Enhanced Lattice Studies on ε_K and ΔM_K

Yikai Huo
RBC-UKQCD Collaborations

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Overview

Motivation

- Weak interaction is least understood section in Standard Model. The weak interaction naturally enables CP violation.
- Kaon is the lightest meson which contains a strange quark. ΔM_K is a very good candidate to test the validity of the Standard Model.
- Analytical methods focus on short distance part of ε_K. A more precise calculation of the long-distance part, ε_K(LD), is necessary.
- Both ΔM_K and ε_K(LD) are highly non-perturbative, good candidates for lattice calculation, having some overlap in calculation.

Previous Results

RBC-UKQCD collaboration has obtained a ΔM_K result with physical quark masses and long-distance contribution to ε_K with unphysical quark masses.

- "Long-distance contribution to ε_K from lattice QCD", Z. Bai, N. H. Christ, J. M. Karpie, C. T. Sachrajda, A. Soni, B. Wang, *Phys.Rev.D* 109 (2024) 5, 054501
- "Lattice calculation of the mass difference between the long- and short-lived K mesons for physical quark masses" Bigeng Wang, *Lattice 2021*

Kaon Mixing

Neutral Kaon Mixing

$$i\frac{d}{dt}\begin{pmatrix} |K^0(t)\rangle \\ |\bar{K}^0(t)\rangle \end{pmatrix} = \left(\begin{pmatrix} M & M_{0\bar{0}} \\ M^*_{0\bar{0}} & M \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma & \Gamma_{0\bar{0}} \\ \Gamma^*_{0\bar{0}} & \Gamma \end{pmatrix}\right)\begin{pmatrix} |K^0(0)\rangle \\ |\bar{K}^0(0)\rangle \end{pmatrix} \quad (1)$$

$$M_{0\bar{0}} = \mathcal{P}\sum_n \frac{\langle\overline{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle}{m_K - E_n}, \quad (2)$$

$$\Gamma_{0\bar{0}} = 2\pi\langle\overline{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle\delta(m_K - E_n) \quad (3)$$

Definition of ΔM_K and ε_K

$$\Delta M_K = 2\,\mathrm{Re}\,M_{0\bar{0}}, \quad (4)$$

$$\varepsilon_K = e^{i\phi_\epsilon}\sin\phi_\epsilon\left(\frac{-\,\mathrm{Im}\,M_{\bar{0}0}}{\Delta M_K} + \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right), \phi_\epsilon = \tan^{-1}\left(\frac{2\Delta M_K}{\Gamma_S - \Gamma_L}\right) = 43.51(5)^\circ. \quad (5)$$

Calculation of $M_{\bar{0}0}$

The $\Delta S = 2$ process can be treated using an effective Hamiltonian, which is the product of a Wilson coefficient and a local $\Delta S = 2$ operator O_{LL}:

$$O_{LL} = (\bar{s}d)_{V-A}(\bar{s}d)_{V-A}. \tag{6}$$

Two weak vertices contribute a factor:

$$\lambda_i = V_{id}V_{is}^*, \tag{7}$$

where $V_{qq'}$ is a CKM matrix element and i stands for three internal up-type quarks: $i = u, c, t$. The unitary property of CKM matrix ensures the orthogonality of its first and second columns:

$$\lambda_u + \lambda_c + \lambda_t = 0 \tag{8}$$

λ_c Elimination

Eliminating the factor λ_c, the modified effective Hamiltonian and the relation between new correction parameters η_i' and conventional ones η_i are shown as follows:

$$H_W^{\Delta S=2} = \frac{G_F^2}{16\pi^2} M_W^2 \left[\lambda_u^2 \eta_1' S_0(0,0,x_c) + \lambda_t^2 \eta_2' S_0(x_t,x_t,x_c) + 2\lambda_u\lambda_t \eta_3' S_0(x_t,0,x_c)\right] O_{LL} + \text{h.c.}$$

- λ_u^2 term has no imaginary part, main content for ΔM_k lattice calculation
- λ_t^2 term can be treated by perturbation theory
- $\lambda_u\lambda_t$ term is then the objective of the ε_K lattice calculation, specifically $c(u-c)$ part.

The effective weak Hamiltonian H_W

Calculate a bi-local product of two local $\Delta S = 1$ operators:
$\langle \overline{K}^0 | T \left\{ H_W^{\Delta S=1}(x) H_W^{\Delta S=1}(y) \right\} | K^0 \rangle$.

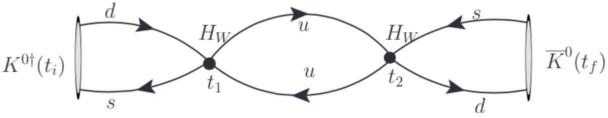

Figure 1: Example of Bi-local Structure Calculated on Lattice

Single Integrated Correlation Function on Lattice

We integrate the product of two $H_W^{\Delta S=1}$ over a time interval $[t-T, t+T]$:

$$\mathcal{A}^s(T) = \frac{1}{2}\sum_{t_2=t_a}^{t_b}\sum_{t_1=t+T}^{t-T}\langle 0|T\left\{\overline{K^0}(t_f)H_W(t_1)H_W(t)\overline{K^0}(t_i)\right\}|0\rangle. \quad (10)$$

inserting intermediate states:

$$\mathcal{A}^s = N_K^2 e^{-M_K(t_f-t_i)}\left\{\sum_n \frac{\langle\overline{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle}{M_K-E_n}\left(-1+e^{(M_K-E_n)(T+1)}\right)\right\}, \quad (11)$$

Intermediate states $|n\rangle = |0\rangle, |\pi\rangle, |\pi\pi\rangle, |\eta\rangle$ which have $E_n < M_K$ or $E_n \simeq M_K$ will contribute exponentially increasing terms.
For $|0\rangle$ and $|\eta\rangle$, we add two operators $c_s\overline{s}d$ and $c_p\overline{s}\gamma_5 d$ to Hamiltonian to subtract their contribution:

$$\langle 0|H_W - c_p\overline{s}\gamma_5 d|K^0\rangle = 0, \langle\eta|H_W - c_s\overline{s}d|K^0\rangle = 0 \quad (12)$$

For $|\pi\rangle$ and $|\pi\pi\rangle$, we calculate correlation functions $\langle\overline{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle$ and subtract them.

The effective weak Hamiltonian H_W for ΔM_K

$$H_W^{\Delta S=1} = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} V^*_{q's} V_{qd} \left(C_1 Q_1^{q'\bar{q}} + C_1 Q_2^{q'\bar{q}} \right) \tag{13}$$

where the $Q_1^{qq'}$ and $Q_2^{qq'}$ are current-current operators:

$$Q_1^{q'\overline{q}} = (\overline{s}_a q'_b)_{V-A} (\overline{q}_b d_a)_{V-A}, \tag{14}$$

$$Q_2^{q'\overline{q}} = (\overline{s}_a q'_a)_{V-A} (\overline{q}_b d_b)_{V-A}. \tag{15}$$

We also add another two operators $c_{s,i}\overline{s}d$ and $c_{p,i}\overline{s}\gamma_5 d$ to remove unphysical contributions:

$$\begin{aligned} \langle 0 | Q_i - c_{p,i}\overline{s}\gamma_5 d | K^0 \rangle &= 0, \\ \langle \eta | Q_i - c_{s,i}\overline{s}d | K^0 \rangle &= 0 \end{aligned} \tag{16}$$

$$Q'_i = Q_i - c_{s,i}\overline{s}d - c_{p,i}\overline{s}\gamma_5 d \tag{17}$$

The effective weak Hamiltonian H_W for ε_K

$$H_W^{\Delta S=1} = \frac{G_F}{\sqrt{2}} \left(\sum_{q,q'=u,c} V_{q's}^* V_{qd} \sum_{i=1,2} C_i Q_i^{q'\bar{q}} - \lambda_t \sum_{i=3}^{6} C_i Q_i \right) \tag{18}$$

where the $Q_1^{qq'}$ and $Q_2^{qq'}$ are current-current operators and $Q_i, 3 \leq i \leq 6$ are QCD penguin operators:

$$Q_1^{q'\bar{q}} = (\bar{s}_a q'_b)_{V-A} (\bar{q}_b d_a)_{V-A}, \tag{19}$$

$$Q_2^{q'\bar{q}} = (\bar{s}_a q'_a)_{V-A} (\bar{q}_b d_b)_{V-A}, \tag{20}$$

$$Q_3 = (\bar{s}_a d_a)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_b q_b)_{V-A}, \tag{21}$$

$$Q_4 = (\bar{s}_a d_b)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_b q_a)_{V-A}, \tag{22}$$

$$Q_5 = (\bar{s}_a d_a)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_b q_b)_{V+A}, \tag{23}$$

$$Q_6 = (\bar{s}_a d_b)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_b q_a)_{V+A}. \tag{24}$$

Four-point Diagrams

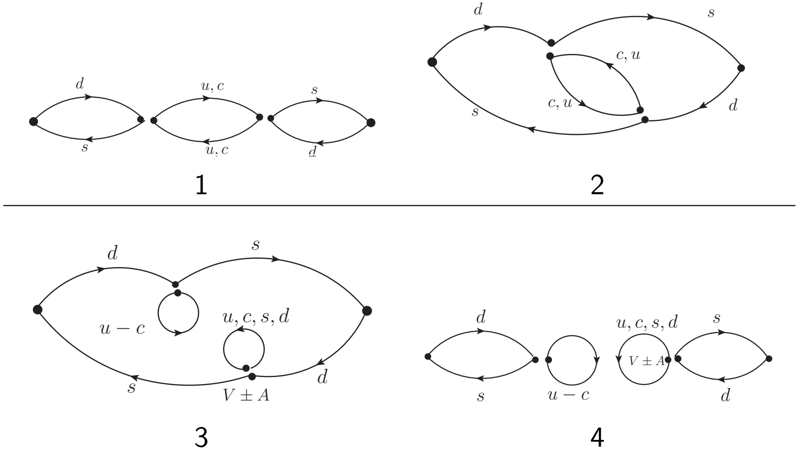

Figure 2: Type-1,2,3 and 4 Diagrams for ΔM_K and ε_K. For ΔM_K, inner quark lines only involve $u - c$ and $(V - A)$ structure. For ε_K, inner quark lines have more combinations of flavors and vertexes

Four-point Diagrams

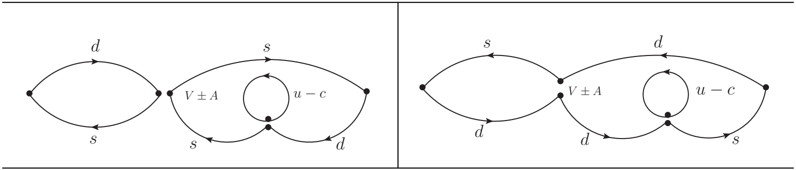

Figure 3: Type-5 4-point diagrams, ε_K only. A current-current operator at one vertex, the other from a penguin operator $(\bar{s}d)_{V-A}(\bar{d}d)_{V\pm A}$ or $(\bar{s}d)_{V-A}(\bar{s}s)_{V\pm A}$.

Short-Distance Divergence and Renormalization

Divergence occurs when two operators coincide with each other.

- GIM mechanism in inner quark lines of ΔM_K removes both quadratic and logarithmic divergences

$$\begin{aligned}&\int d^4p\gamma^\mu(1-\gamma^5)(\frac{\not{p}-m_c}{p^2+m_c^2}-\frac{\not{p}-m_u}{p^2+m_u^2})\gamma^\nu(1-\gamma^5)(\frac{\not{p}-m_c}{p^2+m_c^2})\\ =&\int d^4p\gamma^\mu(1-\gamma^5)\frac{\not{p}(m_u^2-m_c^2)}{(p^2+m_u^2)(p^2+m_c^2)}\gamma^\nu(1-\gamma^5)(\frac{\not{p}(m_u^2-m_c^2)}{p^2+m_c^2})\end{aligned} \tag{25}$$

- For $\varepsilon_K(\mathrm{LD})$, a logarithmic divergence occurs when two operators coincide with each other

$$\begin{aligned}&\int d^4p\gamma^\mu(1-\gamma^5)(\frac{\not{p}-m_c}{p^2+m_c^2}-\frac{\not{p}-m_u}{p^2+m_u^2})\gamma^\nu(1-\gamma^5)(\frac{\not{p}-m_c}{p^2+m_c^2})\\ =&\int d^4p\gamma^\mu(1-\gamma^5)\frac{\not{p}(m_u^2-m_c^2)}{(p^2+m_u^2)(p^2+m_c^2)}\gamma^\nu(1-\gamma^5)(\frac{\not{p}}{p^2+m_c^2})\end{aligned} \tag{26}$$

Short-Distance Divergence and Renormalization of ε_K

- This short-distance divergence can be removed by adding a counter term which is the product of a coefficient and the local operator O_{LL}. Currently, the explicit form has been calculated in the $\overline{\mathrm{MS}}$ scheme

$$\mathcal{H}^{\Delta S=2}_{W,ut} = \frac{G_F^2}{2}\lambda_u\lambda_t \sum_{i=1,2}\left\{\sum_{j=1,6}\int d^4x C_i^{\overline{\mathrm{MS}}} C_j^{\overline{\mathrm{MS}}}[[\widetilde{Q}_i^{\overline{\mathrm{MS}}}(x)\widetilde{Q}_j^{\overline{\mathrm{MS}}}(0)]]^{\overline{\mathrm{MS}}} + C_{7i}^{\overline{\mathrm{MS}}} O_{LL}^{\overline{\mathrm{MS}}}(0)\right\} \tag{27}$$

- In a lattice calculation, this short-distance correction is usually implemented by the regularization-independent(RI/SMOM) method
- We need to bridge the regularization-independent scheme with the $\overline{\mathrm{MS}}$ scheme to obtain the appropriate value.

Renormalized Matrix Elements of ε_K

The final form of the matrix elements calculated on lattice is shown as follows:

$$\begin{aligned}\mathcal{H}_{W,ut}^{\Delta S=2} &= \frac{G_F^2}{2}\lambda_u\lambda_t\sum_{i=1}^{2}\left\{\sum_{j=1}^{6}C_i^{\text{Lat}}C_j^{\text{Lat}}\left(\sum_x[[\widetilde{Q}_i^{\text{Lat}}(x)\widetilde{Q}_j^{\text{Lat}}(0)]]^{\text{Lat}}-X_{ij}^{\text{Lat}}(\mu_{\text{RI}})O_{LL}^{\text{Lat}}(0)\right)\right.\\ &\quad+\left(\sum_{j=1}^{6}C_i^{\overline{\text{MS}}}C_j^{\overline{\text{MS}}}\Delta Y_{ij}^{\overline{\text{MS}}}(\mu_{\overline{\text{MS}}},\mu_{\text{RI}})\right)Z_{LL}^{\text{Lat}\to\overline{\text{MS}}}O_{LL}^{\text{Lat}}(0)\\ &\quad\left.+\left(C_{7i}^{\overline{\text{MS}}}+\sum_{j=1}^{6}C_i^{\overline{\text{MS}}}C_j^{\overline{\text{MS}}}Y_{ij}^{\overline{\text{MS}}}(\mu_{\overline{\text{MS}}},0)\right)Z_{LL}^{\text{Lat}\to\overline{\text{MS}}}O_{LL}^{\text{Lat}}(0)\right\}. \qquad (28)\end{aligned}$$

- The first term removes the divergence from lattice calculation by imposing a RI/SMOM condition.
- The second line stands for matching between the energy scales in the $\overline{\text{MS}}$ scheme.
- The last line stands for the counter term established in the $\overline{\text{MS}}$ scheme and the NNLO matching term from the $\overline{\text{MS}}$ scheme to the RI/SMOM scheme.

Lattice Related Techniques

- Correct finite volume effect of low-energy two-pion intermediate states.
  "Effects of finite volume on the $K_L - K_S$ mass difference",
  N. H. Christ, X. Feng, G. Martinelli, and C. T. Sachrajda, *Phys. Rev. D*, 91(2015), 114510
- More precisely dealing with $\pi\pi$ state, multiple-state fit or GEVP. (We follow the method in "ΔI=3/2 and ΔI=1/2 channels of K$\to\pi\pi$ decay at the physical point with periodic boundary conditions," T. Blum,P. Boyle, D. Hoying, T. Izubuchi, L. Jin, C. JungC. Kelly, C. Lehner, A. Soni, Amarjit and M. Tomii, *PhysRevD*.108(2023).094517).
- Sample all-mode averaging(AMA) method: calculate exact and sloppy propagators with different precision to reduce the computational cost.

Lattice Resources

- Calculations performed on two sets of configurations with physical quark mass
- Frontier: 9,408 nodes, Each Node: 64-core CPU+8GPUs
  - 50k node hours for 64I: 40 exact and 120 sloppy calculation
  - 400k node hours for 96I: 20 exact and 60 sloppy calculation
- Grid: C++ library harnessing the matrix computation potential of the GPU

Name	Action	$a^{-1}(\mathrm{GeV})$	Volume	$m_\pi(\mathrm{MeV})$	Size(fm)
64I	MDWF+I	$2.359(7)$	$64^3 \times 128 \times 12$	139	5.4
96I	MDWF+I	2.708	$96^3 \times 192 \times 12$	140	6.9

Table 1: Dynamical 2+1 flavor domain wall fermion lattices to be used in our calculation, MDWF = Mobius domain wall fermions, I = Iwasaki gauge action.