SymEFT for local tastes of staggered lattice QCD

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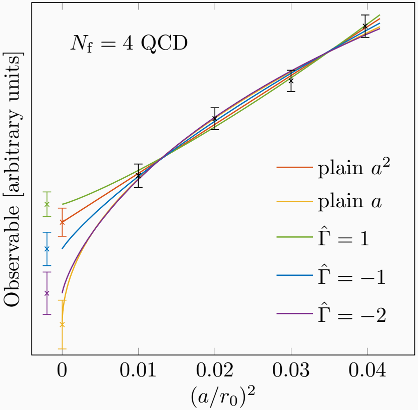
Motivation: Continuum extrapolation



In an asymptotically free theory, like QCD, leading lattice artifacts are of the form (up to factors of $\log \bar{g}(1/a)$)

$$\frac{\mathcal{P}(a)}{\mathcal{P}(0)} = 1 + a^{n_\mathrm{min}} \sum_i \left[\bar{g}^2(1/a)\right]^{\hat{\Gamma}_i} c_i + \mathrm{O}(a^{n_\mathrm{min}+1}, a^{n_\mathrm{min}} \bar{g}^{2\hat{\Gamma}_i+2}(1/a), \ldots)$$

$\hat{\Gamma}_i$ can be negative and distinctly nonzero

$\Rightarrow$ impact on convergence.

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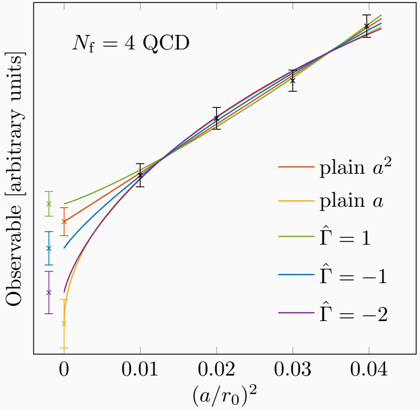

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$\hat{\Gamma}_i$ can be negative and distinctly nonzero
$\Rightarrow$ impact on convergence.

Warning example: 2d O(3) non-linear sigma model **$\min \hat{\Gamma}_i = -3$** [Balog et al., 2009, 2010]

$\Rightarrow$ Compute $\hat{\Gamma}_i$ in QCD to gain better control over continuum extrapolation.

Symanzik Effective Theory (SymEFT) [Symanzik, 1980, 1981, 1983a,b]

Spectral quantities get corrections from the lattice action

$$\Delta S = a^{n_{\mathrm{min}}} \int \mathrm{d}^4 x \sum_i \bar{\omega}_i(g_0) \mathcal{O}_i(x) + \ldots$$

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Example: hadron masses

$$am^X(a) = \lim_{t\to\infty} \log \frac{C^X_{2\mathrm{pt}}(t)}{C^X_{2\mathrm{pt}}(t+a)}$$

$$\frac{m^X(a)}{m^Y(a)} = \frac{m^X}{m^Y}\left\{1 - a^{n_{\min}} \sum_i \hat{c}_i [2b_0\bar{g}^2(1/a)]^{\hat{\Gamma}_i} \left(\frac{m^X_{i;\mathrm{RGI}}}{m^X} - \frac{m^Y_{i;\mathrm{RGI}}}{m^Y}\right) + \dots\right\}$$

where $\hat{\Gamma}_i = (\gamma_0^{\mathcal{B}})_i/(2b_0) + n_i$ can be obtained from 1-loop running of the operators $\mathcal{O}_i$

$$\mu \frac{\mathrm{d}\mathcal{O}_{i;\overline{\mathrm{MS}}}}{\mathrm{d}\mu} = -\bar{g}^2(\mu)\left[\gamma_0^{\mathcal{O}} + \mathrm{O}(\bar{g}^2)\right]_{ij} \mathcal{O}_{j;\overline{\mathrm{MS}}}$$

and a change of basis $\mathcal{O} \to \mathcal{B}$ s.t. $\gamma_0^{\mathcal{B}}$ is diagonal.

Tastes of unrooted staggered quarks

1CR: $S_{\mathrm{F}} = a^4 \sum_x \bar{\chi}(x) \frac{\eta_\mu(x)}{2} \left[\nabla_\mu + \nabla^*_\mu\right] \chi(x), \quad \eta_\mu(x) = (-1)^{\sum_{\nu<\mu} x_\nu / a}$ [Kogut, Susskind, 1975]

Need notion of flavours for connection to continuum theory $\Rightarrow$ taste representation

Local tastes

$$\Phi(y) = \frac{1}{8} \sum_{\xi \in \{0,1\}^4} T[U](y,\xi)\chi(y + a\xi), \; y \in 2a\mathbb{Z}^4$$

Strictly local ✓

Momentum-space tastes [Golterman, Smit, 1984]

$$\tilde{\Phi}_A(p) = \tilde{\chi}(p + \pi_A), \; a\pi_A \in \left\{ \begin{matrix} (0,0,0,0),(\pi,0,0,0), \\ (0,\pi,0,0),\ldots \end{matrix} \right\}$$

Non-local interactions [Kluberg-Stern et al., 1983; Daniel, Kieu, 1986; Jolicoeur et al., 1986] ✗

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Applying SymEFT requires a local description of the lattice theory!

Postponing introduction of gauge-links to TR yields natural discretisation of Kähler-Dirac fermions [Graf, 1978; Rabin, 1982] but breaks Shift-symmetry [Mitra, Weisz, 1983].

$\Rightarrow$ Requires additive mass renormalisation.

Tastes of unrooted staggered quarks free theory cf. [Mitra, Weisz, 1983; Verstegen, 1985]

We choose $T[U](y,\xi) = \prod_{\mu=0}^{3} U_\mu^{\xi_\mu}(y + a\sum_{\nu<\mu}\xi_\nu\hat{\nu})\gamma_\mu^{\xi_\mu}$ with $TT^\dagger \propto \mathbb{1} \Rightarrow \mathcal{D}\bar{\chi}\mathcal{D}\chi \propto \mathcal{D}\bar{\Phi}\mathcal{D}\Phi$

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Taste symmetries in the interacting theory

Gauge invariance	$\Phi(y) \to \Omega(y)\Phi(y), \quad \Omega(y) \in \mathrm{SU}(N)$
$\mathrm{U}(1)_\mathrm{B}$	$\Phi(y) \to e^{i\varphi}\Phi(y), \quad \varphi \in \mathbb{R}$
Remnant chiral	$\Phi_{\mathrm{R,L}} \to \exp(i\vartheta_{\mathrm{R,L}}\gamma_5 \otimes \tau_5)\Phi_{\mathrm{R,L}}, \quad \Phi_{\mathrm{R,L}} = \frac{1\pm\gamma_5\otimes\tau_5}{2}\Phi, \quad \vartheta_{\mathrm{R,L}} \in \mathbb{R}$
Mod. charge conjugation	$\bar{\Phi}(y) \to -\Phi^T(y)C \otimes (C^{-1})^T, \quad \Phi(y) \to C^{-1} \otimes C^T\bar{\Phi}^T(y),$ $U_\mu(x) \to U_\mu^*(x), \quad C\gamma_\mu C^{-1} = -\gamma_\mu^T$

Transformation of $\bar{\Phi}$ mostly omitted for compactness!

Taste symmetries in the interacting theory (involving field-redefinitions)

Mod. Euclidean reflections

$$\Phi(y) \to \gamma_\mu\gamma_5 \otimes \tau_5 \left\{1 + a^2 \ldots\right\} \Phi(y - 2y_\mu\hat{\mu}),$$

$$U_\nu(x) \to \begin{cases} U_\mu^\dagger(x - 2x_\mu\hat{\mu}) & \text{if } \mu = \nu \\ U_\nu(x - 2x_\mu\hat{\mu}) & \text{else} \end{cases}$$

Mod. discrete rotations

$$\Phi(y) \to \tfrac{1}{2}(1 - \gamma_\rho\gamma_\sigma) \otimes (\tau_\rho - \tau_\sigma) \left\{1 + a^2 \ldots\right\} \Phi(R^{-1}y)$$

$$U_\nu(x) \to \begin{cases} U_\rho(R^{-1}x) & \nu = \sigma \\ U_\sigma^\dagger(R^{-1}x) & \nu = \rho \\ U_\nu(R^{-1}x) & \text{else} \end{cases} \qquad \begin{aligned} (R^{-1}x)_\sigma &= -x_\rho, \\ (R^{-1}x)_\rho &= x_\sigma, \\ (R^{-1}x)_{\mu\neq\rho,\sigma} &= x_\mu \end{aligned}$$

Shift symmetry

$$\bar{\Phi}(y) \to \bar{\Phi}(y) 1 \otimes \tau_\mu \left\{1 + 2a\hat{P}_-^{(\mu)} \hat{\overleftarrow{\nabla}}_\mu^\dagger + a^2 \ldots\right\}$$

$$\Phi(y) \to 1 \otimes \tau_\mu \left\{1 + 2a\hat{P}_+^{(\mu)} \hat{\overleftarrow{\nabla}}_\mu + a^2 \ldots\right\} \Phi(y)$$

with projector $\hat{P}_\pm^{(\mu)} = \frac{1 \pm \gamma_\mu\gamma_5 \otimes \tau_\mu\tau_5}{2}$

Transformation of $\bar{\Phi}$ mostly omitted for compactness!

Minimal massless on-shell basis at $O(a^2)$

$\frac{1}{g^2}\mathrm{tr}(D_\mu F_{\nu\rho} D_\mu F_{\nu\rho})$	$\frac{1}{g^2}\sum_\mu \mathrm{tr}(D_\mu F_{\mu\nu} D_\mu F_{\mu\nu})$	$\sum_\mu \bar\Psi\gamma_\mu \otimes 1 D_\mu^3 \Psi$	$g^2(\bar\Psi\gamma_\mu \otimes 1\Psi)^2$
$g^2(\bar\Psi\gamma_\mu\gamma_5 \otimes 1\Psi)^2$	$g^2(\bar\Psi\gamma_\mu \otimes 1 T^a\Psi)^2$	$g^2(\bar\Psi\gamma_\mu\gamma_5 \otimes 1 T^a\Psi)^2$	pure gauge [Lüscher, Weisz, 1985] GW [Sheikholeslami, Wohlert, 1985]
$g^2(\bar\Psi 1 \otimes \tau_\mu\Psi)^2$	$g^2(\bar\Psi 1 \otimes \tau_\mu\tau_5\Psi)^2$	$g^2(\bar\Psi 1 \otimes \tau_\mu T^a\Psi)^2$	$g^2(\bar\Psi 1 \otimes \tau_\mu\tau_5 T^a\Psi)^2$
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$g^2(\bar\Psi\gamma_{\mu\nu} \otimes \tau_\rho\Psi)^2$	$g^2(\bar\Psi\gamma_{\mu\nu} \otimes \tau_\rho\tau_5\Psi)^2$	$g^2(\bar\Psi\gamma_{\mu\nu} \otimes \tau_\rho T^a\Psi)^2$	$g^2(\bar\Psi\gamma_{\mu\nu} \otimes \tau_\rho\tau_5 T^a\Psi)^2$
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$g^2\sum_\mu(\bar\Psi\gamma_{\mu\nu} \otimes \tau_\mu\Psi)^2$	$g^2\sum_\mu(\bar\Psi\gamma_{\mu\nu} \otimes \tau_\mu\tau_5\Psi)^2$	$g^2\sum_\mu(\bar\Psi\gamma_{\mu\nu} \otimes \tau_\mu T^a\Psi)^2$	$g^2\sum_\mu(\bar\Psi\gamma_{\mu\nu} \otimes \tau_\mu\tau_5 T^a\Psi)^2$
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In agreement with [Follana et al., 2007] we find 28 additional 4-quark operators.

Spectral quantities

Reminder: $\mu \frac{\mathrm{d}\mathcal{O}_{i;\overline{\mathsf{MS}}}}{\mathrm{d}\mu} = -\bar{g}^2(\mu) \left[\gamma_0^{\mathcal{O}} + \mathrm{O}(\bar{g}^2)\right]_{ij} \mathcal{O}_{j;\overline{\mathsf{MS}}}$

Diagonalise the 1-loop mixing matrix through change of basis $\mathcal{O} \to \mathcal{B}$ and rewrite

$$\mathcal{B}_{i;\overline{\mathsf{MS}}}(\mu) = [2b_0\bar{g}^2(\mu)]^{\hat{\gamma}_i} \mathcal{B}_{i;\mathsf{RGI}} \times \left\{1 + \mathrm{O}(\bar{g}^2)\right\}, \quad \hat{\gamma}_i = \frac{(\gamma_0^{\mathcal{B}})_i}{2b_0},$$

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We then find for our initial example of hadron masses

$$\frac{m^X(a)}{m^Y(a)} = \frac{m^X}{m^Y}\left\{1 - a^2\sum_i \hat{c}_i[2b_0\bar{g}^2(1/a)]^{\hat{\Gamma}_i}\left(\frac{m^X_{i;\mathrm{RGI}}}{m^X} - \frac{m^Y_{i;\mathrm{RGI}}}{m^Y}\right) + \mathrm{O}(a^3, a^2\bar{g}^{2\hat{\Gamma}_i+2}(1/a))\right\}$$

with matching coefficient $\hat{c}_i$ non-vanishing at n_i-th loop order and $\hat{\Gamma}_i = \hat{\gamma}_i + n_i$.

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with matching coefficient $\hat{c}_i$ non-vanishing at n_i-th loop order and $\hat{\Gamma}_i = \hat{\gamma}_i + n_i$.

$\boldsymbol{\hat{\gamma}_i \in \{-0.301, -0.04, 0.04, 0.209, 0.419, 0.52, 0.56, 0.698, 0.817, 0.92, 0.941, 0.96, 1.12, 1.14, 1.16, 1.24, 1.32, 1.487, 1.661, 1.852\}}$ **remnant chiral** for $N_\mathrm{f} = 4$

What about local fields?

Symmetries involving field-redefinitions rule out on-shell operators in the SymEFT action at $\mathrm{O}(a)$ but **require** EOM-vanishing operators, such as

$$\sum_{\mu}\left[\bar{\Psi}\gamma_5\otimes\tau_\mu\tau_5\{\overleftarrow{D}_\mu^2+D_\mu^2\}\Psi+i\bar{\Psi}\gamma_5\gamma_{\mu\nu}\otimes\tau_\mu\tau_5 F_{\mu\nu}\Psi\right]$$

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Discrete flavour-symmetry of mass-degenerate $N_\mathrm{f}=4$ **continuum** QCD

$$\bar{\Psi}\to\bar{\Psi}1\otimes\tau_5,\quad\Psi\to 1\otimes\tau_5\Psi$$

will ensure automatic $\mathrm{O}(a)$ improvement.

Conclusion

- Extension to asymptotic lattice spacing dependence for spectral quantities of unrooted staggered quarks, see [Husung, 2023] for Wilson, GW quarks.
- No troublesome powers $a^2[2b_0\bar{g}^2(1/a)]^{\hat{\Gamma}_i}$ for spectral quantities at $N_\mathrm{f}=4$: $\hat{\Gamma}_i \gtrsim -0.301$. Loop suppression expected for matching of remnant-chiral 4-quark operators.
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- $N_\mathrm{f} = 8$: $\hat{\Gamma}_i \gtrsim -0.913$
  $N_\mathrm{f} = 12$: $\hat{\Gamma}_i \gtrsim -2.614 \Rightarrow$ 1-loop suppression will become insufficient!

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- **Rooted staggered quarks?**
  - Non-locality puts SymEFT description in question.
  - Feel free to work out correct perturbative prescription to translate $\hat{\Gamma}_i$ into rooted theory!

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Backup: Symmetries of local tastes

Derive symmetries in the *interacting theory* by intermediately switching back to 1CR

$$
\begin{pmatrix} \vdots \\ \Phi(y) \\ \vdots \end{pmatrix} = \frac{1}{8} \begin{pmatrix} \ddots & 0 & 0 \\ 0 & T[U](y,\xi) & 0 \\ 0 & 0 & \ddots \end{pmatrix} \begin{pmatrix} \vdots \\ \chi(y + a\xi) \\ \vdots \end{pmatrix}
$$

$$
\to \frac{1}{8} \begin{pmatrix} \ddots & 0 & 0 \\ 0 & T[U'](y,\xi)\zeta(y + a\xi) & 0 \\ 0 & 0 & \ddots \end{pmatrix} \begin{pmatrix} \vdots \\ \chi(y' + a\xi') \\ \vdots \end{pmatrix}
$$

$$
= \begin{pmatrix} \ddots & 0 & 0 \\ 0 & T[U'](y,\xi)\zeta(y + a\xi) & 0 \\ 0 & 0 & \ddots \end{pmatrix} \mathbb{S} \begin{pmatrix} \ddots & 0 & 0 \\ 0 & T^\dagger[U](y',\xi') & 0 \\ 0 & 0 & \ddots \end{pmatrix} \begin{pmatrix} \vdots \\ \Phi(y') \\ \vdots \end{pmatrix}
$$