Reconstruction of the vector meson propagator using a generalized eigenvalue problem

Fabian J. Frech for the BMW collaboration

Bergische Universität Wuppertal

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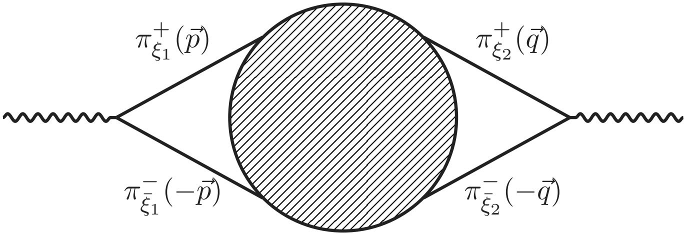
Introduction

- HVP contribution to $g_\mu - 2$ given by ρ



$\Rightarrow$ Use GEVP to extract $\pi\pi$-contributions and reconstruct the long-distance behavior **[1910.01083, 2406.19193, 2406.19194]**

The staggered symmetry group

- Clebsch-Gordan coefficients of the staggered symmetry group

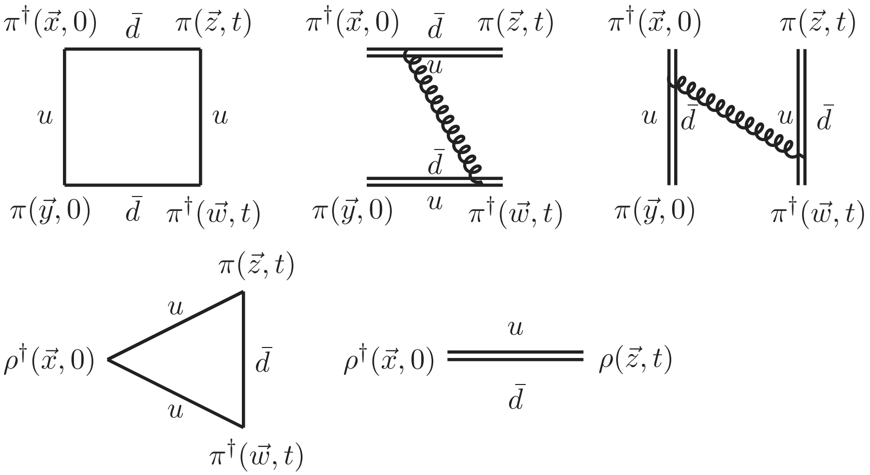
$$\mathbb{Z}_N^3 \rtimes \left[\Gamma^{4,1} \rtimes W_3\right]$$

- Two-pion state with the same quantum numbers as the ρ-meson

$$|s,m,\alpha\rangle_\rho = \sum_{(\vec{p}^a,\xi^a_\mu,\vec{p}^b,\xi^b_\mu)\in s} |\vec{p}^a,\vec{\xi}^a,\xi^a_4;\vec{p}^b,\vec{\xi}^b,\xi^b_4\rangle \langle \vec{p}^a,\vec{\xi}^a,\xi^a_4;\vec{p}^b,\vec{\xi}^b,\xi^b_4|s,m,\alpha\rangle_\rho$$

Derivation in **[2401.00514, 2112.11647]**

Propagators



Implementation

- Random wall sources ξ at t_0 and additional low-mode averaging for ρ
- Sequential inversion for the connected diagramm (square)

$$\langle M^{-1} O_t^B {\color{red}M^{-1}} O_t^B | M^{-1} O_0^A M^{-1} O_0^A \rangle$$

- Number of inversion:

$$T \times \#O \times \#\xi \quad \text{times}$$

Implementation (LMA for connected diagramm)

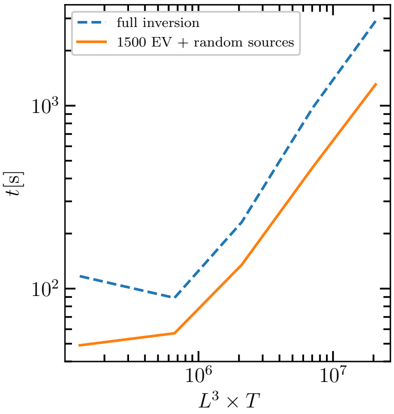
$$M^{-1} = \sum_{i=1}^{N_{\text{eig}}} \frac{1}{\lambda_i} |v_i\rangle \langle v_i| + M_{\text{rest}}^{-1}$$

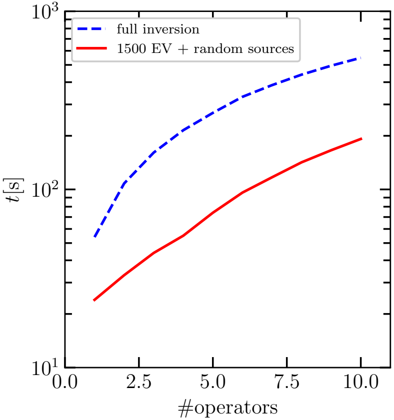
Get rid of T-factor in the total number of inversions

$$\langle M^{-1} O_t^B M^{-1} O_t^B | M^{-1} O_0^A M^{-1} O_0^A \rangle \to \sum_i \frac{1}{\lambda_i} \langle M^{-1} O_t^B |v_i\rangle \langle v_i| O_t^B | M^{-1} O_0^A M^{-1} O_0^A \rangle$$
$$+ \sum_\sigma \langle M^{-1} O_t^B |\sigma\rangle \langle \sigma| M_r^{-1} O_t^B | M^{-1} O_0^A M^{-1} O_0^A \rangle$$

Computational time of LMA and full inversion

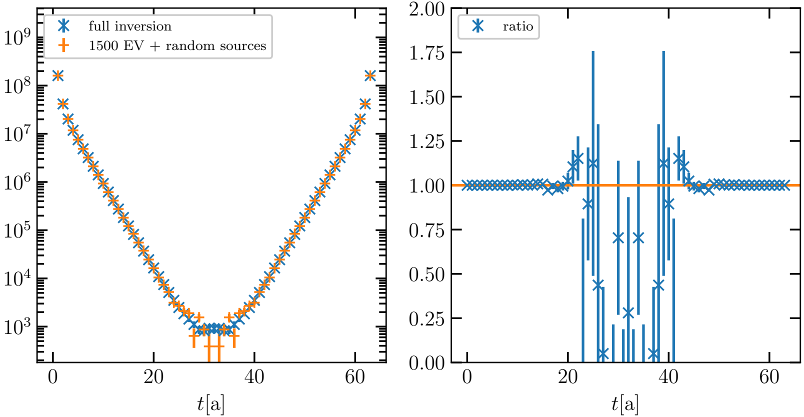
- 1500 eigenvectors, 18 wall sources, $T/2$ sources for the rest part





Comparision LMA vs full inversion

- $48^3 \times 64$ box, 1500 eigenvectors, 18 wall sources, $T/2$ sources for the rest part



Simulation setup

- $L^3 \times T = 48^3 \times 64$
- $a = 0.1315\,\mathrm{fm}$
- Staggered fermions with four steps of stout smearing
- 48 lattice configurations

- Vector meson with LMA **[2407.10913]**
- $\pi\pi$-orbits (with $E < m_\rho$):
  - $|\vec{p}| = 1$: pseudoscalar, axial vector and tensor taste
  - $|\vec{p}| = 2$: pseudoscalar and axial vector taste
  - $|\vec{p}| = 3$: pseudoscalar taste

Analysis strategy

- Solve GEVP to extract controbuting modes $a_i \exp\left(-m_i t\right)$

$$\mathbf{C}(t) v_i(t_0) = \lambda_i(t,t_0) \mathbf{C}(t_0) v_i(t_0)$$
$$\lambda_i(t,t_0) \propto \exp\left(-m_i(t-t_0)\right)$$

$$\mathbf{C}(t) = \left(\begin{array}{ccc|ccc}
C_{t-4}^{\rho\to\rho} & C_{t-3}^{\rho\to\rho} & C_{t-2}^{\rho\to\rho} & C_{t-2}^{\rho\to\pi\pi^1} & C_{t-2}^{\rho\to\pi\pi^2} & \dots \\
C_{t-3}^{\rho\to\rho} & C_{t-2}^{\rho\to\rho} & C_{t-1}^{\rho\to\rho} & C_{t-1}^{\rho\to\pi\pi^1} & C_{t-1}^{\rho\to\pi\pi^2} & \dots \\
C_{t-2}^{\rho\to\rho} & C_{t-1}^{\rho\to\rho} & C_{t}^{\rho\to\rho} & C_{t}^{\rho\to\pi\pi^1} & C_{t}^{\rho\to\pi\pi^2} & \dots \\
\hline
C_{t-2}^{\pi\pi^1\to\rho} & C_{t-1}^{\pi\pi^1\to\rho} & C_{t}^{\pi\pi^1\to\rho} & C_{t}^{\pi\pi^1\to\pi\pi^1} & C_{t}^{\pi\pi^1\to\pi\pi^2} & \dots \\
C_{t-2}^{\pi\pi^2\to\rho} & C_{t-1}^{\pi\pi^2\to\rho} & C_{t}^{\pi\pi^2\to\rho} & C_{t}^{\pi\pi^2\to\pi\pi^1} & C_{t}^{\pi\pi^2\to\pi\pi^2} & \dots \\
\dots & \dots & \dots & \dots & \dots & \dots
\end{array}\right)$$

Analysis strategy

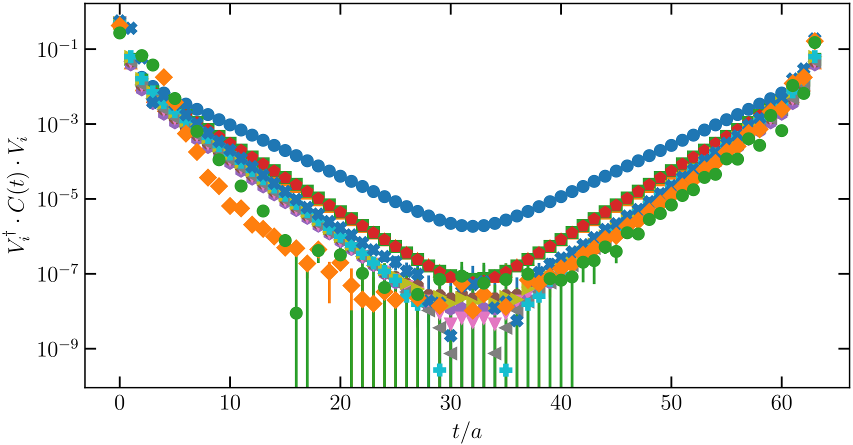
- Fit plateaus to effective masses
- Reconstruct the vector meson propagator

$$\tilde{\lambda}_i(t, t_0) = v_i^T(t_0)\mathbf{C}(t)v_i(t_0)$$

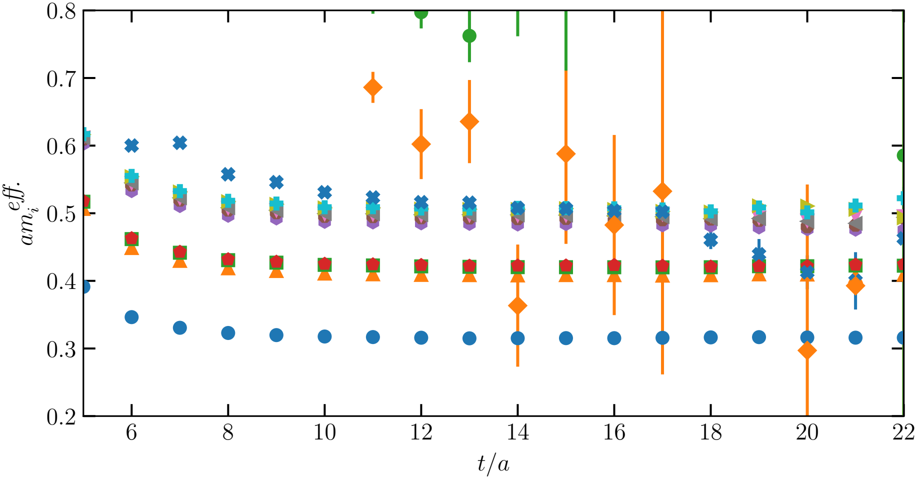
$$\rho_{\text{rec.}}(t) = \sum_i a_i \exp\left(-m_i t\right)$$

$$a_i^{\text{eff.}}(t) = \frac{\left(v_i^T \cdot \mathbf{C}_{\rho\bullet}(t)\right)^2}{v_i^T \cdot \mathbf{C}(t) \cdot v_i} \exp\left(m_i^{\text{eff.}}(t) \cdot t\right)$$

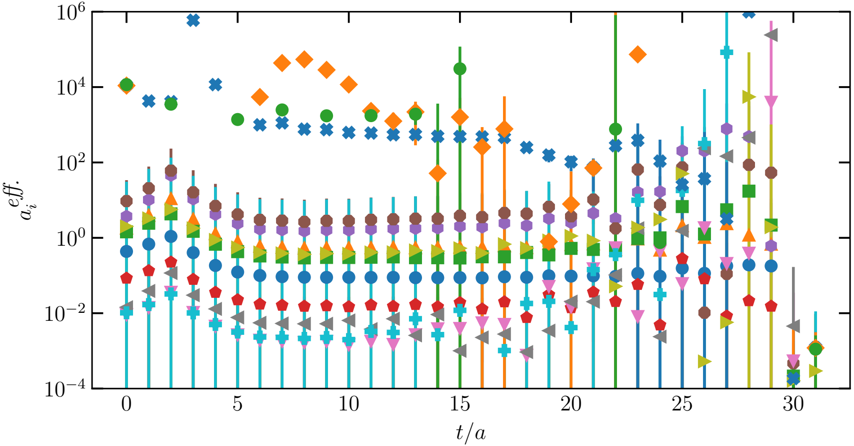
Reconstructed states



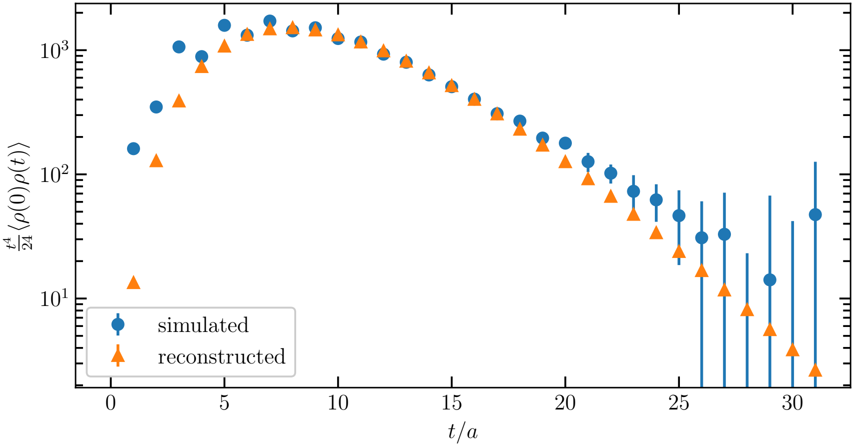
Effective energies of reconstructed states



Plateau fits of the contributions



Reconstruction of the vector meson propagator



Conclusion

Application of the formalism from **[2401.00514]**

Efficient method to compute the connected diagram for two pions

GEVP and reconstruction have to be investigated further

Appendix

Multiplicites of the $\pi\pi$-states

	$\|\vec{p}\|^2 = 0$	$\|\vec{p}\|^2 = 1$	$\|\vec{p}\|^2 = 2$	$\|\vec{p}\|^2 = 3$	$\|\vec{p}\|^2 = 4$
$\|\vec{\xi}\|^2 = 0$	0	1	1	1	1
$\|\vec{\xi}\|^2 = 1$	0	2	3	2	2
$\|\vec{\xi}\|^2 = 2$	0	2	3	2	2
$\|\vec{\xi}\|^2 = 3$	0	1	1	1	1