Novel Lattice Formulation of 2D Chiral Gauge Theory via Bosonization

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- O. Morikawa, S.O., and H. Suzuki, PTEP **2024** (2024) 6, 063B01, [arXiv:2403.03420]

Introduction

- Chiral gauge theories are very important.

  e.g. Standard model, GUT...

- Open problem : Latice formulation of Chiral gauge theories

  $U(1)$ [Lüscher, 1998]..., $SU(2) \times U(1)$ [Kikukawa, Nakayama, 2000]...

Recently, Bosonization-based approach

in 2D $U(1)$, [Berkowitz, Cherman, Jacobson, 2023], [DeMarco, Lake, Wen, 2023],...

Fermion $\Longleftrightarrow$ Boson $\Longrightarrow$ Lattice formulation

Advantages

- Exact chiral symmetry
- Simple formulation

Continuum theory : 2D fermion action via Bosonization

According to Bosonization rule,

massless Dirac fermions ψ and compact boson ϕ [Coleman,1975], [Mandelstam, 1975]

$$\int_{M_2} \mathrm{d}^2 x \, \bar{\psi} i \partial\!\!\!/ \psi \sim \frac{1}{8\pi} \int_{M_2} \mathrm{d}^2 x \, \partial_\mu \phi \partial_\mu \phi$$

and, conservation laws of axial and vector currents :

$\partial_\mu j_{A,\mu} \sim \partial_\mu (\bar{\psi} \gamma_\mu \gamma_5 \psi) \sim \partial_\mu \partial_\mu \phi = 0$ (Equation of motion),

$\partial_\mu j_{V,\mu} \sim \partial_\mu (\bar{\psi} \gamma_\mu \psi) \sim \epsilon_{\mu\nu} \partial_\mu \partial_\nu \phi = 0$ (Bianchi identity).

Chaged objects (Sources of currents)

Axial trans : $e^{i\phi} \to e^{i\phi} e^{i\xi}$, Vector trans : $M(x) \to M(x) e^{i\Lambda}$

$M(x)$ is an operator imposes $\partial_\mu j_{V,\mu} \neq 0$ at x. (Roughly, "monopole".)

Representing singularity $M(x)$ on the lattice is a non-trivial task.

Lattice field contents

Let us consider a 2D compact boson on the lattice.

The fundamental d.o.f on site : $e^{i\phi(n)}$(Compact : $\phi(n) \sim \phi(n) + 2\pi$)

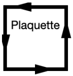
"Invariant derivative"

$\partial\phi(n,\mu) \equiv \frac{1}{i}\ln[e^{-i\phi(n)}e^{i\phi(n+\hat{\mu})}] = \Delta_\mu\phi(n) + 2\pi\ell_{\mu,\alpha}(n), \ell_{\mu,\alpha}(n) \in \mathbb{Z}$

$-\pi \leq \frac{1}{i}\ln e^{i\phi(n)} < \pi$, $\phi(n) = \frac{1}{i}\ln e^{i\phi(n)}$

Under the **admissibility condition** $\sup_{n,\mu}|\partial\phi(n,\mu)| < \epsilon < \frac{\pi}{2}$,

$\Longrightarrow \epsilon_{\mu\nu}\Delta_\mu\partial\phi(n,\nu) = 0$ (Bianchi identity) e.g. [Fujiwara, Suzuki, Wu, 2000]



$= 0$

Admissibility imposes vector-like symmetry on the lattice.

Excision method [Abe, et.al., 2023]

Now, at anywhere, $\epsilon_{\mu\nu}\Delta_\mu\partial\phi(n,\nu) = 0$.

However, we want vector charged objects.

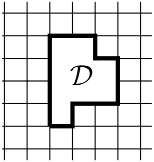
Naively, this is $\epsilon_{\mu\nu}\Delta_\mu\partial\phi(n,\nu) \neq 0$, we cannot (?)

$\Longrightarrow$ **Excision method (making a "hole")**

Excision method

We can define **vector charge** $m \equiv \frac{1}{2\pi}\sum_{(n,\mu)\in\partial\mathcal{D}} \partial\phi(n,\mu) \in \mathbb{Z}$

When $|\partial\mathcal{D}| > 2\pi/\epsilon$, m can be non-zero value.



Background gauging

Our goal : Construct anomaly-free chiral gauge theory

$\Longrightarrow$We want to derive gauge anomaly.

Our background gauged action (Ele-mag gauged action in [Abe, et.al., 2023])

$$S_{\mathrm{B}} = \sum_{\text{flavor : } \alpha} \sum_{n \in M_2} \left[\frac{R^2}{4\pi} D\phi_\alpha(n,\mu) D\phi_\alpha(n,\mu) + \frac{i}{2\pi} q_{V,\alpha} \epsilon_{\mu\nu} A_\mu(\tilde{n}) D\phi_\alpha(n+\hat{\mu},\nu) \right.$$
$$\left. + \frac{i}{2} q_{V,\alpha} \epsilon_{\mu\nu} N_{\mu\nu}(\tilde{n}) \phi_\alpha(n+\hat{\mu}+\hat{\nu}) \right]$$

$$D\phi(n,\mu) \equiv \frac{1}{i} \ln \left[e^{-i\phi_\alpha(n)} U(n,\mu)^{2q_{A,\alpha}} e^{i\phi_\alpha(n+\hat{\mu})} \right]$$
$$= \Delta_\mu \phi_\alpha(n) + 2q_{A,\alpha} A_\mu(n) + 2\pi \ell_{\alpha,\mu}(n)$$

$$F_{\mu\nu}(n) \equiv \frac{1}{i} \ln \left[U(n,\mu) U(n+\hat{\mu},\nu) U(n+\hat{\nu},\mu)^{-1} U(n,\nu)^{-1} \right]$$
$$= \Delta_\mu A_\nu(n) - \Delta_\nu A_\mu(n) + 2\pi N_{\mu\nu}(n)$$

For technical reasons, we put copies on dual lattice. $U(n,\mu) = U(\tilde{n},\mu)$

Gauge anomaly

Under the gauge transformation,

$$\phi_\alpha(n) \to \phi_\alpha(n) - 2q_{A,\alpha}\Lambda(n),$$
$$U(n,\mu) \to e^{-i\Lambda(n)}U(n,\mu)e^{i\Lambda(n+\hat{\mu})},$$
$$U(\tilde{n},\mu) \to e^{-i\Lambda(\tilde{n})}U(\tilde{n},\mu)e^{i\Lambda(\tilde{n}+\hat{\mu})}$$

Then, we can check

$\Delta S_B =$ (gauge anomaly) $\propto \sum_\alpha q_{A,\alpha}q_{V,\alpha}(\sum_{n\in M_2}\Lambda(\tilde{n})F(n) + \cdots)$

Anomaly cancellation condition : $\sum_\alpha q_{A,\alpha}q_{V,\alpha} = 0$

$\Longrightarrow$ The gauge field can be dynamical.

$\Longrightarrow$ We can construct **anomaly-free lattice chiral gauge theory!**

Selection rule I

In the presence of the "hole" (labeled by $\tilde{I}$),

ΔS_B gets additional term : $\Delta S_B = \cdots - i \sum_{\tilde{I},\alpha} q_{V,\alpha} m_{\tilde{I},\alpha} \Lambda(\tilde{n}_{*,\tilde{I}})$

$\boldsymbol{-i \sum_{\tilde{I},\alpha} q_{V,\alpha} m_{\tilde{I},\alpha} \Lambda(\tilde{n}_{*,\tilde{I}})}$ **means vector trans of vector charges.**

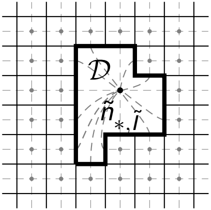
Where, $m_{\tilde{I},\alpha} \equiv \frac{1}{2\pi} \sum_{(n,\mu)\in\partial\mathcal{D}} D\phi_\alpha(n,\mu) - \frac{2q_{A,\alpha}}{2\pi} F(\partial\mathcal{D}) \in \mathbb{Z}$.

$\Longrightarrow$ gauge invariant modification of the vector charge

Considering consistency,

$\sum_{\tilde{I}} m_{\tilde{I},\alpha} = -\frac{2q_{A,\alpha}}{2\pi} \left(\sum_{p\in\Gamma-\sum_{\tilde{I}}\mathcal{D}_{\tilde{I}}} F_{12}(p) + \sum_{\tilde{I}} F(\partial\mathcal{D}_{\tilde{I}}) \right) = -2q_{A,\alpha} Q$

Vector charges saturate 1st Chern number Q. $\Longrightarrow$ **index theorem**!



Selection rule II

The case of axial charged objects ?

Axial charged objects are vertex operator : $V_{\{n_\alpha\}}(n) \equiv e^{i\sum_\alpha n_\alpha \phi_\alpha(n)}$

For a non-zero correlation function, (under global shift : $\phi_\alpha(n) \to \phi_\alpha(n) + \xi_\alpha$)

$\sum_I n_{I,\alpha} = \frac{q_{V,\alpha}}{2\pi} \sum_{\tilde{p}\in\tilde{\Gamma}} F_{12}(\tilde{p}) = q_{V,\alpha}\tilde{Q}$ (I labels vertex operators.)

Two $Q, \tilde{Q}$? If $|F| < \delta, \delta \to$ small enough, then $Q = \tilde{Q}$

Weyl fermions case, for example, $P_L\psi_\alpha \sim e^{-i\phi_\alpha(n)/2} M_{m_\alpha=-1}(\mathcal{D})$.

$P_L\psi_\alpha$ saturate only when $q_{L,\alpha} Q < 0$.

As a result, **it is consistent with the fermion number anomaly** :

$\int_{M_2} \mathrm{d}^2x \partial_\mu J_\mu^{L,R}(x) = \mp q_{L,R} Q \quad (Q = \frac{1}{2\pi}\int_{M_2} F)$

Conclusion

Message

We construct 2D U(1) chiral gauge theory on the lattice with respecting the admissibility condition.

$\Longrightarrow$ Excision method (Vector charges are "hole".)

(*)Related Work : [Berkowitz, Cherman, Jacobson, 2023]
$\Longrightarrow$ Vector charges represent by using dual scalar.(Modified Villain)

In this sense, ours is "yet another".

(*)Limitation : Our formulation cannot be used when $q_{V,\alpha}$ is a half-integer

Future direction

- Generalize Excision method to higher dimension (to appear)
- Lattice formulation via non-abelian bosonization (on going)

Back up

$$\sup_{n,\mu} |D\phi_\alpha(n,\mu)| < \epsilon, \qquad \sup_{n,\mu,\nu} |2q_{A,\alpha} F_{\mu\nu}(n)| < \delta$$

$$\left| \varepsilon_{\mu\nu} \left[\Delta_\mu \ell_{\alpha,\nu(n)} - \frac{2q_{A,\alpha}}{2} N_{\mu\nu}(n) \right] \right| = \frac{1}{2\pi} \left| \varepsilon_{\mu\nu} \left[\Delta_\mu D\phi(n,\nu) - \frac{2q_{A,\alpha}}{2} F_{\mu\nu}(n) \right] \right|$$
$$< \frac{2}{\pi}\epsilon + \frac{1}{2\pi}\delta < 1$$

$\sum_{(n,\mu)\in p} D\phi_\alpha(n,\mu) = \Delta_\mu D\phi_\alpha(n,\nu) - \Delta_\nu D\phi_\alpha(n,\mu) = 2q_{A,\alpha} F_{\mu\nu}(n)$

(generalization of the Bianchi identity)

$F(\partial\mathcal{D}) \equiv \frac{1}{i} \ln \prod_{(n,\mu)\in\partial\mathcal{D}} U(n,\mu) = \sum_{(n,\nu)\in\partial\mathcal{D}} A_\mu(n) + 2\pi N(\partial\mathcal{D})$

In the presence of the "hole", we can "avoid" the bound

$\sum_{(n,\mu)\in\partial D} D\phi_\alpha(n,\mu) = m_\alpha + 2q_{A,\alpha} F(\partial D)$

(generalization of the vector charge)