Discrete symmetry and ‘t Hooft anomalies for 3450 model

Tetsuya Onogi

with Hiroki Wada, Tatsuya Yamaoka

Department of Physics

Osaka University

大阪大学
OSAKA UNIVERSITY

Lattice 2024@Liverpool

30th July, 2024

LATTICE 2024

LIVERPOOL

Outline

1. Introduction
2. Review of Symmetric gapping with Domain-Wall fermion
3. Review of 3450 model in continuum and on the lattice
4. Discrete symmetry
5. ‘t Hooft anomaly
6. Summary

(4–5: our work)

1. Introduction

Chiral gauge theory on the lattice is needed.

- Standard Model is chiral
- Interesting dynamics, which are not fully understood yet.

Status of Lattice formulation

- Chiral projection with Overlap fermion

  Successful for abelian gauge theory [Luscher 1998]

  Not completed for non-abelian gauge theory [Luscher 1999]

Our topic

- Domain-wall (DW) fermion with symmetric gapping interaction

  With gapping interaction and gap out mirror edge mode.

  Many proposals: e.g. lattice 3450 model [Wang-Wen 2013]

Remarks

- For a given symmetric gapping interaction, whether mirror edge mode can really be gapped out or not is still unknown. Numerical test is needed.

- Hint: Existence of edge mode in domain-wall fermion

  $\Longleftrightarrow$ anomaly inflow

  $\Downarrow$

<u>Symmetry & anomaly</u> give
<u>strong restrictions</u> on the scenario
for symmetric gapping with DW fermion.

Questions and our goal

Question:

Is symmetric gapping consistent with anomaly inflow?

(no edge-mode ⟺ no bulk anomaly)

In 3-4-5-0 model

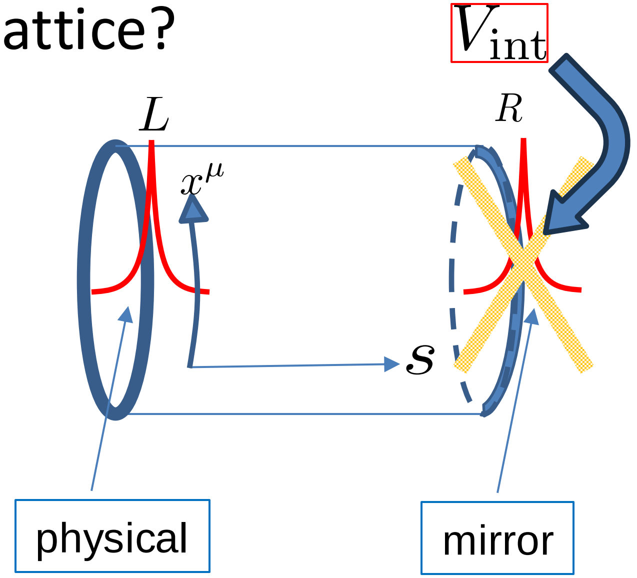
- Continuous symmetries: Already considered.
- Discrete symmetries: Not yet considered.

Our goal:

We study the bulk anomaly including discrete symmetries and examine the consistency of the mirror edge mode gapping scenario for lattice 3-4-5-0 model.

2. Review of symmetric gapping with Domain-Wall fermion

How can we define chiral gauge theory on the lattice?



Gap out mirror edge mode from DW fermion by symmetric interactions $V_{\rm int}$ at mirror edge.

- Promising proposals:

  SO(10), SM in 4dim, 3-4-5-0 model in 2dim …

  Y. Bentov 2014, Wang-Wen 2013, Y. Kikukawa 2017, ….

- Numerical test of mirror edge mode decoupling has been carried out.

What conditions are needed for Gapping interactions?

- Free DW fermion has exact vector symmetries G.
- Interaction $V_{\rm int}$ breaks G to a subgroup H $\quad G \xrightarrow{V_{\rm int}} H$

Necessary condition for symmetry H and $V_{\rm int}$

1. Absence of bulk anomaly for the symmetry H
   Otherwise, anomaly inflow requires massless edge-mode.
   ⟺ obstruction against gapping of mirror edge mode

2. Instanton Saturation
   U(1) instanton sector → chiral zero modes at the mirror edge.
   $V_{\rm int}$ must saturate mirror zero modes to have nonzero path-integral.

3. The 3450 model (review)

Target continuum theory:

Chiral U(1) gauge theory in 1+1 dim with 4 Weyl fermions

chirality : L, L, R, R
charge Q : 3, 4, 5, 0

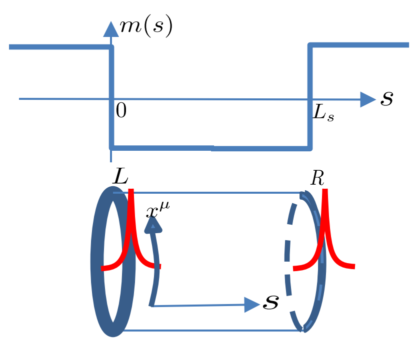
$$S = \int dtdx[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_{Q=3,4} \bar{\psi}_L^Q \gamma^\mu D_\mu^Q \psi_L^Q + \sum_{Q=5,0} \bar{\psi}_R^Q \gamma^\mu D_\mu^Q \psi_R^Q]$$

$$D_\mu^Q = \partial_\mu + iQA_\mu$$

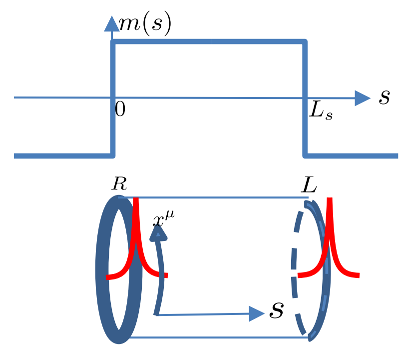
- U(1) gauge anomaly free : $3^2 + 4^2 - 5^2 - 0^2 = 0$
- gravitational anomaly free : $1 + 1 - 1 - 1 = 0$

Domain-wall realization Lattice 3-4-5-0 model (Wang-Wen)

- Four species of Domain-wall fermions with charge Q =3,4,5,0



L−type: Q=3,4



R−type: Q=5,0

- Introduce two 6-fermion interactions $V_{\rm int}$ at the mirror edge.

$$V_{\rm int} \sim g_1 V_1 + g_2 V_2 + h.c. \qquad V_1 = \psi^3 (\bar{\psi}^4)^2 \psi^5 (\psi^0)^2, \quad V_2 = (\psi^3)^2 \psi^4 (\bar{\psi}^5)^2 \psi^0$$

$$G = U(1)_{\psi^3} \times U(1)_{\psi^4} \times U(1)_{\psi^5} \times U(1)_{\psi^0} \xrightarrow{V_{\rm int}} H = U(1)_q \times U(1)_{q'}$$

$$q = (-1, 2, 1, 2), \quad q' = (2, 1, 2, -1)$$

$$\text{c.f.} \quad q_{em} = (3, 4, 5, 0) = q + 2q'$$

Domain-wall realization

Lattice 3-4-5-0 model
[Wang-Wen 2013]

1. Bulk anomaly is zero

$$q_3^2 + q_4^2 - (q_5^2 + q_0^2) = 0$$

$$(q_3')^2 + (q_4')^2 - ((q_5')^2 + (q_0')^2) = 0$$

$$q_3 q_3' + q_4 q_4' - (q_5 q_5' + q_0 q_0') = 0$$

2. Instanton saturation is satisfied.

$$V_1^\dagger (V_2)^2 \sim (\psi^3)^3 (\psi^4)^4 (\bar{\psi}^5)^5$$

't Hooft vertex can be generated

Moreover, both semi-classical analysis after bosonization and discussion based on FQHE shows that symmetric gapping does take place with the gapping interaction.[Wang-Wen 2015]

4. Discrete symmetry in 3-4-5-0 model

1) Discrete symmetries for single flavor case

What is the discrete symmetry allowed by $\mathrm{V_{int}}$?

$$V_1 = \psi^3(\bar{\psi}^4)^2\psi^5(\psi^0)^2, \quad V_2 = (\psi^3)^2\psi^4(\bar{\psi}^5)^2\psi^0$$

Under the transformation $\psi^Q \to \exp(i\alpha_Q)\psi^Q \quad (Q = 3, 4, 5, 0)$,
interaction terms transform as

$$V_1 \to \exp(i(\alpha_3 - 2\alpha_4 + \alpha_5 + 2\alpha_0))V_1$$
$$V_2 \to \exp(i(2\alpha_3 + \alpha_4 - 2\alpha_5 + \alpha_0))V_2$$

Invariance of $\mathrm{V_{int}}$ requires

$$\alpha_3 - 2\alpha_4 + \alpha_5 + 2\alpha_0 = 2\pi n,$$
$$2\alpha_3 + \alpha_4 - 2\alpha_5 + \alpha_0 = 2\pi m \qquad n, m \in \mathrm{Z}$$

Solutions of $\alpha = (\alpha_3, \alpha_4, \alpha_5, \alpha_0)$

Using the transformation $U(1)_q \times U(1)_{q'}$, $q = (-1, 2, 1, 2)$, $q' = (2, 1, 2, -1)$
you can make two of α_Q 's vanish.

Solutions $\alpha^{(1)} = \frac{2\pi}{3}(1, 0, 0, 1)$ $\alpha^{(2)} = \frac{2\pi}{3}(0, 1, -1, 0)$ $\alpha^{(3)} = \frac{2\pi}{4}(0, 1, 0, -1)$

$\alpha^{(4)} = \frac{2\pi}{4}(1, 0, -1, 0)$ $\alpha^{(5)} = \frac{2\pi}{5}(0, 0, 1, 2)$ $\alpha^{(6)} = \frac{2\pi}{5}(2, 1, 0, 0)$

e.g. $\alpha^{(1)}$: $\psi_3 \to \omega_3 \psi_3$, $\psi_0 \to \omega_3 \psi_0$
$\psi_{4,5}$ invariant. $(\omega_3 = \exp(2\pi i/3))$

$\mathbb{Z}_3$ transformation

However, they are all subgroups of $U(1)_q \times U(1)_{q'}$

e.g. $\frac{2\pi}{3}(q + q') = \frac{2\pi}{3}(1, 3, 3, 1) = \alpha^{(1)} \pmod{2\pi\mathbb{Z}}$

⇒ No 0-form discrete symmetries exist for single flavor.

2) Discrete symmetry for multi-flavor case

Symmetry without V_{int}: $G = U(N_f)_{\psi^3} \times U(N_f)_{\psi^4} \times U(N_f)_{\psi^5} \times U(N_f)_{\psi^0}$

With N_f flavors, V_{int} can be given as $V_{\text{int}} = \sum_{I=1}^{N_F} (g_1 V_1(\psi_I) + g_2 V_2(\psi_I))$

⇓

The symmetry reduces

$$G \xrightarrow[V_{\text{int}}]{} H = U(1)_{em} \times U(1)_q \times S_{N_F}$$

Permutation group

$$S_{N_F} : \psi_Q^I \to \psi_Q^{\sigma(I)} \quad (I = 1, \cdots, N_F,\ Q = 3, 4, 5, 0)$$

Multi-flavor system has 0-form discrete symmetry.

5. ‘t Hooft anomaly

Eample: $N_f = 2$ case

‘t Hooft anomalies of 2-flavor lattice 3-4-5-0 model.

Symmetry $$H = U(1)_{em} \times U(1)_q \times S_2$$

$S_2 \cong Z_2$

Gauge fields

$a:$ dynamical $U(1)_{\rm em}$ 1-form gauge field

$A:$ external $U(1)_q$ 1-form gauge field

$(B^{(1)}, B^{(0)}):$ external $U(1)$ (1-form,0-form) gauge field ← gauge fields for 0-form $\mathbb{Z}_2$ sym.

Constraints: $2B^{(1)} = dB^{(0)}$

$$\frac{1}{2\pi}\int_{\Sigma_1} dB^{(0)} \in \mathbb{Z} \longrightarrow \frac{1}{2\pi}\int_{\Sigma_1} B^{(1)} \in \frac{1}{2}\mathbb{Z}$$

Stora-Zumino procedure to obtain ‘t Hooft anomalies

1. Introduce gauge invariant kinetic term for fermions.
2. Assign topological term $S_{\rm SPT}$ in 2+2 dim.

$$S_{\rm SPT} = \frac{2\pi}{2!(2\pi)^2} \int_{\mathcal{M}_4} \Big(\sum_{Q=3,4} (\mathcal{F}_Q)^2 - \sum_{Q=5,0} (\mathcal{F}_Q)^2 \Big)$$

$\mathcal{F}_Q = $ field strength involving all gauge fields

3. Obtain anomaly using descent equations.

$$S_{\rm SPT} = \int_{\mathcal{M}_3} \omega_3^1 \longrightarrow \delta\omega_3^1 = \text{ anomaly}$$

We find ‘t Hooft anomalies involving discrete symmetry

1) cancel for mixed-anomaly,
2) formally cancel for self-anomaly (except for mathematical subtleties).

a more mathematically rigorous analysis is in progress

→ Further consistency check from discrete symmetry

6. Summary

- Lattice 3-4-5-0 model is expected to realize 3-4-5-0 chiral gauge theory in the continuum by symmetric gapping.
- If bulk anomaly exist, it contradicts with symmetric gapping because anomaly inflow requires edge modes.
- We discovered that there is a discrete symmetry for lattice 3-4-5-0 model with multiple flavors.
- We examined ‘t Hooft anomalies including discrete symmetry via Stora-Zumino procedure and found that they seem to vanish.
- This gives further evidence that symmetric gapping works for lattice 3-4-5-0 model.
- We would like to apply this kind of analysis to other interesting models.

Thank you for your attention.

Backup Slides

Symmetry for Nf=2 case

Denoting $\psi_Q = \left(\begin{array}{c} \psi_Q^1 \\ \psi_Q^2 \end{array} \right)$

Symmetry transformation
$$U(1)_Q \times U(1)q \quad : \quad \psi_Q \to \exp(\imath Q\theta_Q + iq_Q\theta_q)\psi_Q$$
$$S_2 \quad : \quad \psi_Q \to \left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right) \psi_Q$$

Change of basis $\tilde{\psi}_Q = \frac{1}{\sqrt{2}} \left(\begin{array}{cc} 1 & 1 \\ 1 & -1 \end{array} \right) \psi$

Symmetry transformation

$$U(1)_Q \times U(1)q : \tilde{\psi}_Q \to \exp(iQ\theta_Q + iq_Q\theta_q)\tilde{\psi}_Q$$
$$S_2 : \tilde{\psi}_Q \to \left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right) \tilde{\psi}_Q$$

Only lower component transforms non-trivially under Z_2

‘t Hooft anomaly

$$S_{SPT} = \frac{2\pi}{2!(2\pi)^2} \int_{M_4} \left(\sum_{Q=3,4} tr(\mathcal{F}_Q^2) - \sum_{Q=5,0} tr(\mathcal{F}_Q^2) \right)$$

$$\mathcal{F}_Q = \begin{pmatrix} Qf + q_Q F & 0 \\ 0 & Qf + q_Q F + dB^{(1)} \end{pmatrix}$$

$$S_{3dim} = \frac{2\pi}{2!(2\pi)^2} \int_{M_3} 4B^{(1)}(f + F)$$

$$\mathcal{A} = \frac{2\pi}{2!(2\pi)^2} \int_{M_2} 4\frac{2\pi}{2}(f + F)$$

$$= 2\pi \frac{1}{2\pi} \int_{M_2} (f + F) \in 2\pi\mathbb{Z}$$

Anomaly vanishes