Tensor network toolbox for probing dynamics of non-Abelian gauge theories

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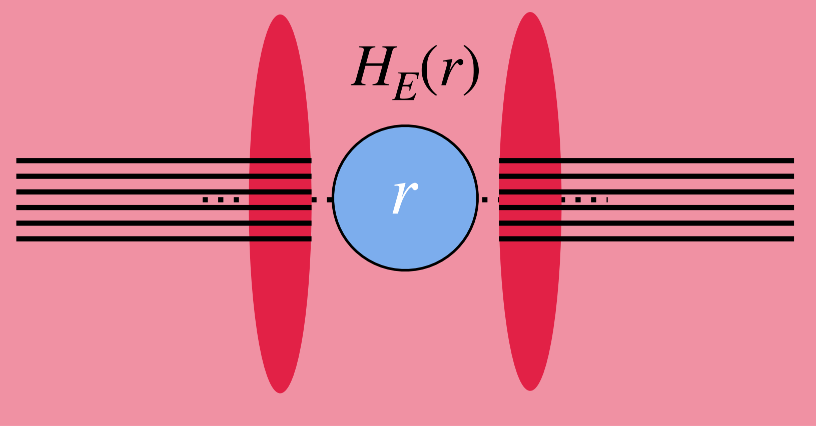
Motivation

- Renewed interest in Hamiltonian formulation due to quantum computing technologies.
- Tensor networks opens up sign problem free approach to study non-perturbative regimes of strongly coupled field theories ie QCD.
- A toolbox to help benchmark and compare quantum simulations.

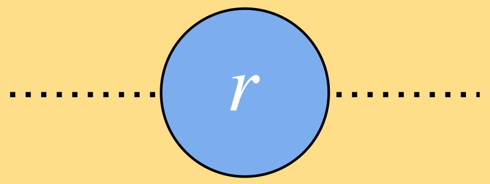
Loop-String-Hadron (LSH) formulation of SU(2) lattice gauge theory in 1+1D

$$\hat{H}^{(1+1)D}_{SU(2)} = \hat{H}_E + \hat{H}_M + \hat{H}_I$$

$$\hat{H}_E = \frac{g_0^2}{4}\sum_r \frac{\hat{\mathcal{N}}_L(r)}{2}\left[\hat{\mathcal{N}}_L(r)+1\right] + (L \to R)$$

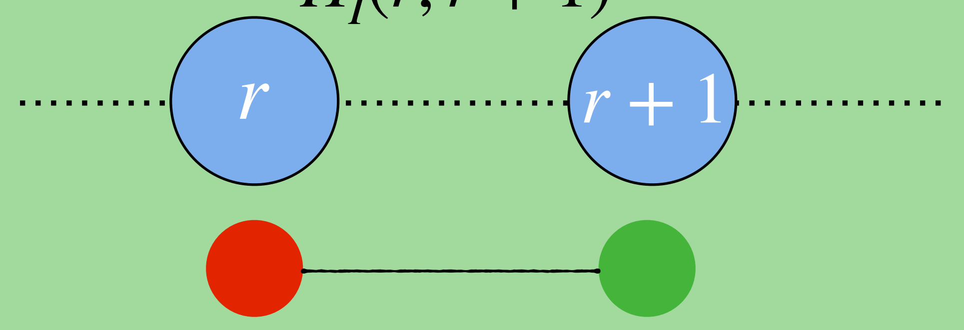


$$\hat{H}_M = m_0 \sum_x (-)^x\left(\hat{\mathcal{N}}_i(x) + \hat{\mathcal{N}}_o(x)\right)$$

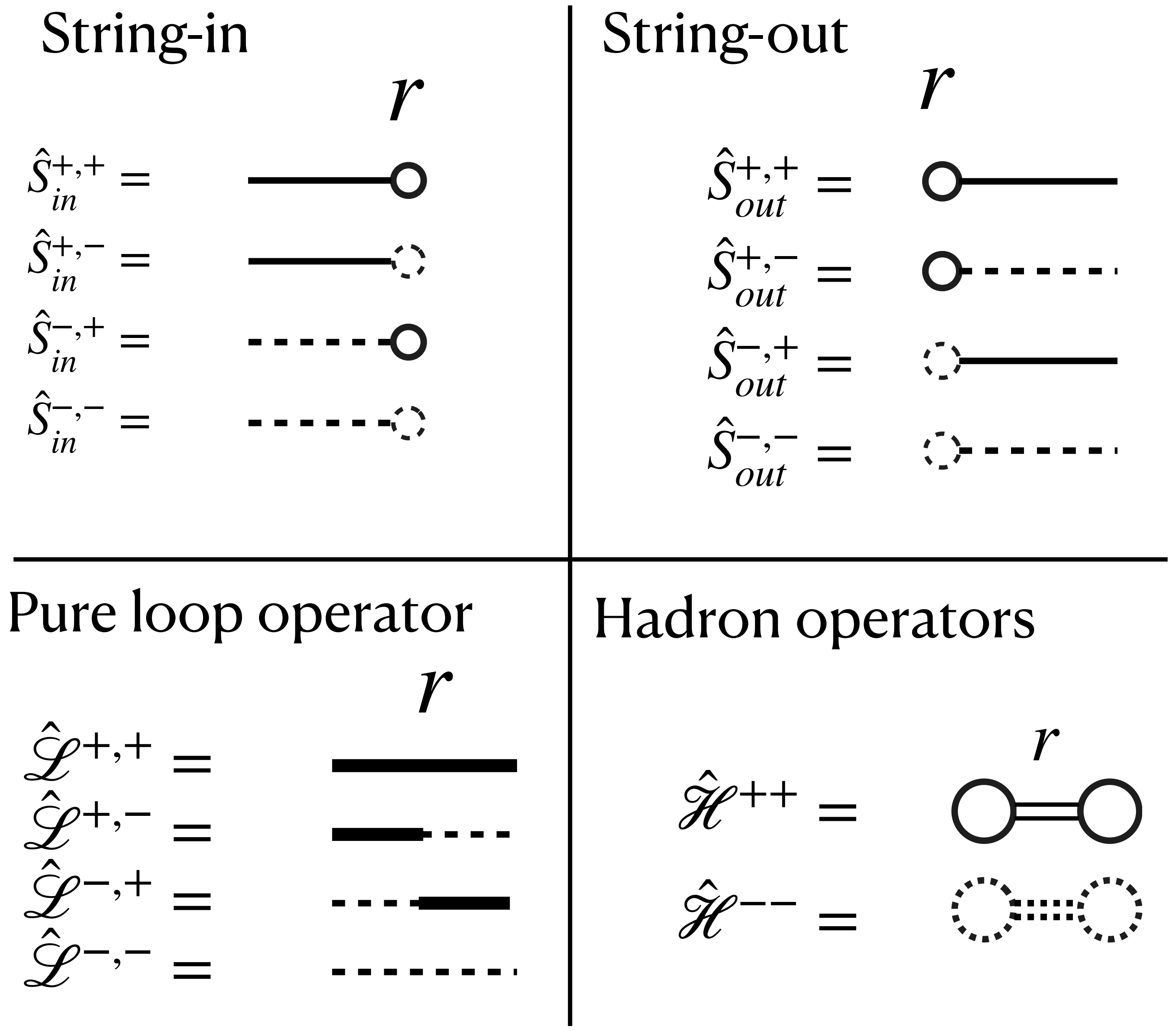


sum fermion occupation number

$$\hat{H}_I = \sum_r \left[\sum_{\sigma=\pm} \hat{\bar{S}}^{+,\sigma}_{\text{out}}(r)\hat{\bar{S}}^{\sigma,-}_{\text{in}}(r+1)\right] + \text{H.c.}$$



- Graphical representation of operators



- Local Abelian Gauss Law

$$n_l + n_o(1-n_i)\big|_r = n_l + n_i(1-n_o)\big|_{r+1}$$

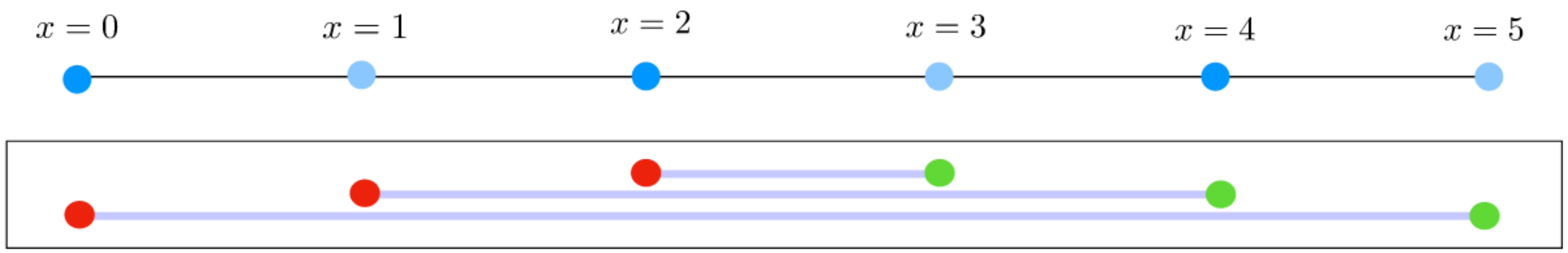
- Local Basis $|n_l, n_i, n_o\rangle_r \quad n_l \in (0, \mathbb{Z}_+) \quad n_i, n_o = 0,1$

* $\hat{\mathcal{N}}_{L/R} = \hat{\mathcal{N}}_l + \hat{\mathcal{N}}_{o/i}\left(1 - \hat{\mathcal{N}}_{i/o}\right)$

Loop-String-Hadron (LSH) formulation of SU(2) lattice gauge theory in 1+1D

- $|n_l, n_i, n_o\rangle$ at each site is glued together throughout the lattice via Abelian Gauss Law (AGL)

$$|\Psi\rangle_{LSH} = \prod_{x=0}^{N-1} |n_l, n_i, n_o\rangle_x$$



$$|0,0,1\rangle^{(0)} \otimes |1,0,1\rangle^{(1)} \otimes |2,0,1\rangle^{(2)} \otimes |2,1,0\rangle^{(3)} \otimes |1,1,0\rangle^{(4)} \otimes |0,1,0\rangle^{(5)}$$

Davoudi et. al *Physical Review D* 104, no. 7 (2021): 074505

The statics and dynamics of string breaking

- String breaking is a consequence of confinement which prohibits existence of spatially isolated charges.
- Gauge-invariance dictates the overall state must be color neutral
- Energetically favourable to generate particle -antiparticle out of vacuum fluctuations.

Progress so far

Abelian LGTs

- Hebenstreit et al. *Physical review letters, 111*(20), 201601
- Hebenstreit et al. *Physical Review D—Particles, Fields, Gravitation, and Cosmology* 87, no. 10 (2013): 105006
- Buyens et a.l *Physical Review D* 96, no. 11 (2017): 114501.
- Pichler, Thomas, et al. Physical Review X 6.1 (2016): 011023.

And many more..

Non-Abelian LGTs

- Kühn et al. *Journal of High Energy Physics* 2015, no. 7 (2015): 1-26
- Sala et al. *Physical Review D* 98, no. 3 (2018): 034505

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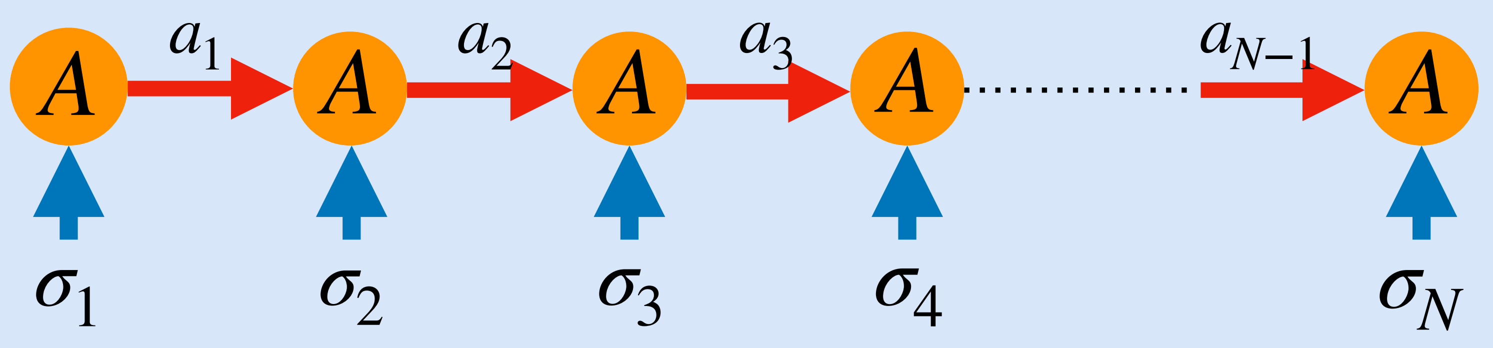
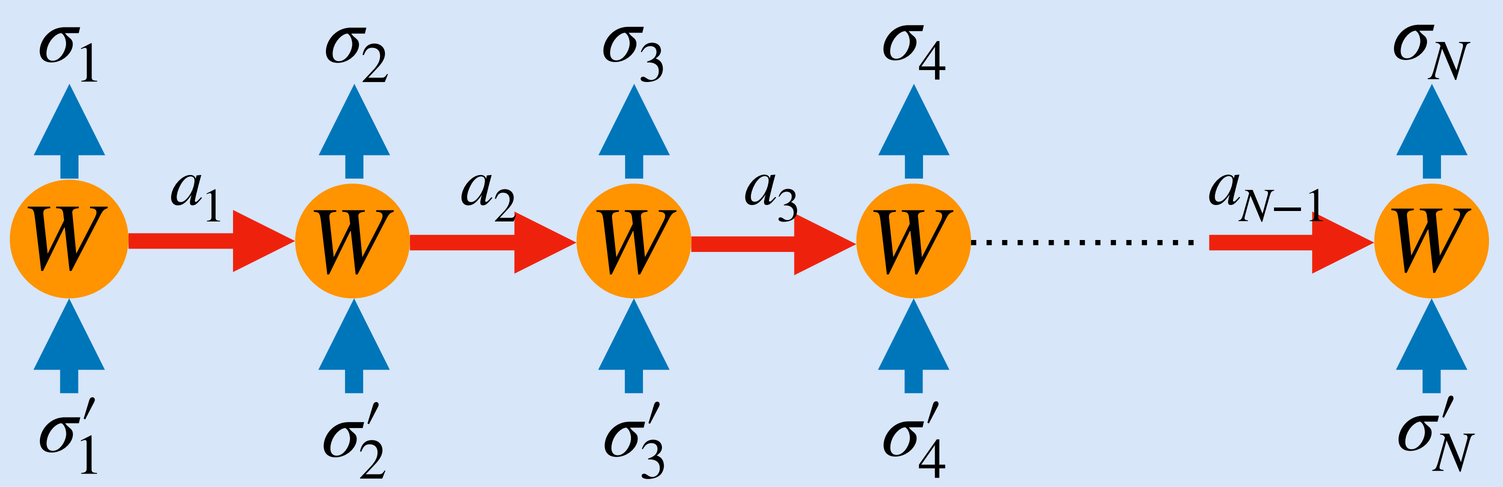
The statics and dynamics of string breaking

A Matrix Product State Ansatz for LSH



$$|\Psi\rangle_{MPS} = \sum_{\sigma_1,\sigma_2,\ldots,\sigma_N} A^{\sigma_1}_{a_1} A^{a_1,\sigma_2}_{a_2} \ldots A^{a_{N-1},\sigma_N} \Big[|\sigma_1\rangle \otimes |\sigma_2\rangle \otimes \ldots \otimes |\sigma_N\rangle \Big]$$

$$\hat{O} = \sum_{\sigma'\sigma} |\sigma'\rangle \Big[\prod_l W^{\sigma'_l}_{\sigma_l} \Big] \langle \sigma |$$



Fishman, Matthew, Steven White, and Edwin Miles Stoudenmire. SciPost Physics Codebases (2022): 004.

- Local state $\sigma_i \equiv |n_l, n_i, n_o\rangle$
- Imposing a cut-off Λ on n_l
- Local Hilbert space : 4Λ
- Constraint on matrix elements of operators such $\mathcal{N}_L, \mathcal{N}_R \leq \Lambda + 1$*
- Global symmetry: $\sum_r \left(\mathcal{N}_i + \mathcal{N}_o\right)$, $\sum_r \left(\mathcal{N}_o - \mathcal{N}_i\right)$

Ground-state calculations via DMRG

$$\frac{2}{ag^2} H = H_E + \mu H_M + x H_I + \Lambda_p H_p$$

$$\Lambda_p H_p = \Lambda_p \sum_r \Big[\mathcal{N}_L(r) - \mathcal{N}_R(r+1) \Big]^2 \equiv \text{Penalty Term}$$

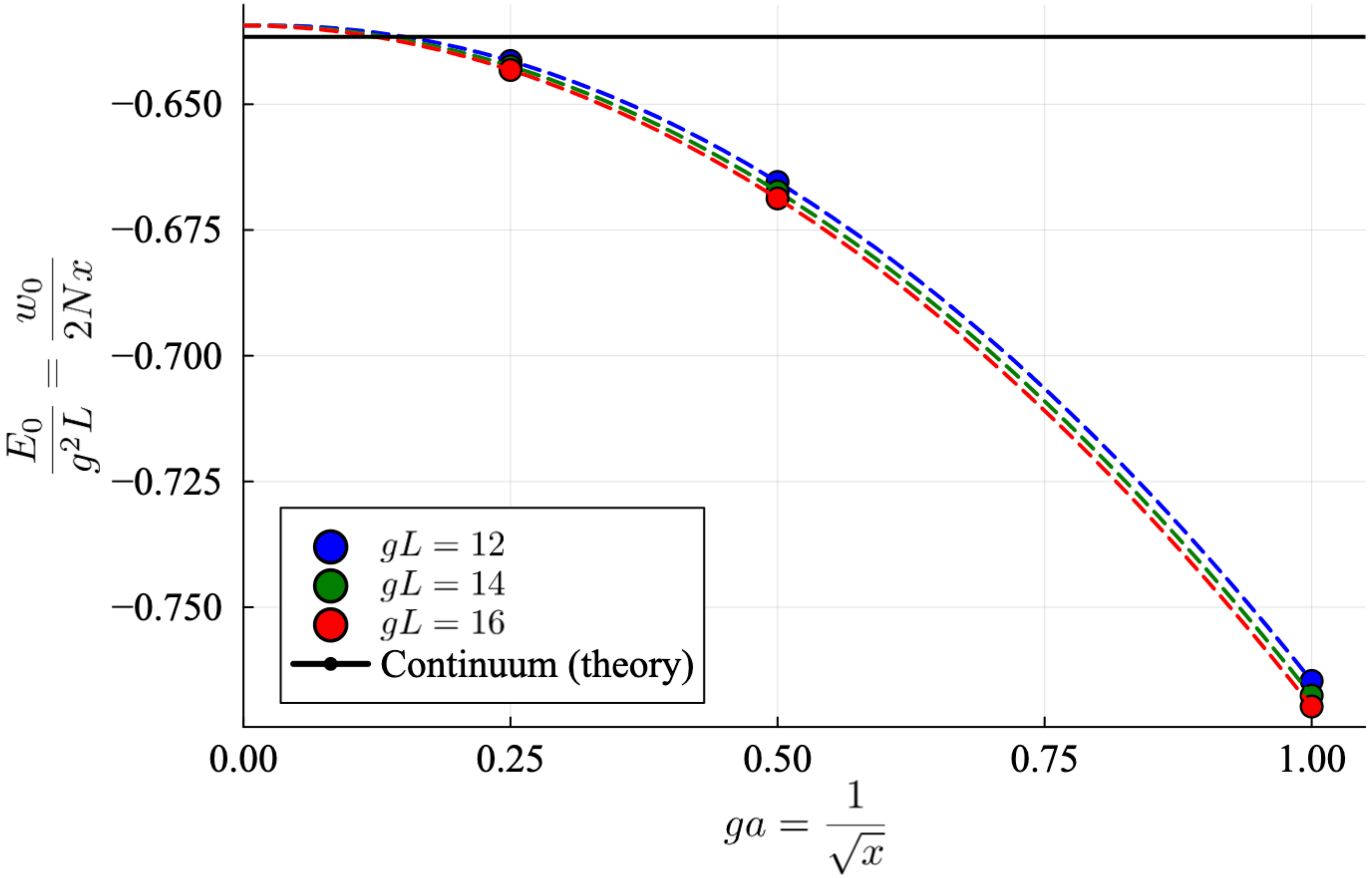
$* \ \mathcal{N}_{L/R} = n_l + n_{o/i}(1 - n_{i/o})$

The statics and dynamics of string breaking

Some preliminary checks

$$\frac{m}{g} = 0.5, \ j^*_{max} = 2$$

- Hamer, C. J. *Nuclear Physics B* 195.3 (1982): 503-521.
- Bañuls et al. *Physical Review X* 7.4 (2017): 041046.



$$^* j_{max} = \frac{\mathcal{N}_L}{2} = \frac{1}{2}\left(n_l + n_o(1 - n_i)\right)$$

Ground-state calculations visa DMRG

$$\frac{2}{ag^2} H = H_E + \mu H_M + x H_I + \Lambda_p H_p$$

$$\mu = \frac{2m}{g}\sqrt{x}$$

$$x = \frac{1}{g^2 a^2}$$

$$\Lambda_p H_p = \Lambda_p \sum_r \left[\mathcal{N}_L(r) - \mathcal{N}_R(r+1)\right]^2 \equiv \text{Penalty Term}$$

Physical results are obtained via

- Taking infinite volume limit, ie N $\to \infty$
- Continuum limit, ie x $\to \infty$
- Infinite cutoff-limit, ie $\Lambda \to \infty$
- Large bond-dimension limit

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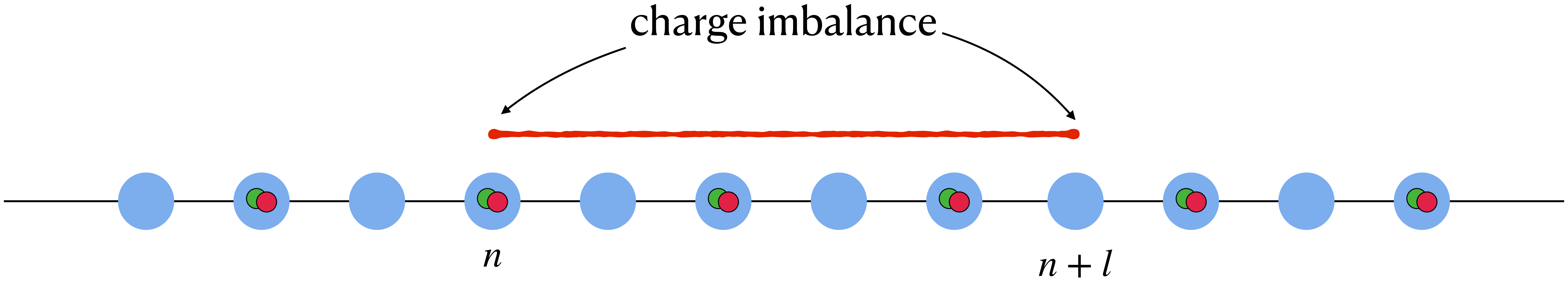
The statics and dynamics of string breaking

Static string

- Initialize state that has a flux tube connecting sites n and $n + l$



$$U(n)U(n+1)\ldots U(n+l-1)\,|\,\text{s.c.v}\rangle$$

- Modify AGL to account for static charges at n and $n + l$.
- DMRG converges to the correct ground state in the external static charge sector

*s.c.v ≡strong-coupling vacuum

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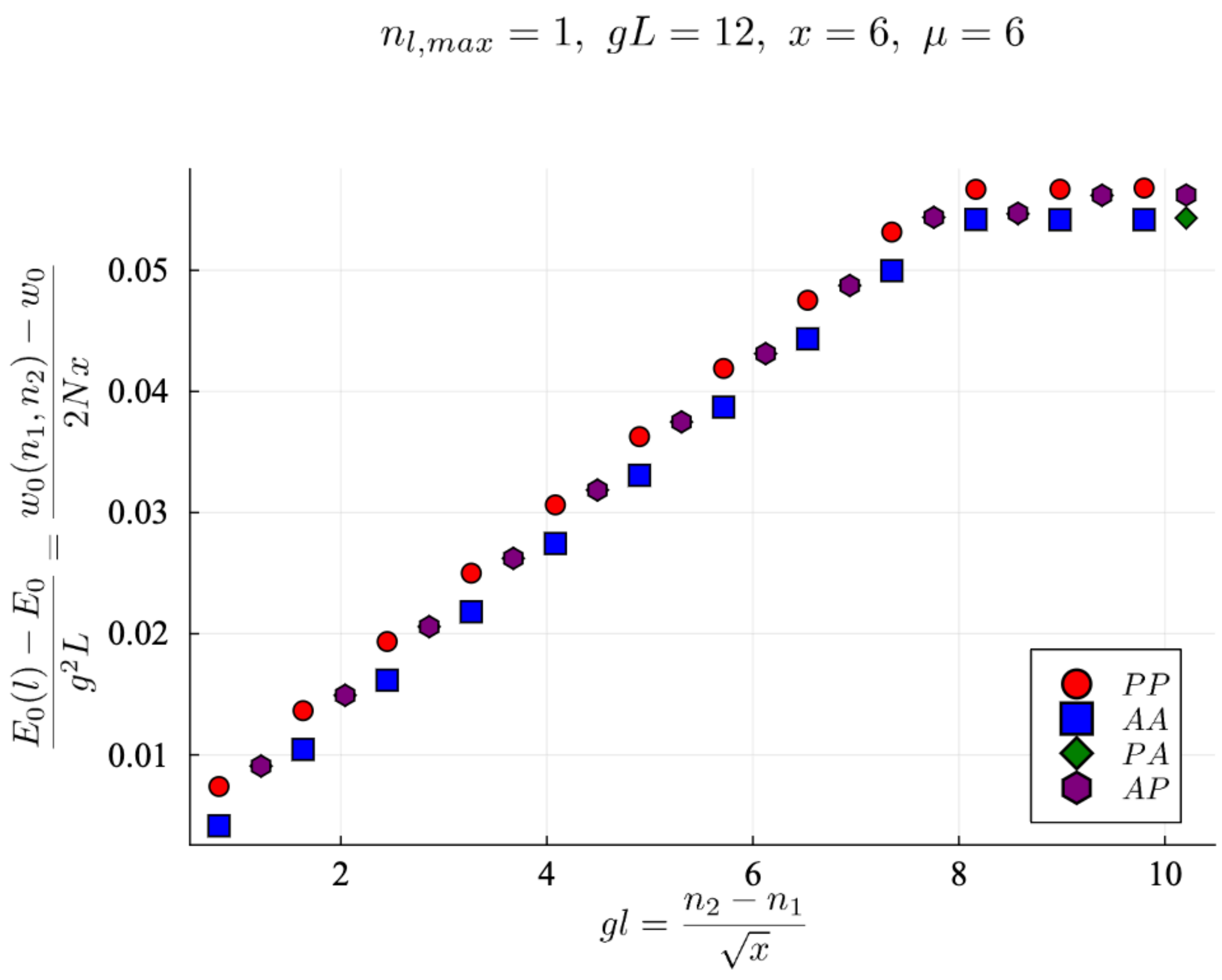
The statics and dynamics of string breaking

Static string



$$*j_{max} = \frac{\mathcal{N}_L}{2} = \frac{1}{2}\Big(n_l + n_o(1 - n_i)\Big)$$

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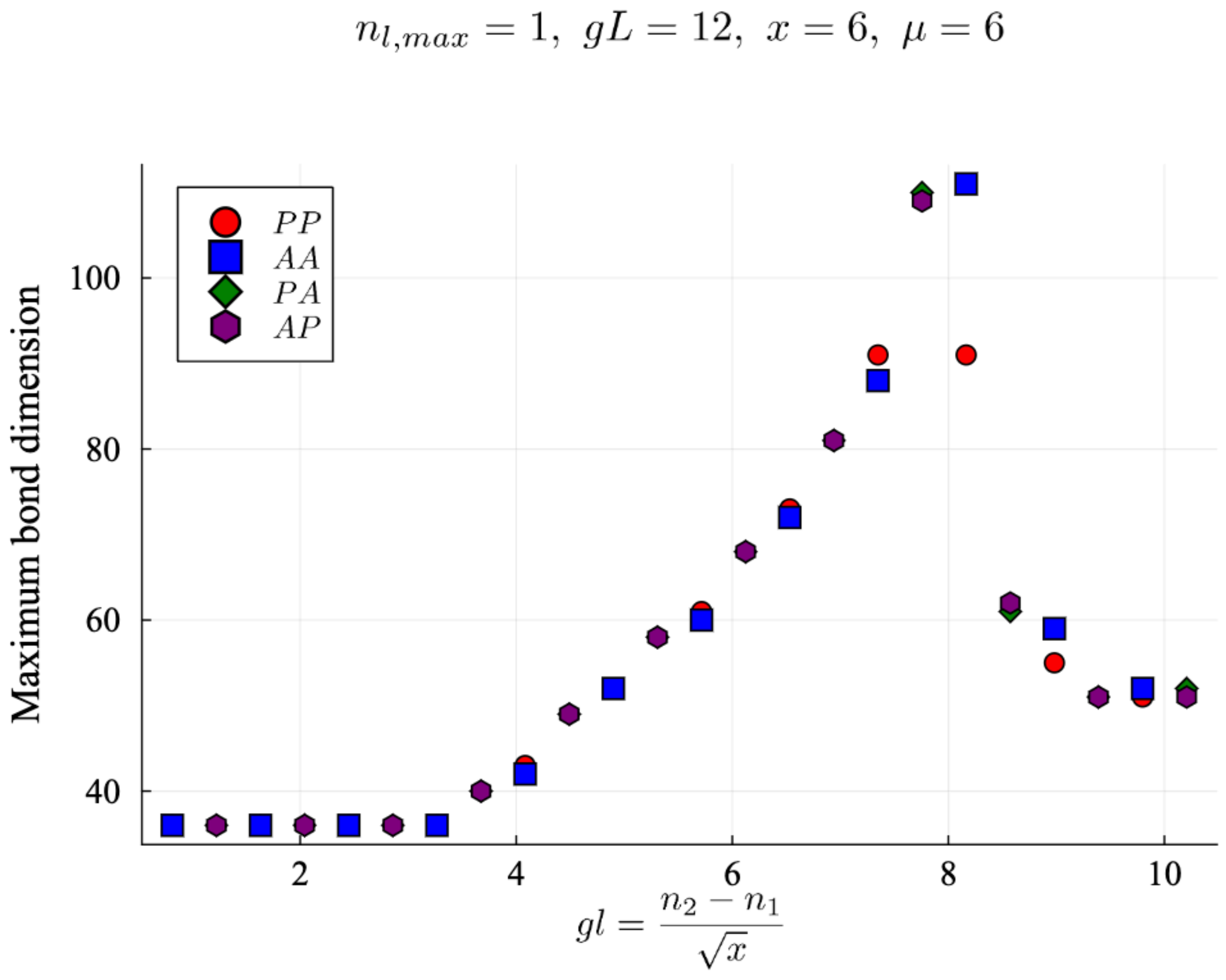
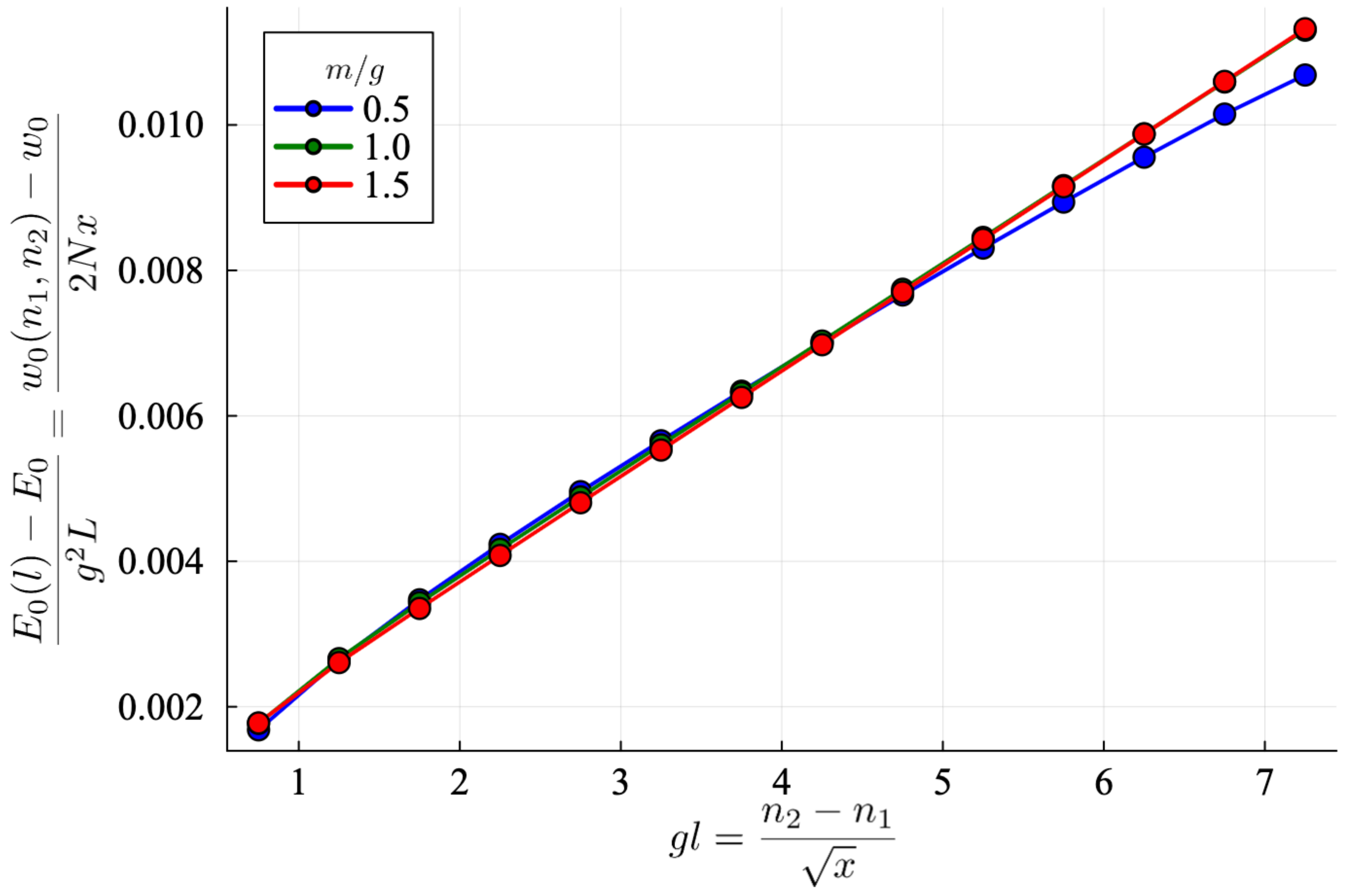
- Linear rise in the static potential as a function of the physical string length, indicating string breaking
- Plateau for larger string lengths, indicating the unbroken regime.



The statics and dynamics of string breaking

Static string

$j_{max} = 2,\ gL = 16,\ x = 16$



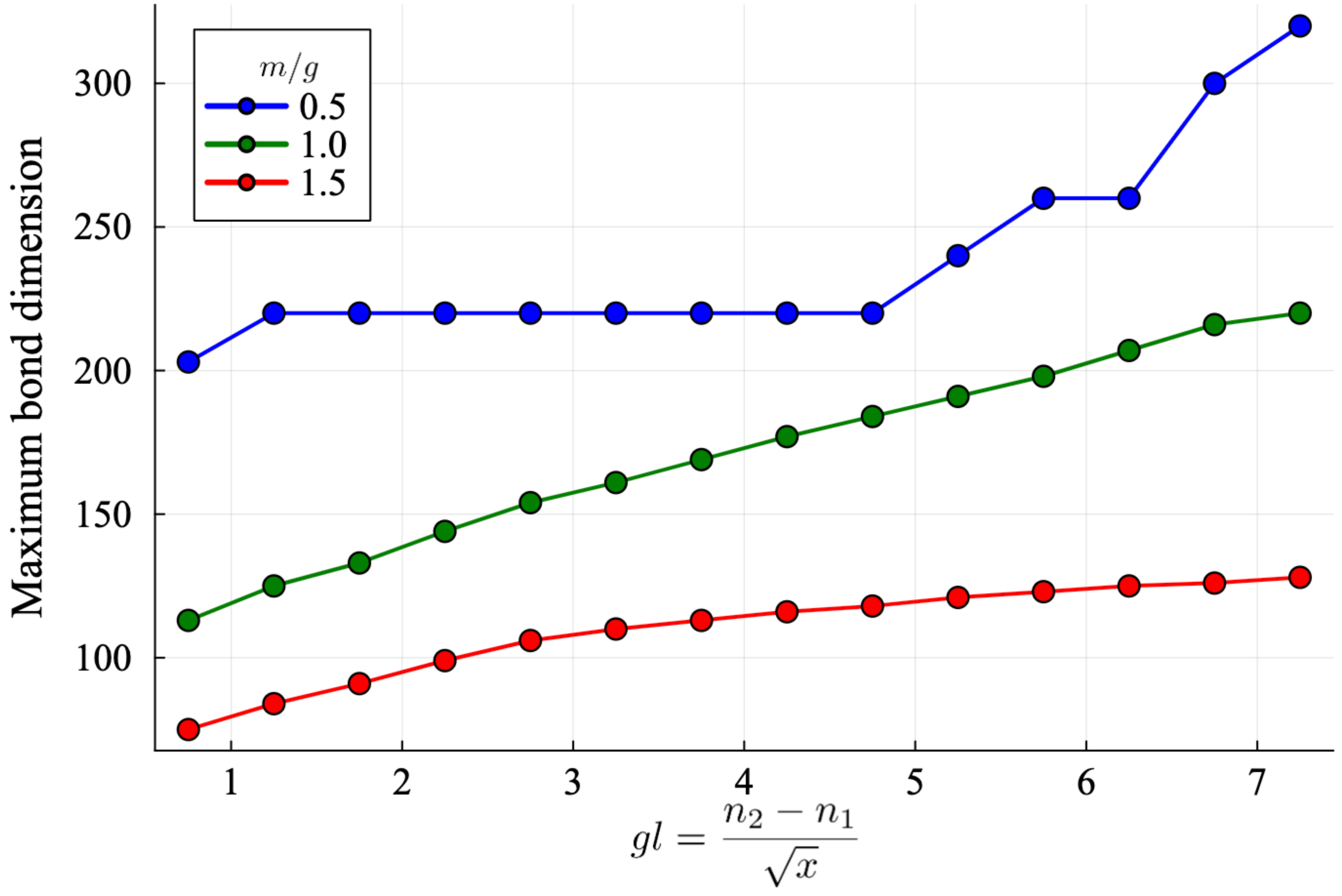
- Linear rise in the static potential as a function of the physical string length, indicating string breaking
- Expect to see a plateau for larger string lengths, indicating the unbroken regime.

$$*\, j_{max} = \frac{\mathcal{N}_L}{2} = \frac{1}{2}\Big(n_l + n_o(1 - n_i)\Big)$$

The statics and dynamics of string breaking

Static string

$$j_{max} = 2,\ gL = 16,\ x = 16$$



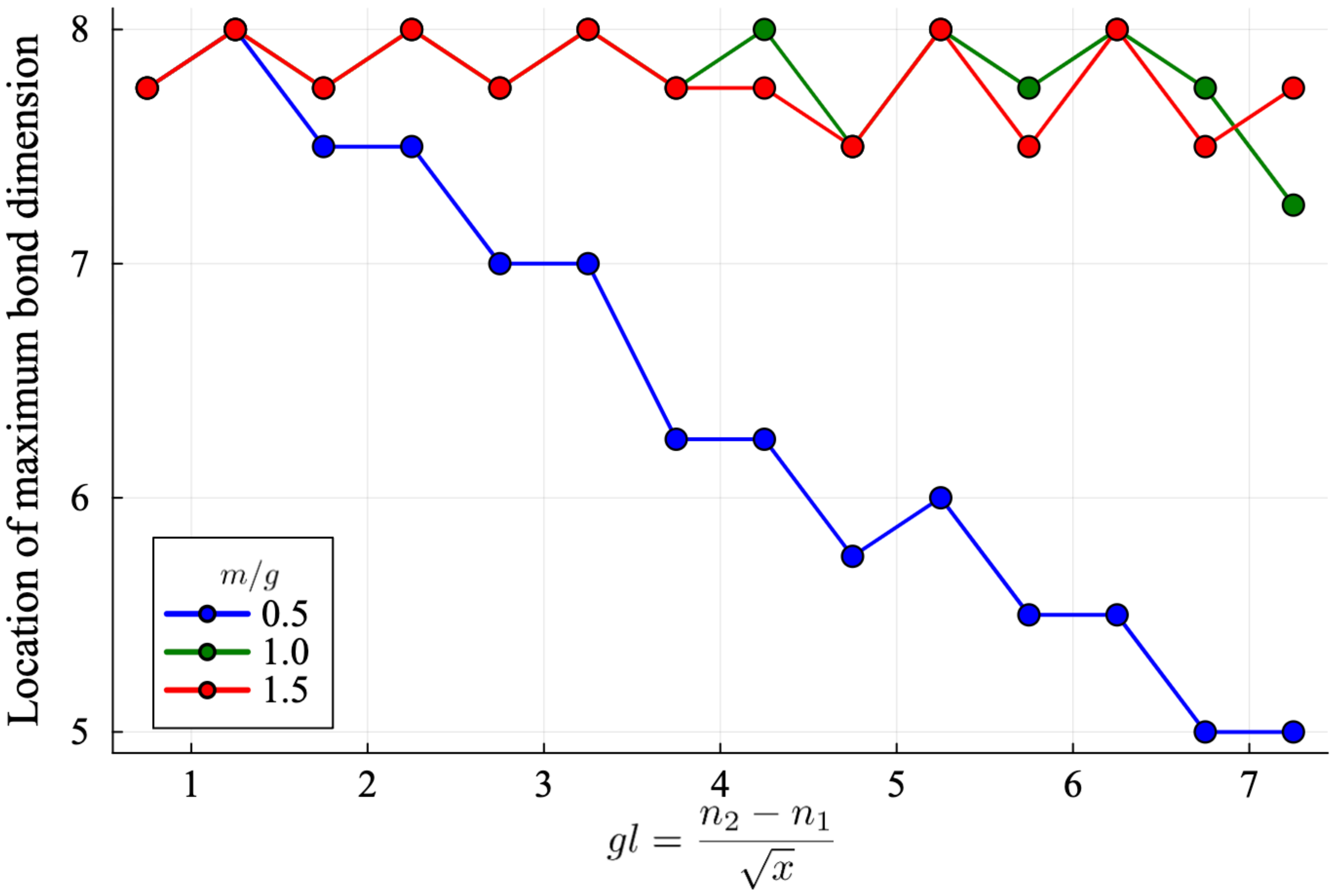
- Smaller m/g value has larger bond dimension requirement for convergence due to stretching and breaking of string.
- Larger m/g requires comparatively lesser bond-dimensions to ensure convergence since string doesn't break.

$$^{*}j_{max} = \frac{\mathcal{N}_L}{2} = \frac{1}{2}\Big(n_l + n_o(1 - n_i)\Big)$$

The statics and dynamics of string breaking

Static string

$j_{max} = 2,\ gL = 16,\ x = 16$



- Lower m/g values allows for string stretching and breaking $\Longrightarrow$hopping of fermions results in shift in location of max bond dimensions

$$*j_{max} = \frac{\mathcal{N}_L}{2} = \frac{1}{2}\Big(n_l + n_o(1 - n_i)\Big)$$

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The statics and dynamics of string breaking

Dynamics with Tensor Networks

- We need to solve the Schrödinger equation for a given MPS representation of the wavefunction.

$$i\frac{d}{dt}|\Psi[A(t)]\rangle = \hat{H}|\Psi[A(t)]\rangle \longrightarrow i\frac{d}{dt}|\Psi[A(t)]\rangle \approx \hat{P}^{1s}\hat{H}|\Psi[A(t)]\rangle$$

$$\hat{P}^{1s} = \sum_{r=1}^{L} \hat{P}^{L,|\Psi\rangle}_{j-1} \otimes \hat{I}_j \otimes \hat{P}^{R,|\Psi\rangle}_{j+1} - \sum_{j=1}^{L-1} \hat{P}^{L,|\Psi\rangle}_{j-1} \otimes \hat{P}^{R,|\Psi\rangle}_{j+1}$$

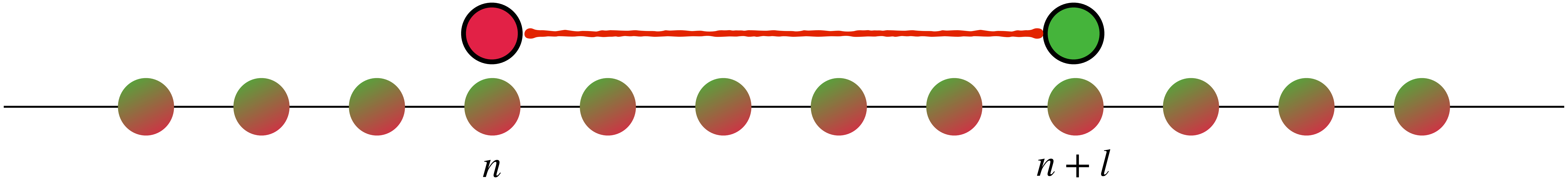
- Using the projector, the LHS of the Schrodinger equation gets mapped to two sets of local equations that can be numerically integrated.

$$\frac{\partial}{\partial t}A_j = -i\hat{H}^{eff}_j A_j \qquad \frac{\partial}{\partial t}C_j = i\hat{H}^{eff}_j C_j$$

- We opt for the 2-site TDVP algorithm

The statics and dynamics of string breaking

Dynamical Strings



String operator $S_{nl} = \psi_n^\dagger U_n \ldots U_{n+l-1} \psi_{n+l}$ $\longrightarrow$ Gauge-invariant operator

String Operator in the LSH picture

$$S_{n,l} = \frac{1}{\sqrt{N_L + 1}} \begin{bmatrix} \hat{\mathcal{S}}^{++}_{out} & \hat{\mathcal{S}}^{+-}_{out} \end{bmatrix}_n \begin{bmatrix} \hat{L}^{++} & \hat{L}^{+-} \\ \hat{L}^{-+} & \hat{L}^{--} \end{bmatrix}_{n+1} \cdots \begin{bmatrix} \hat{L}^{++} & \hat{L}^{+-} \\ \hat{L}^{-+} & \hat{L}^{--} \end{bmatrix}_{n+l-1} \begin{bmatrix} \hat{\mathcal{S}}^{+-}_{in} \\ \hat{\mathcal{S}}^{--}_{in} \end{bmatrix}_{n+l} \frac{1}{\sqrt{N_R + 1}}$$

$$S_{n,l} = \sum_{\sigma_1, \sigma_2, \ldots, \sigma_{2l}} \bar{\mathcal{S}}^{+,\sigma_1}_{out}(n) \bar{\mathcal{L}}^{\sigma_1,\sigma_2}(n+1) \ldots \bar{\mathcal{L}}^{\sigma_{2l-1},\sigma_{2l}}(n+l-1) \bar{\mathcal{S}}^{\sigma_{2l},-}_{in}(n+l)$$

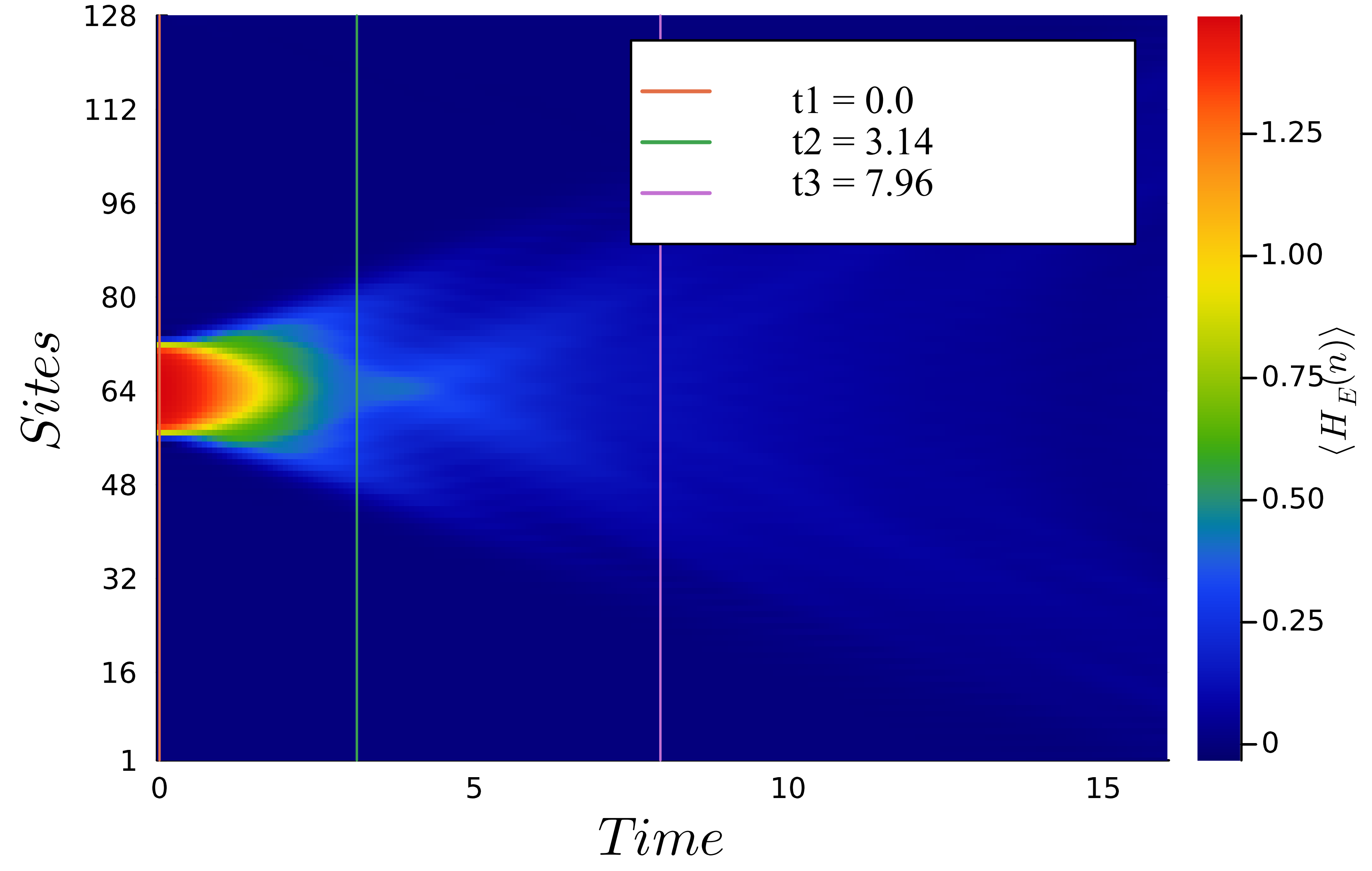
Full expansion gives rise to 2^l terms

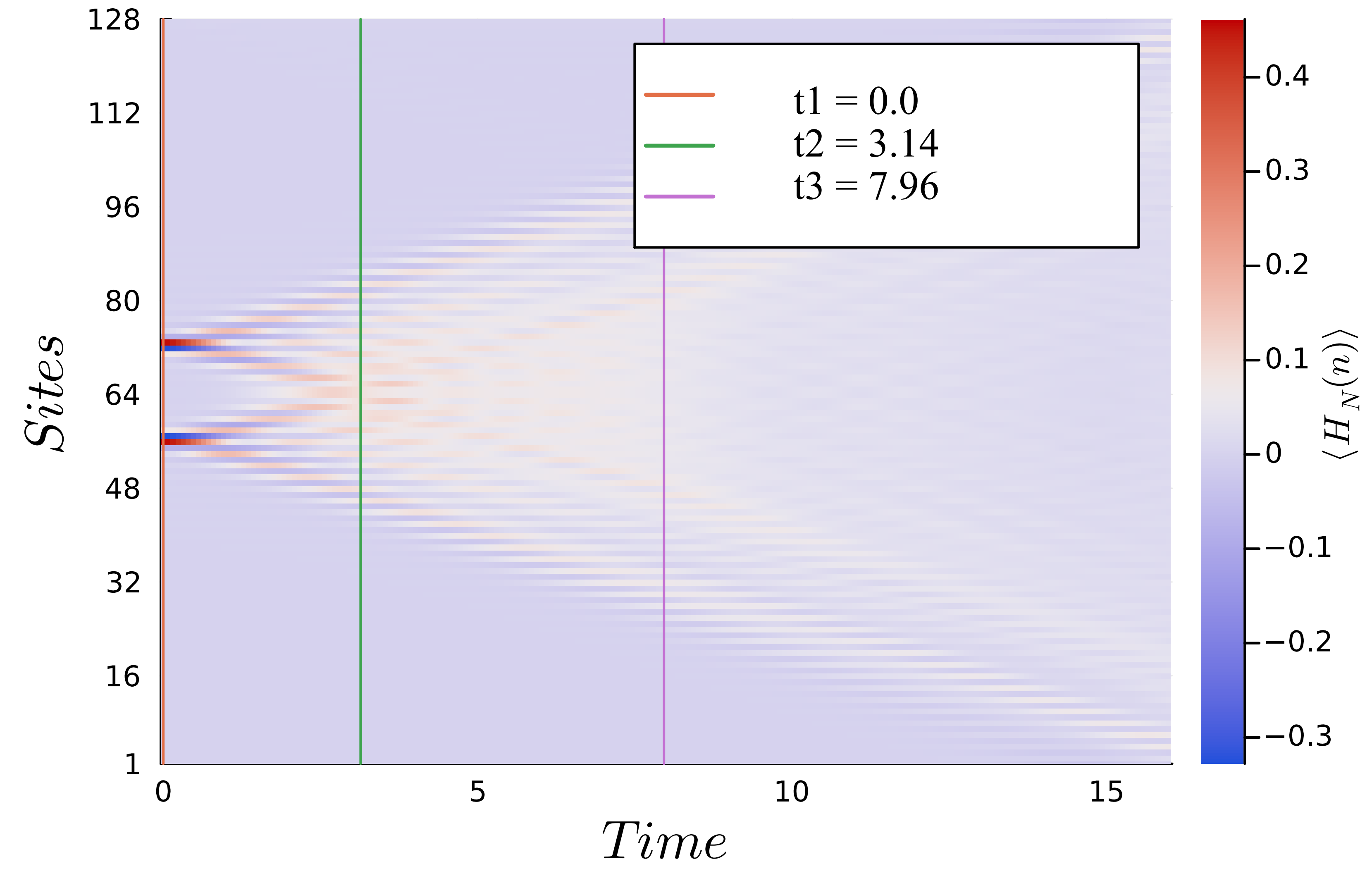
$|\psi_{ini}\rangle = S_{nl}|\psi_{gs}\rangle$ $\longrightarrow$ (Time evolve) $|\psi(t)\rangle = e^{-iH_{LSH}t}|\psi_{ini}\rangle$

$H_{LSH} = H_E + \mu H_m + x H_I$

The statics and dynamics of string breaking

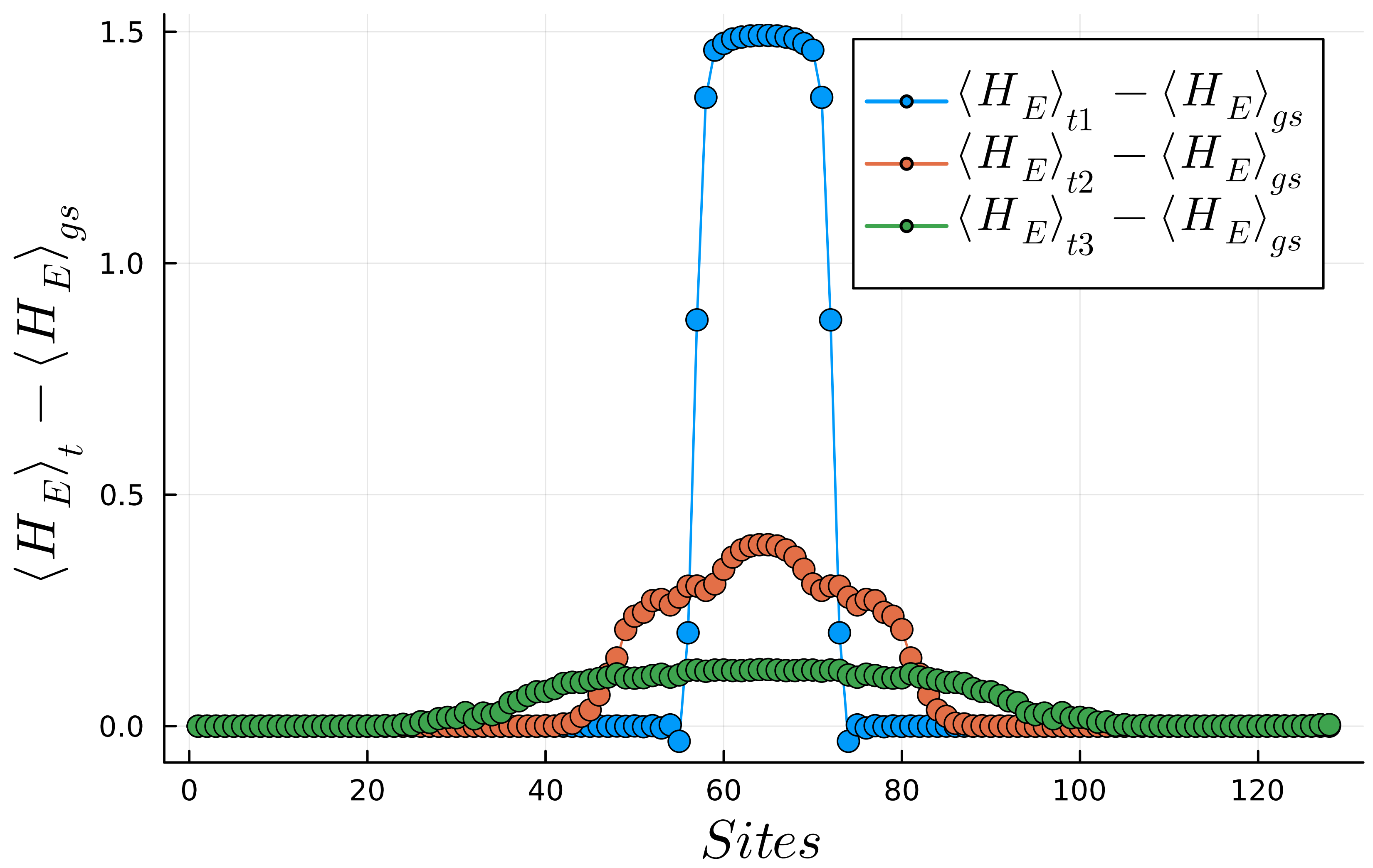
Dynamical Strings

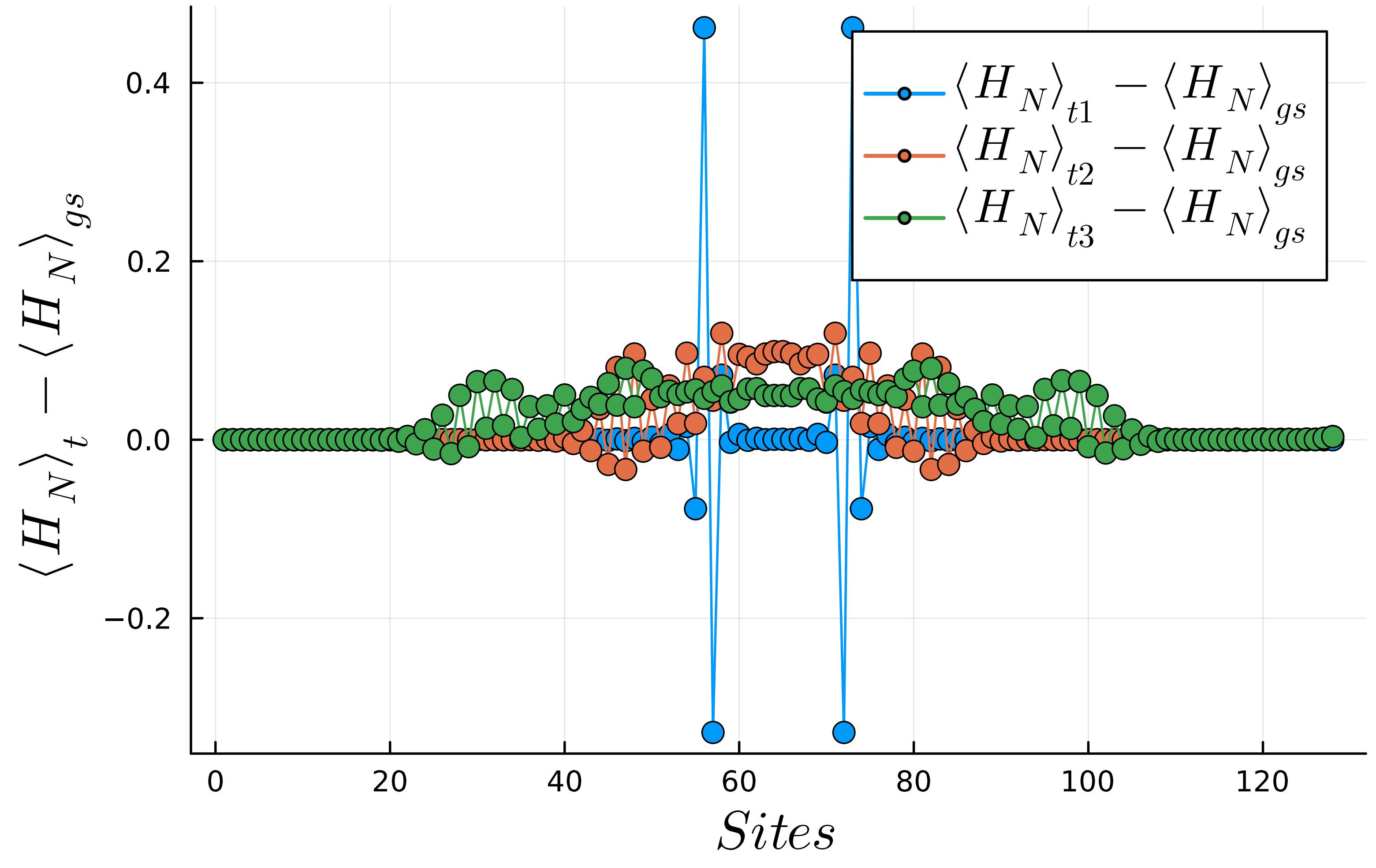




Parameter

- $j_{max} = 3$
- $\mu = 1.6$
- $x = 16.0$
- Bond dimension = 200
- Time = $2axt_{comp}$





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The statics and dynamics of string breaking

Dynamical Strings: Towards a continuum limit

- Is there a way to take the continuum limit for dynamical fermions?
- We look at a scalar function which shows signature of string breaking
- Loschmidt echo is one such candidate

$$\lambda(t) = \frac{-1}{N} log(|\langle\Psi(t)|\Psi(0)\rangle|^2)$$

$$|\Psi(0)\rangle = \frac{1}{a}\left[|string\rangle_{odd-odd} - |string\rangle_{even-even}\right]$$

- Sharp peaks in the rate function $\lambda(t) \implies$ least overlap with the initial string state $\implies$ string breaking

- Fix physical length of string, $L_{phys} = 4$
- Place string symmetrically about centre
- Choose large enough lattice to avoid boundary effects
- Fix m/g value for fermion
- $a \in \left(1, \frac{1}{2}, \frac{1}{4}\right), N \in \left(32,64,128\right)$, $x \in \left(1,4,16\right), \mu \in \left(0.4,0.8,1.6\right)$, $l_{lat} \in \left(4,8,16\right)$

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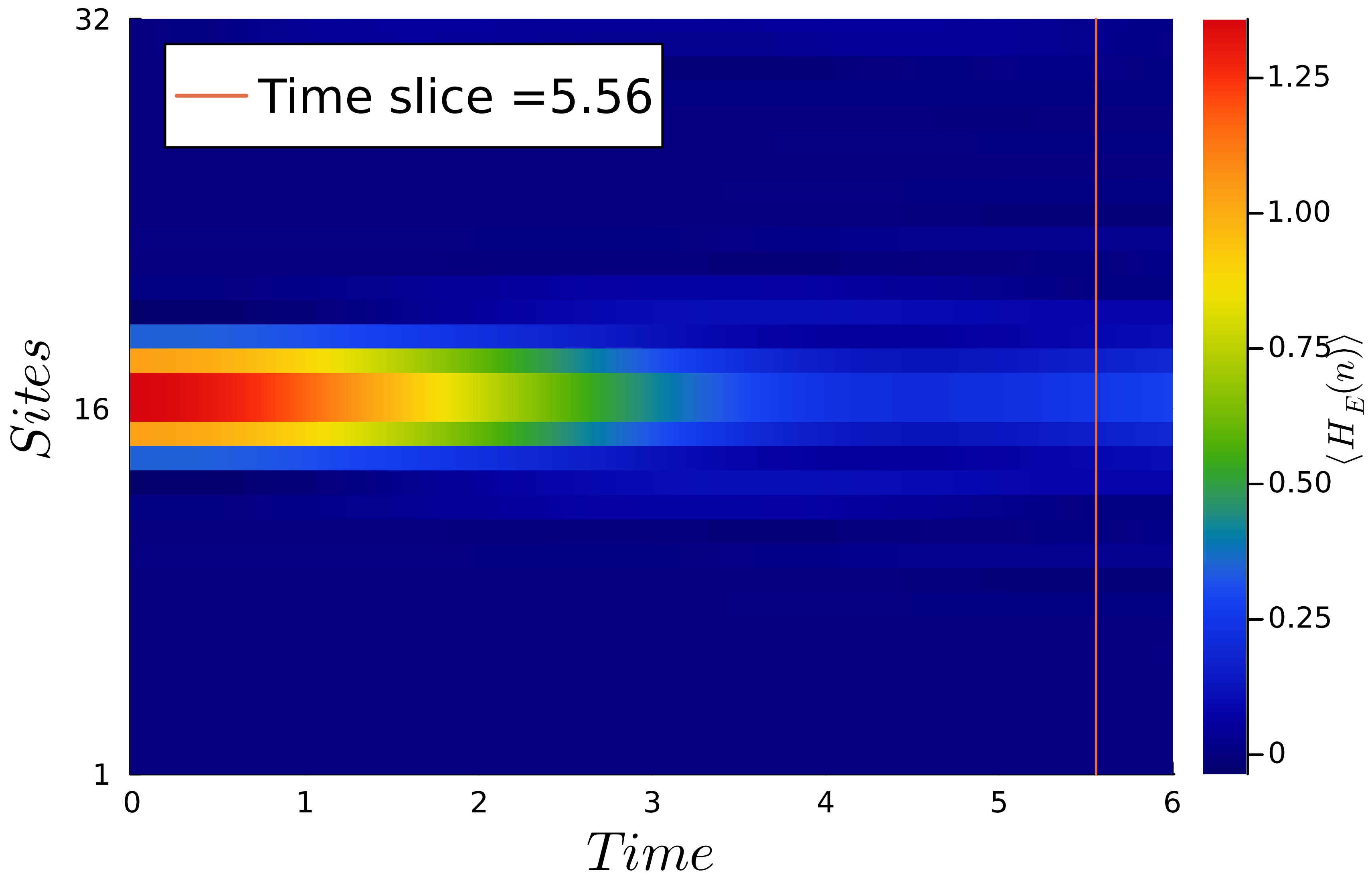
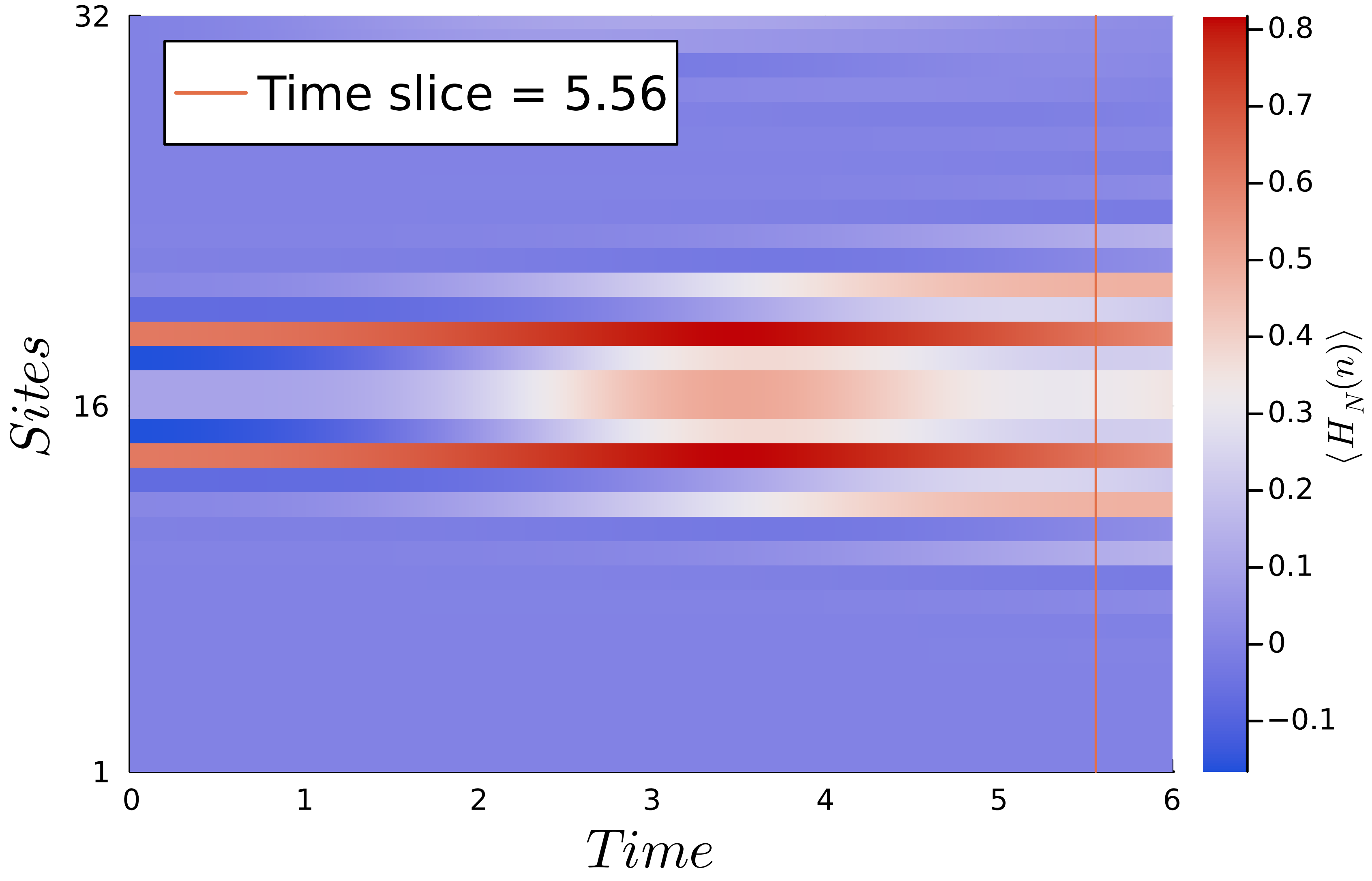
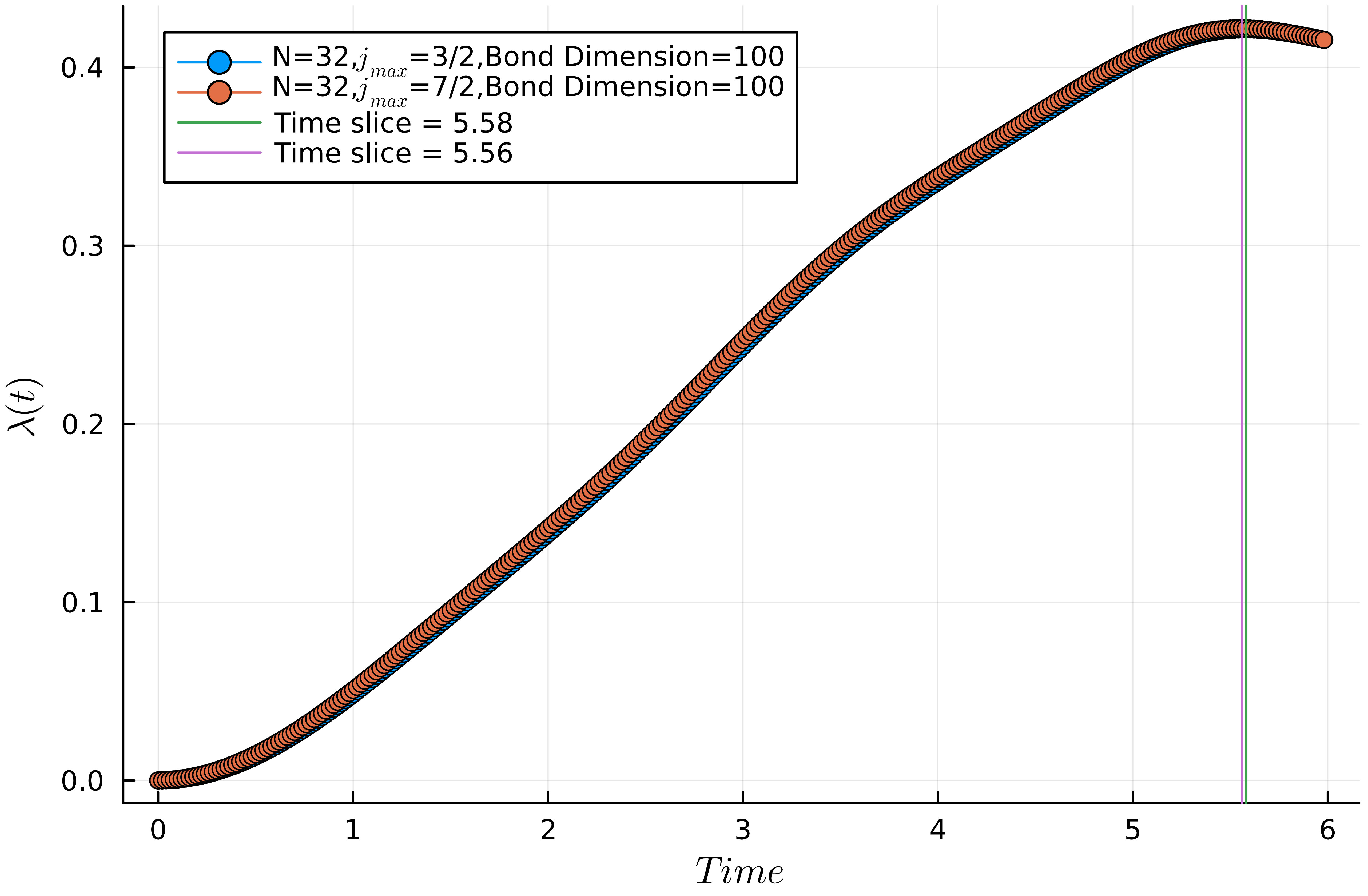
The statics and dynamics of string breaking

Dynamical Strings: Towards a continuum limit







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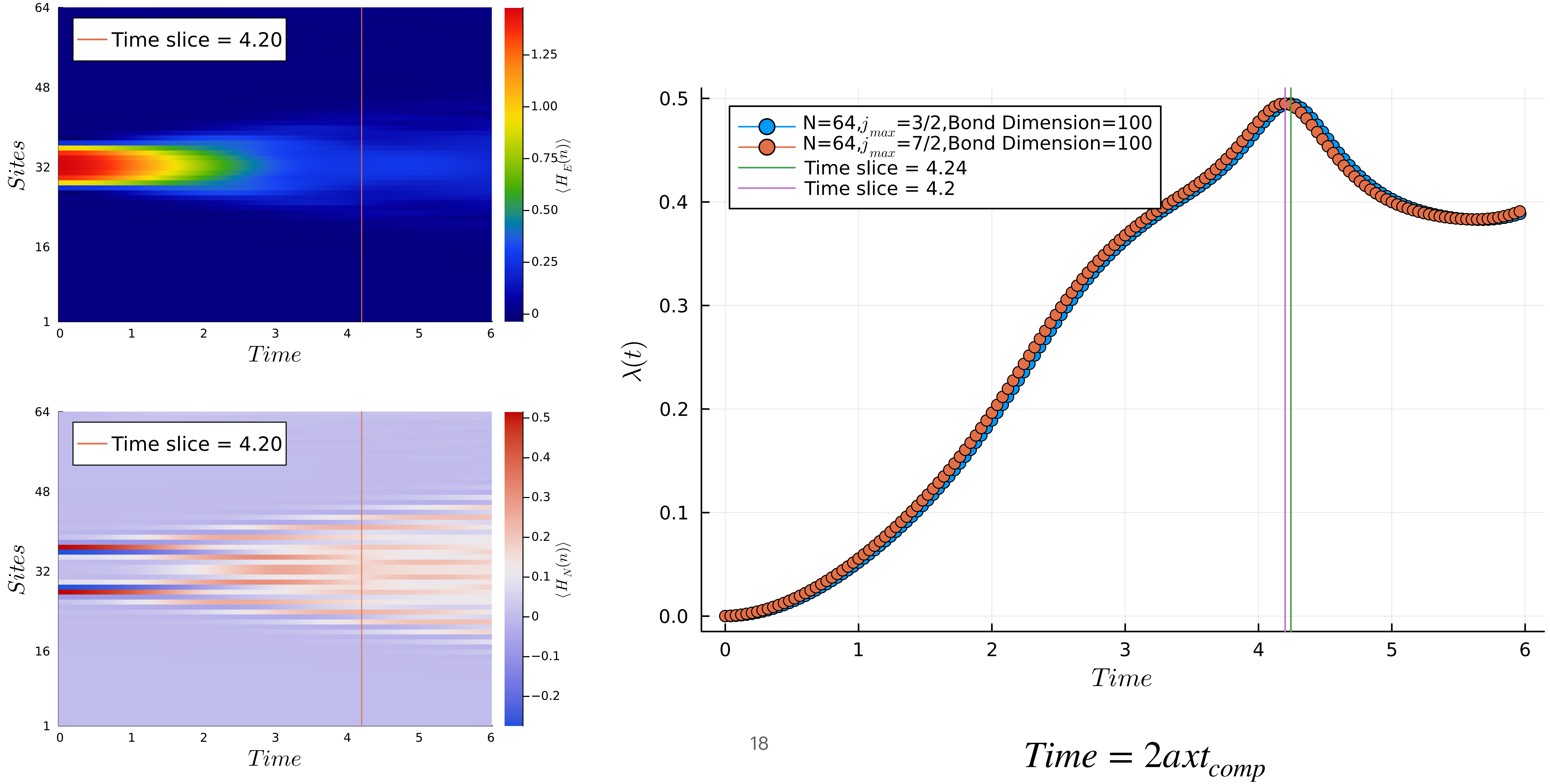
$$Time = 2axt_{comp}$$

The statics and dynamics of string breaking

Dynamical Strings: Towards a continuum limit



$Time = 2axt_{comp}$

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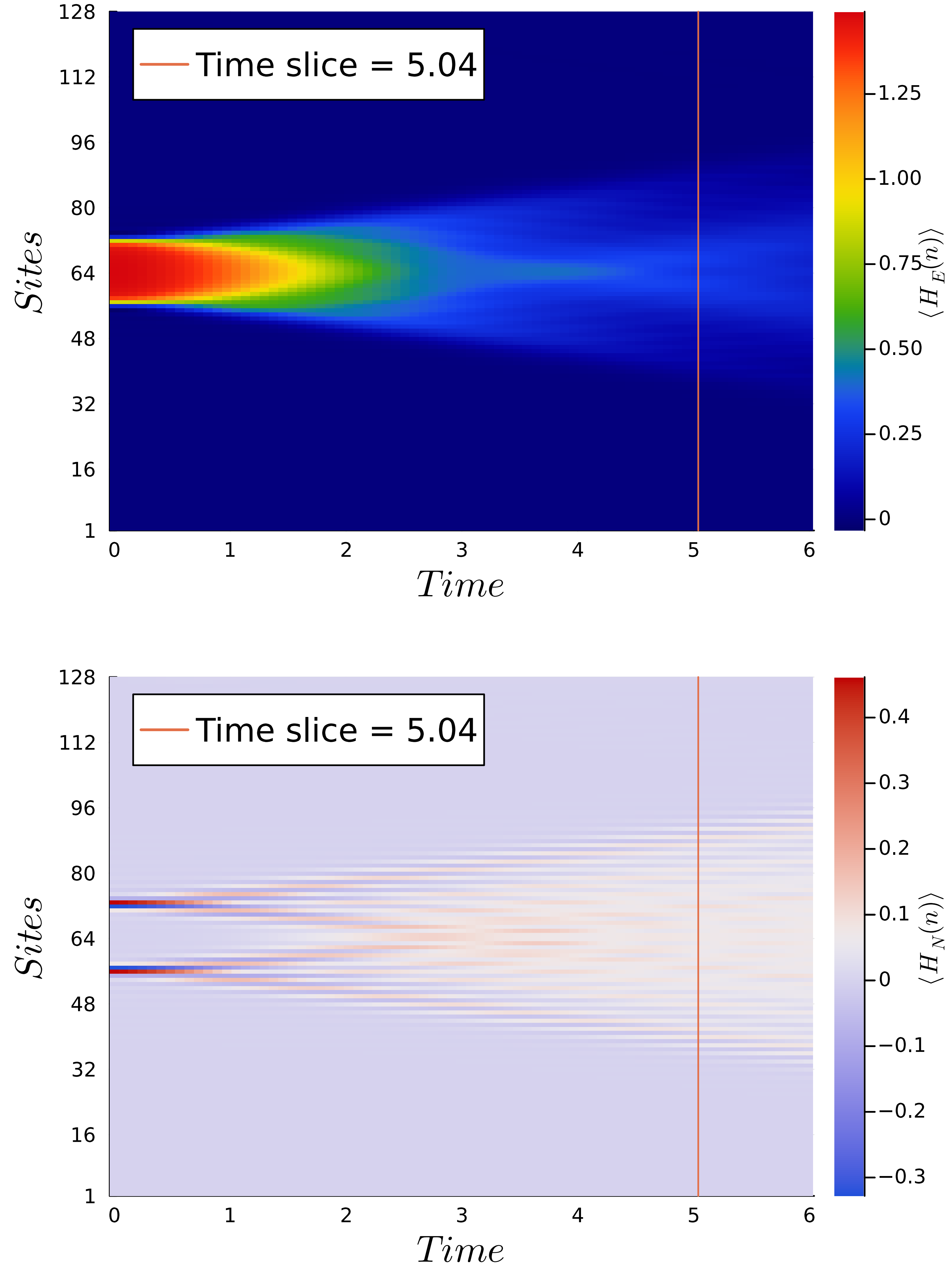
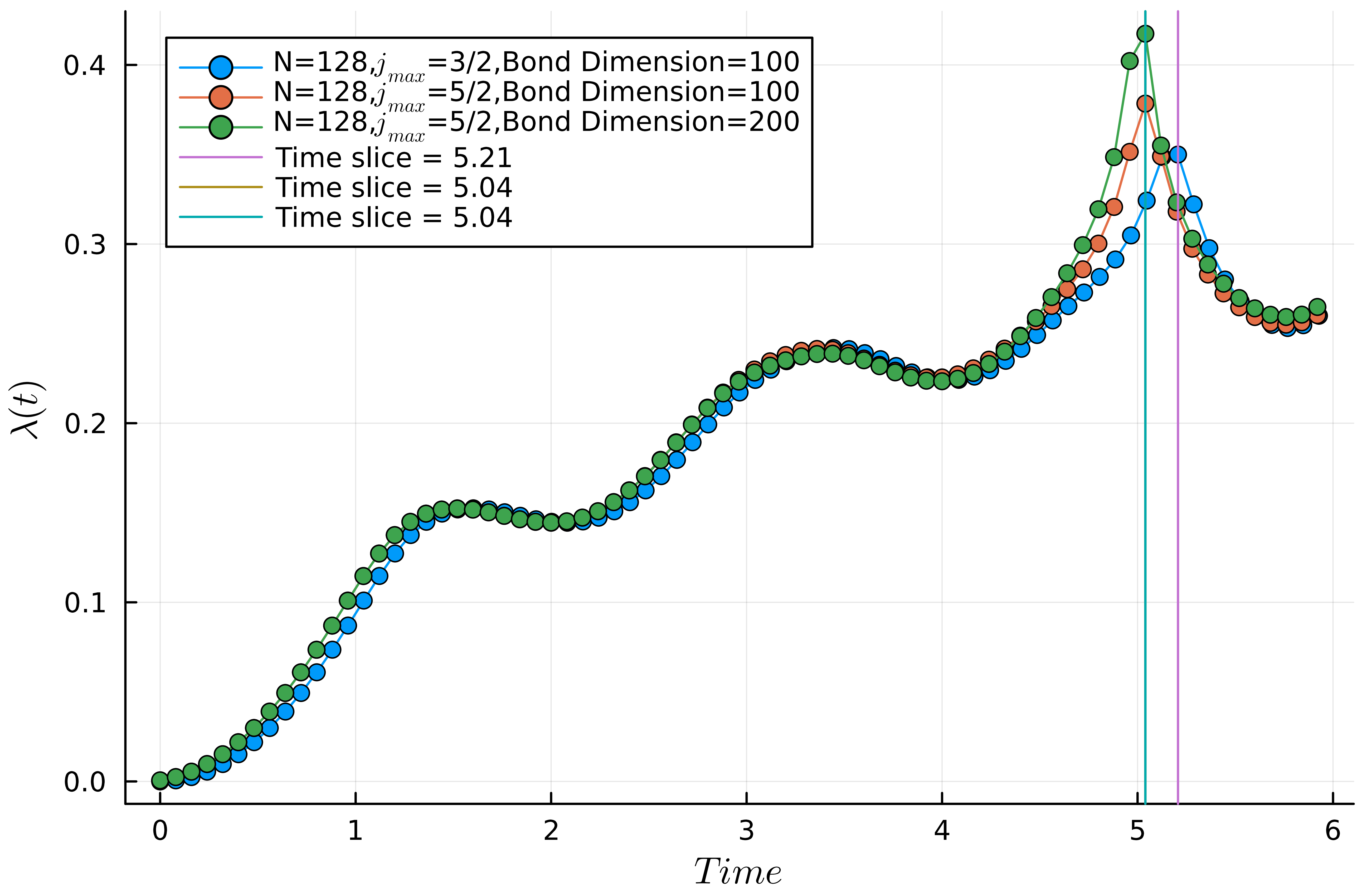
The statics and dynamics of string breaking

Dynamical Strings: Towards a continuum limit





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$$Time = 2axt_{comp}$$

Summary

- Construct an MPS ansatz for the Loop-String-Hadron (LSH) formulation and benchmarked it with existing literature.
- Constructed string states and observed string breaking in low mass regimes.
- Static string potential in good agreement with existing literature.
- Attempted to understand the continuum limit of string breaking in the presence of dynamical fermions

Future directions

- Implementation of quantum circuits/analog simulations for the LSH Hamiltonian to probe real-time dynamics, thermalization.
- Use uniform MPS framework and explore scattering dynamics of non-Abelian gauge theory.
- Extend the tensor network ansatz to 1+1D SU(3) LSH Hamiltonian and 2+1D non-Abelian gauge theories using PEPS.

Thank you!

Appendix 1

$$H_I = \sum_x \frac{1}{\sqrt{\mathcal{N}_L(x)+1}} \left[\sum_{\sigma=\pm} S_{\text{out}}^{+,\sigma}(x) S_{\text{in}}^{\sigma,-}(x+1) \right] \times \frac{1}{\sqrt{\mathcal{N}_R(x+1)+1}} + \text{ H.c.}$$

$$S_{\text{out}}^{++}(x)\mathcal{S}_{\text{in}}^{+-}(x+1) = \left[\chi_o^\dagger\right]_x \left[\chi_o\right]_{x+1} \times \left[\left(1-\mathcal{N}_i\right) + \Lambda^+\mathcal{N}_i\right]_x \left[\mathcal{N}_i + \Lambda^+\left(1-\mathcal{N}_i\right)\right]_{x+1} \times \left[\sqrt{\mathcal{N}_l - \mathcal{N}_i + 2}\right]_x \left[\sqrt{\mathcal{N}_l - \left(1-\mathcal{N}_i\right) + 2}\right]_{x+1},$$

$$S_{\text{out}}^{--}(x)S_{\text{in}}^{-+}(x+1) = \left[\chi_o\right]_x \left[\chi_0^\dagger\right]_{x+1} \times \left[\left(1-\mathcal{N}_i\right) + \Lambda^-\mathcal{N}_i\right]_x \left[\mathcal{N}_i + \Lambda^-\left(1-\mathcal{N}_i\right)\right]_{x+1} \times \left[\sqrt{\mathcal{N}_l + 2\left(1-\mathcal{N}_i\right)}\right]_x \left[\sqrt{\mathcal{N}_l + 2\mathcal{N}_i}\right]_{x+1}$$

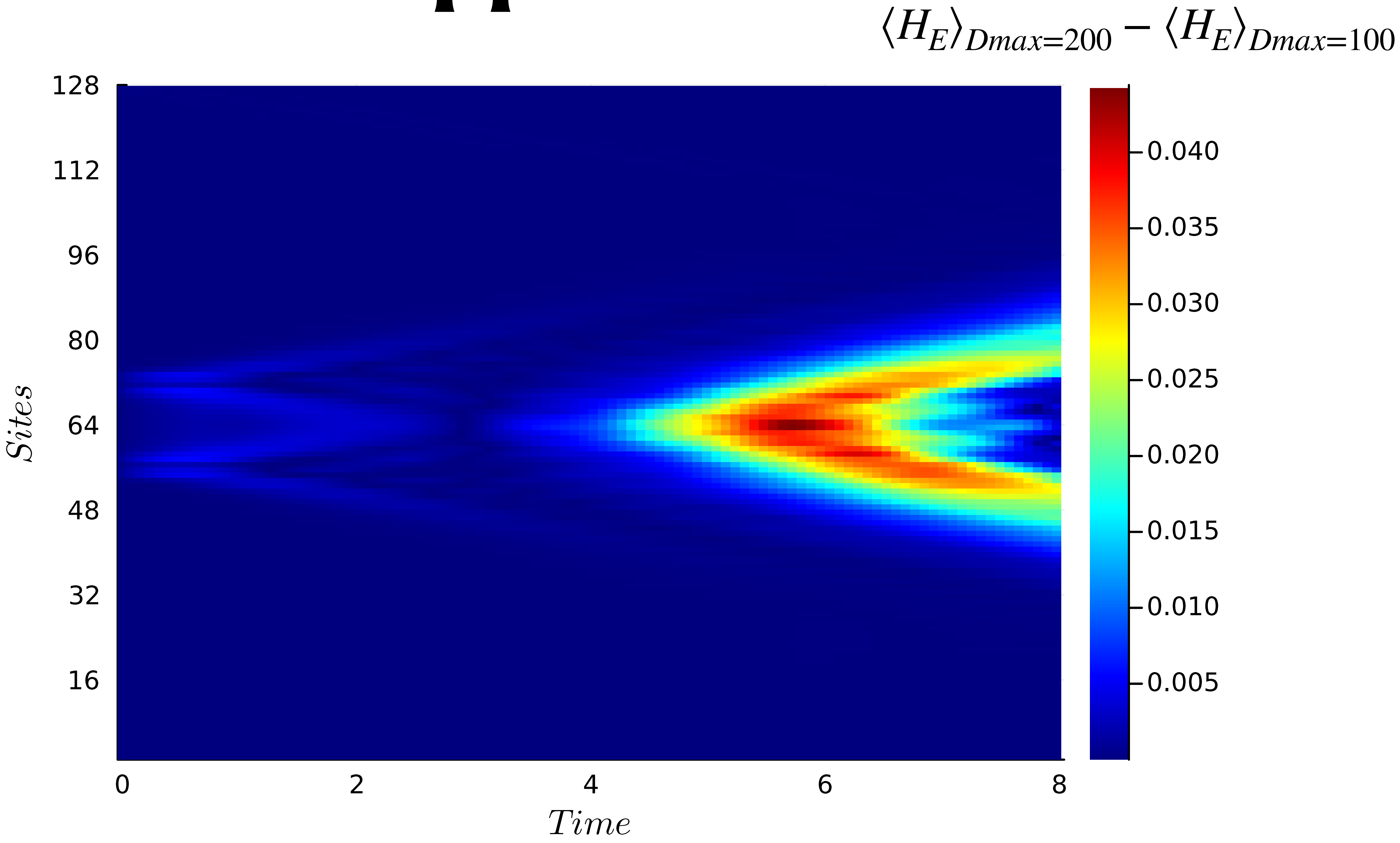
$$S_{\text{out}}^{+-}(x)S_{\text{in}}^{--}(x+1) = \left[\chi_i^\dagger\right]_x \left[\chi_i\right]_{x+1} \times \left[\mathcal{N}_o + \Lambda^-\left(1-\mathcal{N}_o\right)\right]_x \left[\left(1-\mathcal{N}_o\right) + \Lambda^-\mathcal{N}_o\right]_{x+1} \times \left[\sqrt{\mathcal{N}_l + 2\mathcal{N}_o}\right]_x \left[\sqrt{\mathcal{N}_l + 2\left(1-\mathcal{N}_o\right)}\right]_{x+1}$$

$$S_{\text{out}}^{-+}(x)S_{\text{in}}^{++}(x+1) = \left[\chi_i\right]_x \left[\chi_i^\dagger\right]_{x+1} \times \left[\mathcal{N}_o + \Lambda^+\left(1-\mathcal{N}_o\right)\right]_x \left[\left(1-\mathcal{N}_o\right) + \Lambda^+\mathcal{N}_o \right. \times \left[\sqrt{\mathcal{N}_l + \mathcal{N}_o + 1}\right]_x \left[\sqrt{\mathcal{N}_l + \left(1-\mathcal{N}_o\right) + 1} \right.$$

Loop-string-hadron operator factorizations

$$\mathcal{L}^{++} = \Lambda^+\sqrt{(\mathcal{N}_l+1)(\mathcal{N}_l+2+(\mathcal{N}_i \oplus \mathcal{N}_o))}$$
$$\mathcal{L}^{--} = \Lambda^-\sqrt{\mathcal{N}_l(\mathcal{N}_l+1+(\mathcal{N}_i \oplus \mathcal{N}_o))}$$
$$\mathcal{L}^{+-} = -\chi_i^\dagger\chi_o$$
$$\mathcal{L}^{-+} = \chi_i\chi_o^\dagger$$
$$\mathcal{S}_{\text{in}}^{++} = \chi_i^\dagger(\Lambda^+)^{\mathcal{N}_o}\sqrt{\mathcal{N}_l+2-\mathcal{N}_o}$$
$$\mathcal{S}_{\text{in}}^{--} = \chi_i(\Lambda^-)^{\mathcal{N}_o}\sqrt{\mathcal{N}_l+2(1-\mathcal{N}_o)}$$
$$\mathcal{S}_{\text{out}}^{++} = \chi_o^\dagger(\Lambda^+)^{\mathcal{N}_i}\sqrt{\mathcal{N}_l+2-\mathcal{N}_i}$$
$$\mathcal{S}_{\text{out}}^{--} = \chi_o(\Lambda^-)^{\mathcal{N}_i}\sqrt{\mathcal{N}_l+2(1-\mathcal{N}_i)}$$
$$\mathcal{S}_{\text{in}}^{-+} = \chi_o^\dagger(\Lambda^-)^{1-\mathcal{N}_i}\sqrt{\mathcal{N}_l+2\mathcal{N}_i}$$
$$\mathcal{S}_{\text{in}}^{+-} = \chi_o(\Lambda^+)^{1-\mathcal{N}_i}\sqrt{\mathcal{N}_l+1+\mathcal{N}_i}$$
$$\mathcal{S}_{\text{out}}^{+-} = \chi_i^\dagger(\Lambda^-)^{1-\mathcal{N}_o}\sqrt{\mathcal{N}_l+2\mathcal{N}_o}$$
$$\mathcal{S}_{\text{out}}^{-+} = \chi_i(\Lambda^+)^{1-\mathcal{N}_o}\sqrt{\mathcal{N}_l+1+\mathcal{N}_o}$$
$$\mathcal{H}^{++} = \chi_i^\dagger\chi_o^\dagger$$
$$\mathcal{H}^{--} = -\chi_i\chi_o$$

Appendix 2



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