QUANTUM THERMODYNAMICS, LATTICE GAUGE THEORIES, AND QUANTUM SIMULATION

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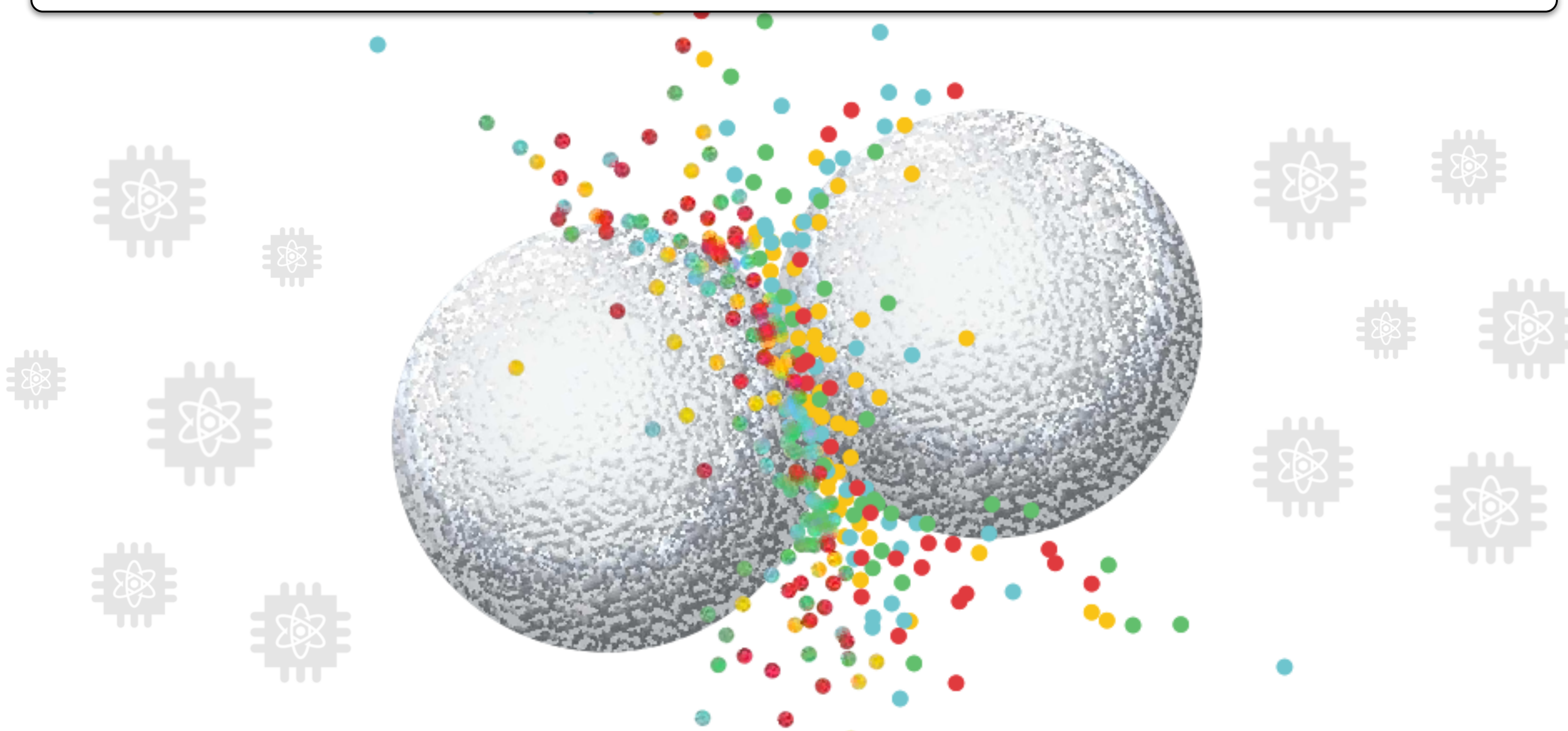
Based on:

ZD, Christopher Jarzynski, Niklas Mueller, Greeshma Oruganti, Connor Powers, and Nicole Yunger Halpern, arXiv:2404.02965 [quant-ph] plus a manuscript in progress (2024).



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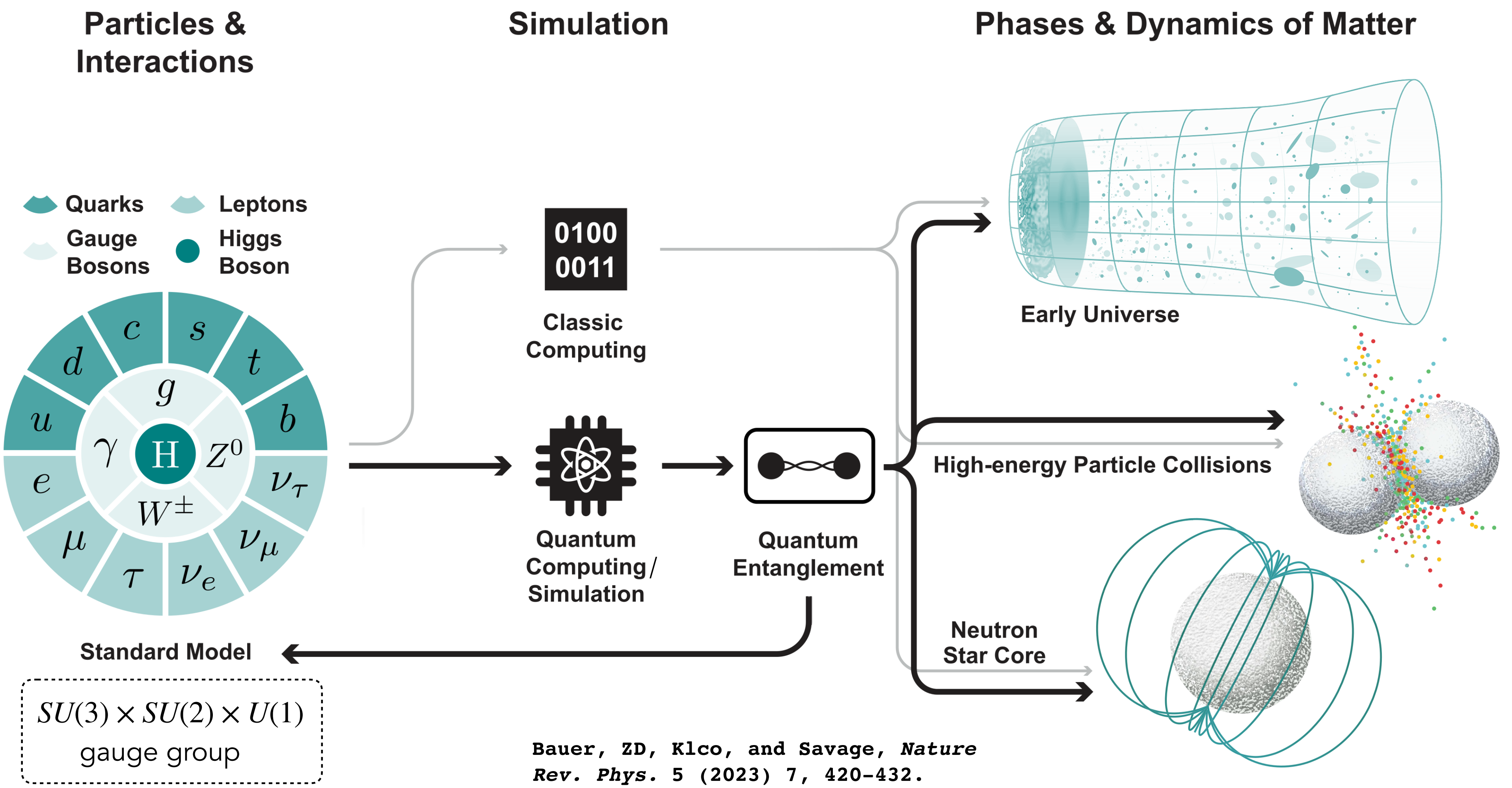
Ultimate goal: Quantum simulation of Standard Model



Bauer, ZD, Klco, and Savage, *Nature Rev. Phys.* 5 (2023) 7, 420-432.

We need to develop equilibrium and non-equilibrium thermodynamics within the Hamiltonian framework of gauge theories.

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PART II:
WORK AND HEAT EXCHANGED IN (NON-EQUILIBRIUM) PROCESSES AND THEIR QUANTUM SIMULATION

PART III:
QUANTUM THERMODYNAMICS OF LATTICE GAUGE THEORIES

PART IV:
THE VALUE OF THE FRAMEWORK: AN EXAMPLE

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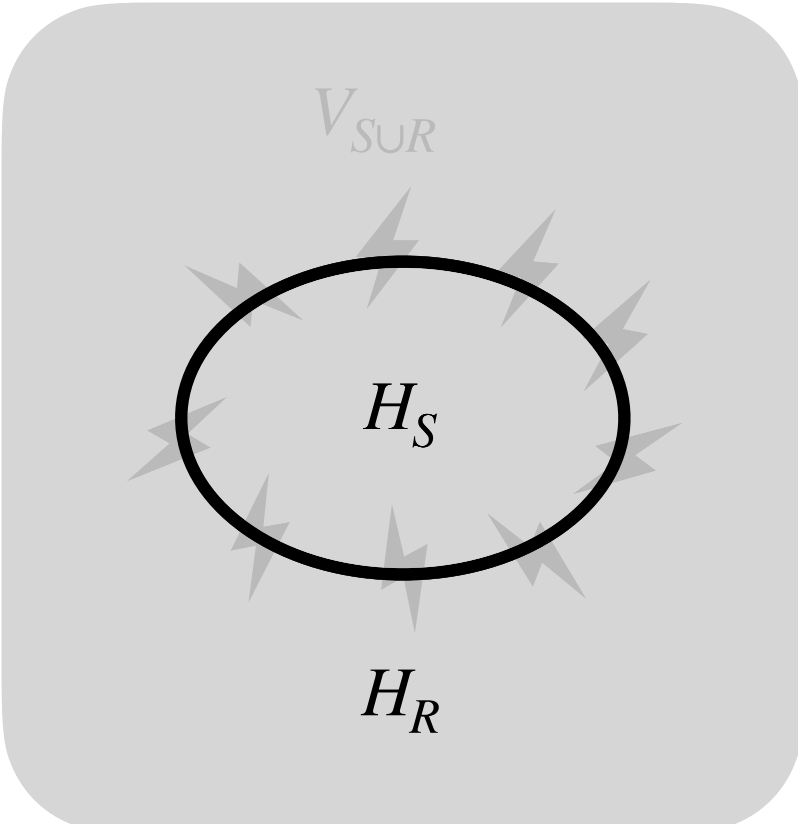
Weak-coupling thermodynamics



Total Hamiltonian: $H_{S\cup R} = H_S + H_R + V_{S\cup R}$

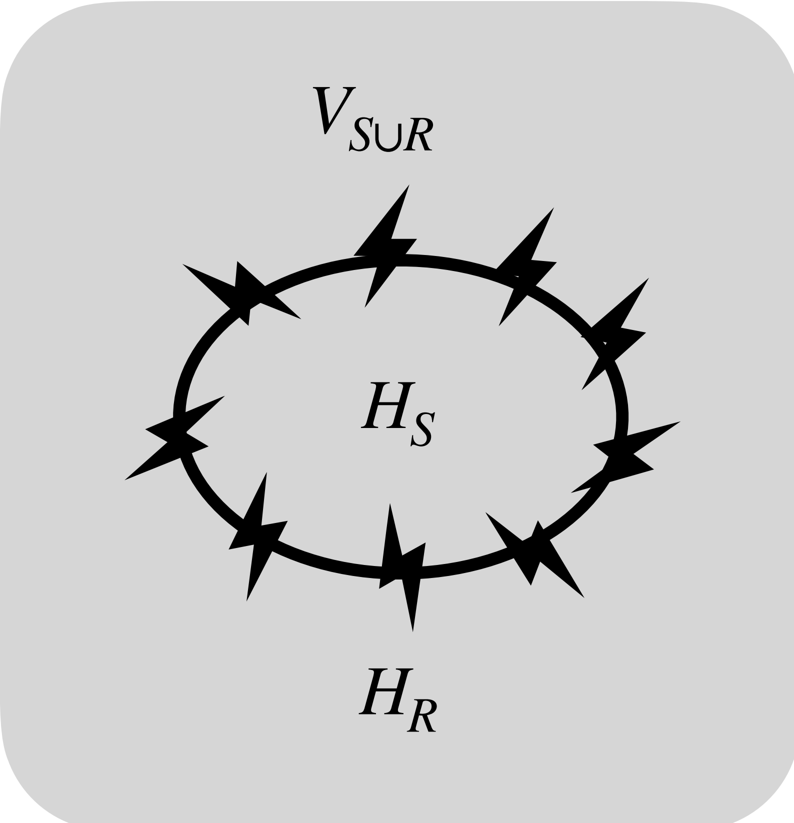
Interactions between system and reservoir contribute negligibly to system's internal energy: $U_S = \langle H_S \rangle$.

Free energy: $F_S = U_S - \beta^{-1}\mathcal{S} = -\beta^{-1}\ln(Z_S)$ with $Z_S = \mathrm{Tr}_S(e^{-\beta H_S})$.

Need to satisfy:

- First law of thermodynamics: $\Delta U_S = W + Q$.
- Second law of thermodynamics: $\Delta F_S \leq W$.

Strong-coupling thermodynamics



Total Hamiltonian: $H_{S\cup R} = H_S + H_R + V_{S\cup R}$

Interactions between system and reservoir contribute non-negligibly to system's internal energy: $U_S = ?$

A solution: Define a Hamiltonian of mean force: $H_S^* = -\beta^{-1}\ln\left(\frac{\mathrm{Tr}_R(e^{-\beta H_{S\cup R}})}{\mathrm{Tr}_R(e^{-\beta H_R})}\right)$.

Then define:

i) $U_S = \langle H_S^* \rangle$,

ii) $F_S = -\beta^{-1}\ln(Z_S^*)$ with $Z_S^* = \mathrm{Tr}_S(e^{-\beta H_S^*})$.

Other choices exist too. Equivalent classically, but not necessarily quantumly!

ZD, Jarzynski, Mueller, Oruganti, Powers, and Yunger Halpern, manuscript in progress (2024).

Rivas, Phys. Rev. Lett. 124, 160601 (2020), Anto-Sztrikacs et. al. PRX Quantum 4, 020307 (2023), Miller and Anders Phys. Rev. E 95, 062123 (2017), Strasberg and Esposito, Phys. Rev. E 99, 012120 (2019).

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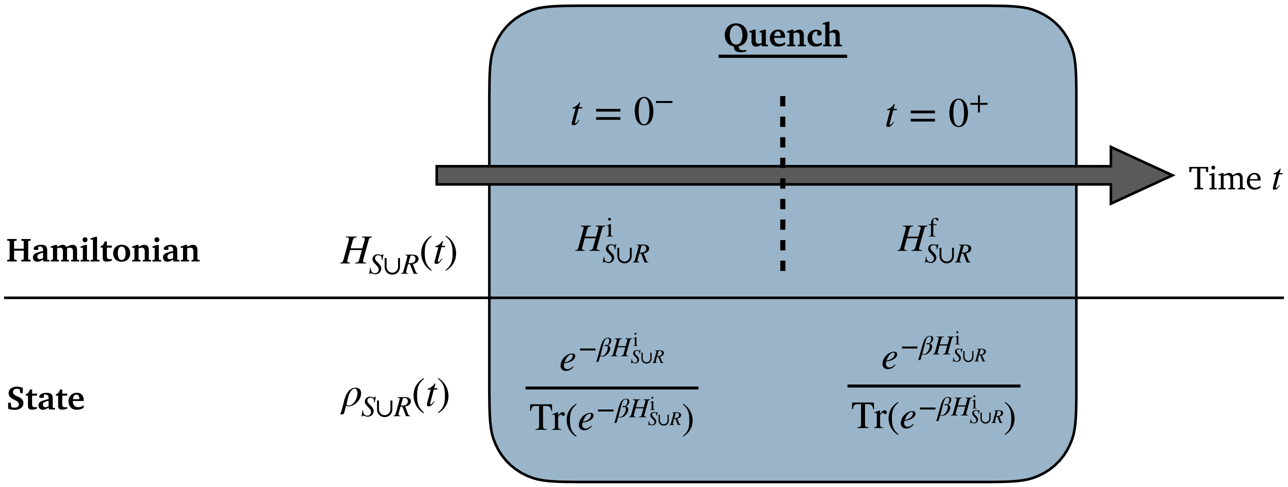
Quantum quenches: Simplest non-equilibrium experiments.

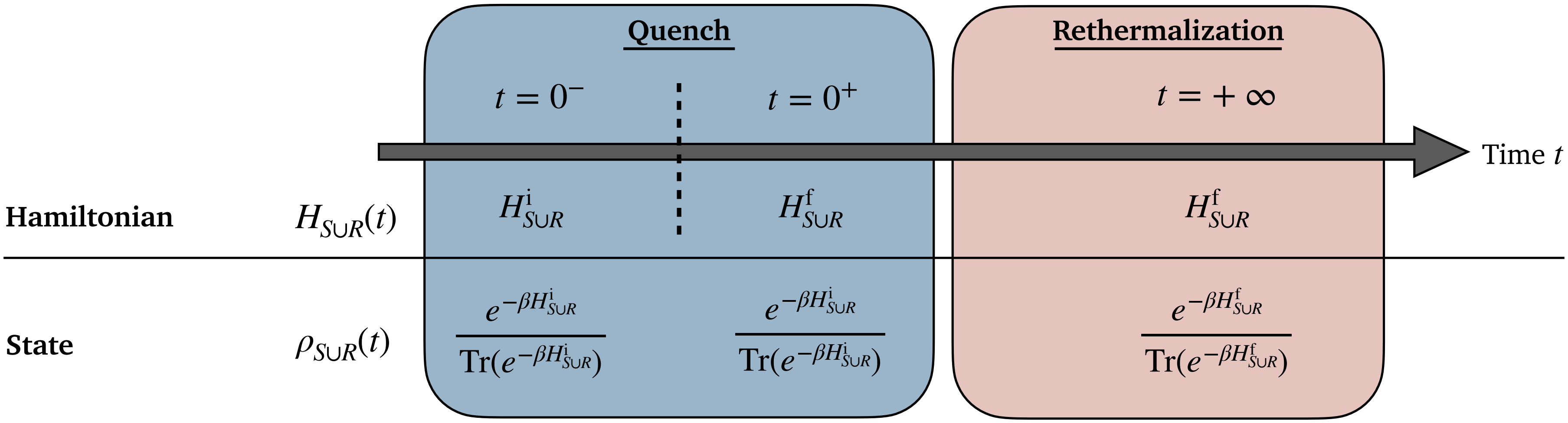
Time t

		$t = 0^-$
Hamiltonian	$H_{S\cup R}(t)$	$H^{\mathrm{i}}_{S\cup R}$
State	$\rho_{S\cup R}(t)$	$\dfrac{e^{-\beta H^{\mathrm{i}}_{S\cup R}}}{\mathrm{Tr}(e^{-\beta H^{\mathrm{i}}_{S\cup R}})}$

Quantum quenches: Simplest non-equilibrium experiments.



Quantum quenches: Simplest non-equilibrium experiments.



Quantum quenches: Simplest non-equilibrium experiments.

		Quench		Rethermalization
	Time t	$t = 0^-$	$t = 0^+$	$t = +\infty$
Hamiltonian	$H_{S \cup R}(t)$	$H^{\mathrm{i}}_{S \cup R}$	$H^{\mathrm{f}}_{S \cup R}$	$H^{\mathrm{f}}_{S \cup R}$
State	$\rho_{S \cup R}(t)$	$\frac{e^{-\beta H^{\mathrm{i}}_{S \cup R}}}{\mathrm{Tr}(e^{-\beta H^{\mathrm{i}}_{S \cup R}})}$	$\frac{e^{-\beta H^{\mathrm{i}}_{S \cup R}}}{\mathrm{Tr}(e^{-\beta H^{\mathrm{i}}_{S \cup R}})}$	$\frac{e^{-\beta H^{\mathrm{f}}_{S \cup R}}}{\mathrm{Tr}(e^{-\beta H^{\mathrm{f}}_{S \cup R}})}$

Change in system's internal energy $\quad \Delta U_S = W \quad$ (from $t = 0^-$ to $t = 0^+$) $\quad \Delta U_S = Q \quad$ (from $t = 0^+$ to $t = +\infty$)

Work and heat in quantum quenches:

In summary: Work: $W := \mathrm{Tr}_S(\rho_S^{\mathrm{i}} H_S^*(t = 0^+)) - \mathrm{Tr}_S(\rho_S^{\mathrm{i}} H_S^*(t = 0^-))$

Heat: $Q := \mathrm{Tr}_S(\rho_S^{\mathrm{f}} H_S^*(t = 0^+)) - \mathrm{Tr}_S(\rho_S^{\mathrm{i}} H_S^*(t = 0^+))$

These definitions are **consistent with first and second laws**, both classically and quantumly.

ZD, Jarzynski, Mueller, Oruganti, Powers, and Yunger Halpern, manuscript in progress (2024).

Can these be measured in quantum-simulation experiments?

Define the entanglement Hamiltonian: $H_{\text{ent}} = -\ln(\rho_S)$ with $\rho_S = \text{Tr}_R(\rho_{S\cup R})$.

Entanglement spectrum: informs thermalization dynamics or distinguishes topological phases.

See e.g., Mueller, Zache, Ott, Phys. Rev. Lett. 129 (2022) 1, 011601, Mueller, Wang, Katz, ZD, Cetina, arXiv:2408.00069 [quant-ph].

We prove that:

$$H_S^* = \beta^{-1} H_{\text{ent}} + F_S.$$

The quantity dissipated work $W_{\text{diss}} = W - \Delta F_S$ can then be shown to be equal to $\beta^{-1} H_{\text{ent}}$ in our quench process!

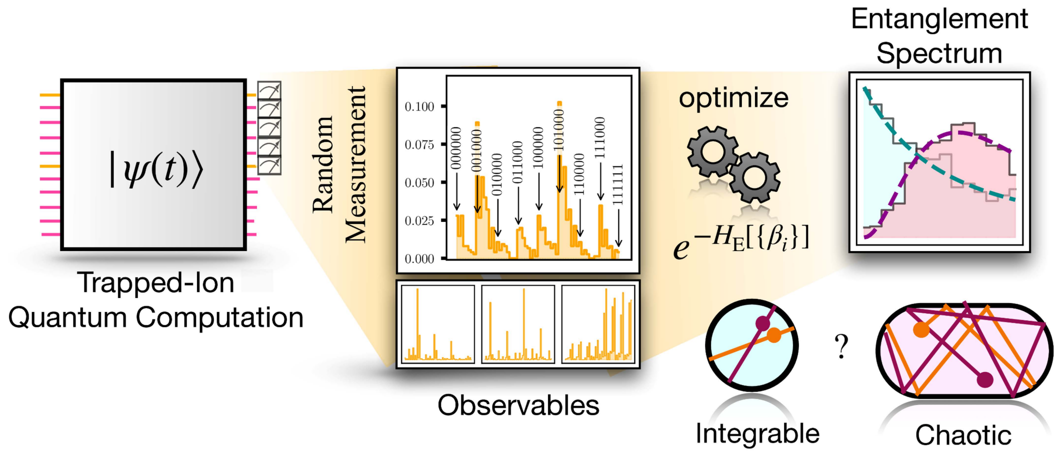
ZD, Jarzynski, Mueller, Oruganti, Powers, and Yunger Halpern, arXiv:2404.02965 [quant-ph].

Why does this matter? Because there are efficient tools for entanglement tomography (at least for ground, excited, and non-equilibrium states).

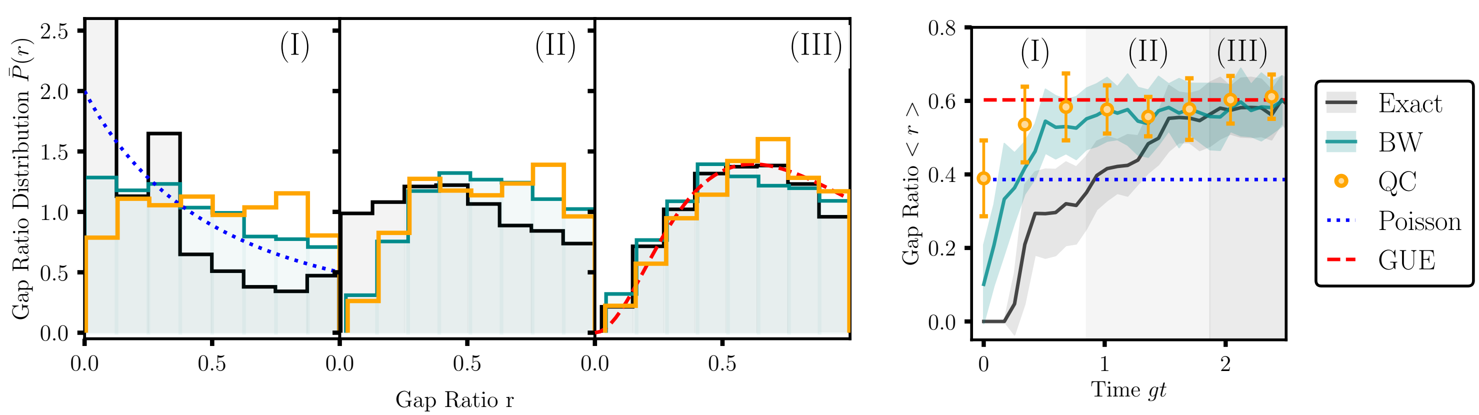
Some examples in: Elben, Flammia, Huang, Kueng, Preskill, Vermersch, and Zoller, Nature Review Physics (2022).

Example: Entanglement tomography in a (2+1)D Z_2 gauge theory

Randomized measurements allow optimizing parameters of a motivated entanglement Hamiltonian ansatz..



…and obtain the gap-ratio distribution of entanglement spectrum,



which signal chaotic behavior, hence thermalization, as system evolves after a quench!

Mueller, Wang, Katz, ZD, Cetina, arXiv:2408.00069 [quant-ph].

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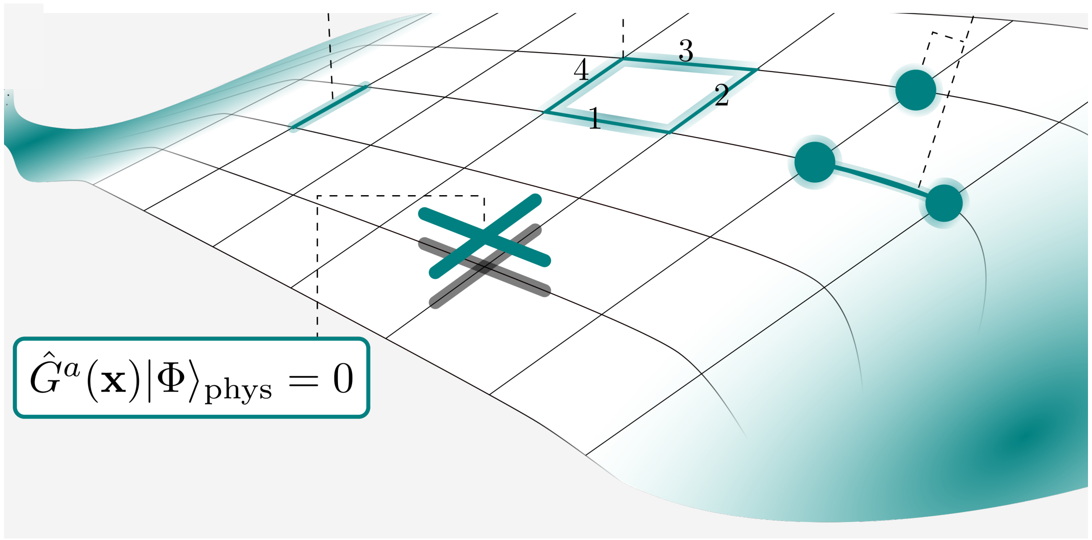
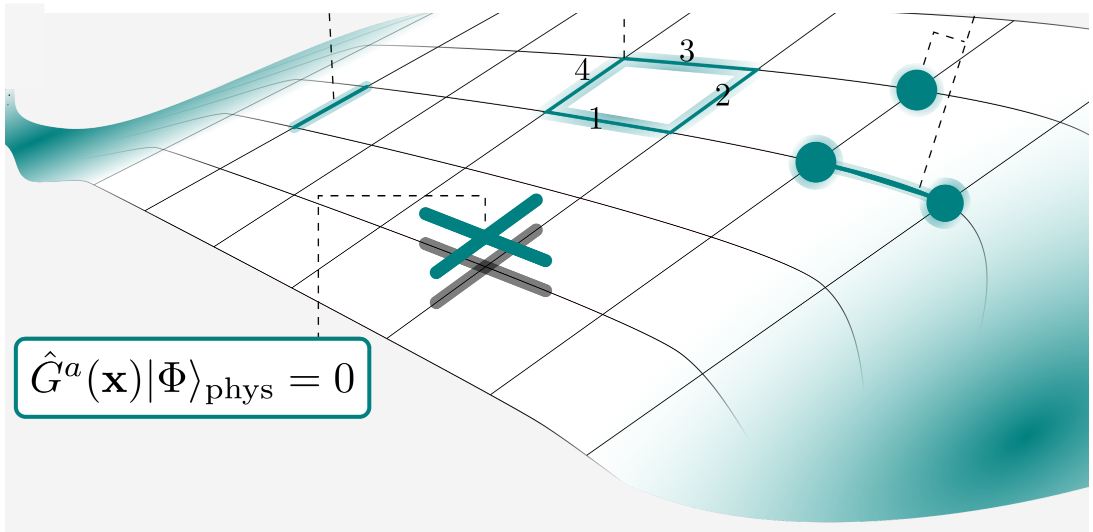
Hamiltonian formulation of lattice gauge theories



Bauer, ZD, Klco, and Savage, *Nature Rev. Phys.* 5 (2023) 7, 420-432.

Hamiltonian formulation of lattice gauge theories



Bauer, ZD, Klco, and Savage, *Nature Rev. Phys.* 5 (2023) 7, 420-432.

Bi-partitining the system into subsystem and reservoir is non-trivial in LGTs. Think about imposing the constraint via

$$H'_{S\cup R} = H_{S\cup R} + \kappa \sum_{\mathbf{x}} f(G(\mathbf{x}))$$

by choosing κ such that dynamics under $H'_{S\cup R}$ is constrained to the physical sector.

Hence, we posit that LGT thermodynamics may need to be studied within the strong-coupling framework.

ZD, Jarzynski, Mueller, Oruganti, Powers, and Yunger Halpern, arXiv:2404.02965 [quant-ph].

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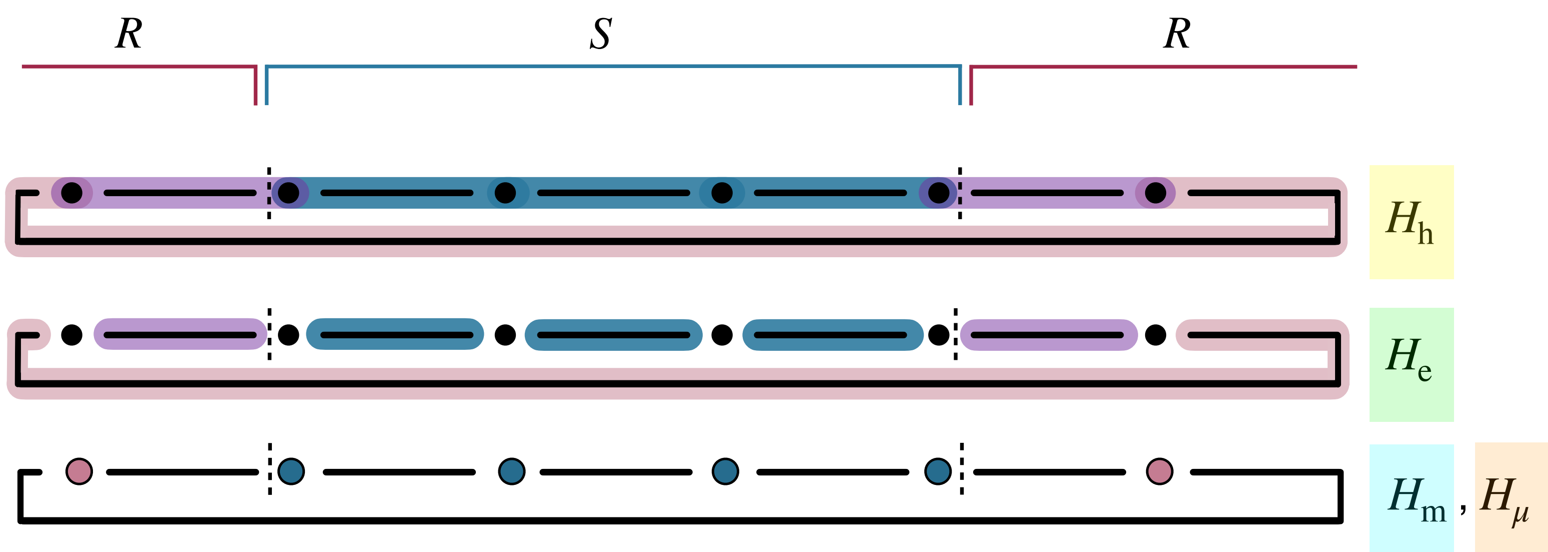
Example: Z_2 LGT in (1+1)D coupled to hardcore bosonic matter

Consider the Hamiltonian:

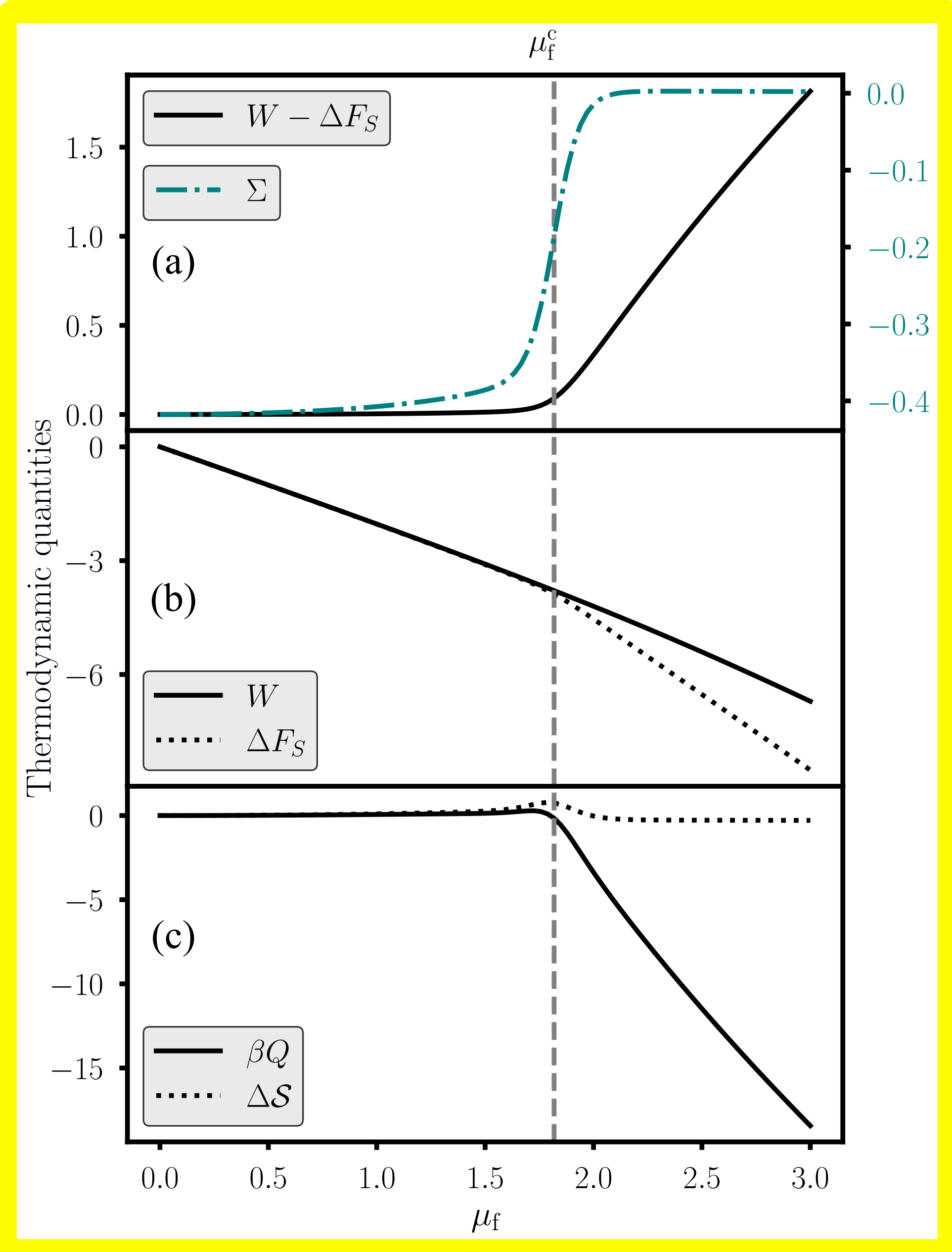
$$H = \underbrace{-t\sum_{n=0}^{N-1}\left(\sigma_n^+\tilde{\sigma}_n^z\sigma_{n+1}^- + \text{h.c.}\right)}_{H_\text{h}} \underbrace{- \epsilon\sum_{n=0}^{N-1}\tilde{\sigma}_n^x}_{H_\text{e}} \underbrace{+ m\sum_{n=0}^{N-1}(-1)^n\sigma_n^+\sigma_n^-}_{H_\text{m}} \underbrace{- \mu\sum_{n=0}^{N-1}\sigma_n^+\sigma_n^-}_{H_\mu} + c\sum_{n=0}^{N-1}\mathbb{I}_n$$

Bi-partition the full system to system and reservoir as:



ZD, Jarzynski, Mueller, Oruganti, Powers, and Yunger Halpern, arXiv:2404.02965 [quant-ph].

A chemical-potential quench

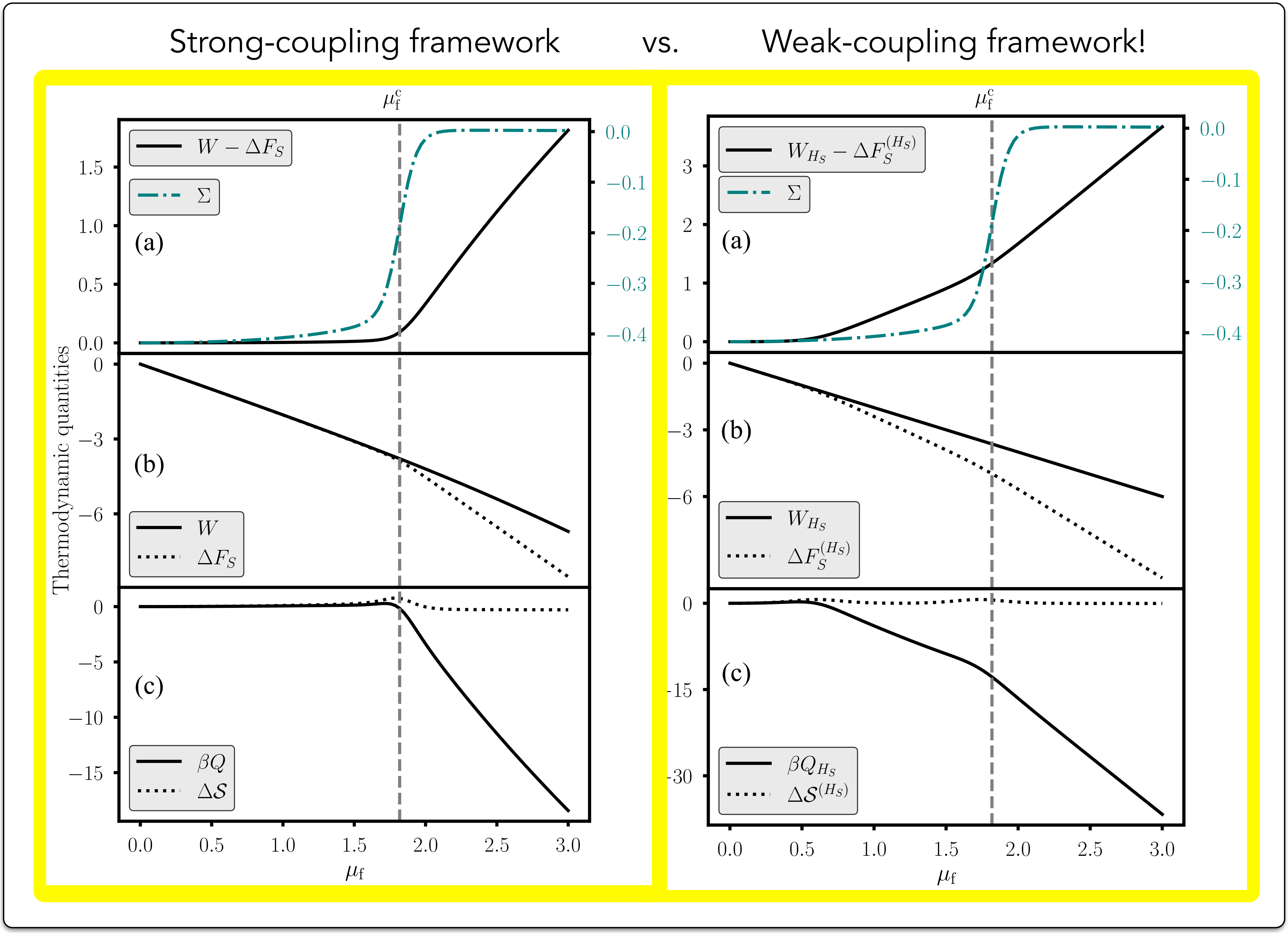


Quench the system chemical potential form $\mu_i = 0 \to \mu_f$:

Compute thermodynamical quantities: $\Delta F_S, W, Q, \Delta \mathcal{S}$ with strong-coupling framework.

Notice the distinct behavior in these quantities in the "chirally symmetric" vs. "chirally broken" phases (compare with order parameter $\Sigma = \frac{1}{N_s} \sum_n (-1)^n \langle \sigma_n^+ \sigma_n^- \rangle$).

ZD, Jarzynski, Mueller, Oruganti, Powers, and Yunger Halpern, arXiv:2404.02965 [quant-ph].



Strong-coupling framework
vs.
Weak-coupling framework!
μ_f^c
$W - \Delta F_S$
Σ
(a)
(b)
(c)
W
ΔF_S
βQ
$\Delta \mathcal{S}$
Thermodynamic quantities
μ_f
$W_{H_S} - \Delta F_S^{(H_S)}$
W_{H_S}
$\Delta F_S^{(H_S)}$
βQ_{H_S}
$\Delta \mathcal{S}^{(H_S)}$

Summary

- Thermodynamic quantities must be defined with care in strong-coupling quantum thermodynamics.
- We define work and heat in non-equilibrium quench processes in a way consistent with the first and second laws of thermodynamics.
- We apply this framework to gauge-theory thermodynamics and demonstrate the sensitivity of thermodynamic quantities to phase transitions.
- We show how these thermodynamic quantities can be extracted using entanglement-tomography tools in quantum simulations.

outlook

- Explore thermodynamic and continuum limits?
- Extension of the formalism to other non-equilibrium processes such as high-energy collisions?
- Demonstration in a quantum-simulation experiment? Need equilibrium-state entanglement Hamiltonian tomography.