Update of kaon semileptonic form factor
using $N_f = 2 + 1$ PACS10 configurations

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Outline

Introduction

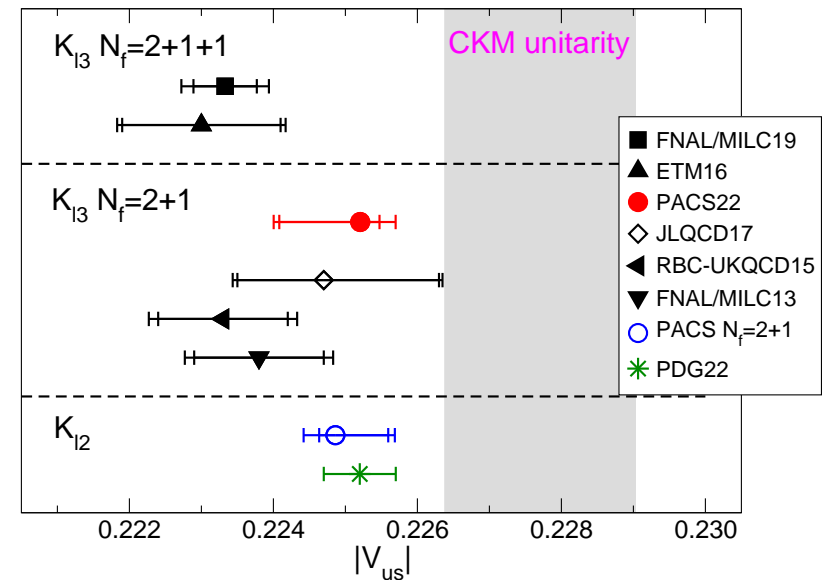Indirect search for physics beyond standard model through $|V_{us}|$

$|V_{us}|$ determinations

  CKM unitarity

  $|V_{us}| \approx \sqrt{1 - |V_{ud}|^2}$ w/ $|V_{ud}|$ ['20 Hardy, Towner]

  $K_{\ell 2}$ decay [PDG22]

  $K_{\ell 3}$ decay Most accurate ['19 FNAL/MILC]

    Tensions among three $|V_{us}|$

    Independent and precise calculations are important.

$K_{\ell 3}$ form factors with two PACS10 configurations ['20 PACS, '22 PACS]
  $L \gtrsim 10$[fm] at very close to physical point
  Negligible finite L effect, tiny q^2 region, tiny chiral extrapolation
    Largest uncertainty form finite a effect

  This talk:
    Updated preliminary result with 3rd PACS10 configuration

2

Simulation parameters

PACS10 configurations: $L \gtrsim 10\,[\text{fm}]$ at very close to physical point

$N_f = 2 + 1$ six-stout-smeared non-perturbative $\mathcal{O}(a)$-improved Wilson action

$+$ Iwasaki gauge action

	$L^3 \cdot T$	$L[\text{fm}]$	$a[\text{fm}]$	$a^{-1}[\text{GeV}]$	$M_\pi[\text{MeV}]$	$M_K[\text{MeV}]$	N_{conf}	$t_{\text{sep}}[\text{fm}]$
PACS10/L256	256^4	10.5	0.041	4.792	142	514	20	3.1–4.0
PACS10/L160	160^4	10.2	0.063	3.111	137	501	20	2.3–4.1
PACS10/L128	128^4	10.9	0.085	2.316	135	497	20	3.4–3.9

(1000–2500 measurements in each t_{sep})

All the results on PACS10/L256 are preliminary.

$|V_{us}|$ using $|V_{us}|f_+(0) = 0.21635(39)$ ['22 Seng *et al.*]

$K_{\ell 3}$ form factors $f_+(q^2), f_0(q^2)$ from 3-point function

w/ $Z(2) \otimes Z(2)$ random source spread in L^3, color, spin ['08 RBC-UKQCD]

random smeared source also adopted in $L = 160, 256$ calculations

$$C_{V_\mu}(t, p) = \langle 0|O_K(t_{\text{sep}}, \mathbf{0})V_\mu(t, \mathbf{p})O_\pi^\dagger(0, \mathbf{p})|0\rangle$$

$V_\mu : \begin{cases} \text{Local vector current with } Z_V \\ \text{Conserved vector current} \end{cases}$

$$\langle \pi(p)|V_\mu|K(0)\rangle = (p_K + p_\pi)_\mu f_+(q^2) + (p_K - p_\pi)_\mu f_-(q^2)$$

$$f_0(q^2) = f_+(q^2) - \frac{q^2}{M_K^2 - M_\pi^2}f_-(q^2)$$

$p_K = (M_K, \vec{0}), p_\pi = (E_\pi, \vec{p})$

$q^2 = -(M_K - E_\pi)^2 + p^2$

Program for Promoting Researches on the Supercomputer Fugaku

Large-scale lattice QCD simulation and development of AI technology

Resources: Fugaku in HPCI System Research Project

(hp200062, hp200167, hp210112, hp220079, hp230199, hp240207)

3

Calculation method w/ local vector current

2-point function* $X = \pi, K$

$$C_X(t, p) = \langle 0|O_X(t, \mathbf{p})O_X^\dagger(0, \mathbf{p})|0\rangle = \frac{Z_X^2}{2E_X}\left(e^{-E_X t} + e^{-E_X(2T-t)}\right) + \cdots$$

$$p \equiv |\mathbf{p}| = \left|\frac{2\pi}{L}\mathbf{n}\right|, \quad |\mathbf{n}| = 0\text{--}6 \text{ with periodic boundary}$$

3-point function*

$$C_{V_\mu}(t, t_{\sf sep}, p) = \langle 0|O_K(t_{\sf sep}, \mathbf{0})V_\mu(t, \mathbf{p})O_\pi^\dagger(0, \mathbf{p})|0\rangle = \frac{Z_\pi Z_K}{Z_V}\frac{M_\mu(p)}{4E_\pi M_K}e^{-E_\pi t}e^{-M_K(t_{\sf sep}-t)} + \cdots$$

*Averaging ones with periodic, anti-periodic temporal boundary conditions
reducing wrapping around effect in 3pt, and doubling periodicity in 2pt

$Z_V = 1/\sqrt{F_\pi^{\sf bare}(0)F_K^{\sf bare}(0)}$ determined w/ electromagnetic form factor $F_{\pi,K}(0) = 1$

Ratio $(0 \ll t \ll t_{\sf sep})$

$$\frac{Z_\pi Z_K Z_V C_{V_\mu}(t, t_{\sf sep}, p)}{C_\pi(t, p)C_K(t_{\sf sep} - t, 0)} = M_\mu(p) + \underline{A(p)e^{-\Delta_\pi(p)t} + B(p)e^{-\Delta_K(t_{\sf sep}-t)}}$$

1st excited state, π', K', contributions

$$\Delta_\pi(p) = E_{\pi'}(p) - E_\pi(p), \Delta_K = M_{K'} - M_K$$

Extract $M_\mu(p) = \langle\pi(p)|V_\mu|K(0)\rangle$ w/ fit including 1st excited states

Conserved current case: $V_\mu \to \widetilde{V}_\mu$ and $Z_V = 1$
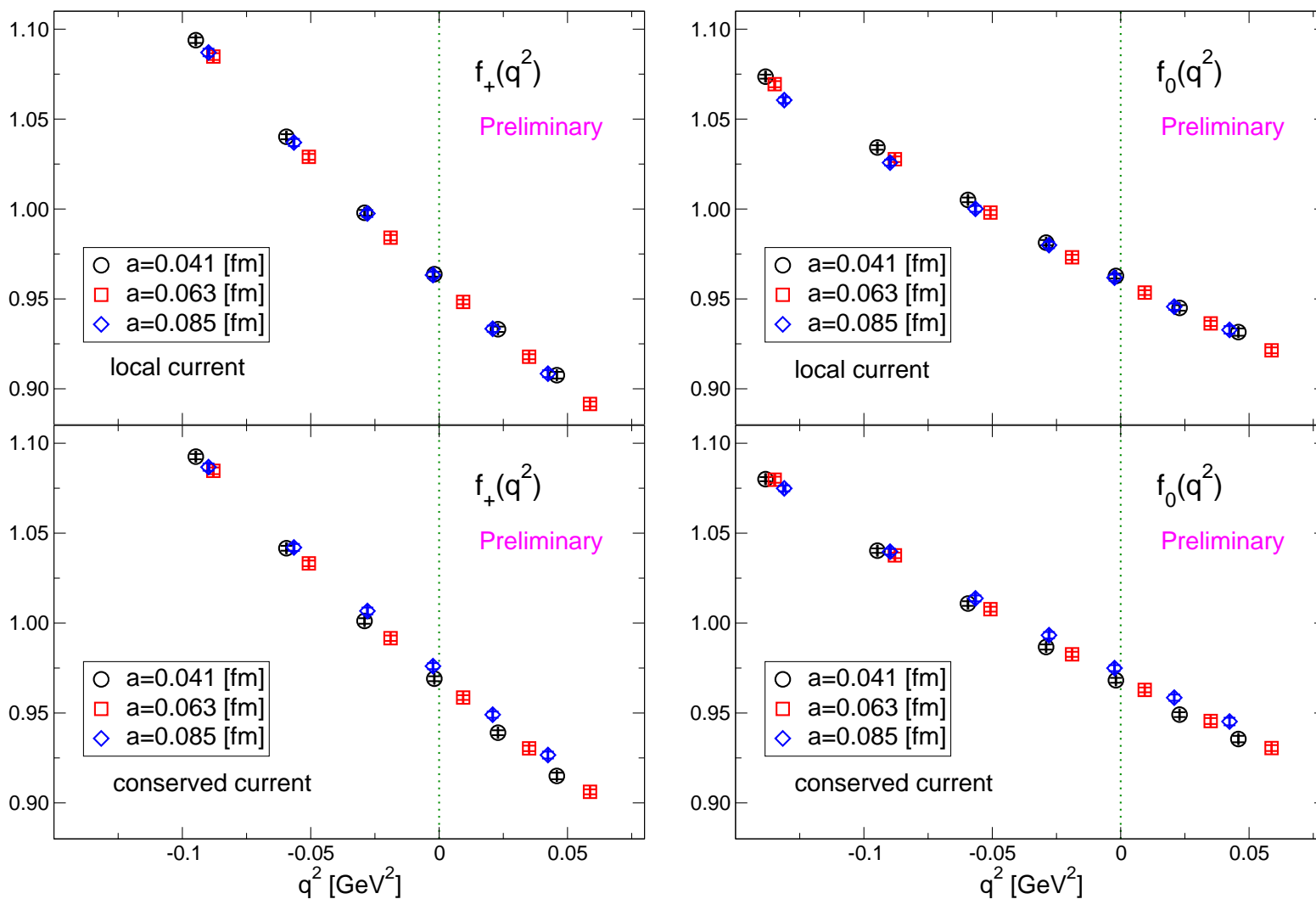
Form factors at each q^2 except for $p = 0$ (only $f_0(q^2)$)

$$M_\mu(p) = (p_K + p_\pi)_\mu f_+(q^2) + (p_K - p_\pi)_\mu f_-(q^2)$$

$$f_0(q^2) = f_+(q^2) - \frac{q^2}{M_K^2 - M_\pi^2}f_-(q^2)$$

$$p_K = (M_K, \vec{0}), p_\pi = (E_\pi, \vec{p})$$
$$q^2 = -(M_K - E_\pi)^2 + p^2$$

4

$f_+(q^2)$ and $f_0(q^2)$ at three lattice spacings

Clear signal at three lattice spacings

Several data in tiny q^2 region thanks to huge volume

Update from Lat23: Removing $\mathcal{O}(a^2)$ factor in conserved $f_+(q^2), f_0(q^2)$ due to point-splitting current

q^2 interpolation $+$ $a \to 0$ extrapolation

Fit based on SU(3) NLO ChPT with $f_+(0) = f_0(0)$ c.f.) PACS:PRD106,9,094501(2022)

$$f_+(q^2) = 1 - \frac{4}{F_0^2}L_9(\mu)q^2 + K_+(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_M\Delta M + c_2^+ q^4 + g_+^{cur}(a, q^2)$$

$$f_0(q^2) = 1 - \frac{8}{F_0^2}L_5(\mu)q^2 + K_0(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_M\Delta M + c_2^0 q^4 + g_0^{cur}(a, q^2)$$

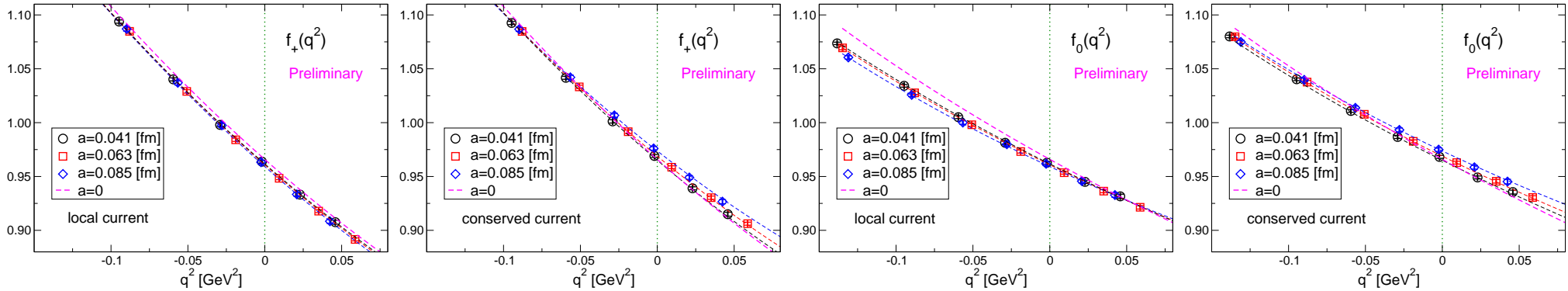K_+, K_0: known functions ['85 Gasser, Leutwyler], $\Delta M = (M_K^2 - M_\pi^2)^2$

$g_{+,0}^{cur} = \sum_{n,m} e_{+,0}^{cur,nm} a^n q^{2m}$, cur = local, conserved: a^2, a extrapolation investigated

free parameters: $L_5(\mu), L_9(\mu), c_0, c_2^+, c_2^0 + e_{+,0}^{cur,nm}$

fixed parameters: $\mu = 0.77$ GeV, $F_0 = 0.11205$ GeV $\qquad F_0$ estimated from FLAG $F^{SU(2)}/F_0$ w/ $F^{SU(2)} = 0.129$ GeV
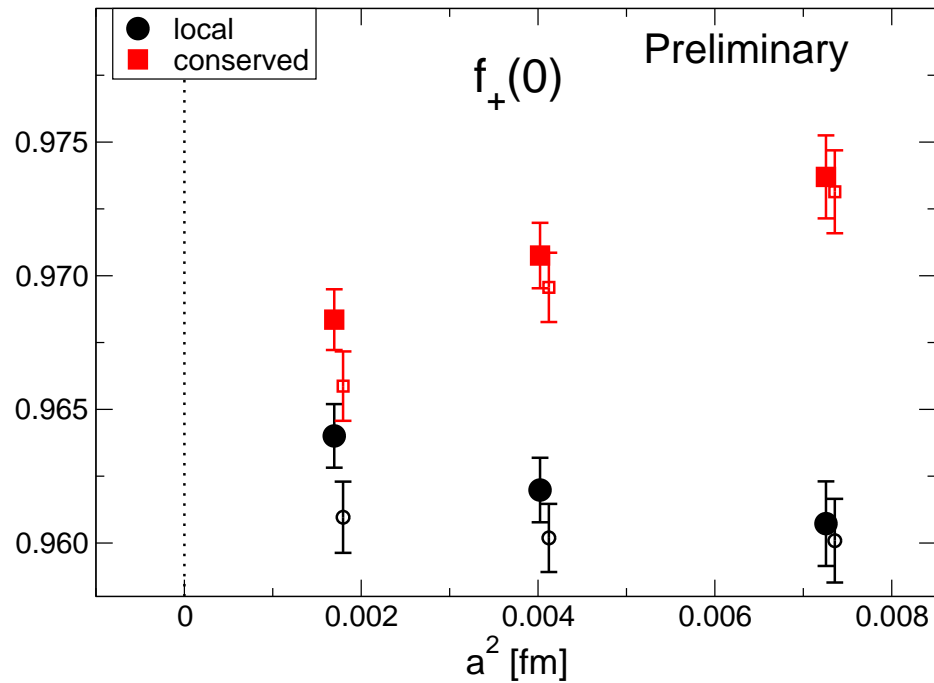

magenta dashed-line : $a = 0$ @ physical point

Simultaneous fit for (f_+, f_0) with (local,conserved)

Tiny extrapolation to physical M_{π^-} and M_{K^0} using same formulas

a[fm]	M_π[MeV]	M_K[MeV]
0.041	142	514
0.063	137	501
0.085	135	497

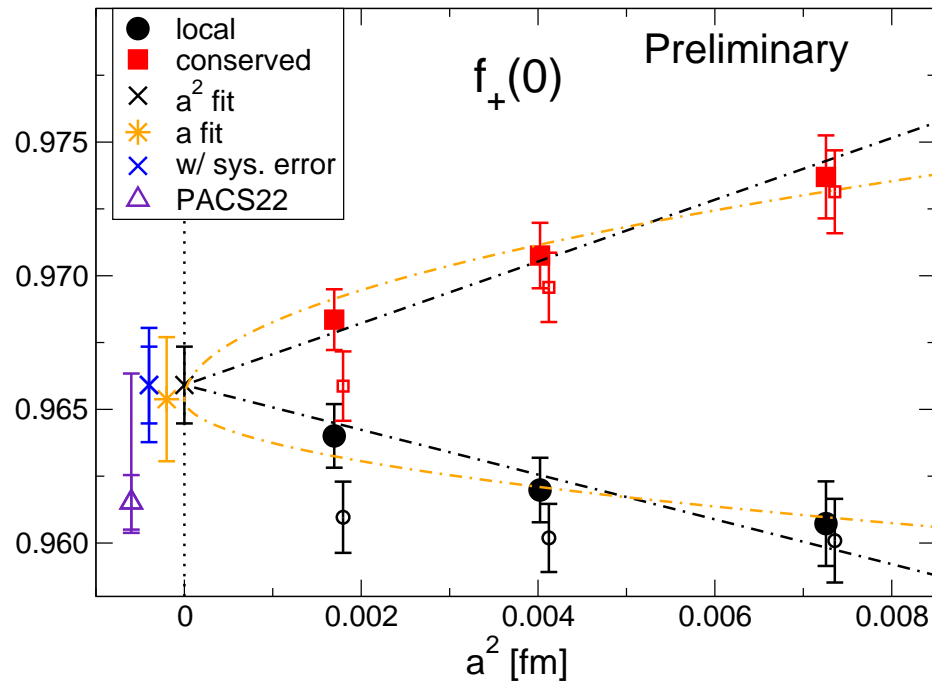$m_{\pi^-} = 139.57061$ MeV, $m_{K^0} = 497.611$ MeV

6

Continuum extrapolation of $f_+(0)$

closed (open) symbol: physical (simulation) point data

Chiral extrapolation: tiny upward shift especially at $a = 0.041$ fm

Continuum extrapolation of $f_+(0)$ (simultaneous fit result at $q^2 = 0$)

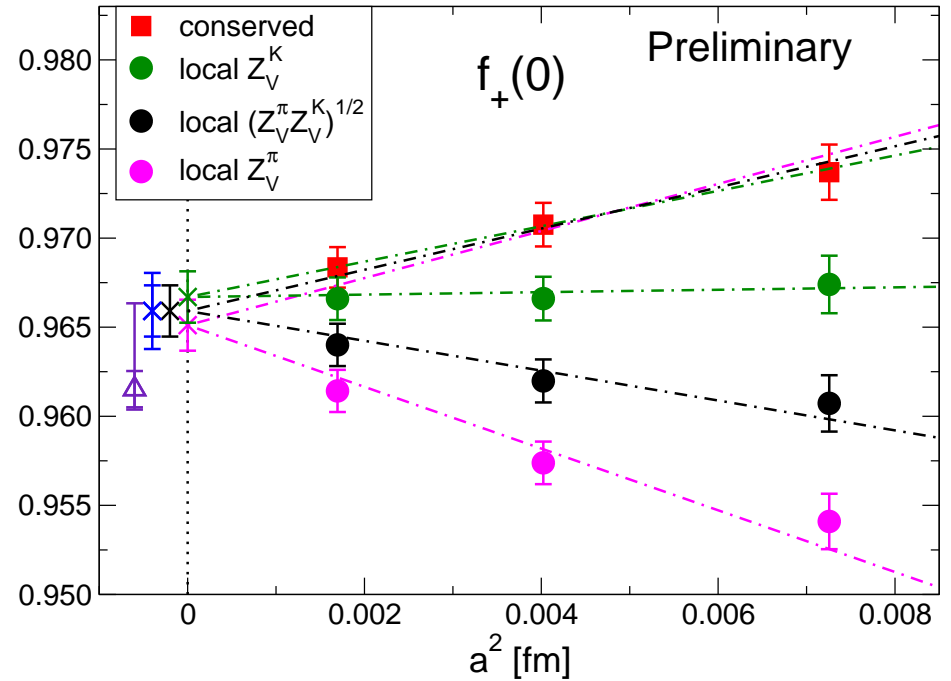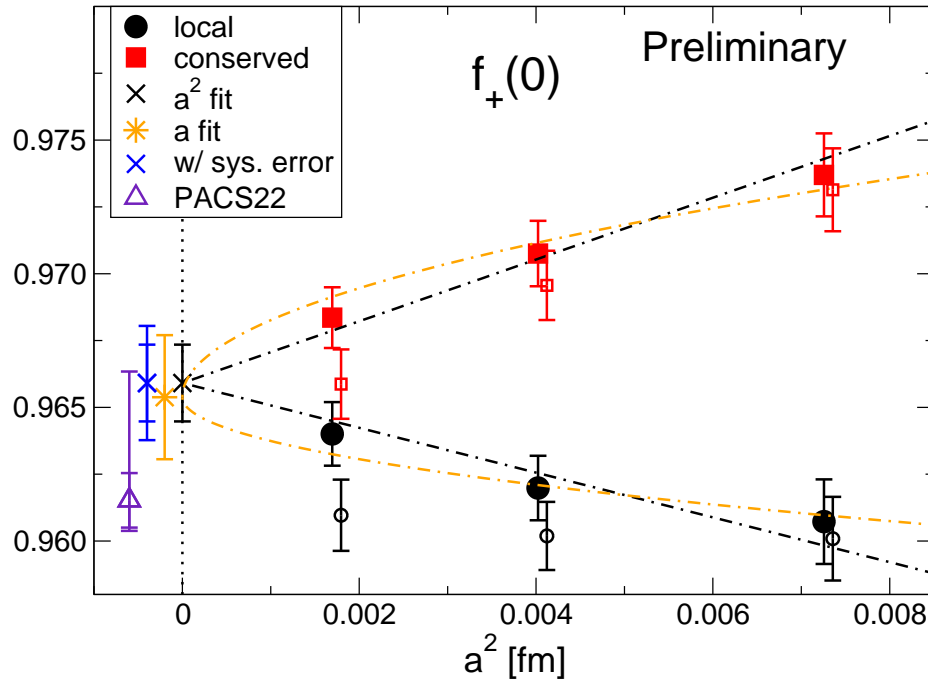closed (open) symbol: physical (simulation) point data

Chiral extrapolation: tiny upward shift especially at $a = 0.041$ fm

a^2 fit result agrees with a fit result and PACS22 result

Systematic error estimated various different analyses

e.g., isospin breaking effect w/ SU(3) ChPT and different Z_V in local data

Continuum extrapolation of $f_+(0)$ (simultaneous fit result at $q^2 = 0$)

Legend (left plot):
- local
- conserved
- a^2 fit
- a fit
- w/ sys. error
- PACS22

$f_+(0)$ Preliminary

Legend (right plot):
- conserved
- local Z_V^K
- local $(Z_V^\pi Z_V^K)^{1/2}$
- local Z_V^π

$f_+(0)$ Preliminary

a^2 [fm]

closed (open) symbol: physical (simulation) point data

$Z_V^K = 1/F_K^{\mathrm{bare}}(0), Z_V^\pi = 1/F_\pi^{\mathrm{bare}}(0)$

Chiral extrapolation: tiny upward shift especially at $a = 0.041$ fm

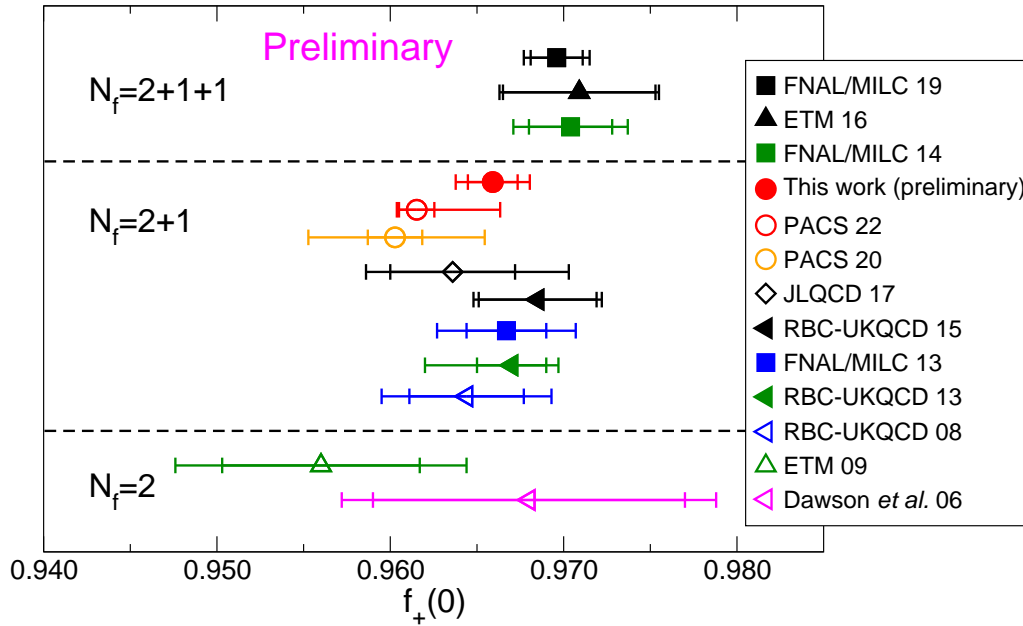a^2 fit result agrees with a fit result and PACS22 result

Systematic error estimated various different analyses

e.g., isospin breaking effect w/ SU(3) ChPT and different Z_V in local data

$\mathcal{O}(a)$-improved current $V_\mu^{\mathrm{imp}} = V_\mu + c_V \partial_\nu T_{\nu\mu}$ does not affect $f_+(0)$

$f_+^{\mathrm{imp}}(q^2) = f_+(q^2) - c_V q^2 f_T(q^2), \ f_0^{\mathrm{imp}}(q^2) = f_0(q^2)$ through $\langle \pi | T_{\nu\mu} | K \rangle = (q_\nu^+ q_\mu - q_\mu^+ q_\nu) f_T(q^2)$

$f_+(0)$ and $|V_{us}|$

inner, outer = statistical, total(stat.+sys.)   inner, outer = lattice, total(lat.+exp.)

grey band: Standard model prediction using $|V_{us}| \approx \sqrt{1 - |V_{ud}|^2}$ ['20 Hardy, Towner]

$f_+(0)$: Reasonably agree with previous lattice calculations $\lesssim 2\sigma$

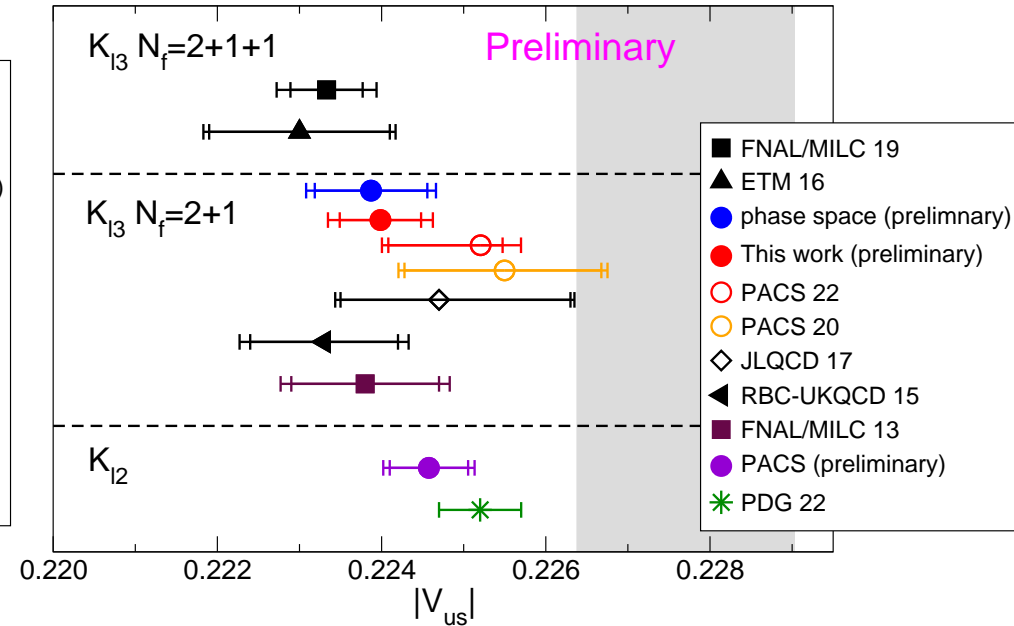$|V_{us}|$: from $|V_{us}| f_+(0) = 0.21635(39)$ ['22 Seng et al.]

agree with $|V_{us}|$ from K_{l2} using f_K/f_π $\dfrac{|V_{us}|}{|V_{ud}|}\dfrac{f_K}{f_\pi} = 0.27683(35)$

['19 Di Carlo et al.]

2σ difference from CKM unitarity (grey band)

consistent with $|V_{us}|$ from phase space integral

8

Summary

Preliminary result of $K_{\ell 3}$ form factors with 3rd PACS10 configuration

$a = 0.041$ fm, $L \gtrsim 10$[fm] very close to physical point

- Clear signal for $f_+(q^2), f_0(q^2)$ in tiny q^2 region

- Continuum extrapolation of $f_+(0)$ with local and conserved data

  Small, clear effect from chiral extrapolation

  Reasonably consistent with PACS22 and previous results

- $|V_{us}|$ from our preliminary result

  Reasonably consistent with previous $K_{\ell 3}$ determinations

  Consistent with $K_{\ell 2}$ determinations

  Different from CKM unitarity by 2 σ

Future works

Isosping breaking effect from lattice calculation

dynamical charm quark effect
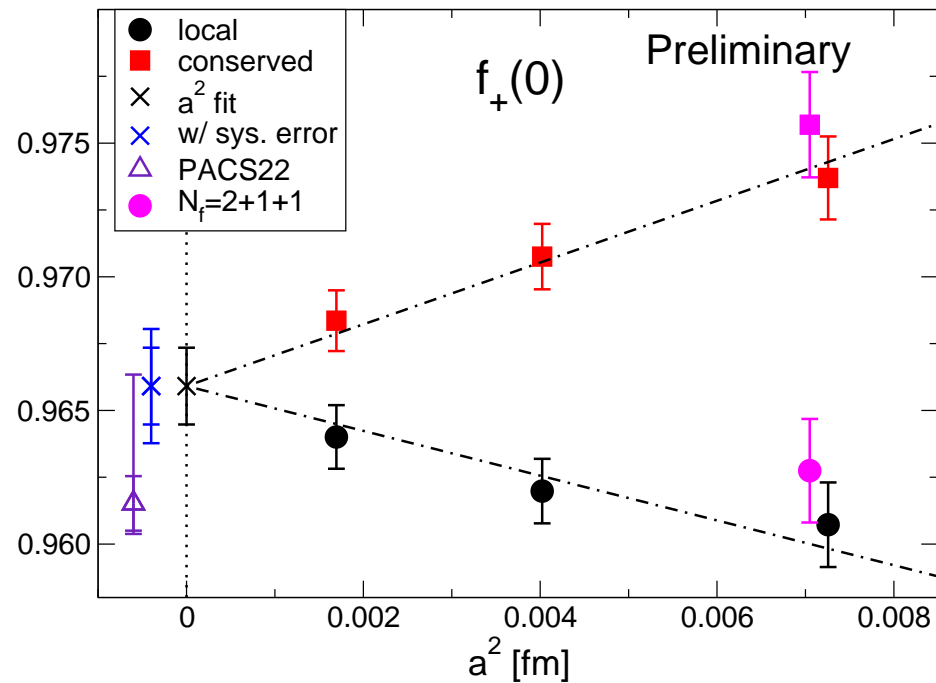
$N_f = 2 + 1 + 1$ simulation

PACS10$_c$ configuration generation

the same actions, but $N_f = 2 + 1 + 1$ quark
(very close to) physical point on more than $(10 \text{ fm})^4$ volume

at three lattice spacings

Preliminary PACS10$_c$/L128 result agrees PACS10/L128 result

Back up

Phase space integral I_K^ℓ
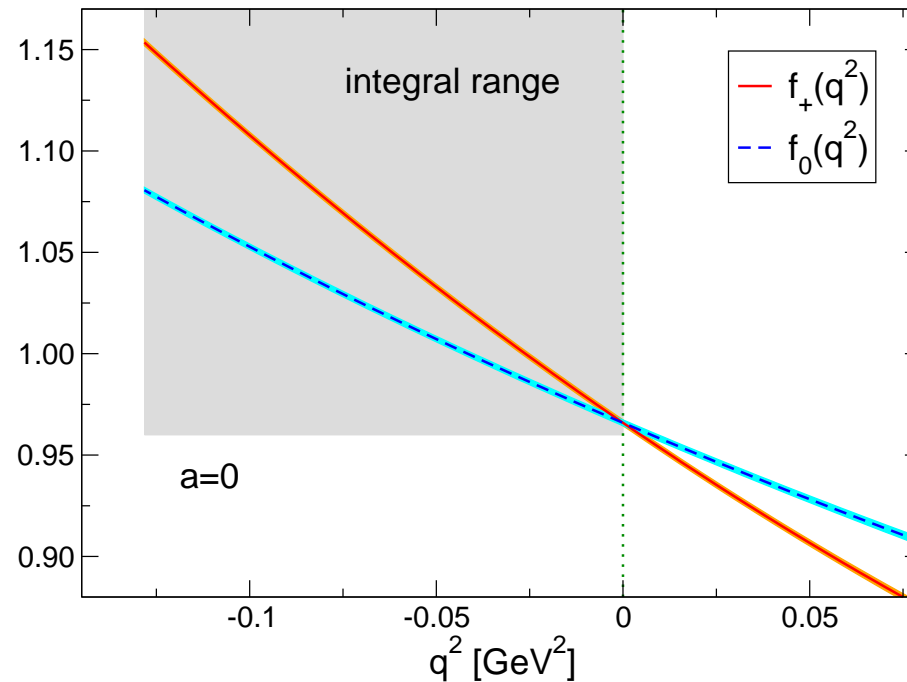
$$\Gamma_{K\ell 3} = C_{K\ell 3}(|V_{us}|f_+(0))^2 I_K^\ell \qquad \Gamma_{K_{\ell 3}}: \text{ decay width, } C_{K_{\ell 3}}: \text{ known factor, } \ell = e, \mu$$

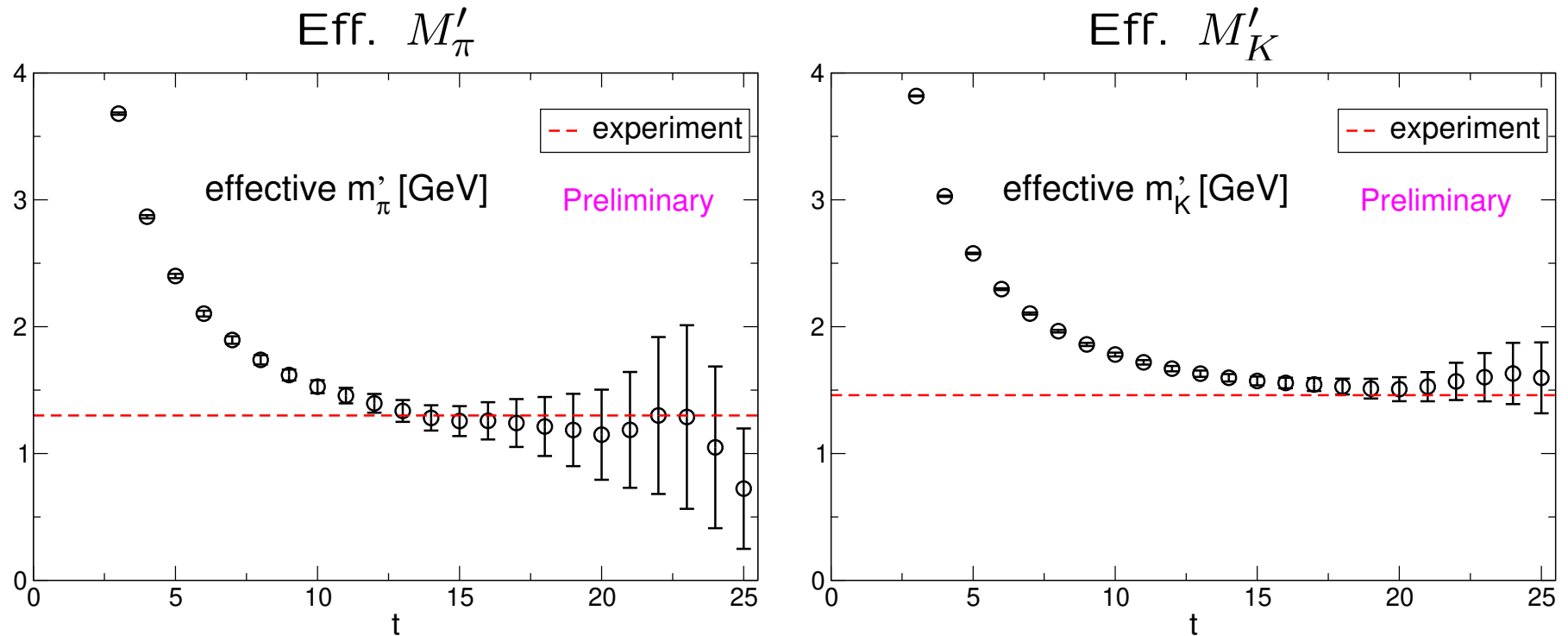$$|V_{us}|f_+(0) = 0.21654(41) \text{ ['17 Moulson]}$$

$$\leftarrow I_K^\ell \text{ from dispersive representation of experimental } \overline{F}_{+,0}(t)$$

$$I_K^\ell = \int_{m_\ell^2}^{(M_K - M_\pi)^2} dt \left(J_+(t)\overline{F}_+^2(t) + J_0(t)\overline{F}_0^2(t) \right), \quad \overline{F}_{+,0}(t) = \frac{f_{+,0}(-t)}{f_+(0)}$$

$$J_{+,0}(t): \text{ known function ['84 Leutwyler, Roos]}$$

Effective mass for 1st excited states π', K' on 256^4

Eff. M'_π

effective m'$_\pi$ [GeV]

Preliminary

--- experiment

Eff. M'_K

effective m'$_K$ [GeV]

Preliminary

--- experiment

Effective mass of 1st excited state (local source)

$$M'(t) = \log\left(\frac{C'(t)}{C'(t+1)}\right), \; C'(t) = C(t) - \underline{A_0 e^{-M_0 t}}$$

ground state contribution from fit in $t \gg 1$

Slope and curvature for $f_+(q^2)$ and $f_0(q^2)$