Scaling results for Charged sectors of near conformal QCD

Based on *Phys.Rev.D* 109 (2024) 12, in collaboration with J. Bersini, C. Gambardella and F. Sannino

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Overview

1. Basic blocks
2. Executive Summary
3. The effective Lagrangian
4. Motivations & Methodologies
5. Results
6. Back up slides

Goal of this talk

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progress in understanding the QCD phase diagram

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progress in understanding the QCD phase diagram

new way to access near conformal information

Talks of other members of the group

- *P. Butti on Tuesday*: $B \to D^{(*)}$ decays from $N_f = 2+1+1$ highly improved staggered quarks and clover b-quark in the Fermilab interpretation.
- *A. Rago on Tuesday*: openQCD on GPU
- *S. Martins on Tuesday*: Progress on the GPU porting of HiRep

Basic blocks

Introduction

Unveiling near conformal properties of QCD

Introduction

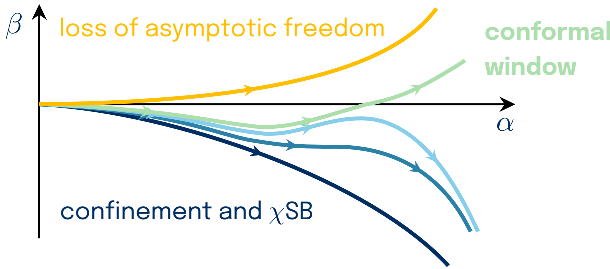
IR dynamics of $SU(N)$ theories
depends on both the matter content
and on the strength of the coupling constant

WHAT do
we mean ?

Unveiling near conformal
properties of QCD

Introduction

the β-function may develop an IR fixed point



WHAT do we mean ?

Unveiling near conformal properties of QCD

Executive Summary

Executive Summary

approaching from below the conformal window: near conformal regime

near conformality is non linearly realised via the dilaton σ and its potential $V(\sigma)$

WHEN ?

HOW ?

Unveiling near conformal properties of QCD

Executive Summary

Unveiling near conformal properties of QCD at nonzero isospin density

WHEN ?

approaching from below the conformal window: near conformal regime

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WHERE ?

on a cylinder

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energy $\simeq$ scaling dimension on $\mathbb{R}^4$

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scaling dimensions of QCD operators carrying isospin charge

WHAT ?

Unveiling near conformal properties of QCD at nonzero isospin density

Main result

We consider the dynamics near the lower edge of the conformal window on a non-trivial background to determine scaling dimensions of QCD operators carrying isospin charge:

$$\Delta_Q \equiv r\, E_Q = \Delta_Q^* + \text{near CFT terms} \tag{1}$$

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$$\Delta_Q \equiv r\, E_Q = \Delta_Q^* + \text{near CFT terms} \tag{1}$$

$$\Delta_Q = \Delta_Q^* + \left(\frac{m_\sigma}{4\pi\nu}\right)^2 Q^{\frac{\Delta}{3}} B_1 + \left(\frac{m_\pi(\theta)}{4\pi\nu}\right)^4 Q^{\frac{2}{3}(1-\gamma)} B_2 + \mathcal{O}\left(m_\sigma^4, m_\pi^8, m_\sigma^2 m_\pi^4\right) \tag{2}$$

J. L. Cardy, *Conformal invariance and universality in finite-size scaling*, S. Hellerman at al., *On the CFT operator spectrum at large global charge*

The effective Lagrangian

Chiral Lagrangian at finite isospin and θ-angle

The low-energy dynamics of the theory is described by the chiral Lagrangian below

$$\mathcal{L} = \nu^2 Tr\{\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\} + m_\pi^2 \nu^2 Tr\{M\Sigma + M^\dagger \Sigma^\dagger\}$$ Goldstones' dynamics

$$+2i\mu\nu^2 Tr\{I\partial_0 \Sigma \Sigma^\dagger - I\Sigma^\dagger \partial_0 \Sigma\} + 2\mu^2\nu^2 Tr\{II - \Sigma^\dagger I \Sigma I\}$$ isospin contribution

$$-a\nu^2 \left(\theta - \frac{i}{2} Tr\{\log \Sigma - \log \Sigma^\dagger\}\right)^2$$ topological term: θ -angle

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Here ν is half the pion decay constant, μ is the (generalized) isospin chemical potential, m_π is the mass of the Goldstones and

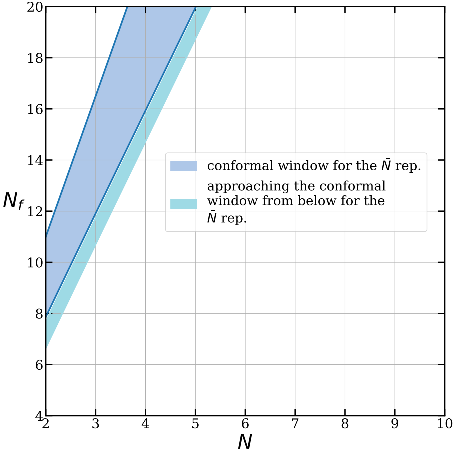
$$\Sigma = e^{i\varphi/\nu}\,, \quad \varphi = \Pi^a T^a + \frac{S}{\sqrt{N_f}}\,, \quad M = \mathbb{1}_{N_f}\,, \quad I = \frac{1}{2}\begin{pmatrix} \mathbb{1}_{N_f/2} & 0 \\ 0 & -\mathbb{1}_{N_f/2} \end{pmatrix} \tag{3}$$

D. T. Son et al., *QCD at finite isospin density*, E. Witten, *Large N chiral dynamics*

Motivations & Methodologies

Motivations: When & How

When



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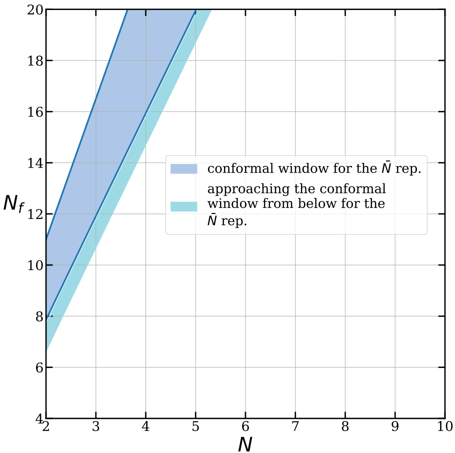

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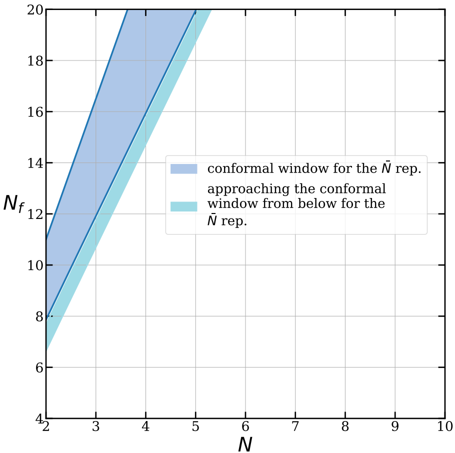
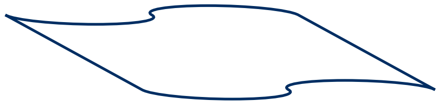


How

$near - conformality$: introduction of a potential $V(\sigma)$ as a source of explicit breaking of conformality

D. D. Dietrich et al., *Conformal window of SU(N) gauge theories with fermions in higher dimensional representations*, M. Golterman et al., *Low-energy effective action for pions and a dilatonic meson*, T. Appelquist et al., *Dilaton effective field theory*

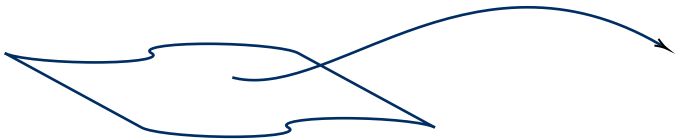
Methodologies: Where, Why & What



We want to unveil near conformal properties of the theory on flat spacetime

Methodologies: Where, Why & What

Weyl map:



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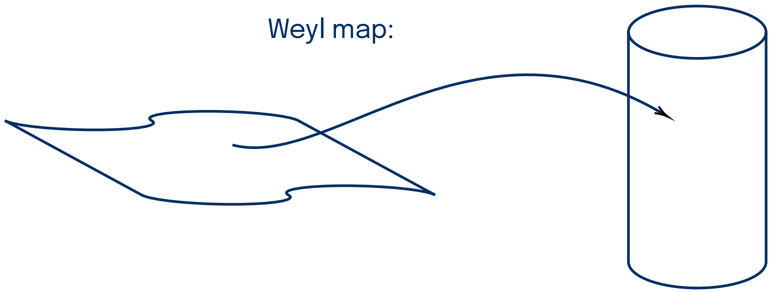

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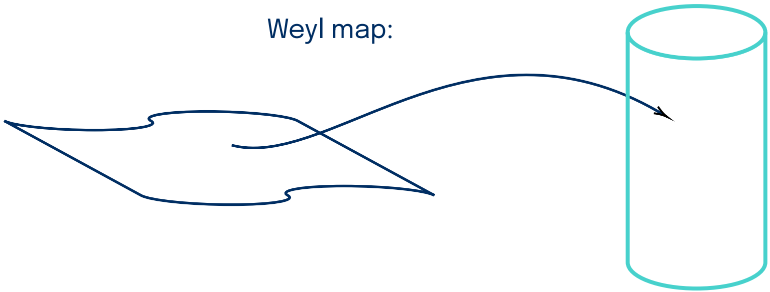
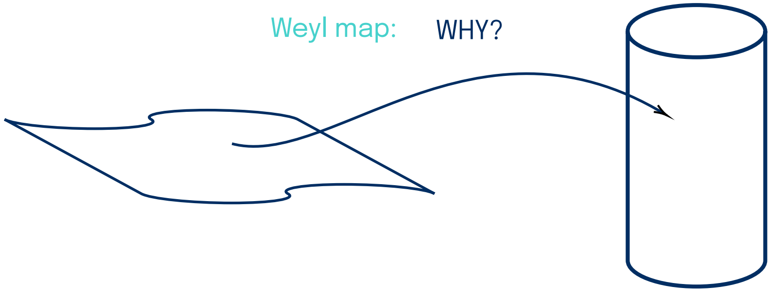


WHERE?

we carry the computations on a $cylinder$ with radius r

Methodologies: Where, Why & What

Weyl map: WHY?



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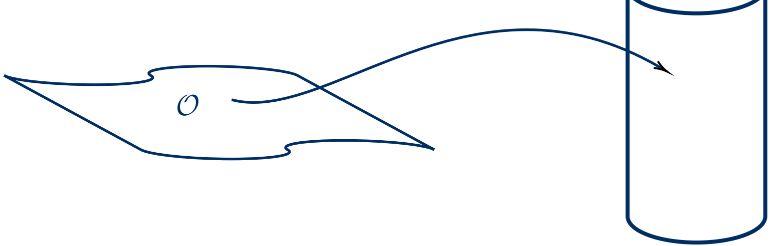
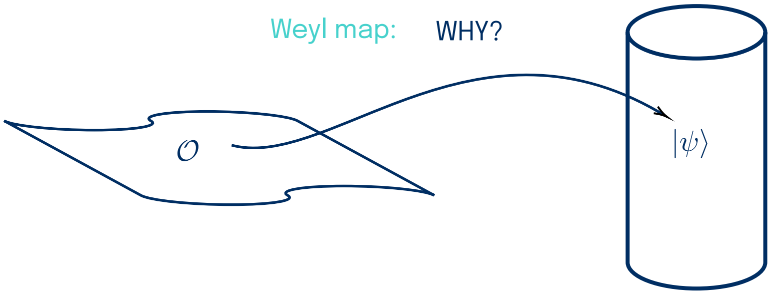


via the $state - operator\ correspondence$ $\Delta = r\, E$

Methodologies: Where, Why & What

Weyl map: WHY?



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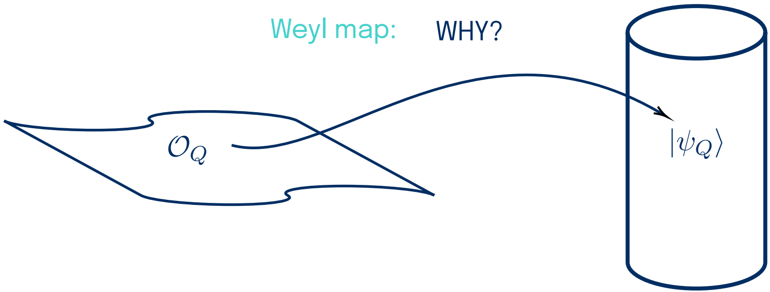

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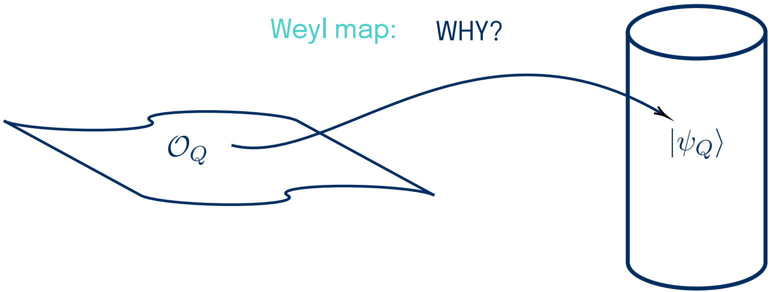

via the *state − operator correspondence* $\Delta_Q = r\, E_Q$

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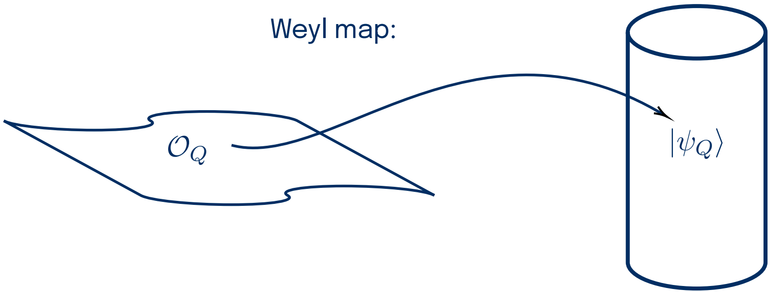

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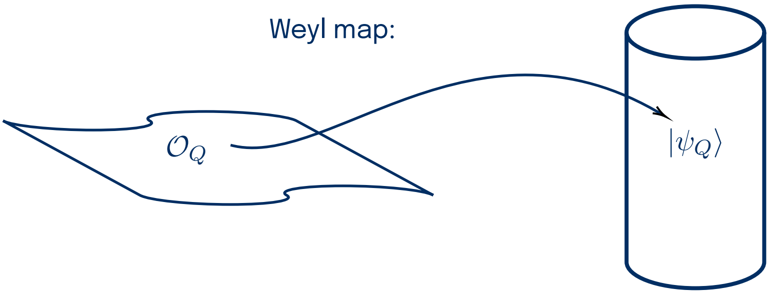

via the $state - operator\ correspondence$ $\Delta_Q = r\, E_Q$

WHAT?

ground state energy on the cylinder $\Longrightarrow$ scaling dimensions of operators carrying charge Q on flat spacetime

The near-conformal Lagrangian: the dilaton

$$\tilde{\mathcal{L}} = \nu^2 Tr\{\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\}\, e^{-2\sigma f} + m_\pi^2 \nu^2 Tr\{M\Sigma + M^\dagger \Sigma^\dagger\}\, e^{-y\sigma f}$$ Goldstones' dynamics

$$+ 2\mu^2\nu^2 Tr\{II - \Sigma^\dagger I \Sigma I\} e^{-2\sigma f} + 2i\mu\nu^2 Tr\{I\partial_0 \Sigma\Sigma^\dagger - I\Sigma^\dagger \partial_0 \Sigma\} e^{-2\sigma f}$$ isospin

$$- a\nu^2 \left(\theta - \frac{i}{2} Tr\{\log\Sigma - \log\Sigma^\dagger\}\right)^2 e^{-4\sigma f}$$ topological contribution

$$+ \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma - \frac{R}{6f^2}\right) e^{-2\sigma f} - \Lambda_0^4\, e^{-4\sigma f}$$ dilaton's dynamics & geometric terms

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The Lagrangian that we use is

$$\mathcal{L}_\sigma = \tilde{\mathcal{L}} - V(\sigma) \tag{4}$$

A. Salam et al., *Nonlinear realizations. II. Conformal symmetry*, M. Golterman et al., *Low-energy effective action for pions and a dilatonic meson*, T. Appelquist et al., *Dilaton effective field theory*

EOMs and Ground State Energy

the classical ground state energy

$$E_Q = \mu Q - \mathcal{L}_\sigma \tag{5}$$

is computed by solving the EOMs

$$\frac{\partial \mathcal{L}}{\partial \alpha} = \frac{\partial \mathcal{L}}{\partial \varphi} = \frac{\partial \mathcal{L}}{\partial \sigma_0} = 0, \quad \frac{\partial \mathcal{L}}{\partial \mu} = \frac{Q}{V}\,, \tag{6}$$

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we solve the EOMs perturbatively in positive powers of the parameters m_σ^2 and m_π^2

Results

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the non-conformal corrections depend on the parameters encoding the explicit breaking of scale invariance

Results: Conformal Contribution

$$\Delta_Q \equiv r\,E_Q = \Delta_Q^* \;+\; \text{near CFT terms} \tag{9}$$

S. Hellerman et al., *On the CFT operator spectrum at large global charge*, A. Monin et al., *Semiclassics, goldstone bosons and CFT data*. J. Bersini et al., *Charging the conformal window at nonzero θ angle*

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$$\Delta_Q^* = c_{4/3} Q^{4/3} + c_{2/3} Q^{2/3} + (Q^0) \tag{10}$$

is the scaling dimension in the conformal limit at the leading order in the large

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$$c_{4/3} = \frac{3}{8}\left(\frac{2\Lambda^2}{\pi N_f \nu^2}\right)^{2/3}, \quad c_{2/3} = \frac{1}{4f^2}\left(\frac{2\pi^2}{N_f \nu^2 \Lambda^4}\right)^{1/3} \tag{11}$$

S. Hellerman et al., *On the CFT operator spectrum at large global charge*, A. Monin et al., *Semiclassics, goldstone bosons and CFT data*. J. Bersini et al., *Charging the conformal window at nonzero θ angle*

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$$B_2 = -\,3^{4-\gamma} 2^{4\gamma-3} \pi^{2\gamma+2} c_{4/3}^{\gamma-4} N_f^{\gamma-1} (\nu r)^{2(\gamma+1)} \left(1 + \frac{(\gamma-4) c_{2/3}}{2c_{4/3}} Q^{-2/3} + \left(Q^{-4/3}\right)\right)$$

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geometry

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geometry *charge expansion*

Conclusions & Take Home Messages

scaling dimensions of QCD operators carrying isospin charge at the lower boundary of the QCD conformal window

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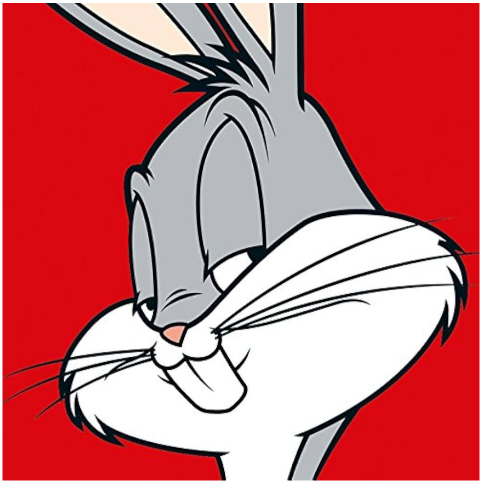
- the near-conformal corrections are proportional to m_σ^2 and m_π^4

novel way to compute the dilaton mass

- the large charge framework allows us to perform analytical calculations

Q works as a new tunable parameter



That's all Folks!

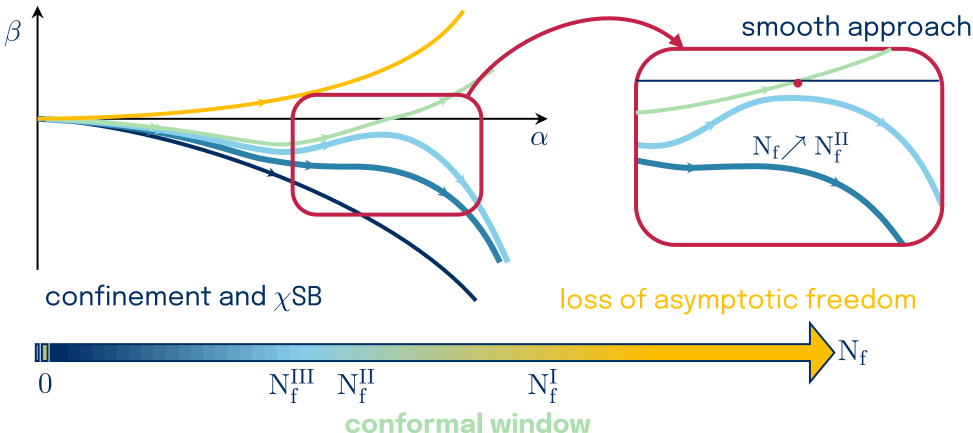
Thank you!

Some related talks:

- *R. Zwicky on Tuesday*: Dilaton effective theory and soft theorems
- *J. Ingoldby on Friday*: Dilaton Forbidden Dark Matter

Back up slides

$SU(2)$: **walking**



β
α
smooth approach
$N_f \nearrow N_f^{II}$
confinement and χSB
loss of asymptotic freedom
N_f
0
N_f^{III}
N_f^{II}
N_f^{I}
conformal window

Symmetries I: spontaneous breaking

We consider the dynamics near the lower edge of the conformal window

To smoothly approach the lower edge of the conformal window a new light scalar is included: *the dilaton*

The IR dynamics is studied within the dilaton effective field theory

The first step is to upgrade the chiral Lagrangian to a conformally invariant theory

via the introduction of a scalar degree of freedom σ, the *dilaton*, which under scale dilations $x \mapsto e^{\lambda} x$ transforms as

$$\sigma \mapsto \sigma - \frac{\lambda}{f} \,. \tag{15}$$

S. Coleman, *Aspects of Symmetry: Selected Erice Lectures*, C. Isham et al., *Spontaneous breakdown of conformal symmetry*, A. Salam et al., *Nonlinear realizations. II. Conformal symmetry*

Symmetries I: spontaneous breaking

Scale invariance can then be enforced at the effective action level by coupling σ to each operator $\mathcal{O}_k$ of dimension k appearing in the Lagrangian as

$$\mathcal{O}_k \mapsto e^{(k-4)\sigma f} \, \mathcal{O}_k \, . \tag{16}$$

Dilaton Effective Field Theory

The resulting theory features non-linearly realized dilation invariance with f and σ being the length scale and the Goldstone boson associated with the spontaneous breaking of conformal symmetry, respectively.

S. Coleman, *Aspects of Symmetry: Selected Erice Lectures*, C. Isham et al., *Spontaneous breakdown of conformal symmetry*, A. Salam et al., *Nonlinear realizations. II. Conformal symmetry*

Symmetries II: explicit sources

Explicit breaking of the latter can be taken into account introducing a potential term for σ.

Consider perturbing a CFT with an operator $\mathcal{O}$ with conformal dimension Δ, i.e.

$$\mathcal{L}_{CFT} \to \mathcal{L}_{CFT} + \lambda_{\mathcal{O}} \mathcal{O} \,, \tag{17}$$

with $\lambda_{\mathcal{O}}$ the corresponding coupling. For $\lambda_{\mathcal{O}} \ll 1$ the perturbation generates the following dilaton potential

$$V(\sigma) = \frac{m_\sigma^2 e^{-4f\sigma}}{4(4-\Delta)f^2} - \frac{m_\sigma^2 e^{-\Delta f\sigma}}{\Delta(4-\Delta)f^2} + \mathcal{O}(\lambda_{\mathcal{O}}^2) \,. \tag{18}$$

W. D. Goldberger et al., *Distinguishing the Higgs boson from the dilaton at the Large Hadron Collider*, T. Appelquist et al., *Dilaton effective field theory*

Methodology

Operators having large internal charge can be associated, via state/operator correspondence, to a superfluid phase on a cylinder.

$Why?$ By virtue of the operator/state correspondence, a scalar operator with $U(1)$ charge Q corresponds to a state with homogeneous charge density in the theory compactified on the cylinder with radius r.
This state will have charge density $\rho \sim Q/r^{d-1}$.

$When$ Q$\gg 1$ there exists a parametric separation between

$$\underbrace{\frac{1}{r}}_{\text{the scale of compactification}} \quad \text{and} \quad \underbrace{\rho^{1/(d-1)} \sim \frac{Q^{1/(d-1)}}{\mathcal{R}}}_{\text{scale associated to the charge density}} \tag{19}$$

In this window of energy the CFT state and its excitations will therefore correspond to some *condensed matter phase*: we consider it to be in a $superfluid\ phase$

Methodology

The derivative and loop expansion are controlled by powers of the ratio between the IR scale, $1/r$, and the UV scale $\rho^{1/(d-1)}$:

Semiclassic methods: Large charge expansion

Inverse powers of the charge Q control the derivative and loop expansion of the theory on the cylinder. Order by order, the non-universal features associated with any specific CFT will be encapsulated by finitely many coefficients in the effective Lagrangian.

as for the pion Lagrangian: $m_{QCD} = 4\pi f_\pi$ is the UV scale while m_π is the IR and the physical observables are controlled by a systematic expansion in powers of m_π / m_{QCD}

Large charge expansion: the leading order

We will consider our system on a manifold $\mathcal{M}$ with volume r^3 such that the underlying new scale of the theory is

$$\Lambda_Q = (Q/r^3)^{1/3} \tag{20}$$

where Q is the fixed isospin charge.
Concretely, we will take our manifold to be

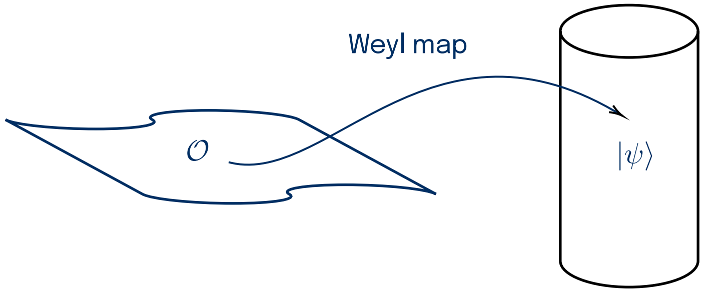
$$\mathcal{M} = \mathbb{R} \times S^{d-1} \tag{21}$$

such that we can consider an approximate state-operator correspondence that implies

$$\Delta_Q = rE_Q \,, \qquad E_Q = \mu Q - \mathcal{L} \tag{22}$$

where Δ_Q is the scaling dimension of the lowest-lying operator with isospin charge Q and E_Q is the ground state energy on $\mathbb{R} \times S^{d-1}$ at fixed charge

Digression I: Large charge setup



In the conformal limit, $m_\pi = m_\sigma = 0$, Δ^*_Q can then be computed via a semiclassical expansion in the double scaling limit

$$\Lambda_0 f \to 0\,, \quad Q \to \infty\,, \quad Q(\Lambda_0 f)^4 = \text{fixed}\,. \tag{23}$$

Digression II: Large charge setup

This can be seen by considering the expectation value of the evolution operator $U = e^{-HT}$ in an arbitrary state $|Q\rangle$ with charge Q

$$\langle U \rangle_Q \equiv \langle Q | e^{-HT} | Q \rangle \underset{T\to\infty}{\longrightarrow} \mathcal{N} e^{-E_Q T} = \mathcal{N} e^{-\frac{\Delta^*_Q}{r} T} \,, \tag{24}$$

with H the Hamiltonian, T the time interval, and $\mathcal{N}$ an unimportant normalization factor. Then one can rescale the fields as $\Sigma \to \nu f \Sigma$ and $e^{-f\sigma} \to \sqrt{Q} e^{-f\sigma}$ to exhibit

Q as a new counting parameter in the path integral expression for $\langle U \rangle_Q$

A. Monin et al., *Semiclassics, goldstone bosons and CFT data*, G. Badel et al., *The epsilon expansion meets semiclassics*

Digression III: Large charge setup

The scaling dimension of the lowest-lying operator assumes the following form

$$rE_Q = \Delta_Q = \sum_{j=-1} \frac{1}{Q^j} \Delta_j \left(Q(\Lambda_0 f)^4 \right) \ . \tag{25}$$

The leading order Δ_{-1} is given by the classical ground state energy on $\mathbb{R} \times S^3_r$

whereas the next-to-leading order Δ_0 is determined by the fluctuations around the classical trajectory.

A. Monin et al., *Semiclassics, goldstone bosons and CFT data*, G. Badel et al., *The epsilon expansion meets semiclassics*

Chiral Lagrangian at finite isospin and θ-angle

The low-energy dynamics of the theory is described by the chiral Lagrangian below

$$\mathcal{L} = \nu^2 Tr\{\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\} + m_\pi^2 \nu^2 Tr\{M\Sigma + M^\dagger \Sigma^\dagger\}$$ Goldstones' dynamics

$$+2i\mu\nu^2 Tr\{I\partial_0\Sigma\Sigma^\dagger - I\Sigma^\dagger\partial_0\Sigma\} + 2\mu^2\nu^2 Tr\{II - \Sigma^\dagger I \Sigma I\}$$ isospin contribution

$$-a\nu^2\left(\theta - \frac{i}{2}Tr\{\log\Sigma - \log\Sigma^\dagger\}\right)^2$$ topological term: θ -angle

vacuum ansatz Σ_0:

$$\Sigma_0 = U(\alpha_i)\,\Sigma_c\,, \text{ with } U(\alpha_i) = \text{diag}\{e^{-i\alpha_1}, \ldots, e^{-i\alpha_{N_f}}\}\,. \quad (26)$$

$$\Sigma_c = \mathbb{1}_{N_f}\cos\varphi + i\left(\begin{pmatrix} 0 & \mathbb{1}_{N_f/2} \\ \mathbb{1}_{N_f/2} & 0 \end{pmatrix}\cos\eta + i\begin{pmatrix} 0 & -\mathbb{1}_{N_f/2} \\ \mathbb{1}_{N_f/2} & 0 \end{pmatrix}\sin\eta\right)\sin\varphi$$

The near-conformal Lagrangian: the dilaton

1. the state-operator correspondence enables us to deduce the scaling dimension for the lowest-lying operator with (generalised) isospin charge Q
2. this is achieved by determining the energy associated with the vacuum structure inducing the superfluid phase transition

we therefore evaluate $\mathcal{L}_\sigma$ on the vacuum ansatz (26), obtaining

$$\mathcal{L}_\sigma\left[\Sigma_0,\sigma_0\right] = -e^{-4f\sigma_0}\Lambda_0^4 - V(\sigma_0) - \frac{R\,e^{-2f\sigma_0}}{12f^2} + 2m_\pi^2\nu^2 X\cos\varphi\, e^{-f\sigma_0 y} + N_f\mu^2\nu^2 e^{-2f\sigma_0}\sin^2\varphi - a\nu^2 e^{-4f\sigma_0}\bar{\theta}^2\,. \tag{27}$$

where σ_0 denotes the classical dilaton solution and

$$\bar{\theta} = \theta - \sum_i^{N_f}\alpha_i\,,\quad X = \sum_{i=1}^{N_f}\cos\alpha_i\,, \tag{28}$$

EOMs

$$\mathcal{L}_\sigma\left[\Sigma_0,\sigma_0\right]=-e^{-4f\sigma_0}\Lambda_0^4-V(\sigma_0)-\frac{R\,e^{-2f\sigma_0}}{12f^2}+2m_\pi^2\nu^2X\cos\varphi\,e^{-f\sigma_0 y}$$
$$+N_f\mu^2\nu^2e^{-2f\sigma_0}\sin^2\varphi-a\nu^2e^{-4f\sigma_0}\bar{\theta}^2 \quad (29)$$

The classical ground state energy is computed by solving the following EOMs

$$\sin\varphi(N_f\mu^2e^{-2f\sigma_0}\cos\varphi-m_\pi^2Xe^{-f\sigma_0 y})=0\,, \quad (30)$$

$$ae^{-4f\sigma_0}\bar{\theta}-m_\pi^2\sin\alpha_i\cos\varphi e^{-f\sigma_0 y}=0\,, \qquad i=1,..,N_f\,, \quad (31)$$

$$\frac{Re^{-2f\sigma_0}}{6f}+4af\nu^2e^{-4f\sigma_0}\bar{\theta}^2+4f\Lambda_0^4e^{-4f\sigma_0}-\left.\frac{\partial V(\sigma)}{\partial\sigma}\right|_{\sigma=\sigma_0}+$$
$$-2fN_f\mu^2\nu^2e^{-2f\sigma_0}\sin^2\varphi-2fym_\pi^2\nu^2X\cos\varphi e^{-f\sigma_0 y}=0\,, \quad (32)$$

$$2N_f\mu\nu^2e^{-2f\sigma_0}\sin^2\varphi=\frac{Q}{V}\,, \quad (33)$$

Solving the EOMs

$$\frac{\partial \mathcal{L}}{\partial \alpha} = \frac{\partial \mathcal{L}}{\partial \varphi} = \frac{\partial \mathcal{L}}{\partial \sigma_0} = 0, \quad \frac{\partial \mathcal{L}}{\partial \mu} = \frac{Q}{V}\,, \tag{34}$$

to determine the classical ground state energy on the cylinder we need to solve the above EOMs in the variables φ, α_i, σ_0 and μ and plug the solution into eq.(29)

we expand the variables as
$x = x_0 + x_1 m_\sigma^2 + x_2 m_\pi^2 + x_3 m_\sigma^4 + x_4 m_\pi^4 + x_5 m_\sigma^2 m_\pi^2 + \left(m_\sigma^6, m_\pi^6, m_\sigma^4 m_\pi^2, m_\sigma^2 m_\pi^4\right)$ where $x = \{\mu, \varphi, \sigma_0, \alpha_i\}$ and determine the coefficients of the expansion by solving the EOMs order by order

we redefines the cosmological constant as

$$\Lambda^4 \equiv \Lambda_0^4 + \frac{m_\sigma^2}{4 f^2 (4-\Delta)}\,.$$

The Phase diagram

The Lagrangian of the theory evaluated on the ground state ansatz reads

$$\mathcal{L}[\Sigma_0] = 2m_\pi^2\nu^2 X\cos\varphi + N_f\mu^2\nu^2\sin^2\varphi - a\nu^2\bar{\theta}^2 \,. \tag{35}$$

The angle φ and the Witten variables α_i are determined by the EOM as

$$\sin\varphi\left(N_f\cos\varphi - \frac{m_\pi^2 X}{\mu^2}\right) = 0 \,, \tag{36}$$

$$m_\pi^2\sin\alpha_i\cos\varphi = a\bar{\theta} \,, \quad i = 1,\ldots,N_f \tag{37}$$

The energy density of the system in the two phases reads

$E(\theta) = -2m_\pi^2\nu^2 X + a\nu^2\bar{\theta}^2$ normal phase $(\varphi = 0)$

$E(\theta) = -\dfrac{m_\pi^4\nu^2}{N_f\mu^2}X^2 - N_f\nu^2\mu^2 + a\nu^2\bar{\theta}^2$ superfluid phase $\left(\cos\varphi = \dfrac{m_\pi^2 X}{N_f\mu^2}\right)$ (38)

Solutions I: Normal Phase

$$\sin\varphi\left(N_f\cos\varphi-\frac{m_\pi^2 X}{\mu^2}\right)=0\,, \tag{39}$$

$$m_\pi^2\sin\alpha_i\cos\varphi=a\bar{\theta}\,,\quad i=1,\ldots,N_f \tag{40}$$

we solve eq.(40) by expanding in powers of m_π^2/a that we take to be small.
Specifically, at the leading order in m_π^2/a, we have

$$\alpha_i=\begin{cases}\pi-\alpha(\theta)\,, & i=1,\ldots,n\\ \alpha(\theta)\,, & i=n+1,\ldots,N_f\,,\end{cases} \tag{41}$$

where

$$\alpha(\theta)=\frac{\theta+(2k-n)\pi}{(N_f-2n)},\quad k=0,\ldots,N_f-2n-1\,,\quad n=0,...,\left[\frac{N_f-1}{2}\right] \tag{42}$$

J. Bersini et al., *The θ-angle and axion physics of two-color QCD at fixed baryon charge*

Normal Phase Ground State Energy

$$\alpha(\theta) = \frac{\theta + (2k-n)\pi}{(N_f - 2n)}, \quad k = 0, \ldots, N_f - 2n - 1\,, \quad n = 0, ..., \left[\frac{N_f - 1}{2}\right] \tag{43}$$

The parameters n and k label the various solutions to the EOMs.
The interval of values for k is constrained because at fixed n the solutions are periodic in k of period $N_f - 2n$
the solution minimizing the energy has $n = 0$ and the following values of $\alpha(\theta)$

$$\alpha(\theta) = \begin{cases} \frac{\theta}{N_f}\,, & \theta \in [0, \pi] \\ \frac{\theta - 2\pi}{N_f}\,, & \theta \in [\pi, 2\pi]\,, \end{cases} \tag{44}$$

which correspond, respectively, to $k = 0$ and $k = N_f - 1$.

J. Bersini et al., *The θ-angle and axion physics of two-color QCD at fixed baryon charge*

Solutions II: Superfluid Phase

The EOM becomes

$$\frac{m_\pi^4}{N_f \mu^2} X \sin \alpha_i = a\bar{\theta}\,, \quad i = 1, \ldots, N_f \tag{45}$$

we solve eq.(40) by expanding in powers of $m_\pi^2/(a\mu^2)$ that we take to be small. Specifically, at the leading order in $m_\pi^2/(a\mu^2)$, we have

$$\alpha_i = \begin{cases} \pi - \alpha(\theta)\,, & i = 1, \ldots, n \\ \alpha(\theta)\,, & i = n+1, \ldots, N_f\,, \end{cases} \tag{46}$$

where

$$\alpha(\theta) = \frac{\theta + (2k - n)\pi}{(N_f - 2n)}, \quad k = 0, \ldots, N_f - 2n - 1\,, \quad n = 0, ..., \left[\frac{N_f - 1}{2}\right] \tag{47}$$

J. Bersini et al., *The θ-angle and axion physics of two-color QCD at fixed baryon charge*

Superfluid Phase Ground State Energy

$$\alpha(\theta) = \frac{\theta + (2k-n)\pi}{(N_f - 2n)}, \quad k = 0, \ldots, N_f - 2n - 1\,, \quad n = 0, ..., \left[\frac{N_f - 1}{2}\right] \tag{48}$$

The parameters n and k label the various solutions to the EOMs.
The interval of values for k is constrained because at fixed n the solutions are periodic in k of period $N_f - 2n$
the solution minimizing the energy has $n = 0$ and the following values of $\alpha(\theta)$

$$\alpha(\theta) = \begin{cases} \frac{\theta}{N_f}\,, & \theta \in [0, \pi] \\ \frac{\theta - 2\pi}{N_f}\,, & \theta \in [\pi, 2\pi]\,, \end{cases} \tag{49}$$

which correspond, respectively, to $k = 0$ and $k = N_f - 1$.

J. Bersini et al., *The θ-angle and axion physics of two-color QCD at fixed baryon charge*

Spectrum

Fixing the generalized isospin charge results in

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_V \overset{N_f^2-1}{\rightsquigarrow} SU(N_f)_V \times U(1)_V \longrightarrow SU\left(\frac{N_f}{2}\right)_u \times SU\left(\frac{N_f}{2}\right)_d \times U(1)_I \times U(1)_V$$

$$\overset{\frac{N_f^2}{4}}{\rightsquigarrow} SU\left(\frac{N_f}{2}\right)_{ud} \times U(1)_V \,, \tag{50}$$

No dilaton: the spectrum of light modes is composed of $N_f^2/4$ massless Goldstone bosons with speed $v_G = 1$ that parameterize the coset

$$G/H = \frac{SU(N_f/2)_u \times SU(N_f/2)_d \times U(1)_I \times U(1)_V}{SU(N_f/2)_{ud} \times U(1)_V}.$$

These modes arrange themselves in: adjoint representation plus a singlet (associated with the spontaneous breaking of $U(1)_I$)

In addition, a pseudo-Goldstone mode stems from the would-be spontaneous breaking of $U(1)_A$ which we call the S (singlet) mode.