Axion QED as a Lattice Gauge Theory and Non-Invertible Symmetry

Yamato Honda (Kyushu University)

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- Y.H, S.Onoda, H.Suzuki, PTEP **2024**, No.7, 073B04 (2024) [arXiv:2403.16752]
- Y.H, S.Onoda, H.Suzuki, [arXiv:2405.07669]

Axion QED in Continuous Theory

- Axion-photon coupling $\sim \int_{\mathcal{M}_4} \phi f \wedge f$
- Ordinary $U(1)$ 0-form gauge symmetry: $a \to a + d\lambda$
- 2π periodicity of ϕ: $\phi + 2\pi \sim \phi$
- This periodicity should be understood as 2π shift in each patch
- It is not so obvious whether $\int_{\mathcal{M}_4} \phi f \wedge f$ respects this periodicity
- What about for singular configurations such as 't Hooft loop or axion string?

$\Longrightarrow$ lattice field theory

Modified Villain Formalism: Photon

- $\mathbb{R}$-valued 1-cochain(form) a, $\mathbb{Z}$-valued 2-cochain z [Sulejmanpasic-Gattringer '19]
- $\mathbb{Z}^{(1)}$ gauge symmetry

$$a \to a + 2\pi m, \qquad z \to z - dm$$

- $\mathbb{R}^{(0)}$ gauge symmetry

$$a \to a + d\lambda$$

$$\Longrightarrow \mathbb{R}/2\pi\mathbb{Z} \simeq U(1) \text{ gauge theory}$$

- Gauge invariant field strength

$$f = da + 2\pi z$$

Modified Villain Formalism: Axion

- $\mathbb{R}$-valued 0-cochain ϕ, $\mathbb{Z}$-valued 1-cochain l
- $\mathbb{Z}^{(0)}$ gauge symmetry

$$\phi \to \phi + 2\pi k, \qquad l \to l - dk$$

$$\Longrightarrow \ 2\pi \text{ periodic scalar}$$

- Gauge invariant "field strength"

$$\partial\phi := d\phi + 2\pi l$$

't Hooft Loop, Axion String

- Impose the Bianchi identity for the topological charges

$$df = 2\pi dz = 0, \qquad d\partial\phi = 2\pi dl = 0$$

- Introduce $\mathbb{R}$-valued Lagrange multipliers $\tilde{a}, \chi$

$$S \subset \sum_{\Gamma} (i\tilde{a} \cup dz + i dl \cup \chi)$$

- Naive magnetic objects

$$T_q(\gamma) = \exp\left[iq \sum_{\gamma} \tilde{a}\right], \qquad S'_q(\sigma) = \exp\left[iq' \sum_{\sigma} \chi\right]$$

- Insertion of $T_q(\gamma)$ leads to a violation of the Bianchi identity

$$dz = q\delta_3[\gamma] \neq 0$$

Lattice Axion-Photon Coupling

- Naive axion-photon coupling is not $\mathbb{Z}^{(0)}$ invariant

$$\frac{ie^2}{8\pi^2}\sum_{\Gamma}\phi\cup f\cup f \xrightarrow[\mathbb{Z}^{(0)}]{} \frac{ie^2}{8\pi^2}\sum_{\Gamma}\phi\cup f\cup f+\frac{ie^2}{8\pi^2}\sum_{\Gamma}2\pi k\cup f\cup f$$

- Refine the coupling

$$S_{\text{int}}=-\frac{ie^2}{8\pi^2}\sum_{\Gamma}\left[\partial\phi\cup(\text{CS})-4\pi^2\phi\cup P_2(z)\right]$$

$$(\text{CS}):=a\cup f+2\pi z\cup a+2\pi a\cup_1 dz$$

[Jacobson-Sulejmanpasic '23]

$$P_2(z):=z\cup z+z\cup_1 dz$$

- Similar approach in BF theory [Gorantla-Lam-Seiberg-Shao '21]

Gauge Invariance and Magnetic Objects

- S_{int} violates $\mathbb{Z}^{(1)}$, $\mathbb{R}^{(0)}$ gauge invariance only at $dz \neq 0$, $dl \neq 0$

$$S_{\text{int}} \xrightarrow[\mathbb{Z}^{(1)}]{} S_{\text{int}} + \frac{ie^2}{2} \sum_{\Gamma} (-2\phi \cup m \cup dz + dl \cup m \cup a)$$

- At the same time, also transform $\tilde{a}$, χ

$$\tilde{a} \xrightarrow[\mathbb{Z}^{(1)}]{} \tilde{a} + e^2 \phi \cup m, \qquad \chi \xrightarrow[\mathbb{Z}^{(1)}]{} \chi - \frac{e^2}{2} m \cup a$$

- As a result, gauge invariant magnetic objects are modified as $\partial \mathcal{R} = \gamma$

$$T_q(\gamma) = \exp \left[iq \left\{ \sum_{\gamma} \left(\tilde{a} - \frac{e^2}{2\pi} \phi \cup a \right) + \sum_{\mathcal{R}} \frac{e^2}{2\pi} (-2\pi l \cup a + \phi \cup f) \right\} \right]$$

Non-Invertible Symmetry Operator

- Non-invertible symmetry exists even in the presence of axial anomaly [Choi-Lam-Shao '22], [Córdova-Ohmori '22]

$$d \star j_5 = \frac{e^2}{4\pi^2} f \cup f \neq 0$$

- Non-invertible symmetry operator generating the axial transformation $\phi \to \phi + \frac{2\pi p}{N}$

$$U_{\frac{2\pi p}{N}}(\mathcal{M}_3) = \exp\left[\frac{i\pi p}{N} \sum_{\mathcal{M}_3} \left\{ \star j_5 - \frac{e^2}{4\pi^2}(a \cup f + 2\pi z \cup a) \right\}\right] \times \mathcal{Z}_{\mathcal{M}_3}[z]$$

- $\mathcal{Z}_{\mathcal{M}_3}[z]$ is 3D lattice BF partition function [Honda-Morikawa-Onoda-Suzuki '24]

$$\mathcal{Z}_{\mathcal{M}_3}[z] = \frac{1}{N^s} \int D[b]D[c] \exp\left[\frac{i\pi p e^2}{N} \sum_{\mathcal{M}_3} \{ b(dc - z) - z \cup c \}\right]$$

Sweep through 't Hooft Loop

- When $q \neq 0 \bmod N$, 't Hooft loop vanishes

$$\mathcal{Z}_{\mathcal{M}_3}[z] = 0, \text{if } dz|_{\mathcal{M}_3} \neq 0 \bmod N$$

$$\Longrightarrow \text{ non-invertibility}$$

- When $q = 0 \bmod N$, act trivially on 't Hooft loop

$$\left\langle U_{\frac{2\pi p}{N}}(\mathcal{M}_3) T_q(\gamma) \right\rangle = \left\langle U_{\frac{2\pi p}{N}}(\mathcal{M}'_3) T_q(\gamma) \right\rangle$$

- In continuous theory, 't Hooft loop acquires field strength f [Choi-Lam-Shao '22, $\cdots$]

't Hooft Loop as the Boundary of Electric Symmetry Operator

- Quick way to find the gauge invariant 't Hooft loop

$$f \cup \star f \to (f - 2\pi B) \cup \star (f - 2\pi B)$$

- Set $B = -q\delta_2[\mathcal{R}]$

$$\int D[\Phi] \exp\left[iq \sum_{\mathcal{R}} \star f \right] e^{-S} = \int D[\Phi] e^{-S_B}$$
$$\underset{z \to z+B}{=} \int D[\Phi] T_q(\gamma) e^{-S}$$

- Half space gauging

Summary

- Construction of lattice axion QED action with modified Villain formalism
- Non-genuine magnetic object for gauge invariance
- 0-form non-invertible symmetry
- Future works
  - Discrepancies with the results from the continuous theory $\leftarrow$ anomaly inflow
  - 1-form non-invertible symmetry

- Thank you for your attention