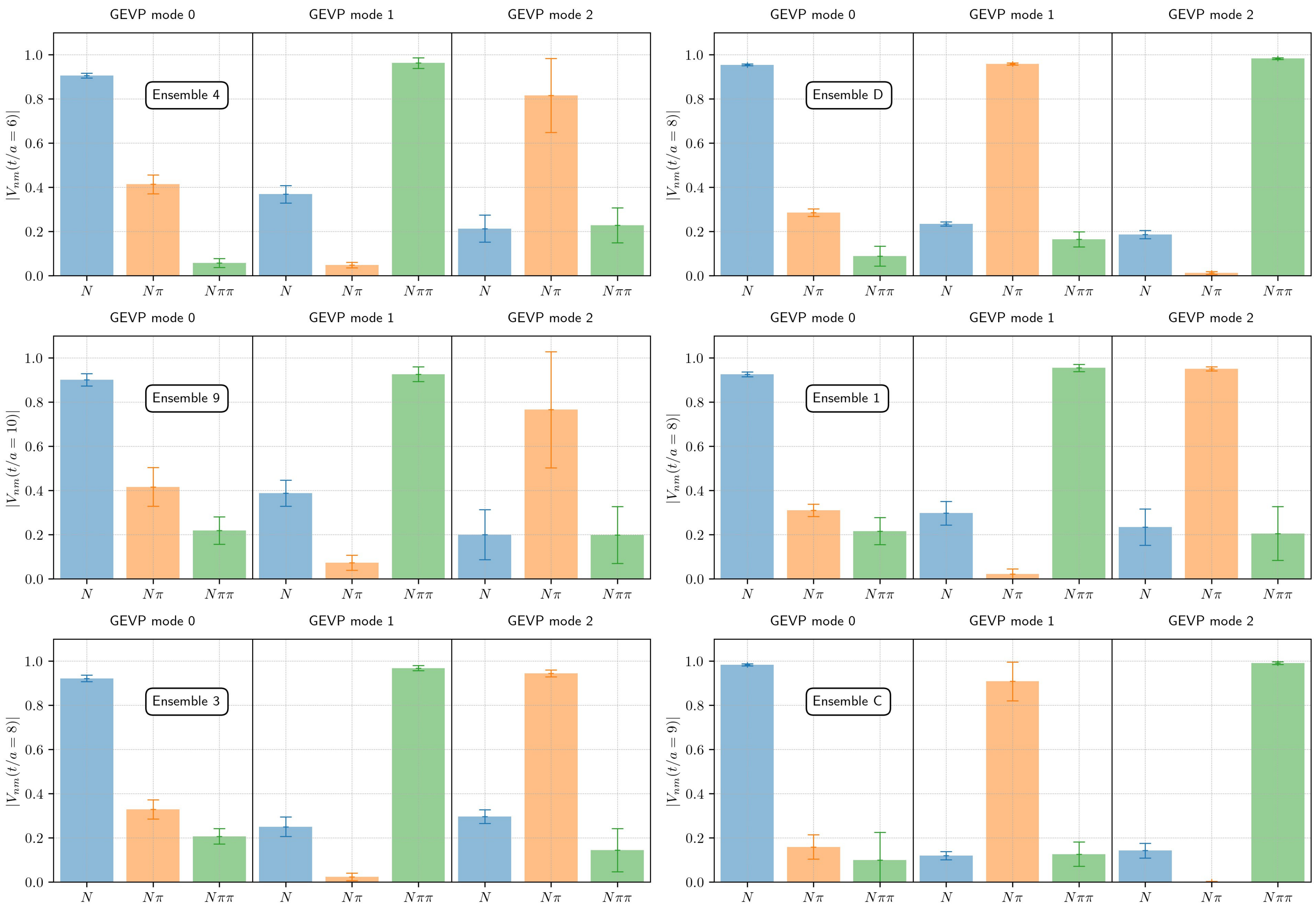

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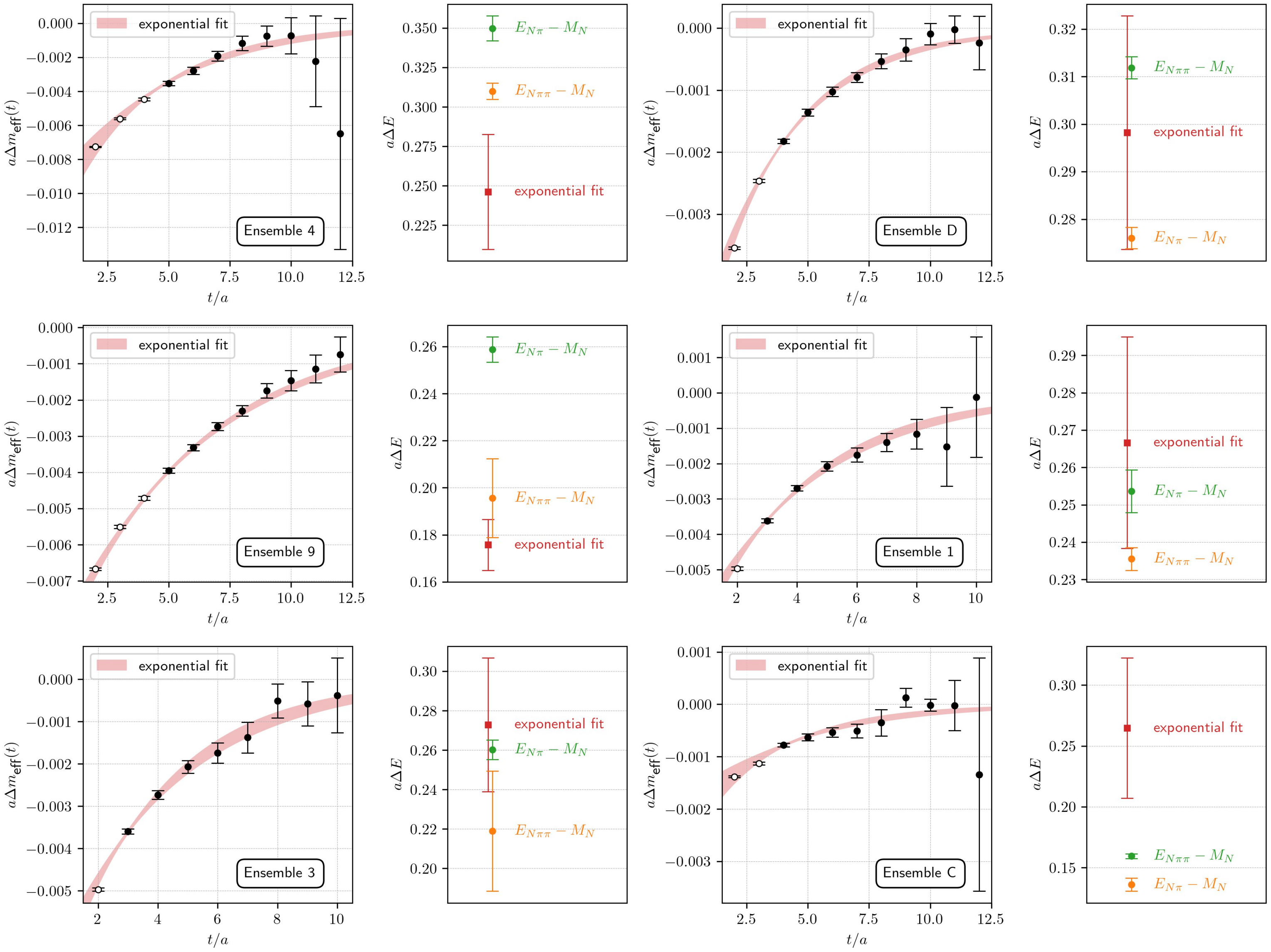
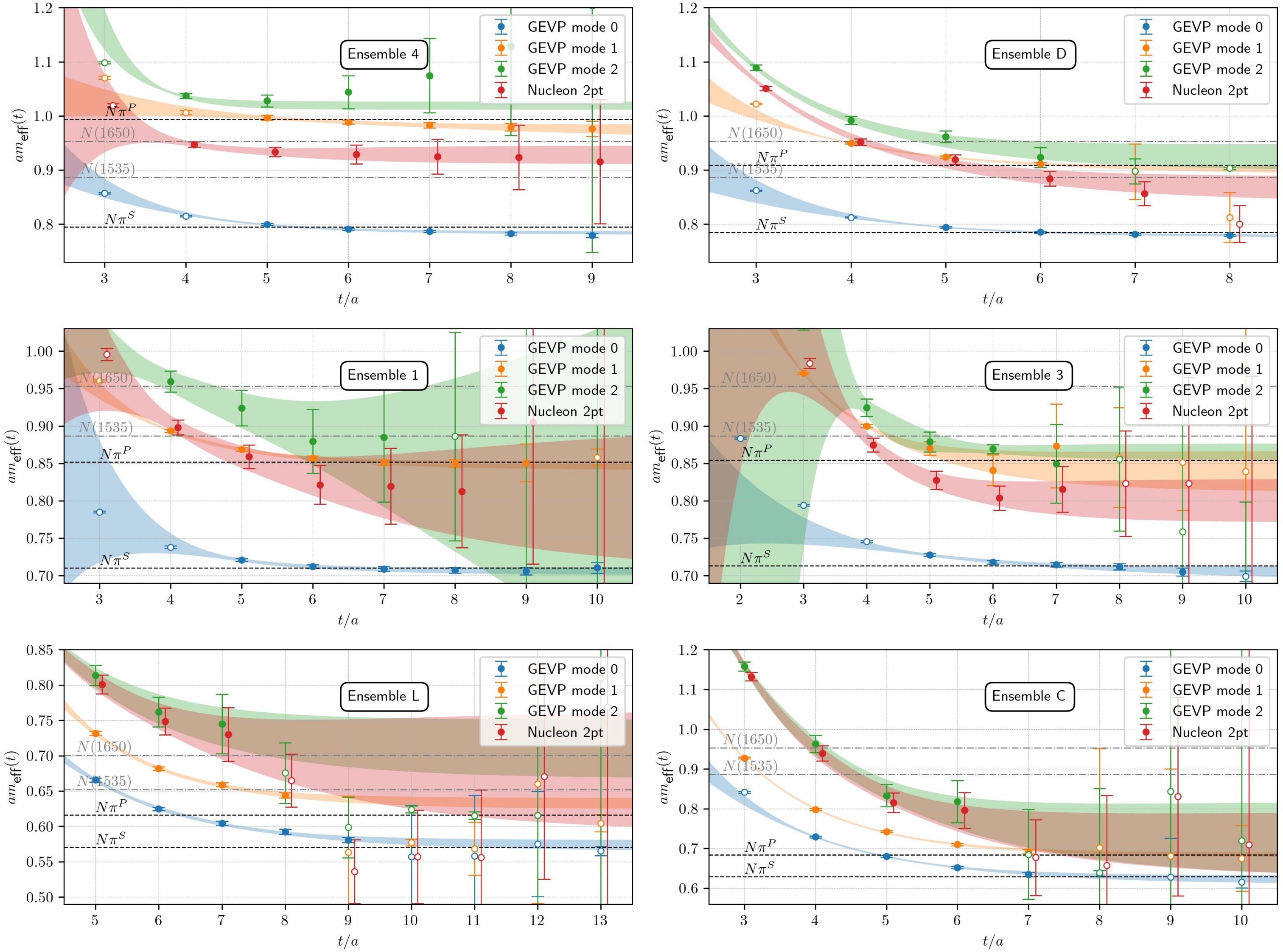


Overview GEVP in the negative parity sector



Overview GEVP in the negative parity sector