Updates on anisotropic pure gauge ensembles with HISQ

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Motivation

- At $T_c = 156$ MeV : Crossover from confined phase into Quark-Gluon Plasma (QGP)
- Suppression of heavy quarkonia yields in heavy-ion collisions indicates formation of QGP
- At finite T the "melting" of such states can be studied through the broadening of peaks of the spectral function

Motivation

- Definition of spectral function $\rho(\omega, \vec{p})$ through real time 2-point correlation functions:

$$\rho(\omega, \vec{p}) = \frac{1}{2\pi}\left[D^{>}(\omega, \vec{p}) - D^{<}(\omega, \vec{p})\right] = \frac{1}{\pi} D^R(\omega, \pi)$$

  It is the link between real and imaginary time (Euclidean) correlation functions

- Euclidean 2-point correlation functions:

$$G(\tau, \vec{p}) = \int_0^\infty d\omega \rho(\omega, \vec{p}) K(\omega, \tau)\,, \quad K(\omega, \tau) = \frac{\cosh\left(\omega T - \omega/2T\right)}{\sinh(\omega/2T)}$$

- Having measured $G(\tau, \vec{p})$ at temperature T for the desired quarkonium, one can then invert this integral transformation to reconstruct $\rho(\omega, \vec{p})$

Motivation

$T = 1/(aN_\tau)$, so we have to achieve two things simultaneously:

- Have large number of temporal direction points N_τ for so that reconstruction of $\rho(\omega, \vec{p})$ from $G(\tau, \vec{p})$ is better-posed
- Keep temperature high enough (around T_c) without needing to resort to expensive fine lattices

Anisotropic lattices: $a_\tau = a_s/\xi$
$\xi = 1, 2, \ldots, 8 \quad a_s = 0.08, \ldots, 0.20$ fm

Goal: Achieve relevant temperatures with $N_\tau \simeq 50$ or even $N_\tau \simeq 100$

Motivation

- Anisotropic simulations with Wilson quarks for spectral reconstruction have been performed (see e.g. Aarts *et. al.* 1703.09246)
- Ensembles with staggered quarks are cheaper to generate and tune, giving access to significantly larger amount of statistics
- Isotropic HISQ (Highly Improved Staggered Quarks) ensembles have been used for quarkonium spectral reconstruction (see e.g. Larsen *et. al.* 1910.07374)
- Our goal: Large-N_τ anisotropic HISQ (aHISQ) ensembles for more reliable reconstruction of quarkonium spectral functions

Anisotropic Pure Gauge Ensembles: Generation

Tree-level Symanzik improved gauge action with anisotropy:

$$S_g = \beta \frac{1}{\xi_g^{(0)}} \left[\mathcal{P}_{ss} + c_{rt} \mathcal{R}_{ss} \right] + \beta \xi_g^{(0)} \left[\mathcal{P}_{st} + c_{rt} \mathcal{R}_{st} \right]$$

$\mathcal{P}$ and $\mathcal{R}$ represent sums of plaquettes and $(2 \times 1, 1 \times 2)$ rectangles respectively, in spatial-spatial (ss) or spatial-temporal orientation (st)

$\xi_g^{(0)}$ is the bare gauge anisotropy. The gauge anisotropy has to be tuned simultaneously with the gauge coupling to achieve a predefined $\xi_g = \xi = a_s/a_\tau$

Anisotropic Pure Gauge Ensembles: ξ_g, a_s tuning

- Simultaneous tuning of the gauge anisotropy and lattice spacing is performed with a method introduced by Borsanyi *et. al.* (1205.0781 and 1802.07718)
- The method is based on the w_0 scale of Gradient Flow (Lüscher 1006.4518)

Anisotropic Pure Gauge Ensembles: ξ_g, a_s tuning

Gradient Flow for gauge links with anisotropy:

$$\frac{\partial U_{x,\mu}}{\partial t} = -\sum_{\nu \neq \mu} \rho_{\mu\nu} \mathcal{P}_{\mathcal{A}} \left[U_{x,\mu} S^{\dagger}_{x,\mu\nu} \right] U_{x,\mu}$$

- t is the flow time
- $\rho_{i4} = \xi_{gf}^2$ and the rest are 1. ξ_{gf} is the "flow anisotropy"
- $S_{x,\mu\nu}$ is the sum of staples attached to $U_{x,\mu}$.
- $\mathcal{P}_{\mathcal{A}}$ is a projector on anti-hermitean traceless matrices

Anisotropic Pure Gauge Ensembles: ξ_g, a_s tuning

- One defines $w_{0,s}$ and $w_{0,t}$ as:

$$\left[t\frac{d}{dt}t^2\langle E_{ss}\rangle\right]_{t=w_{0,s}^2} = 0.15 \quad \left[t\frac{d}{dt}t^2\langle E_{st}\rangle\right]_{t=w_{0,t}^2} = 0.15$$

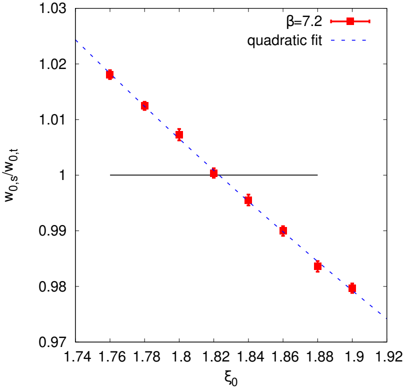
where:

$$E_{ss} = \frac{1}{4}\sum_{x,i\neq j} F_{ij}^2(x) \quad E_{st} = \frac{\xi_g^2}{2}\sum_{x,i} F_{ij}^2(x) \tag{1}$$

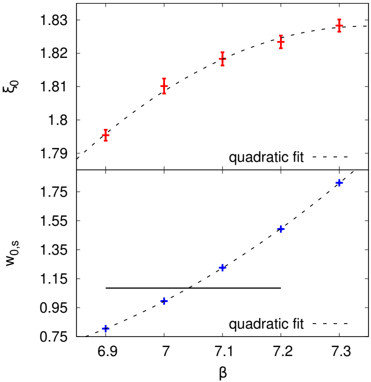
- One generates ensembles at fixed β and several $\xi_g^{(0)}$, applies the gradient flow at $\xi_{gf} = \xi_g$ on each one and finds the $\xi_g^{(0)}$ for which $w_{0,s} = w_{0,t}$. The corresponding ensemble has the desired anisotropy ξ_g but not the desired a_s, defined by $w_{0,s}$.
- One repeats for several values of β until the desired $w_{0,s}$ is achieved.

Anisotropic Pure Gauge Ensembles: ξ_g, a_s tuning

For $\xi = 2$, $a = 0.16$ fm:



(a) First step of tuning

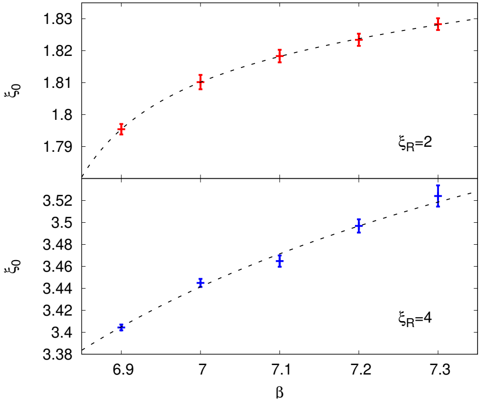


(b) Second step of tuning

Anisotropic Pure Gauge Ensembles: ξ_g, a_s tuning

Fitting $\xi_g^{(0)}(\beta)$ with a Padé formula:

$$\xi_g^{(0)}(\beta) = \xi_g \left(1 + \frac{10}{\beta}\ \frac{c1 + c2/\beta}{1 + c3/\beta}\right)$$



Anisotropic Pure Gauge Ensembles: ξ_g, a_s tuning

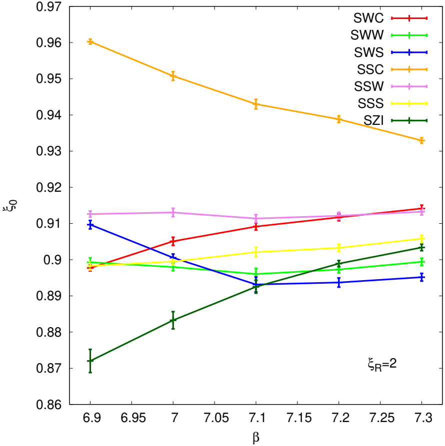
For the fit parameters we find:

$$\xi_g = 2: \quad c_1 = -0.0594 \pm 0.0004 \quad c_2 = 0.396 \pm 0.004 \quad c_3 = -6.73 \pm 0.01$$

$$\xi_g = 4: \quad c_1 = -0.06 \pm 0.03 \quad c_2 = 0.3 \pm 0.3 \quad c_3 = -6.1 \pm 0.8$$

Anisotropic Pure Gauge Ensembles: ξ_g, a_s tuning

- Various combinations of gradient flow staple and observable for the action density were explored.
- SWC scheme was preferred; It has larger discretization effects but correct monotonicity.

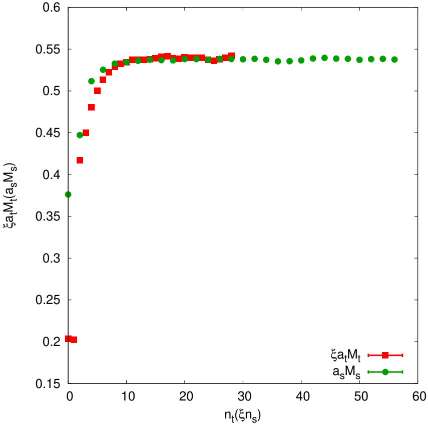


Anisotropic Quenched Spectrum: ξ_f, m_s tuning

- The bare quark anisotropy $\xi_f^{(0)}$ enters the quark action (HISQ$\rightarrow$aHISQ)
- ξ_f and quark masses have to be tuned simultaneously (target is $\xi_f = \xi_g = \xi$)
- In the quenched case, these can be tuned independently of ξ_g, a_s
- We aim at 2+1 dynamical simulations, with physical strange and fixed ratio $m_l = m_s/5$
- Strange is tuned through fictitious $\eta_{s\bar{s}}$ meson with mass 685.8 MeV (HPQCD 0910.1229). This leads to $\sim$ 300 MeV Goldstone pion on HISQ ensembles

Anisotropic Quenched Spectrum: ξ_f, m_s tuning

One can define the fermion anisotropy through the $\eta_{s\bar{s}}$ ground state energies in spatial and temporal directions as $\xi_f = a_s M_s / a_t M_t$. One can then adjust $\xi_f^{(0)}$ until $\xi_f = \xi_g = \xi$



$\xi = 2$, cw-cw,pt-pt

Anisotropic Quenched Spectrum: ξ_f, m_s tuning

Alternatively one can use the dispersion relation for $\eta_{s\bar{s}}$:

$$E^2(p) = M_t^2 + \frac{p^2}{\xi_f^2}$$

- The ground state energy M_t can be determined by $\vec{p} = 0$ correlator fits
- $\vec{p} \neq 0$ correlators yield values for $E(p)$. Then, by fitting the dispersion relation one obtains ξ_f
- One finally adjusts $\xi_f^{(0)}, m_s^{(0)}$ until $\xi_f = \xi_g, M_t = M_{\eta_{s\bar{s}}}$

Anisotropic Quenched Spectrum: ξ_f, m_s tuning

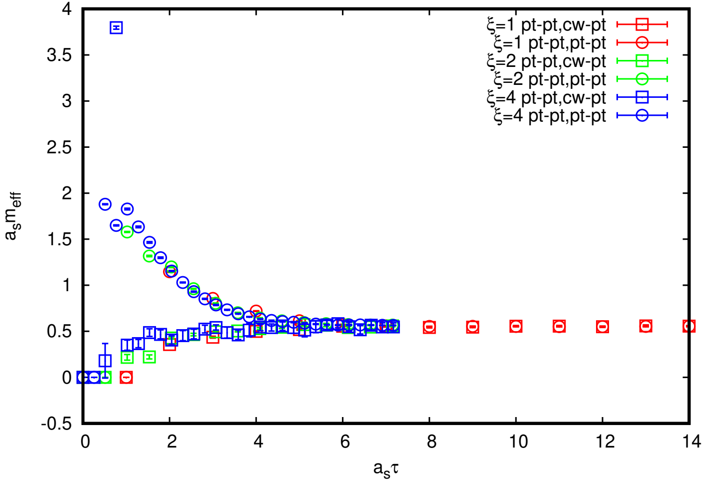
- Staggered fermions: Each fermion appears in a multiplet of 4 tastes
- Mesons therefore come in 16 tastes, with degenerate masses in continuum. On the lattice we get (spin×taste basis):

$$\begin{aligned}\gamma_5 \to & \gamma_5 \times \gamma_5,\ \gamma_5 \times \gamma_0\gamma_5,\ \gamma_5 \times \gamma_i\gamma_5, \\ & \gamma_5 \times \gamma_i\gamma_j,\ \gamma_5 \times \gamma_i\gamma_0,\ \gamma_5 \times \gamma_i, \\ & \gamma_5 \times \gamma_0, \gamma_5 \times 1\end{aligned}$$

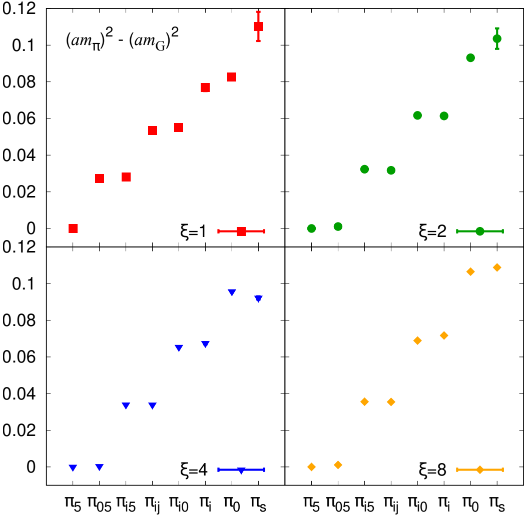
  (Golterman, 1985)
  These have nondegenerate masses at finite a
- In tuning we use the $\gamma_5 \times \gamma_5$ (Goldstone) meson taste (the lightest)

Anisotropic Quenched Spectrum: ξ_f, m_s tuning

Goldstone pion effective mass for different source/sink combinations:



Anisotropic Quenched Spectrum: Pion splittings



Summary/Outlook

- We have studied the anisotropic Gradient flow and tuned ξ_g, a_s for quenched ensembles with
  $\xi = 1, 2, \ldots, 8 \quad a_s = 0.08, \ldots, 0.20$ fm
- We have performed exploratory tuning of ξ_f, m_s for some ensembles, and studied the pion splittings

Next steps:

- Tune ξ_f, m_s for all these ensembles, study the pion splittings
- Start exploring dynamical 2+1 aHISQ
- Produce a library of aHISQ ensembles
- Apply relativistic or nonrelativistic valence quarks