U(1)-gauged 2-flavor spin system in 3-D

Christof Gattringer [1] and Tin Sulejmanpasic [2]

[1] Austrian Science Fund FWF & University of Graz
[2] University of Durham

FWF Österreichischer Wissenschaftsfonds

UNIVERSITÄT GRAZ

Introduction and motivation

Generalized Villain actions for U(1) lattice gauge fields have powerful properties [1]:

4d: Magnetically charged matter, electric-magnetic duality, Witten effect ...

3d: Control over monopole content on the lattice

2d: Control over vortices, natural integer valued θ-term ... (see also [2], [3])

In this contribution we present first (very) preliminary results for a 3d simulation of a U(1)-gauged 2-flavor spin system where lattice monopoles are removed by implementing a suitable constraint on the Villain variables.

[1] Tin Sulejmanpasic, Christof Gattringer, Nuclear Physics B 943 (2019) 114616

[2] Aleksey Cherman, *Exact lattice chiral symmetry* - this conference

[3] Evan Berkowitz, *Generalized BKT transitions and persistent order on the lattice* - this conference

Generalized Villain action

- Gauge fields are coupled to matter via the link variables $e^{iA_{x,\mu}}$
- Shift invariance: $A_{x,\mu} \;\to\; A_{x,\mu} \;+\; 2\pi n_{x,\mu} \qquad n_{x,\mu} \in \mathbb{Z}$
- Exterior derivative: $(dA)_{x,\mu\nu} \;=\; A_{x+\hat{\mu},\nu} \;-\; A_{x,\nu} \;-\; A_{x+\hat{\nu},\mu} \;+\; A_{x,\mu}$
- Shift transformation: $(dA)_{x,\mu\nu} \;\to\; (dA)_{x,\mu\nu} \;+\; 2\pi(dn)_{x,\mu\nu}$
- Define field strength as: $F_{x,\mu\nu} \;=\; (dA)_{x,\mu\nu} \;+\; 2\pi k_{x,\mu\nu}$
- The Villain variables $k_{x,\mu\nu}$ are summed over $\mathbb{Z}$
- Monopole charge:

$$q_x \;\equiv\; (dk)_{x,123} \;=\; k_{x+\hat{3},12} \;-\; k_{x,12} \;-\; k_{x+\hat{2},13} \;+\; k_{x,13} \;+\; k_{x+\hat{1},23} \;-\; k_{x,23}$$

Partition sum and matter field action

$$Z \;=\; \int D[\Phi] \int D[A] \sum_{\{k\}} C[k] \;\; e^{\,-\,\beta\, S_{gauge}[A,k] \,+\, J\, S_{spin}[A,\Phi]}$$

$$S_{spin}[A,\Phi] \;=\; \frac{1}{2} \sum_{x,\mu} \left[\Phi_x^\dagger \, e^{\,i\, A_{x,\mu}} \, \Phi_{x+\hat{\mu}} \;+\; c.c. \right] \qquad\qquad \Phi_x \;=\; \begin{pmatrix} \cos\theta_x \;\, e^{i\alpha_x} \\ \sin\theta_x \;\, e^{i\beta_x} \end{pmatrix}$$

$$D[\Phi] \;=\; \prod_x \int_0^{\pi/2} d\theta_x \; \int_{-\pi}^{\pi} d\alpha_x \; \int_{-\pi}^{\pi} d\beta_x$$

Gauge part with constraint Villain action

$$Z = \int D[\Phi] \int D[A] \sum_{\{k\}} C[k] \; e^{-\beta S_{gauge}[A,k] + J S_{spin}[A,\Phi]}$$

$$S_{gauge}[A,k] = \frac{1}{2} \sum_{x,\mu<\nu} \left((dA)_{x,\mu\nu} + 2\pi k_{x,\mu\nu} \right)^2$$

$$\int D[A] = \prod_{x,\mu} \int_{-\pi}^{\pi} dA_{x,\mu}$$

$$\sum_{\{k\}} C[k] = \prod_{x,\mu<\nu} \sum_{k_{x,\mu\nu} \in \mathbb{Z}} \prod_{x} \delta\left((dk)_{x,123} \right)$$

No monopoles!

$$(dk)_{x,123} = k_{x+\hat{3},12} - k_{x,12} - k_{x+\hat{2},13} + k_{x,13} + k_{x+\hat{1},23} - k_{x,23}$$

Comments on Monte Carlo updates

- The Φ_x and the unconstraint $A_{x,\mu}$ can be simulated with local MC updates.

- $k_{x,\mu\nu}$ on the dual lattice: $k_{x,12} \sim \tilde{k}_{\tilde{x},3}\,, \quad k_{x,13} \sim -\tilde{k}_{\tilde{x},2}\,, \quad k_{x,23} \sim \tilde{k}_{\tilde{x},1}$

$$(dk)_{x,123} \quad \Rightarrow \quad \sum_{\mu} \left[\tilde{k}_{\tilde{x},\mu} - \tilde{k}_{\tilde{x}-\hat{\mu},\mu} \right] = \vec{\nabla} \cdot \vec{\tilde{k}}_{\tilde{x}}$$

- The constraints for the Villain variables imply conserved flux of the $\tilde{k}_{\tilde{x},\mu}$:

$$(dk)_{x,123} = 0 \quad \Rightarrow \quad \sum_{\mu} \left[\tilde{k}_{\tilde{x},\mu} - \tilde{k}_{\tilde{x}-\hat{\mu},\mu} \right] = 0$$

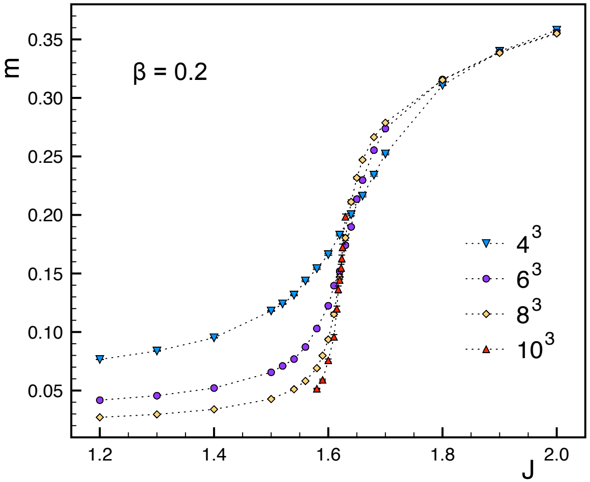
- Use worm algorithms or local moves on dual plaquettes + winding updates.

Order parameter for matter

$$M = \left| \sum_x \vec{S}_x \right|$$

$$\vec{S}_x = \frac{1}{2} \Phi_x^\dagger \, \vec{\sigma} \, \Phi_x$$

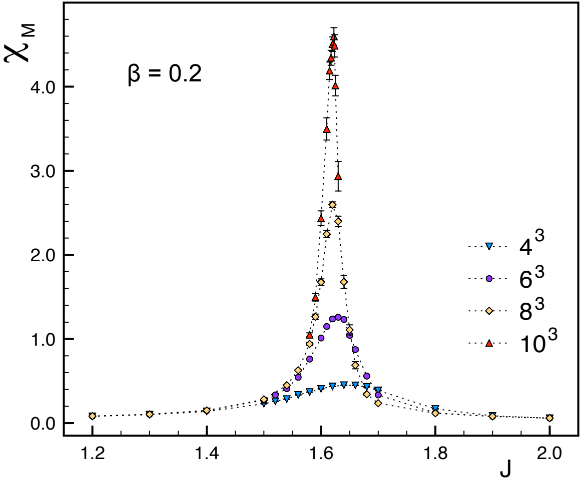
$$m = \frac{1}{L^3} \langle M \rangle$$



Susceptibility for matter

$$\chi_M = \frac{1}{L^3} \left\langle \left(M - \langle M \rangle \right)^2 \right\rangle$$

Results indicate breaking of SO(3) at $J \sim 1.6$

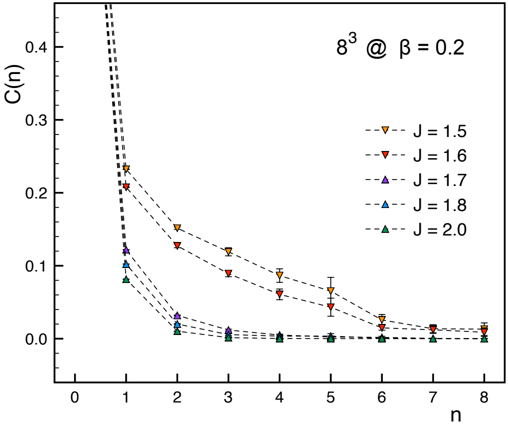


Study gauge dynamics with monopole correlator

To probe the monopole dynamics we violate the Villain constraints for two cubes separated by n links:

$$k_{x,23} \to k_{x,23} + 1$$

$$\forall x = (j,0,0)\ ,\ j = 1,2,\ \dots n$$



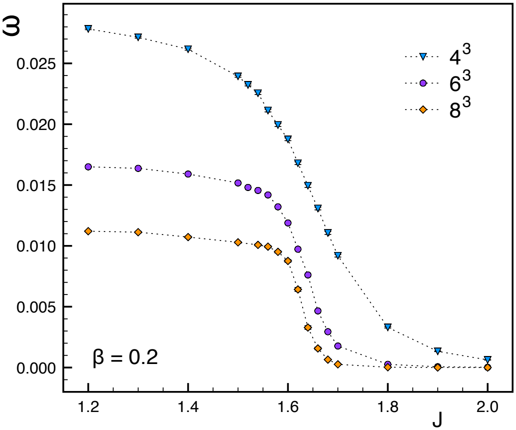
$$C(n) = \left\langle \prod_{j=1}^{n} \frac{e^{-\frac{\beta}{2}\left((dA)_{x,23}+2\pi(k_{x,23}+1)\right)^2}}{e^{-\frac{\beta}{2}\left((dA)_{x,23}+2\pi k_{x,23}\right)^2}} \Bigg|_{x=(j,0,0)} \right\rangle$$

Winding number of dual Villain variables

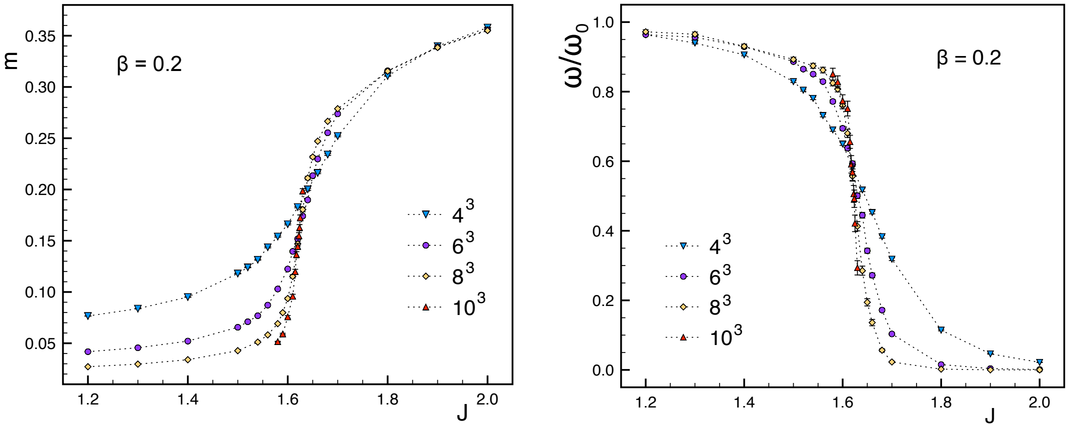
Alternatively we can study the winding number density of dual loops of Villain variables:

$$\omega = \frac{\Omega}{3L^2}$$

$$\Omega = \left|\sum_{x:\, x_1=0} k_{x,23}\right| + \left|\sum_{x:\, x_2=0} k_{x,13}\right| + \left|\sum_{x:\, x_3=0} k_{x,12}\right|$$

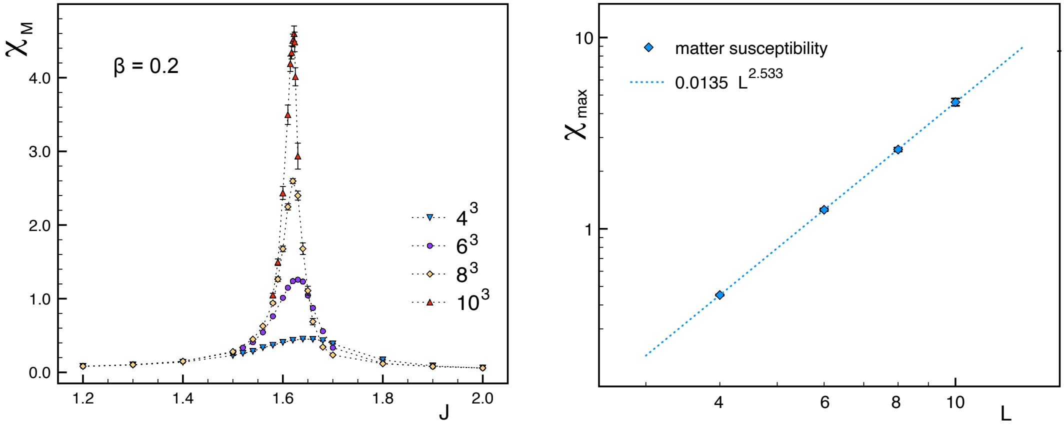


Both order parameters



Transition at $J \sim 1.6$ where SO(3) symmetry breaks and monopoles stop propagating.

Volume scaling of χ_{max}



Scaling slower than volume might hint at a second order transition near $J_c \sim 1.61$.

Summary

- Modified Villain actions allow to control the monopole content of U(1) LGT.
- Using this approach we study a 3d gauged 2-flavor spin model without monopoles.
- The matter order parameter shows a transition at some J_c where SO(3) becomes broken.
- Monopole correlator changes decay properties at that J_c.
- Winding number of dual Villain variables constitutes an order parameter.
- A very preliminary finite volume analysis hints at a continuous transition.