

Entropy in the gravitational collapse of a scalar field

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Consider the collapse of a free, massless scalar field to a black hole.

Semiclassical gravity predicts Hawking radiation for late times.

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One would expect this calculation to be very messy and to depend on the detailed nature of the gravitational collapse. However, as I shall show, one can derive an asymptotic form [...] which depends only on the surface gravity of the resulting black hole.

Can we numerically determine the field evolution during the collapse phase including back reaction?

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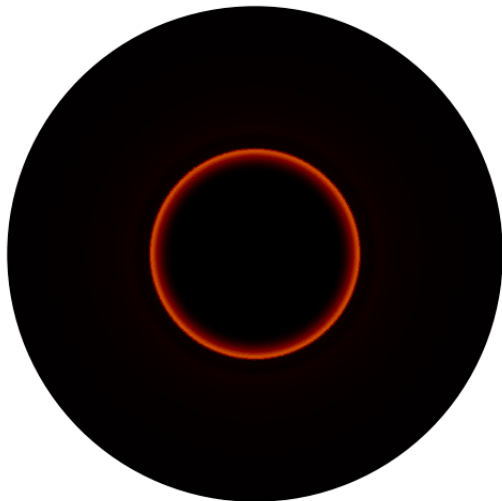
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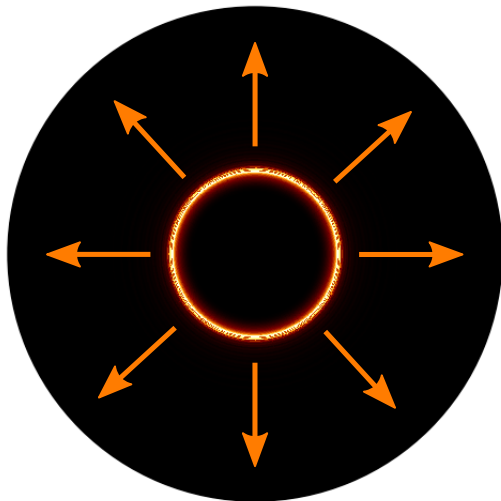
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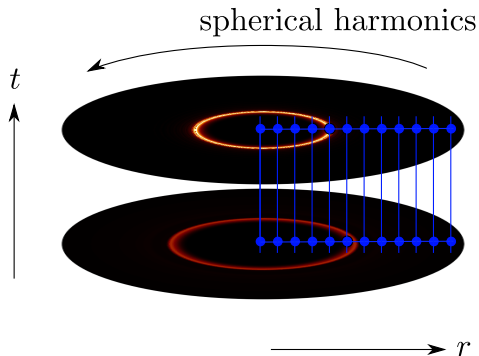
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[1] M. W. Choptuik, "Universality and scaling in gravitational collapse of a massless scalar field," Phys. Rev. Lett. **70** (1993), 9-12

Semiclassical Einstein equation:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \langle \psi | T_{\mu\nu} | \psi \rangle$$

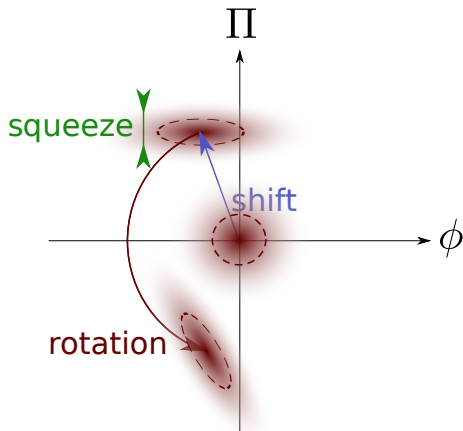
Choose state $|\psi\rangle$ such that:

- Close to a classical state \rightarrow Coherent state.
- Expectation value is spherically symmetric.

Choose spherical symmetric coordinate system [1]

$$g_{\mu\nu} = \begin{pmatrix} \alpha^2(t, r) & & & \\ & -a^2(t, r) & & \\ & & -r^2 & \\ & & & -r^2 \cos^2 \theta \end{pmatrix}$$

Gaussian states



System can be understood in terms of Gaussian states.

Gaussian states: State that can be written as

$$\exp(iF) | 0 \rangle$$

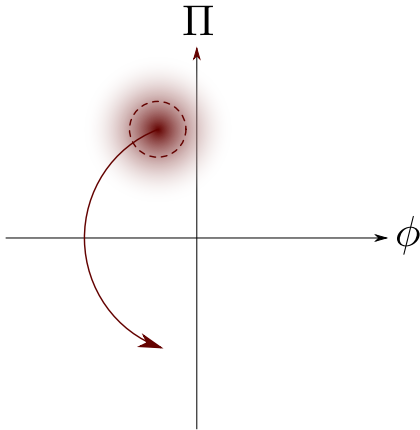
where $| 0 \rangle$ is the ground state a harmonic Osz. and F is a quadratic function of a^\dagger and a .

Operation can be decomposed into

- shift
- rotation
- squeeze

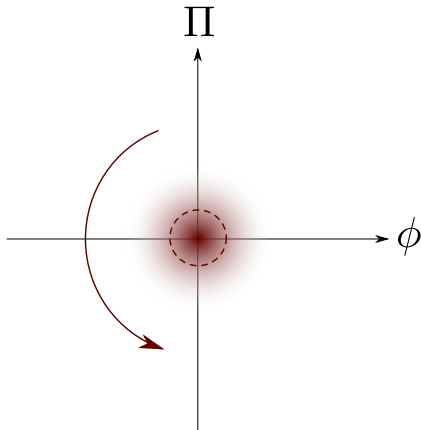
State can be described by mean value $\vec{\mu}$ and covariance matrix C .

Formalism important in quantum optics.



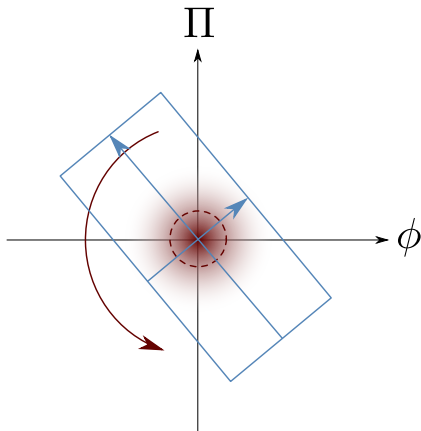
- Gaussian state parametrized by classical central value and covariance matrix

Gaussian states



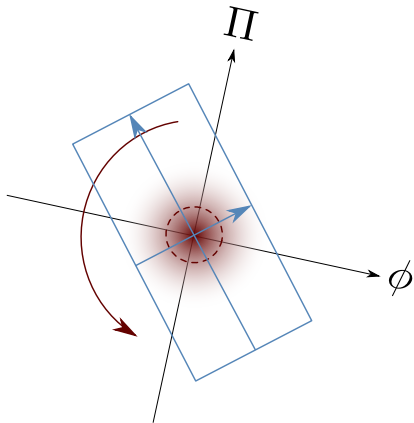
- Gaussian state parametrized by classical central value and covariance matrix
- Time evolution given by rigid rotation in phase space

Gaussian states



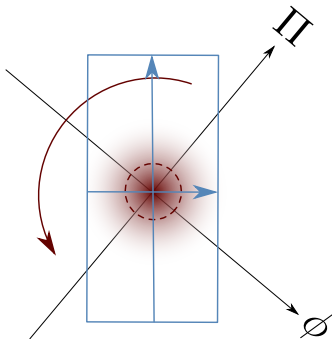
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- Evolution of metric \rightarrow Change of the oscillator parameters

Gaussian states



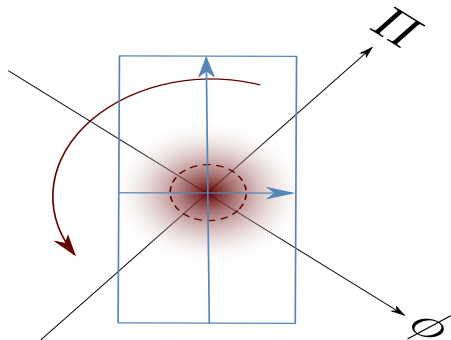
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- Evolution of metric \rightarrow Change of the oscillator parameters
- Reinterpret same state in new oscillator basis

Gaussian states



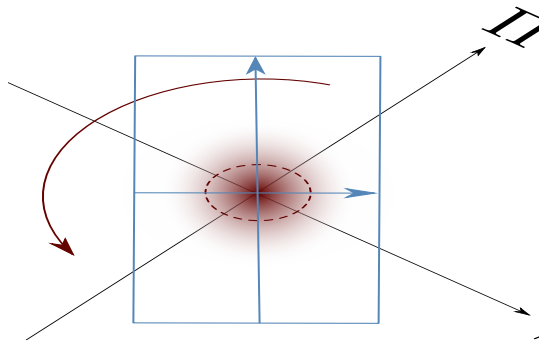
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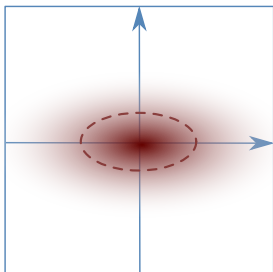


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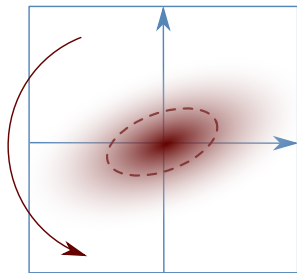


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- Gaussian state parametrized by classical central value and covariance matrix
- Time evolution given by rigid rotation in phase space
- Evolution of metric \rightarrow Change of the oscillator parameters
- Reinterpret same state in new oscillator basis
- System is in a squeezed coherent state

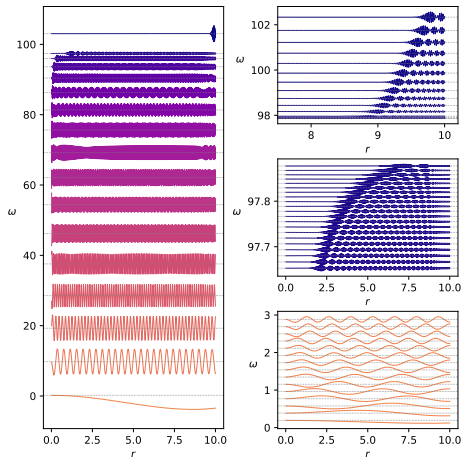
Gaussian states



- Gaussian state parametrized by classical central value and covariance matrix
- Time evolution given by rigid rotation in phase space
- Evolution of metric \rightarrow Change of the oscillator parameters
- Reinterpret same state in new oscillator basis
- System is in a squeezed coherent state
- Field evolution not trivial

Scalar field decomposition

$l = 0$:



Hamiltonian of the field can be written as

$$\mathcal{H} = \sum_{l=0}^{\infty} (2l+1) \left(\Pi_l \Pi_l^\dagger + \phi_l^\dagger K \phi_l \right)$$

with

$$K = \nabla_r^T \nabla_r + \frac{l(l+1)}{r^2} \alpha^2$$

where ∇_r is a discretized derivative.

Eigenmodes decomposition:

$$K = V \omega^2 V^T$$

- V : Mode functions.
- ω : Mode frequency

Phase space evolution

$$H = \Pi\Pi^\dagger + \phi^\dagger K\phi = \vec{x}^\dagger M\vec{x} \quad x = (\Pi_1^\dagger \quad \Pi_2^\dagger \quad \dots \quad \phi_1 \quad \phi_2 \quad \dots)^T$$

Scalar field time evolution: $\vec{\mu}(t + \Delta t) = S\vec{\mu}(t)$ and $C(t + \Delta t) = SC(t)S^T$ where $S = T^{-1}RT$

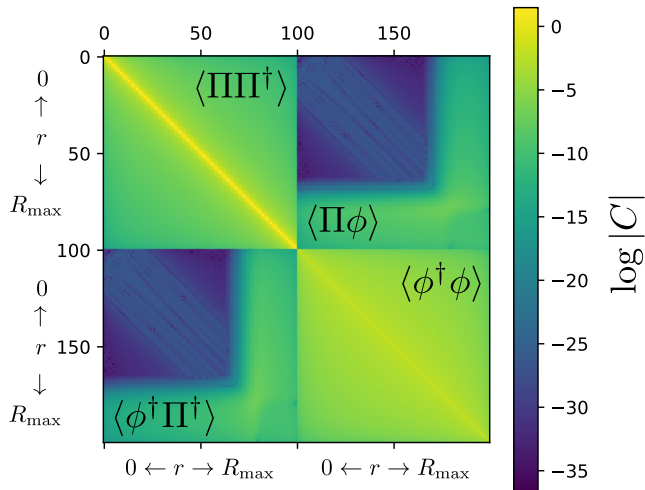
$$T = \begin{pmatrix} \frac{1}{\sqrt{\omega}} V^T & \\ & \sqrt{\omega} V^T \end{pmatrix}$$

$$R = \begin{pmatrix} \cos(\omega_1 \Delta t) & & \sin(\omega_1 \Delta t) & & \\ & \cos(\omega_1 \Delta t) & & \sin(\omega_1 \Delta t) & \\ & & \ddots & & \ddots \\ -\sin(\omega_1 \Delta t) & & & \cos(\omega_1 \Delta t) & \\ & -\sin(\omega_1 \Delta t) & & & \cos(\omega_1 \Delta t) \\ & & \ddots & & \ddots \end{pmatrix}$$

- T transforms from position space to oscillator basis
- R is a rigid rotation in phase space for each oscillator

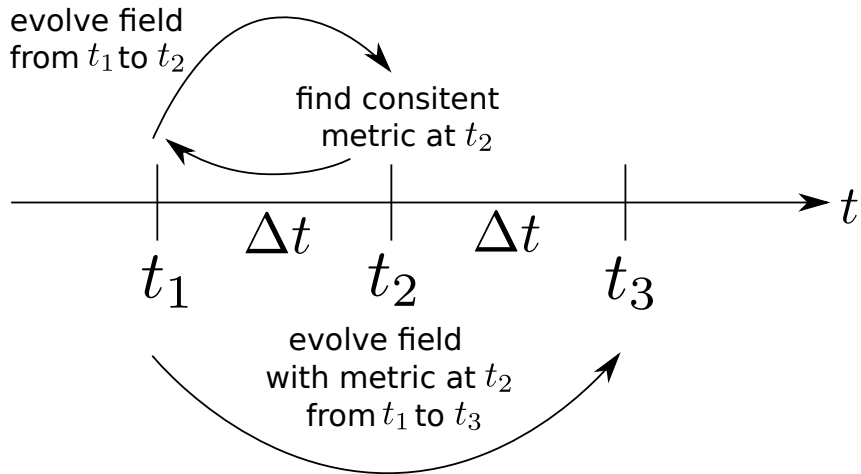
Metric change enters only through T and ω

Covariance matrix



- Covariance matrix contains full information of the state
- Blocks are the equal time correlation functions
- Entanglement is contained in that matrix

Combined evolution



Divergence structure of the energy momentum tensor

Energy momentum tensor in curved space time can be calculated from the coincidence limit of the two-point function $G(x, x')$.

For well behaved Hadamard states $|\psi\rangle$, the divergence structure is [1]

$$\lim_{x' \rightarrow x} \langle \psi | G(x, x') | \psi \rangle = \frac{u(x, x')}{\sigma(x, x')} + v(x, x') \ln \sigma(x, x') + w(x, x')$$

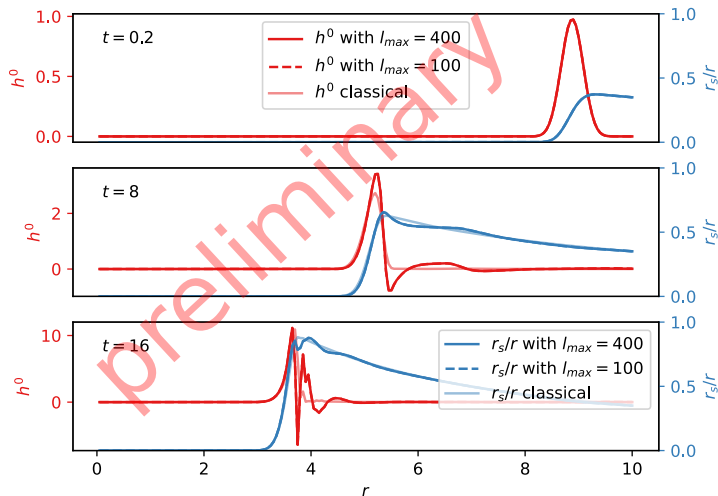
Here, $\sigma(x, x')$ is the geodesic distance between x and x' and $u(x, x')$, $v(x, x')$ are state-independent function that depend only on the metric and $w(x, x')$ is regular.

How does that translate to the sum of angular l modes?

Can one use a $1/l$ expansion to relate them? \rightarrow Work in progress.

[1] S. A. Fulling, M. Sweeny and R. M. Wald, "Singularity Structure of the Two Point Function in Quantum Field Theory in Curved Space-Time," *Commun. Math. Phys.* **63** (1978), 257-264

Pauli-Villars fields



As suggested by [1], using Pauli-Villars fields with alternating signs and masses

$$m_2^2 + m_4^2 = m_1^2 + m_3^2 + m_5^2$$

$$m_2^4 + m_4^4 = m_1^4 + m_3^4 + m_5^4$$

removes divergences for finite values of the masses.

Left side: $m_1 = 1$.

[1] B. Berczi, P. M. Saffin and S. Y. Zhou, "Gravitational collapse of quantum fields and Choptuik scaling," JHEP **02** (2022), 183 doi:10.1007/JHEP02(2022)183 [arXiv:2111.11400 [hep-th]].

Symplectic eigenvalues

Every positive definite symmetric matrix M can be brought to the Williamson form W via a symplectic transformation S :

$$M = SWS^T$$

where

$$W = \bigoplus_{i=1}^N \sigma_i \mathbf{1}_{2 \times 2}$$

σ_i : Symplectic eigenvalues.

Matrix S deentangles modes of uncoupled harmonic oscillators, then applies squeezes to transform sub ellipsoids to circles.

If M is the covariance matrix of a pure state, all $\sigma_i = 1$. For mixed states $\sigma_i > 1$.

→ For covariance matrices of a subsystem, the σ_i give a measure of the entanglement.

Entanglement Entropy

We divide the system into an inner and outer part at some radius r_{cut} .

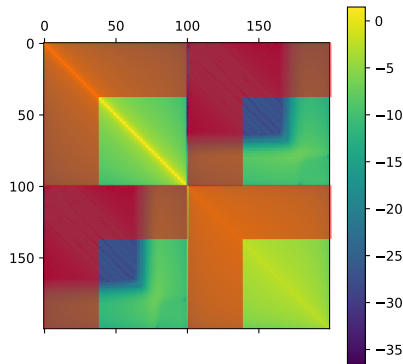
We construct two correlation matrices C_{inner} and C_{outer} by removing lines and columns corresponding to outer or inner points.

Entanglement entropy is defined by

$$S = \sum_i [(\sigma_i + 1) \log(\sigma + 1) - (\sigma_i - 1) \log(\sigma_i - 1)]$$

σ_i : Symplectic eigenvalues of either C_{inner} or C_{outer} .

Entanglement entropy measures entanglement between two subsystems.



red: cut away

Flat space case

Entanglement entropy was considered in flat space [1].

Entanglement entropy behaves as

$$S(r) = \kappa M^2 A^2(r)$$

κ : Constant depending on regulator

M : Scale of the regulator (i.e. inverse lattice spacing)

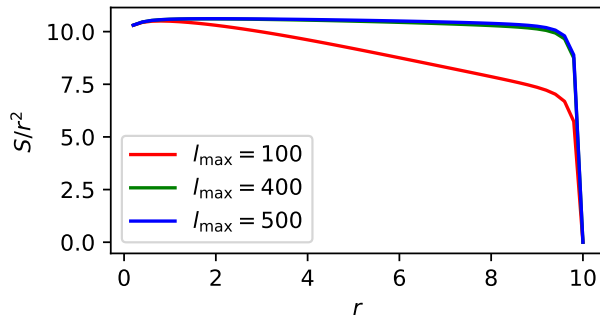
$A(r)$: Surface of the boundary

In curved spacetime:

$$M^{-2} \sim a^2(r) dr^2 = \frac{r}{d} dr^2$$

[1] M. Srednicki, "Entropy and area," Phys. Rev. Lett. **71** (1993), 666-669 doi:10.1103/PhysRevLett.71.666 [arXiv:hep-th/9303048 [hep-th]].

preliminary



Symplectic eigenvalues

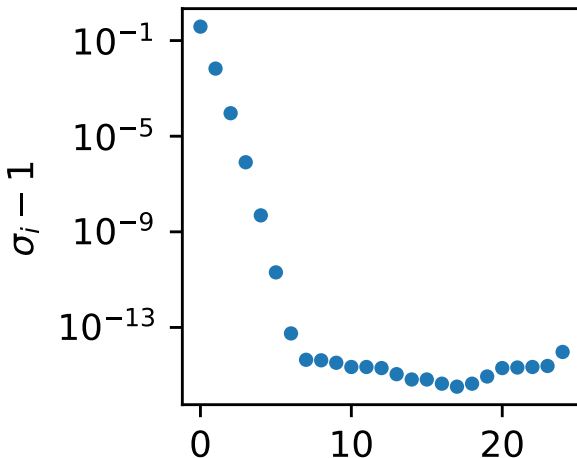
preliminary

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$$S = \sum_i [(\sigma_i + 1) \log(\sigma_i + 1) - (\sigma_i - 1) \log(\sigma_i - 1)]$$

Few symplectic eigenvalues are dominant.

Corresponding modes have support peaked at the horizon.



Symplectic eigenvalues

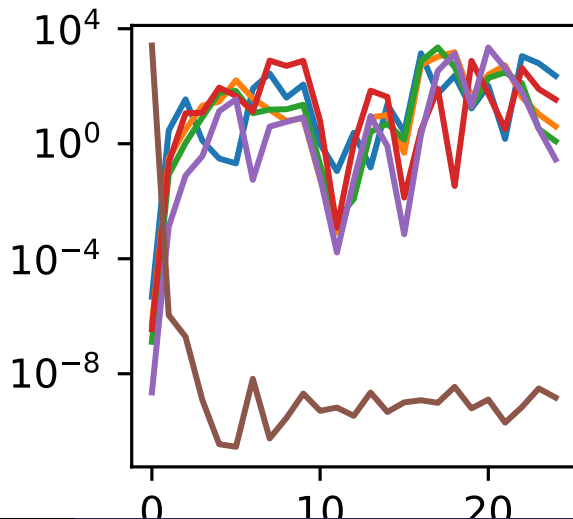
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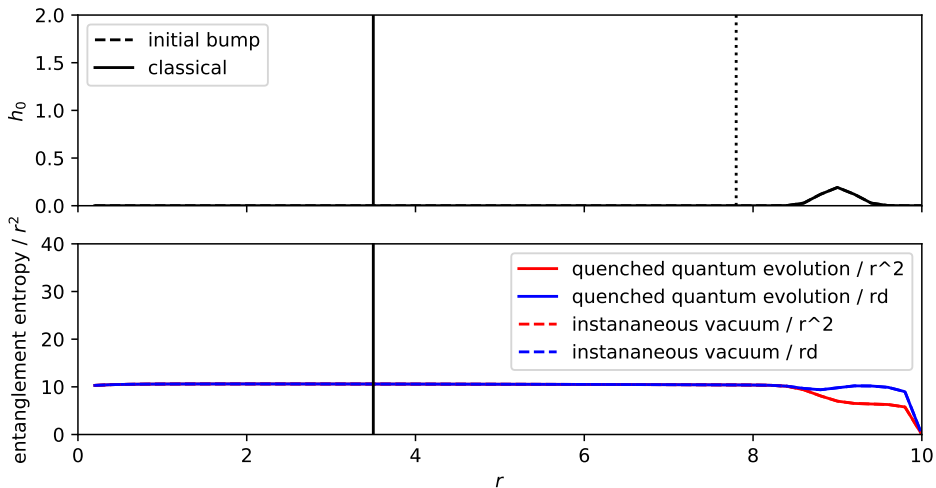
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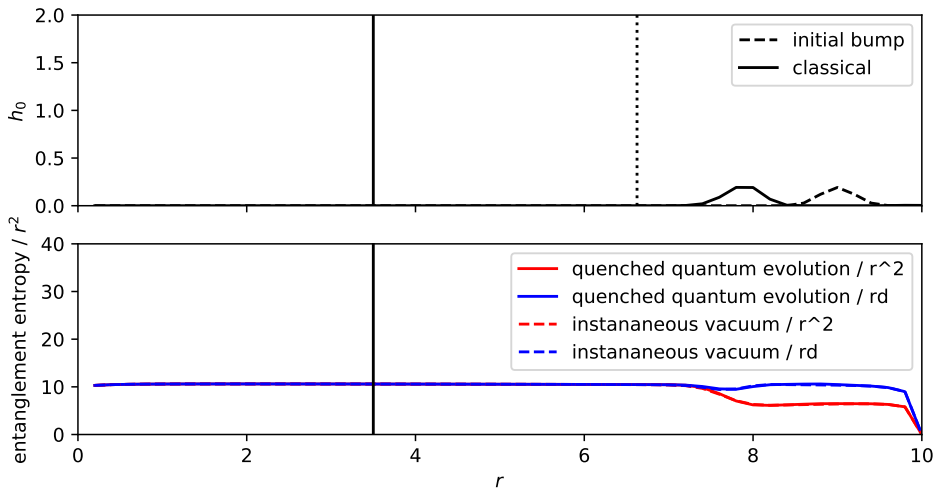
Quenched evolution

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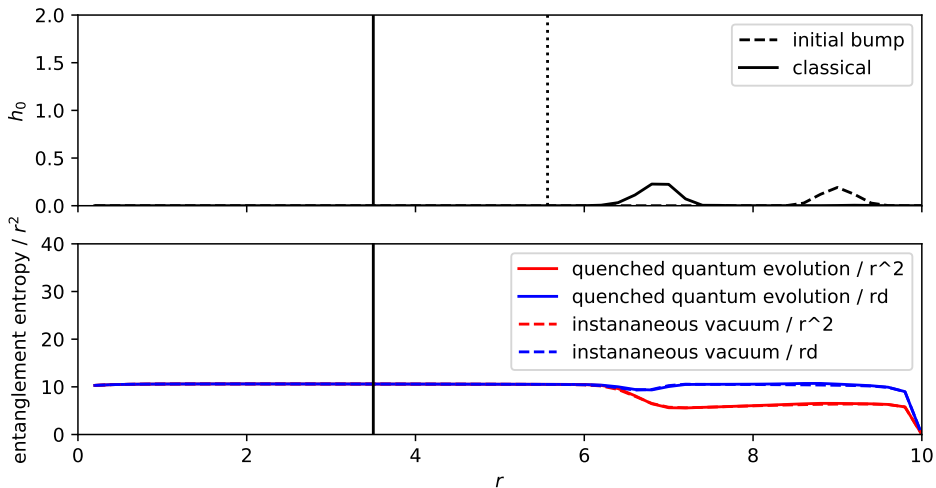
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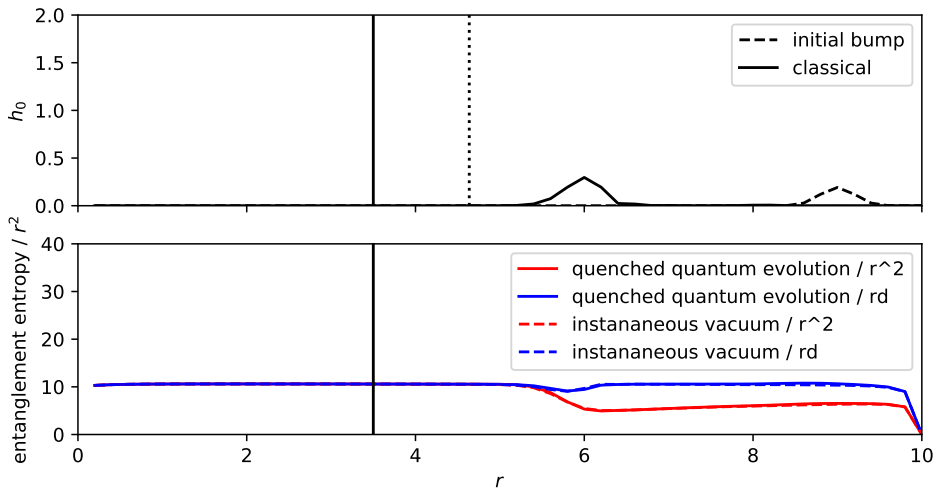
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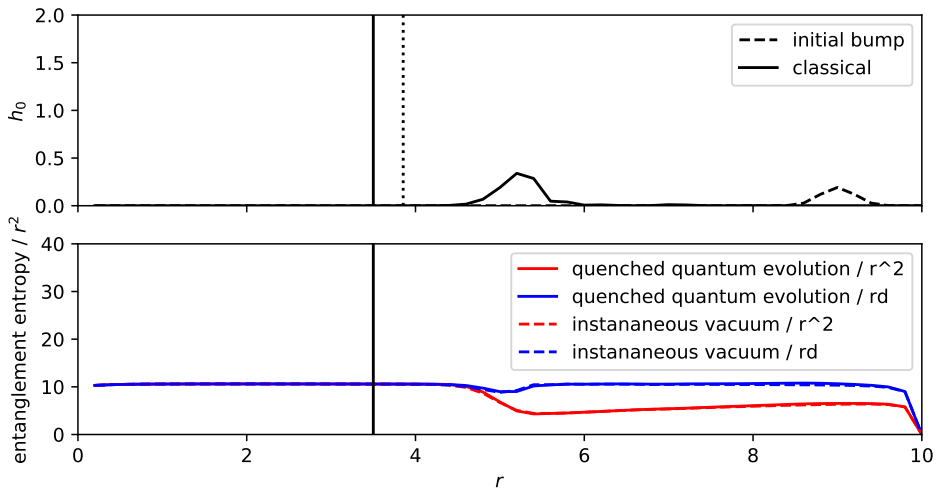
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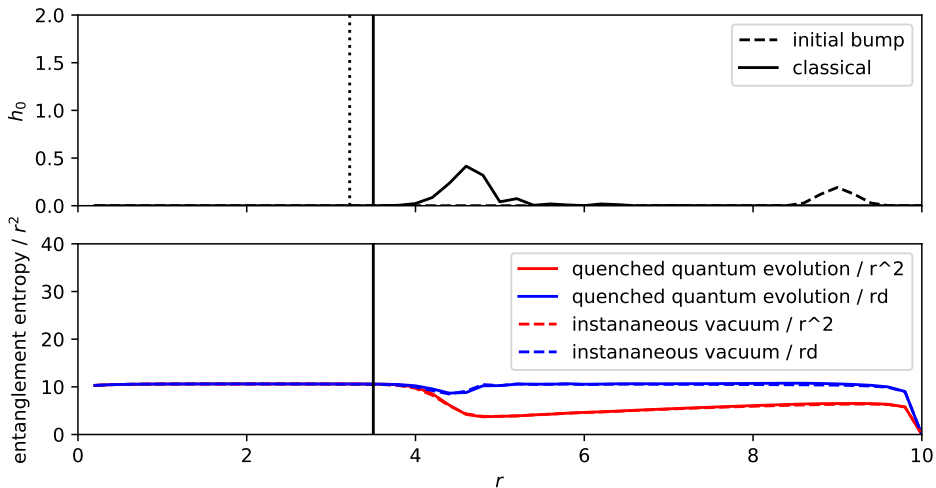
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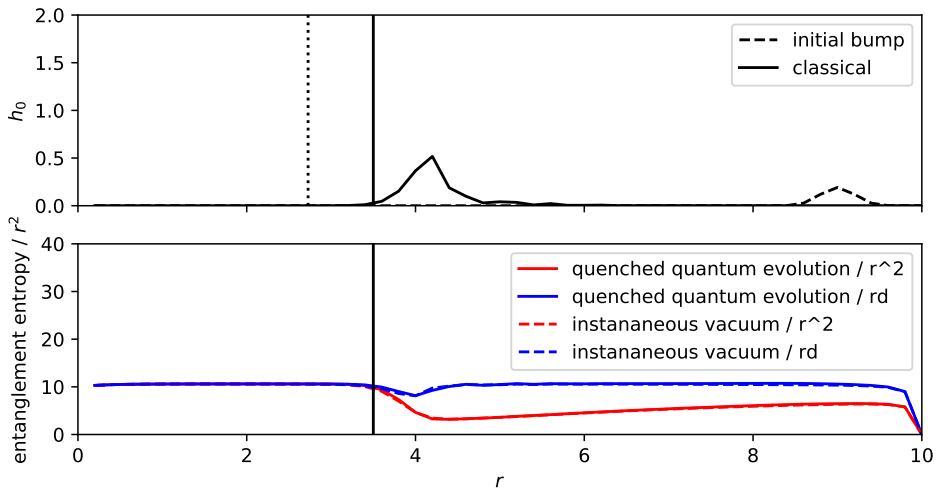
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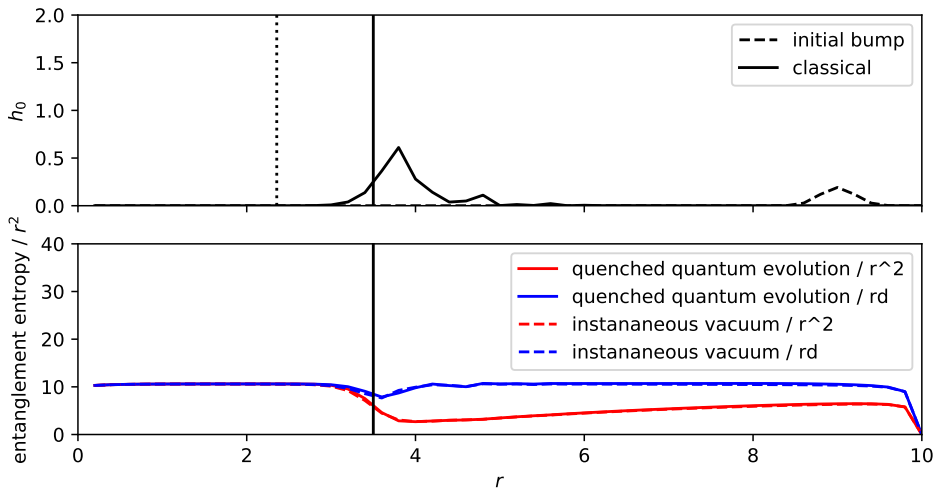
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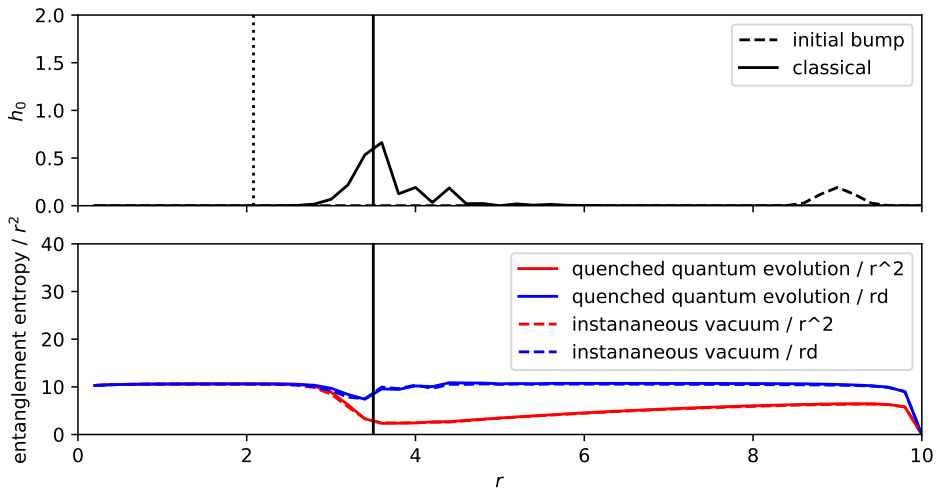
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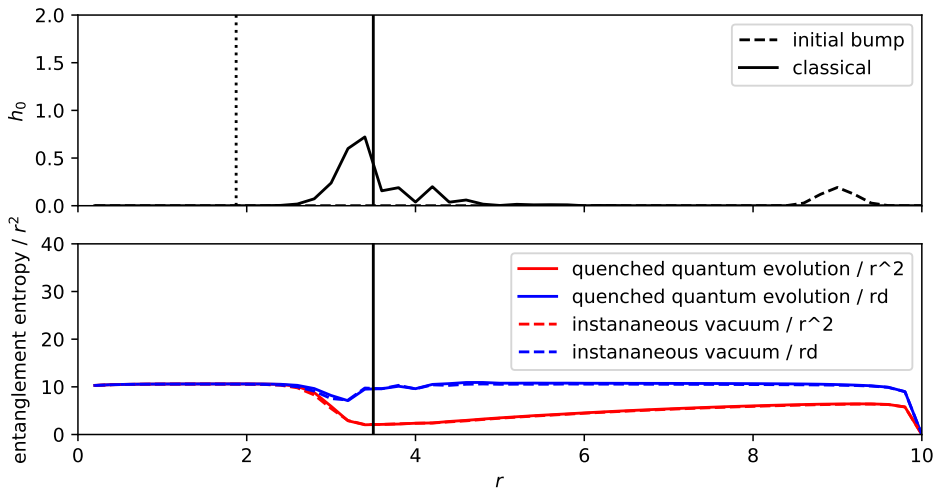
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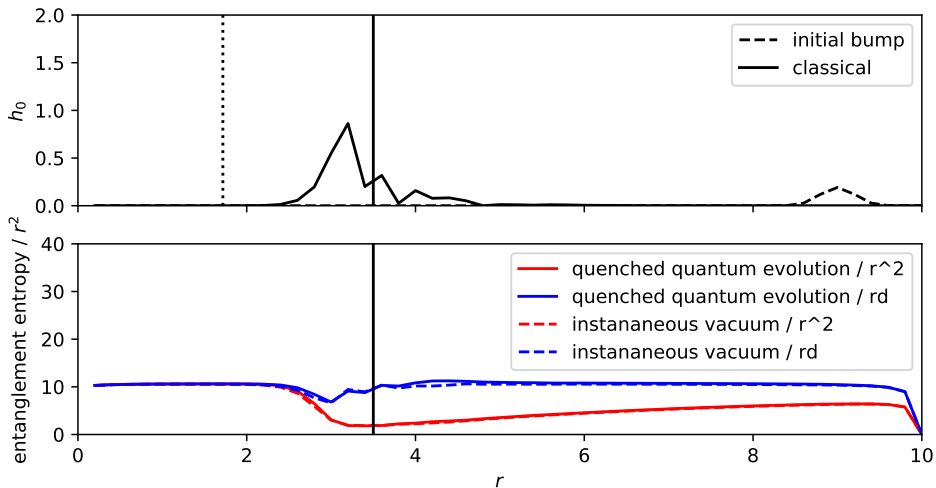
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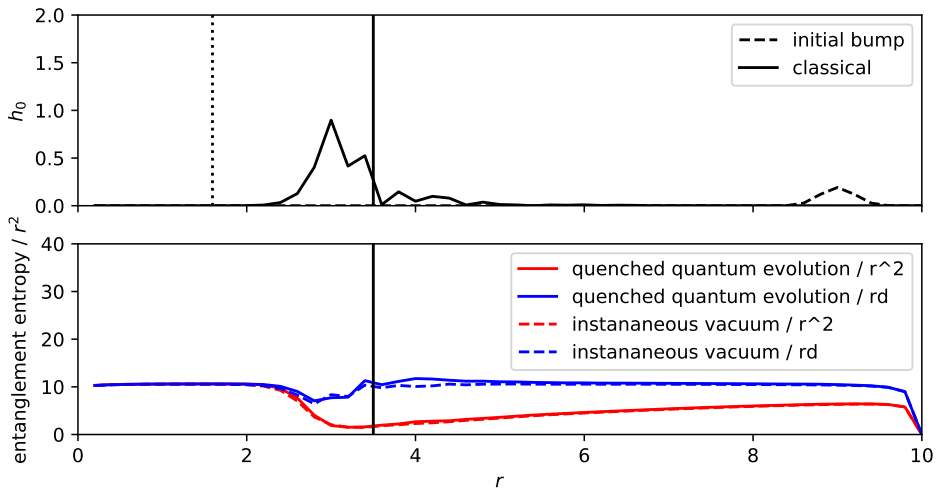
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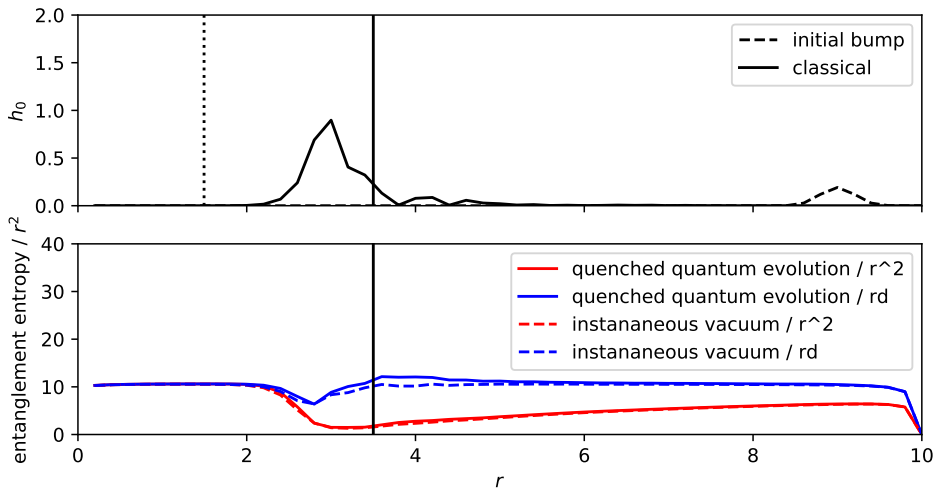
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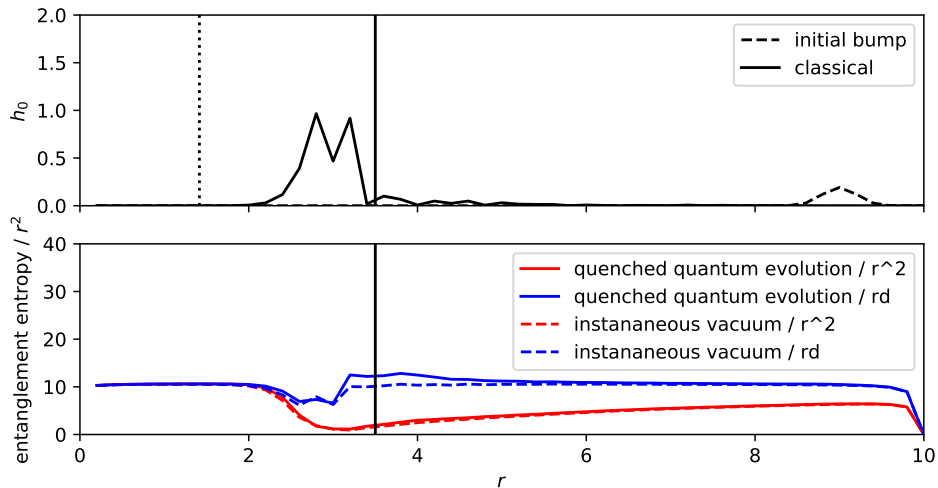
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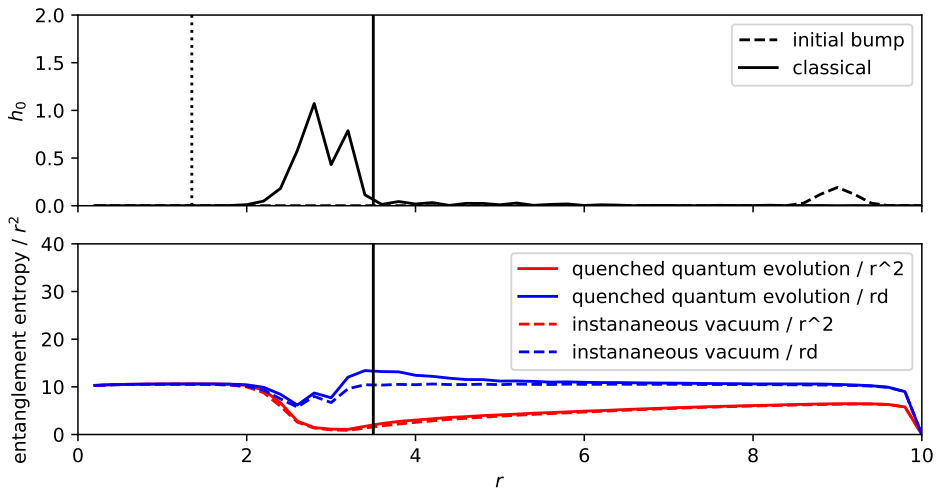
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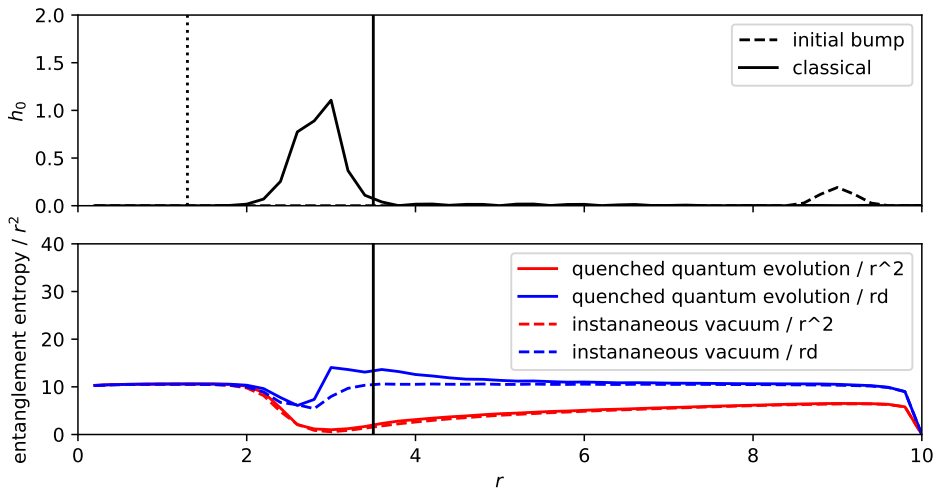
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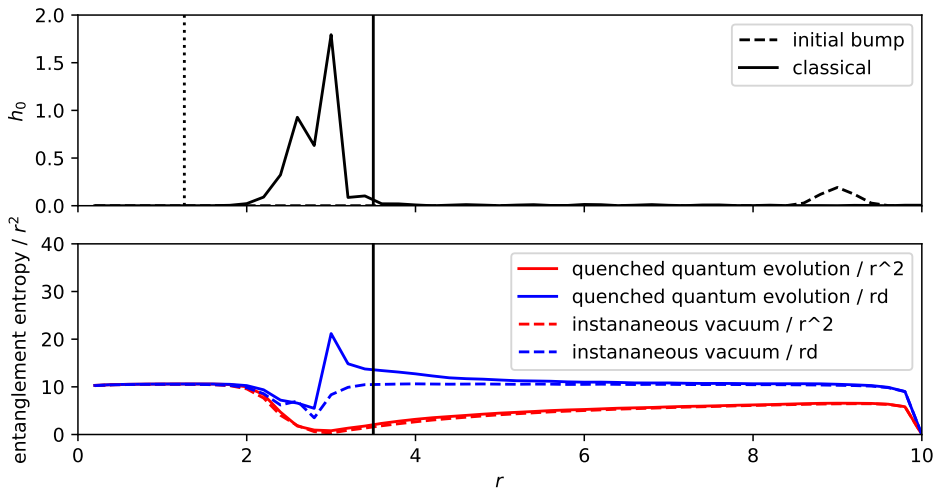
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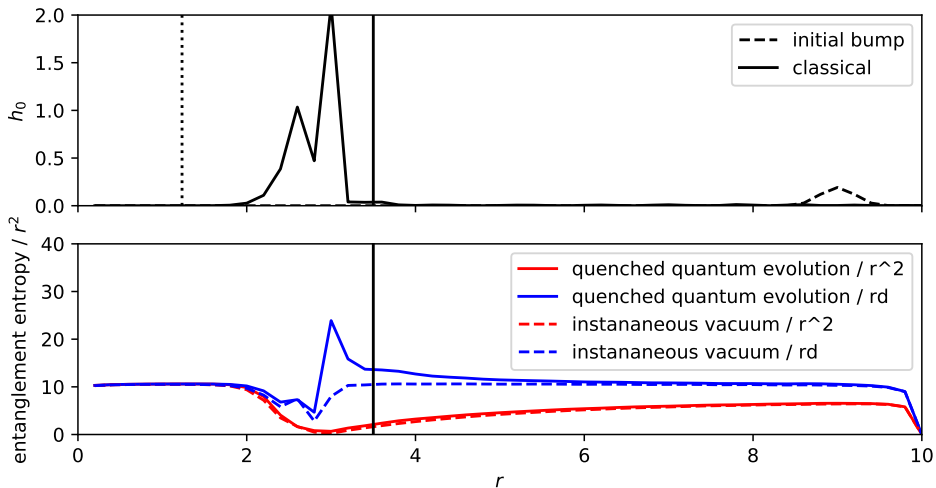
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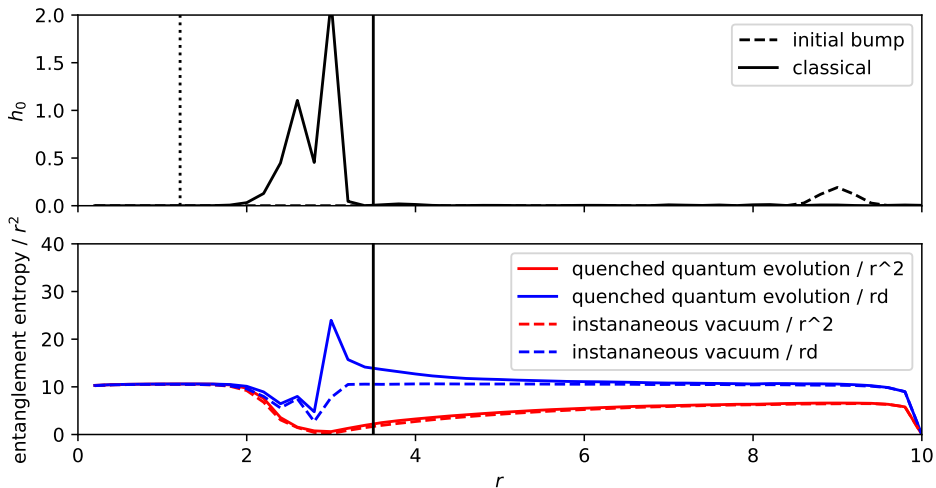
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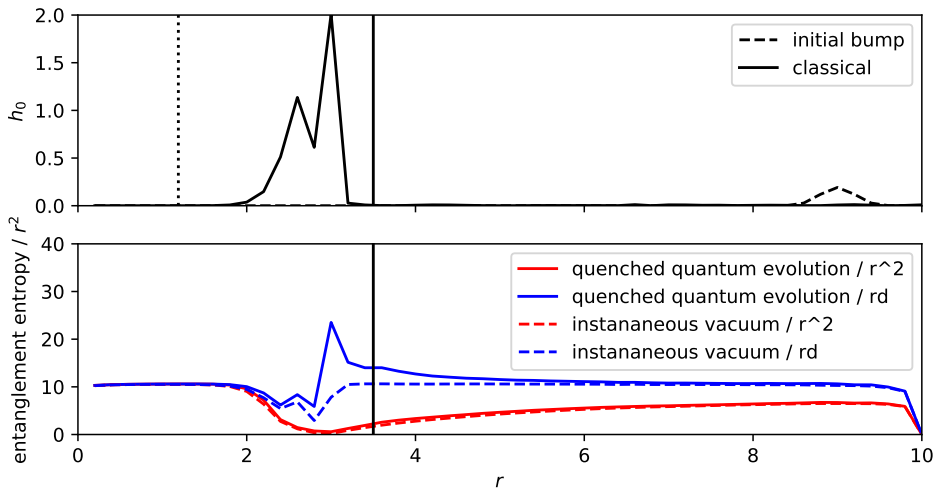
Quenched evolution

preliminary



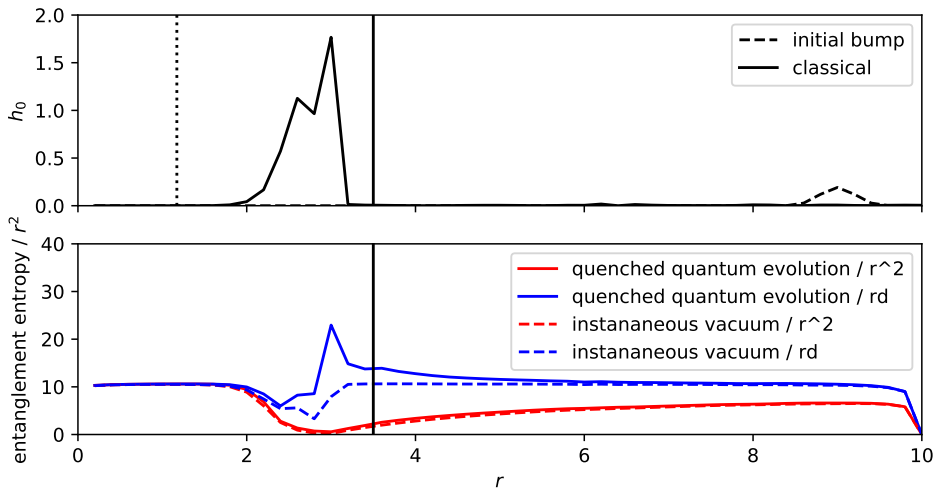
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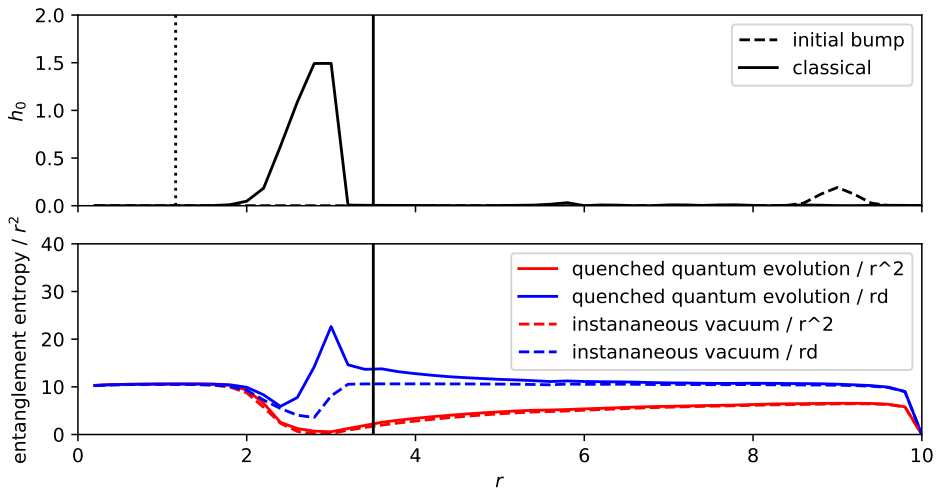
Quenched evolution

preliminary



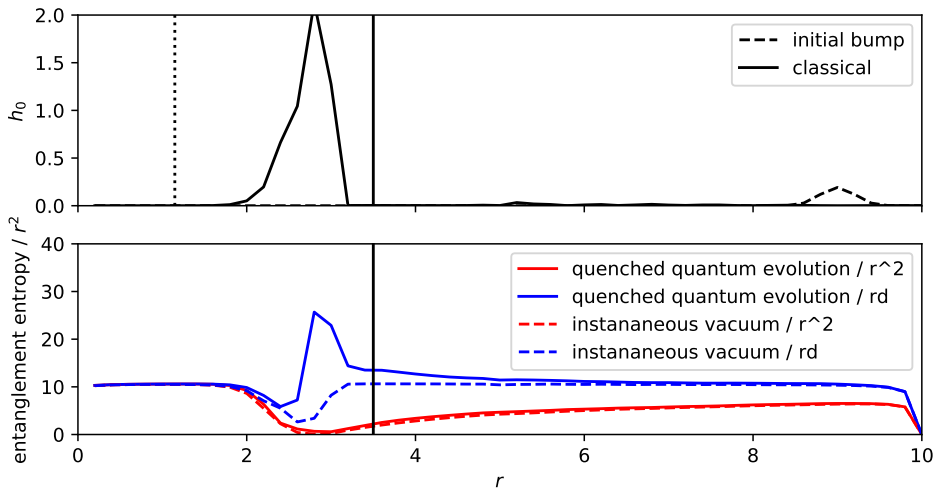
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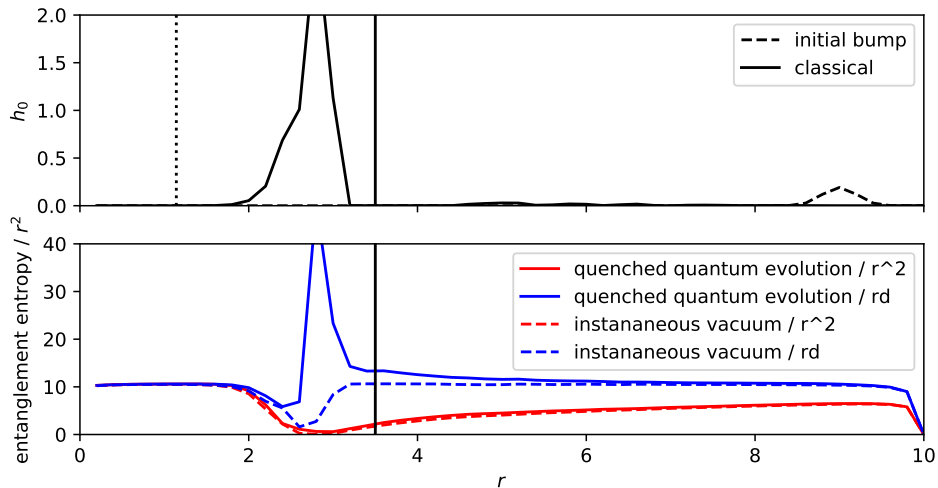
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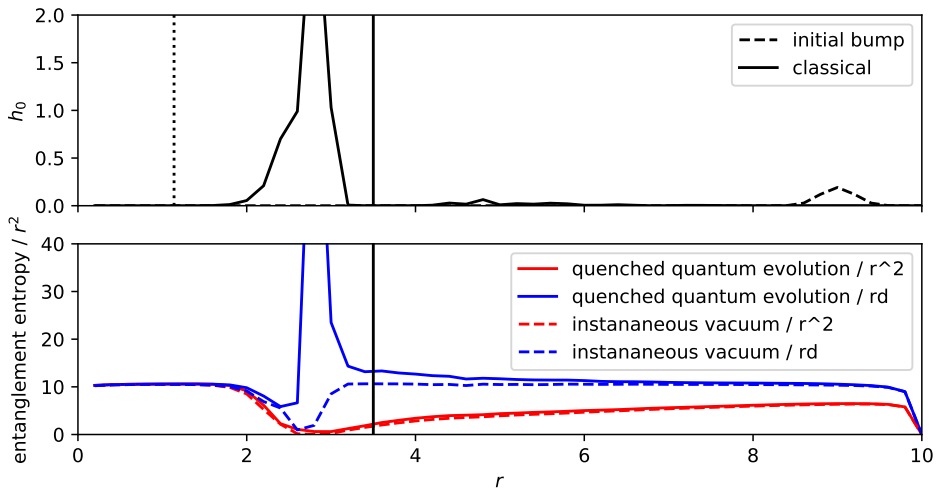
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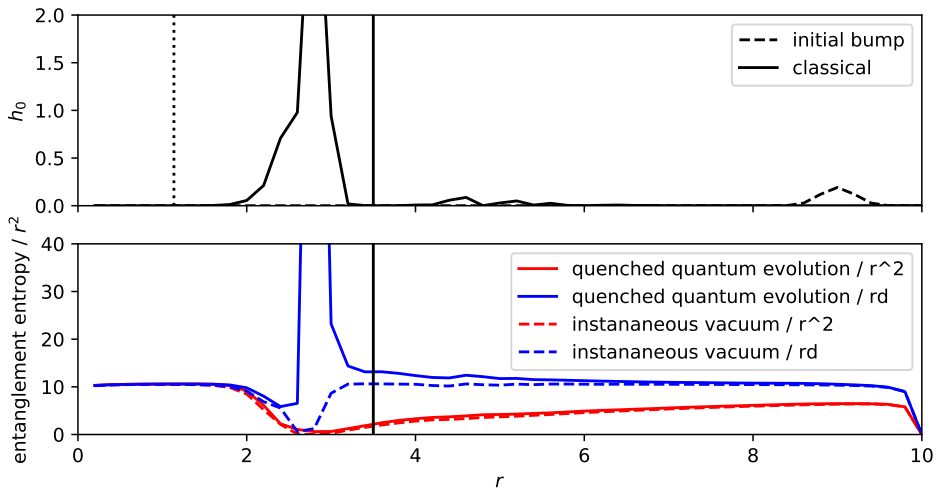
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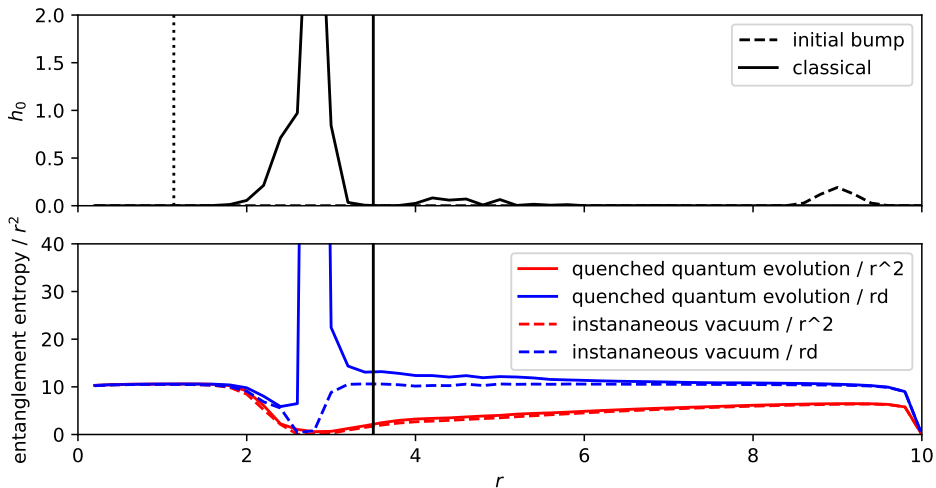
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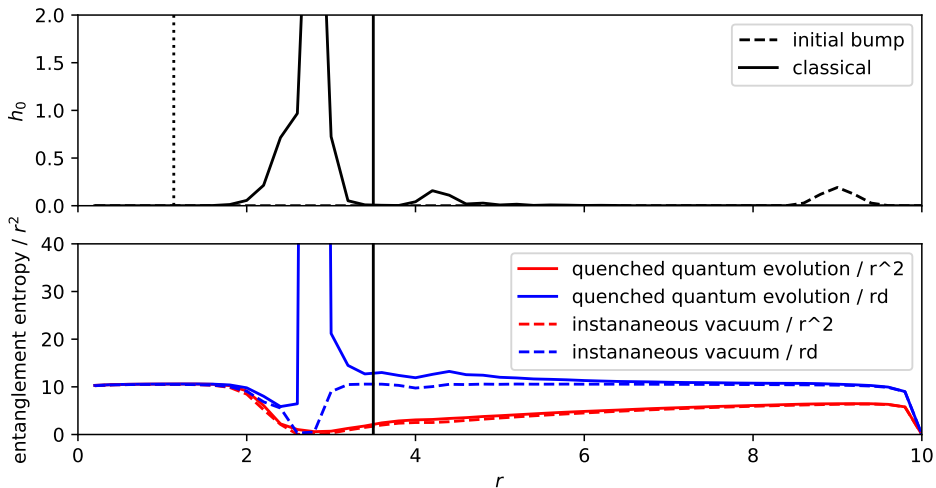
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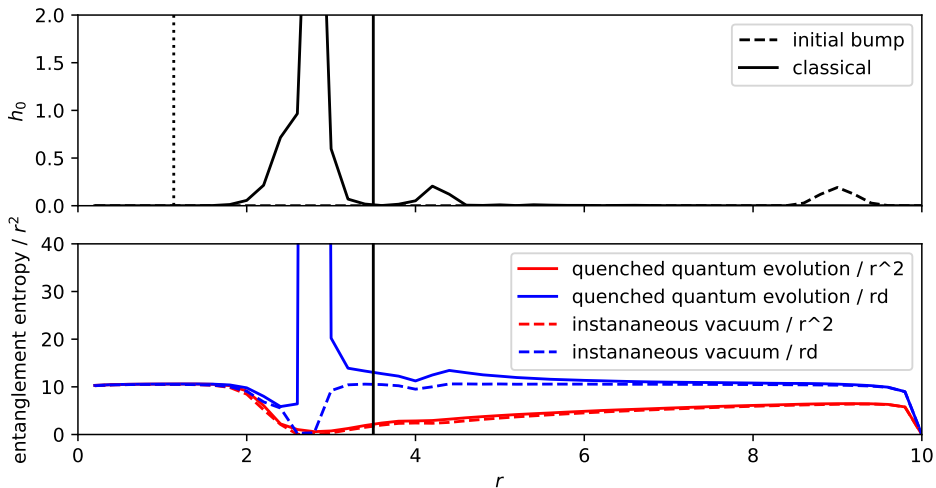
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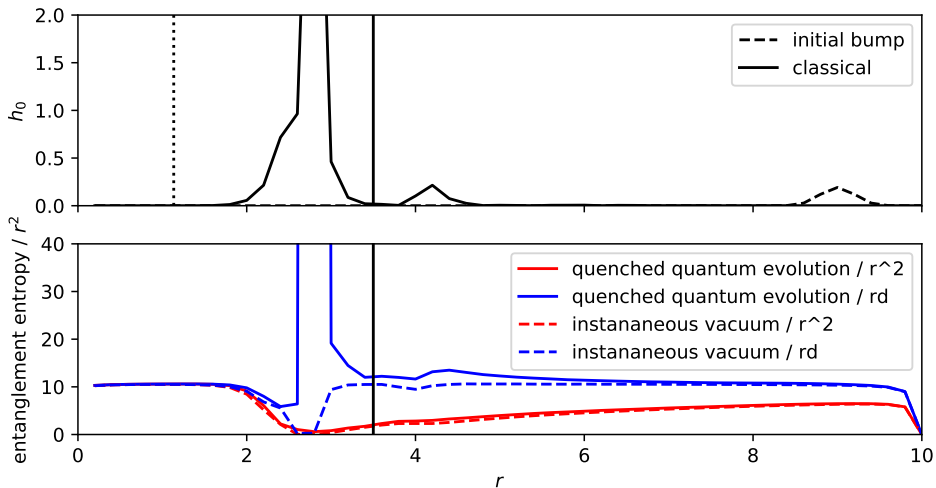
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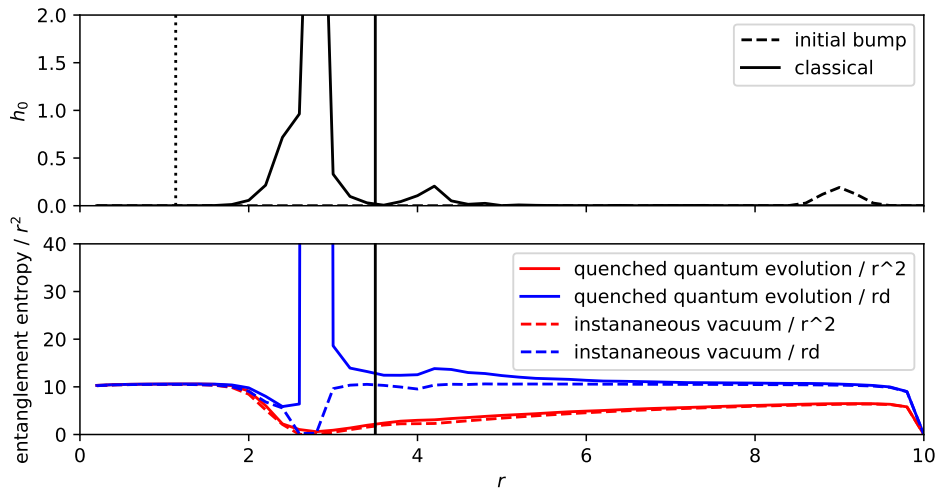
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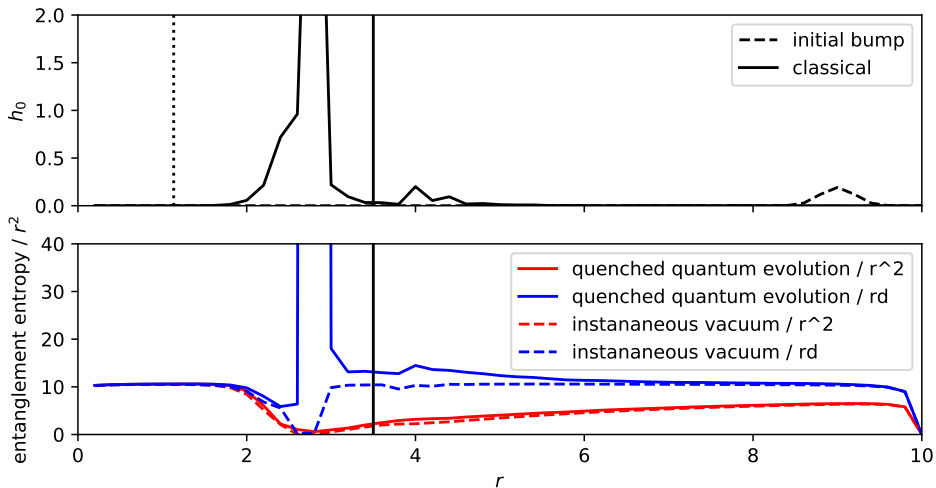
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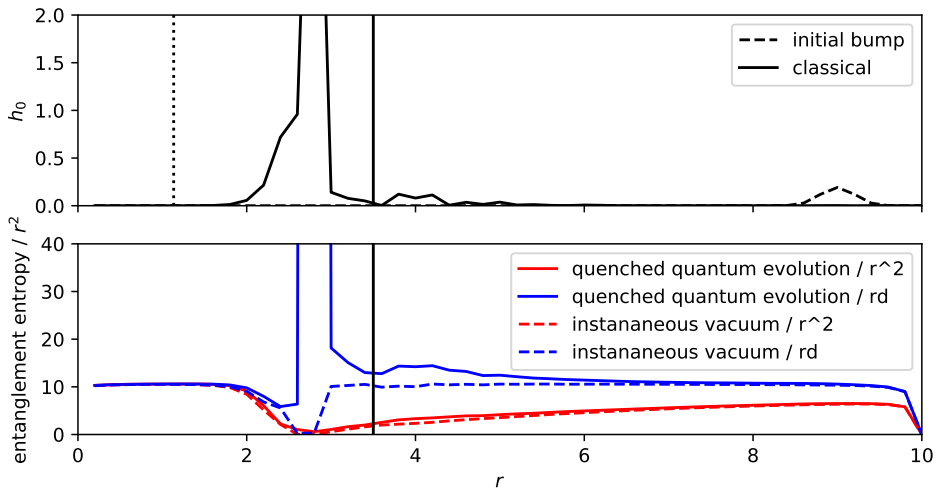
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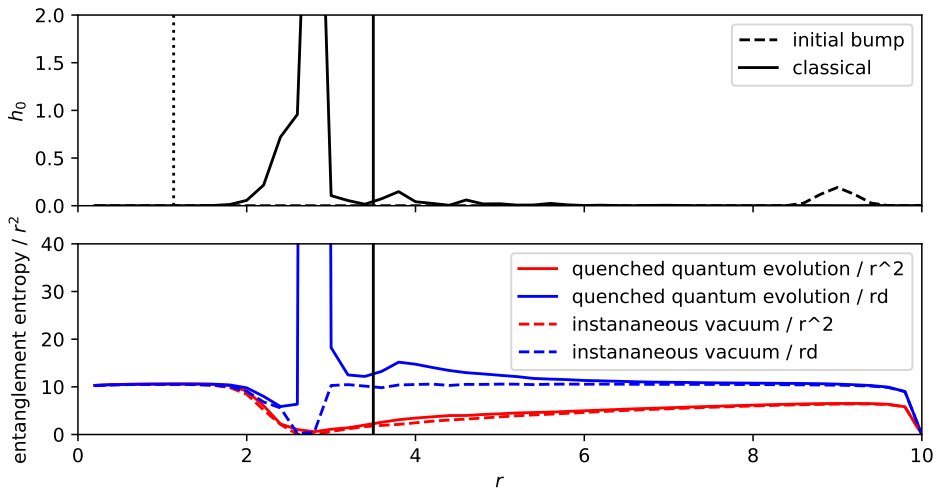
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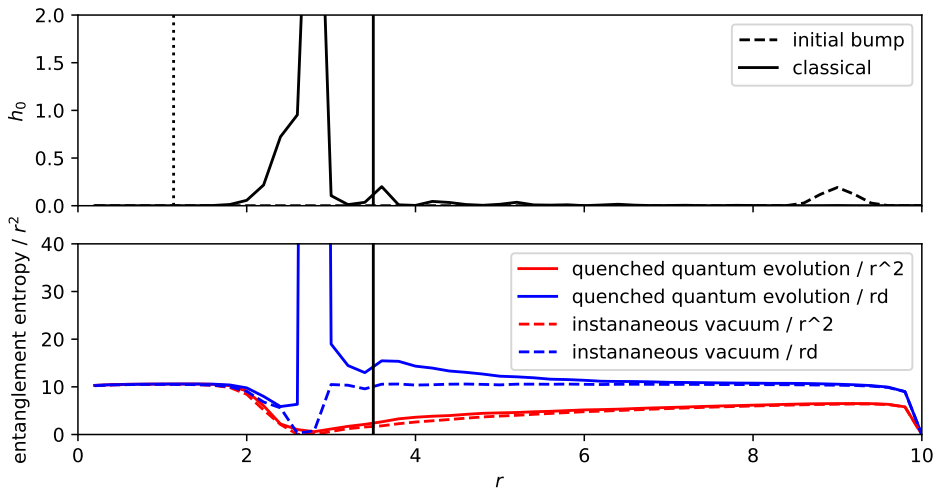
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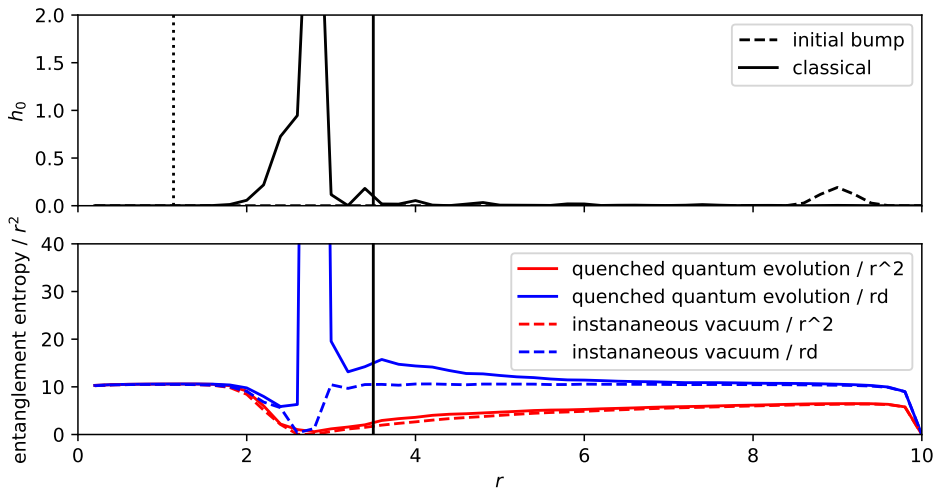
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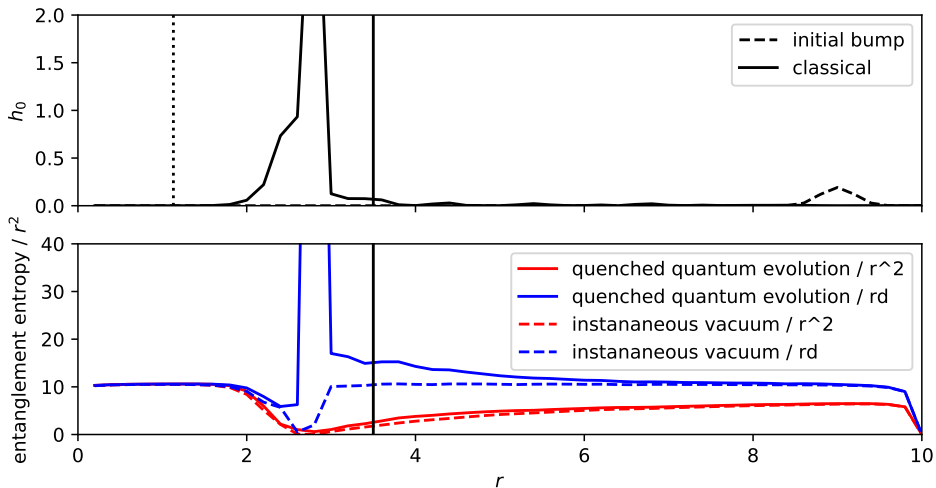
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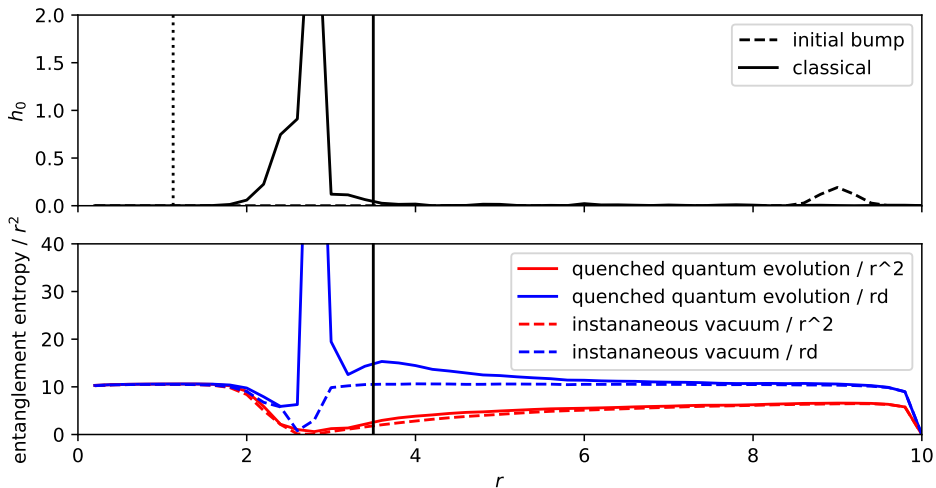
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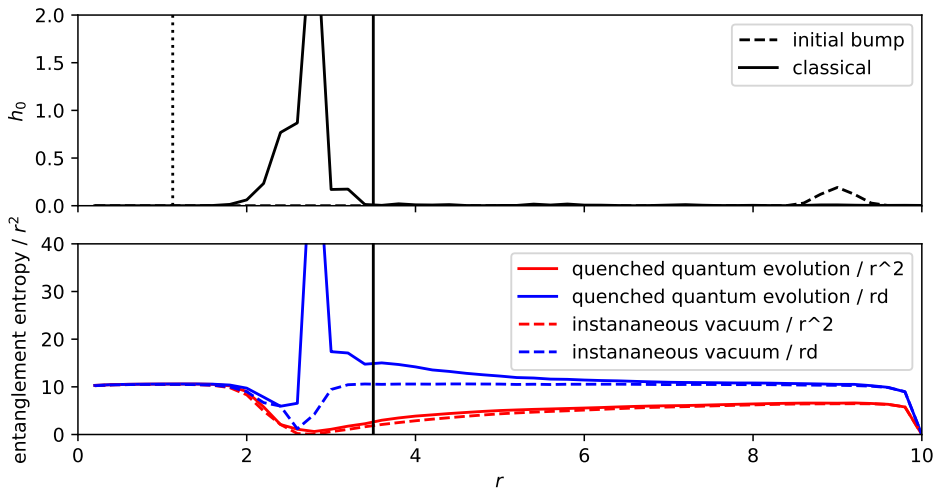
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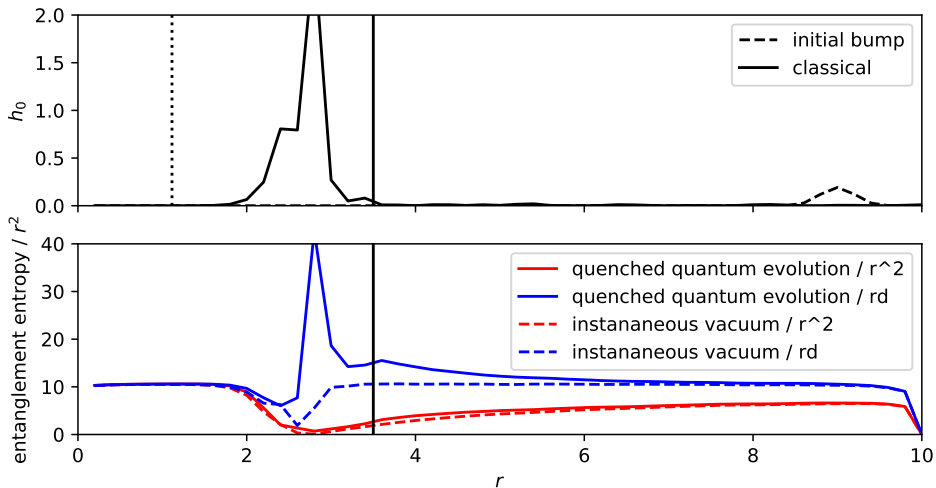
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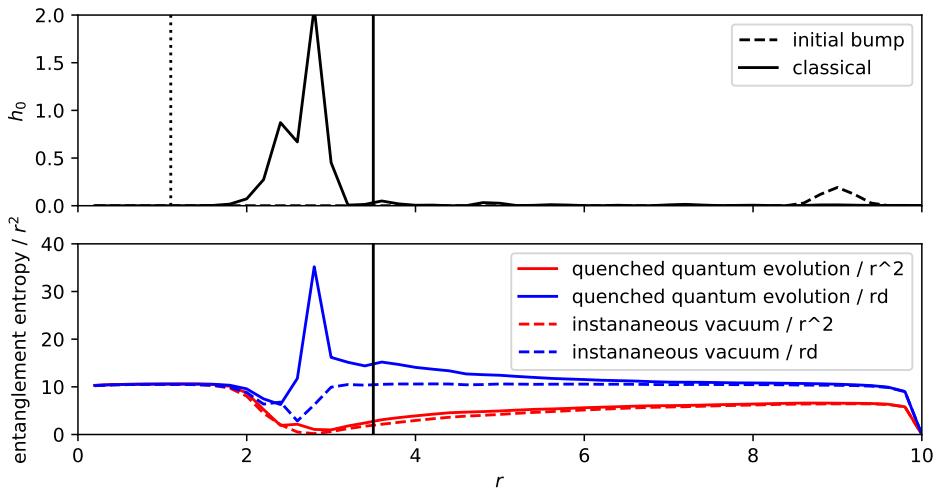
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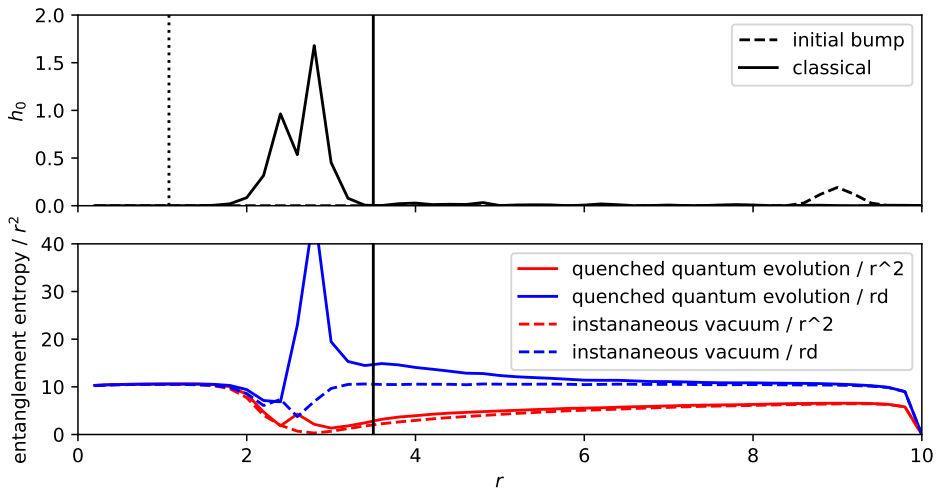
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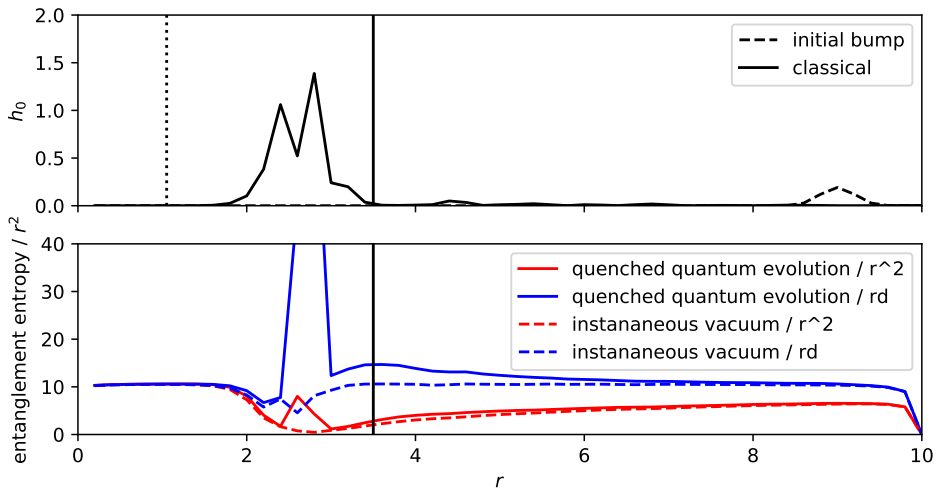
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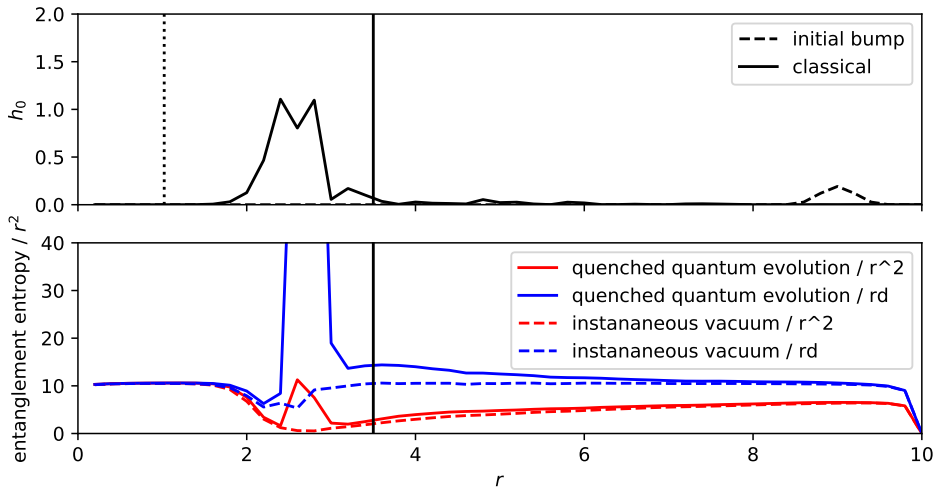
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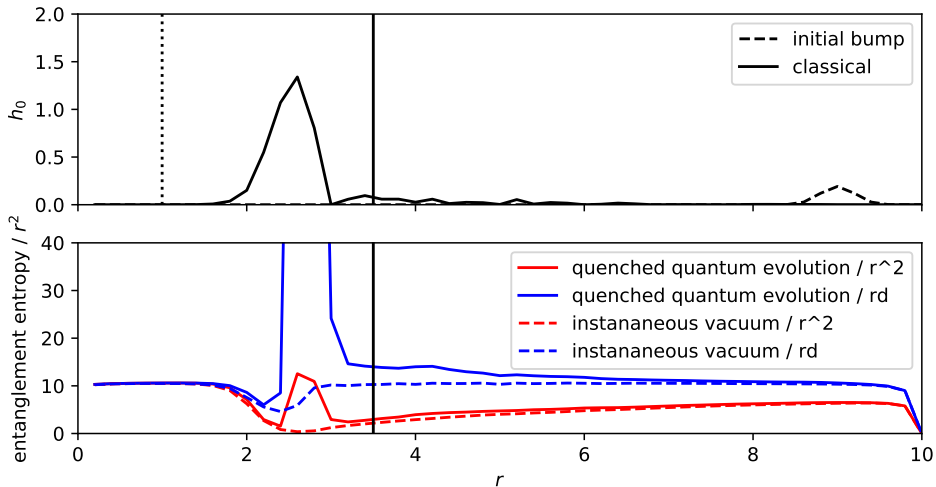
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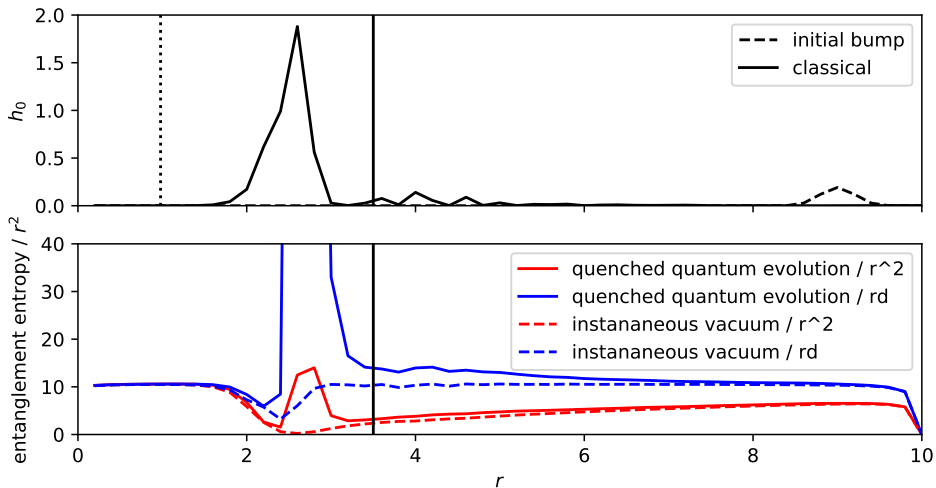
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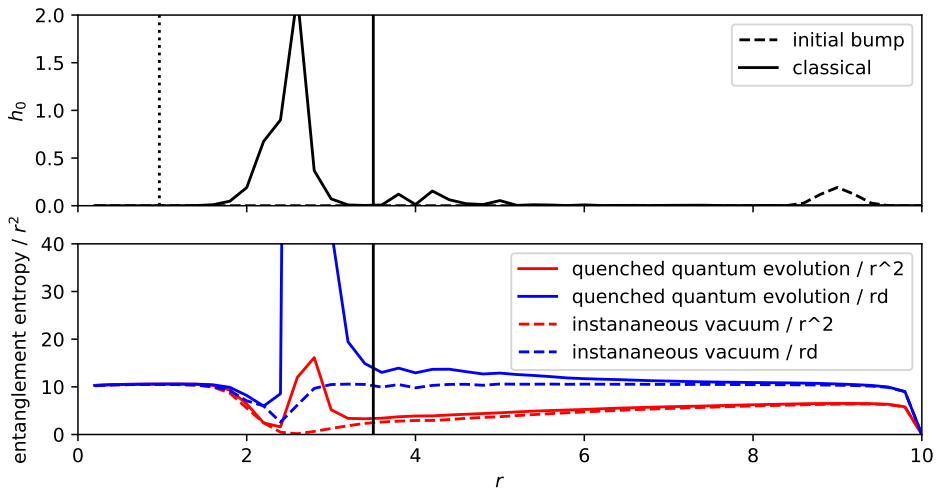
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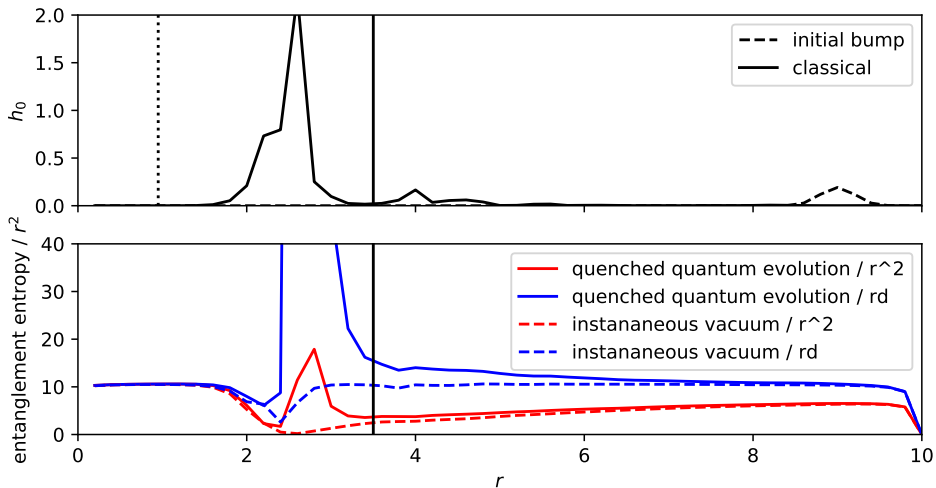
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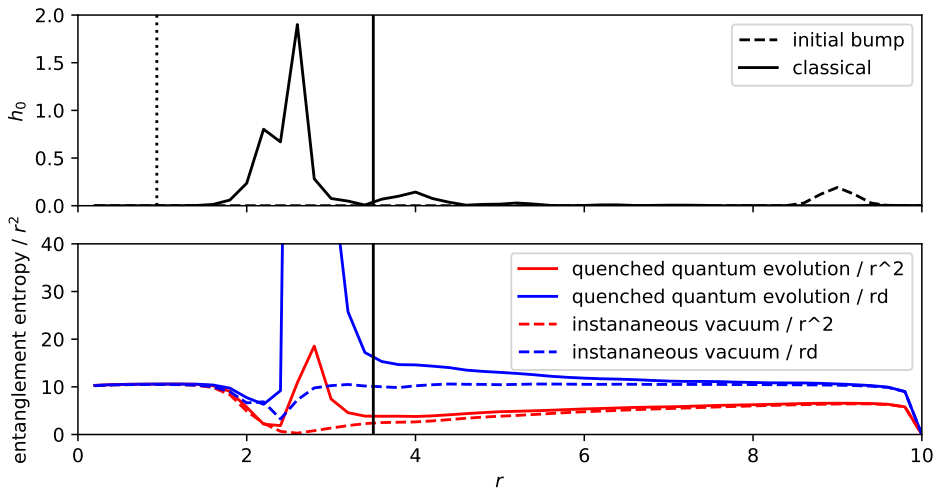
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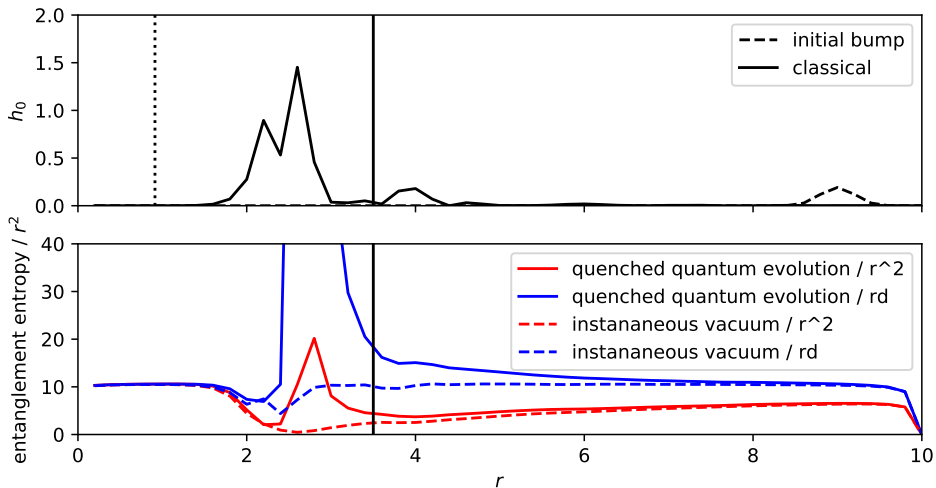
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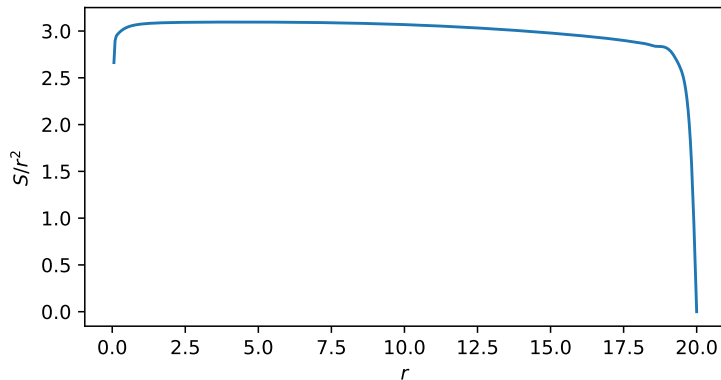


Entropy with Pauli-Villars regularization

Use PV regularization to define finite entropy.

Tune masses so that $S/r^2 \simeq \pi$.

preliminary

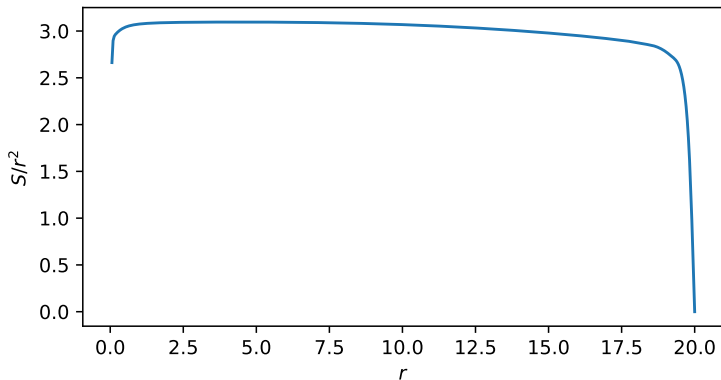


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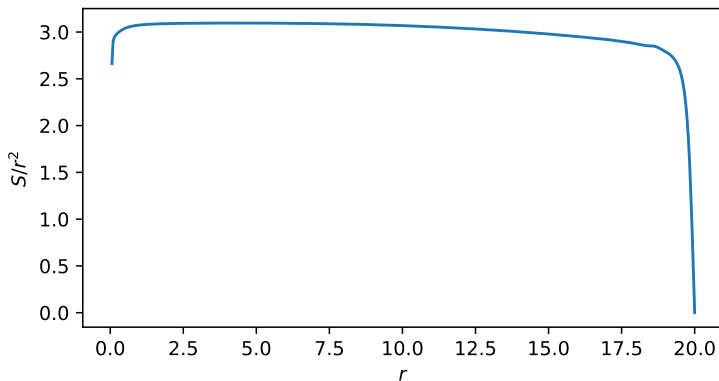


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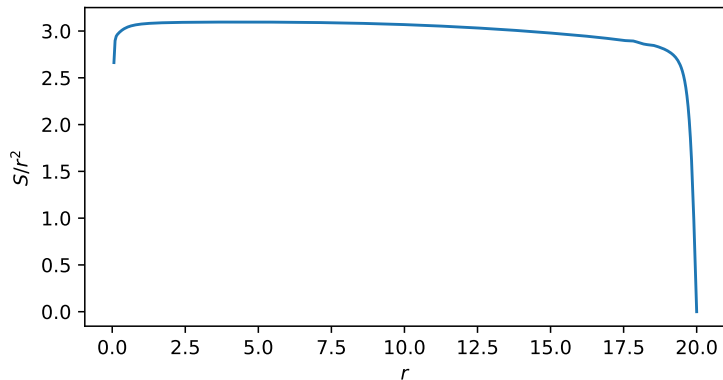


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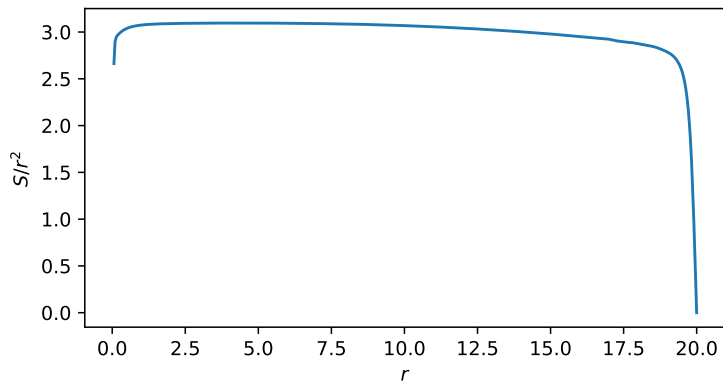


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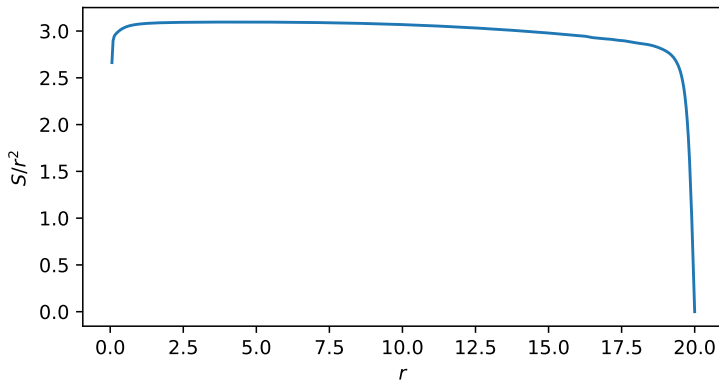


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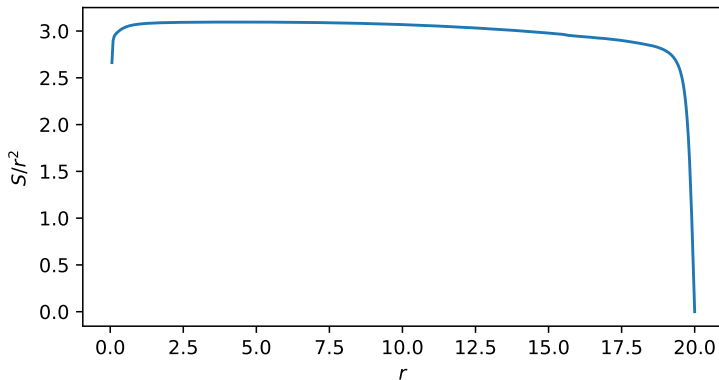


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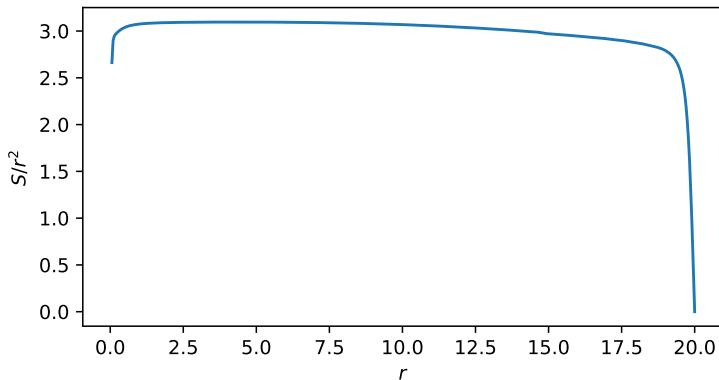


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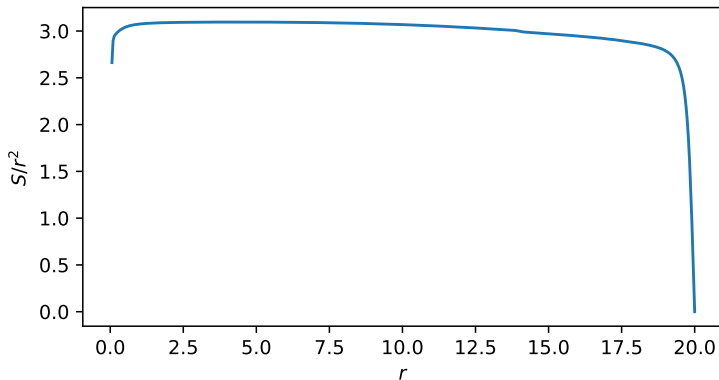


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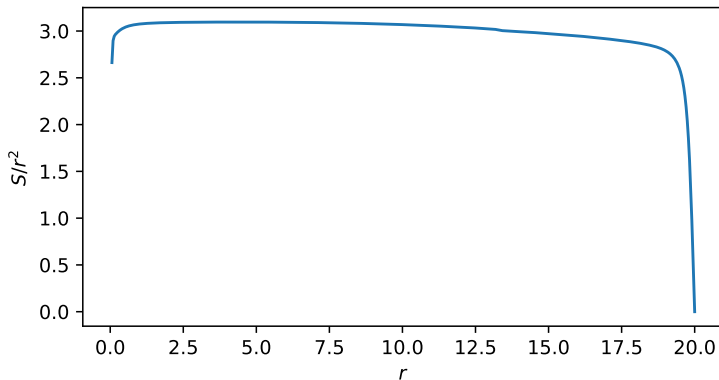


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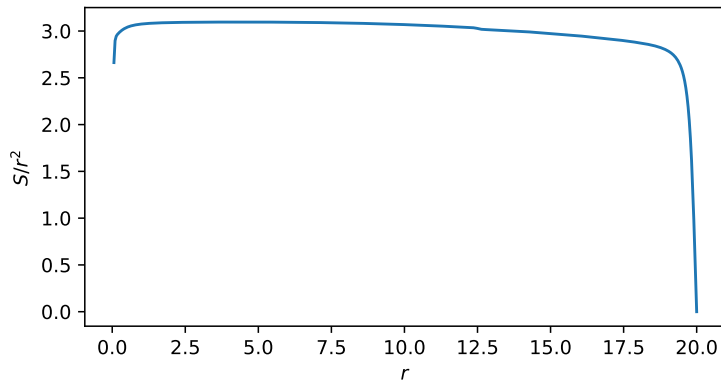


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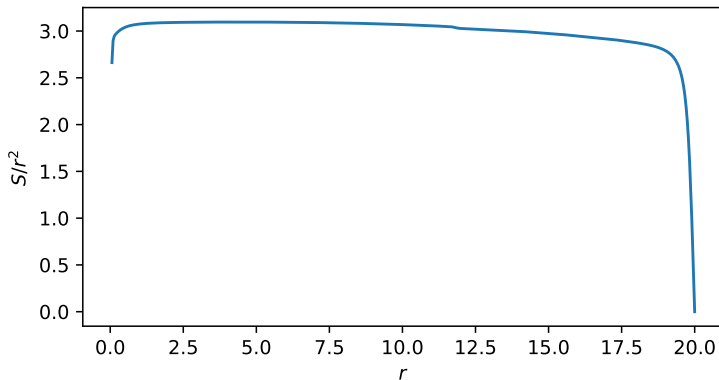


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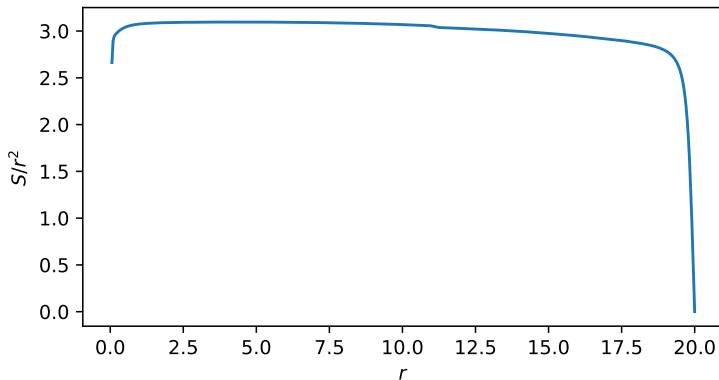


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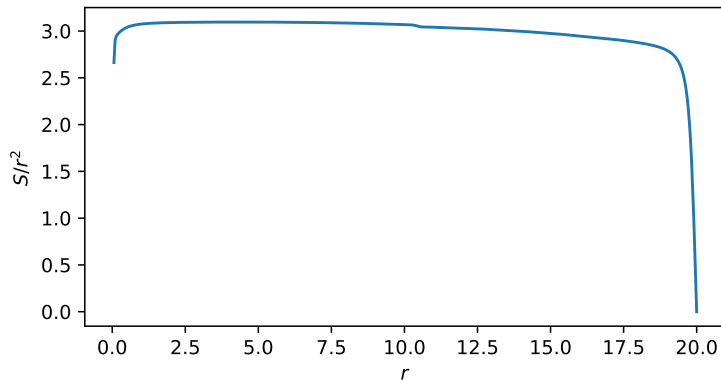


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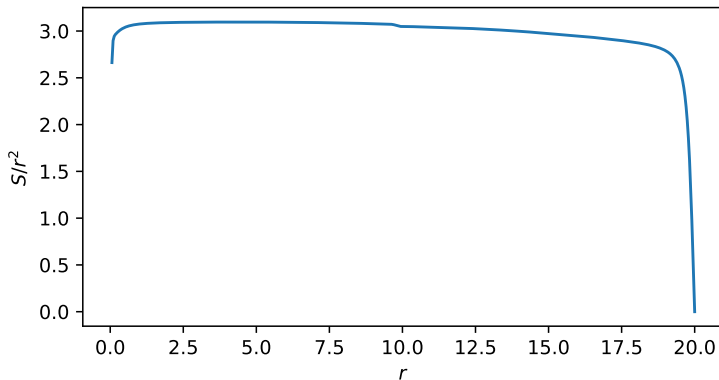


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preliminary



Temperature

Williamson transformations of covariance matrix gives a basis of harmonic oscillators, which are in thermal states.

$\sigma_i = 1 \rightarrow$ Oscillator in ground state

$\sigma_i > 0 \rightarrow$ Oscillator in thermal state

Inverse temperatures β_i can be defined via

$$\beta_i \omega_i = \log \left(\frac{\sigma_i + 1}{\sigma_i - 1} \right)$$

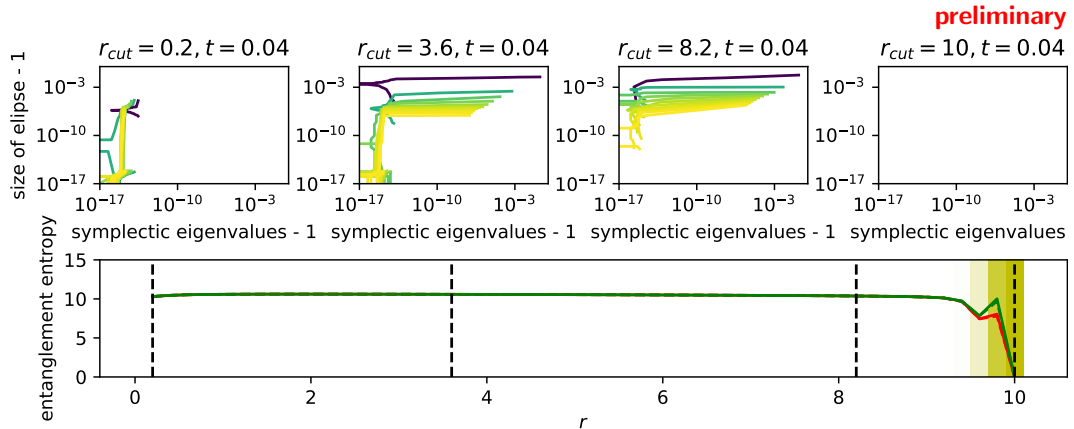
But what is ω_i ? Oscillator basis from Williamson form \neq Oscillators in the Hamiltonian.

Hamiltonian contains modes that mix inner and outer system, except if a horizon forms.

Compare with the $\sqrt{\text{surface}}$ of the ellipses defined by the covariance matrix projected to eigenmodes of the Hamiltonian.

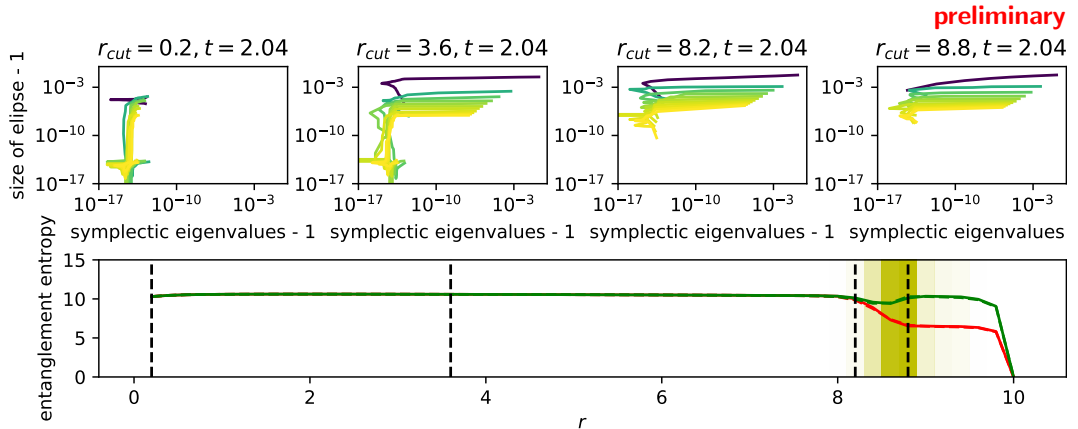
Should agree in the case of a thermal state.

Temperature



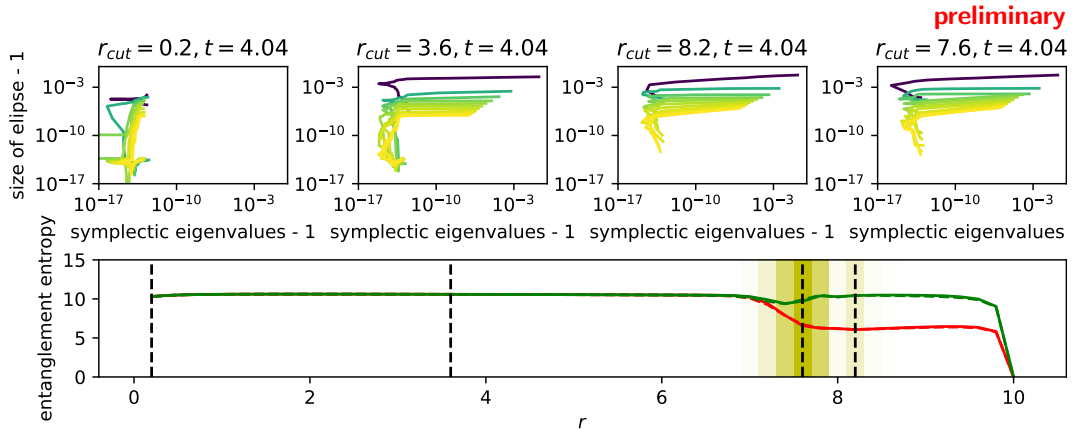
red: Entropy normalized with r^2 , green: Entropy normalized with rd

Temperature



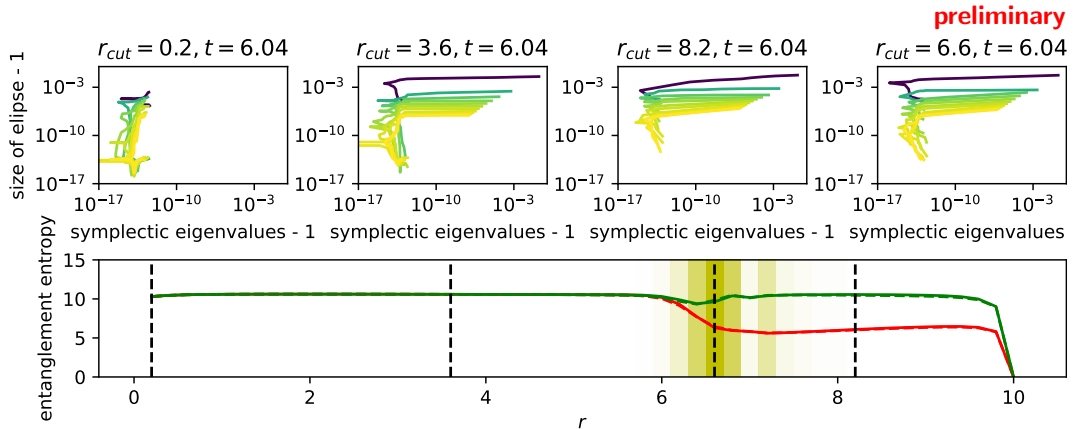
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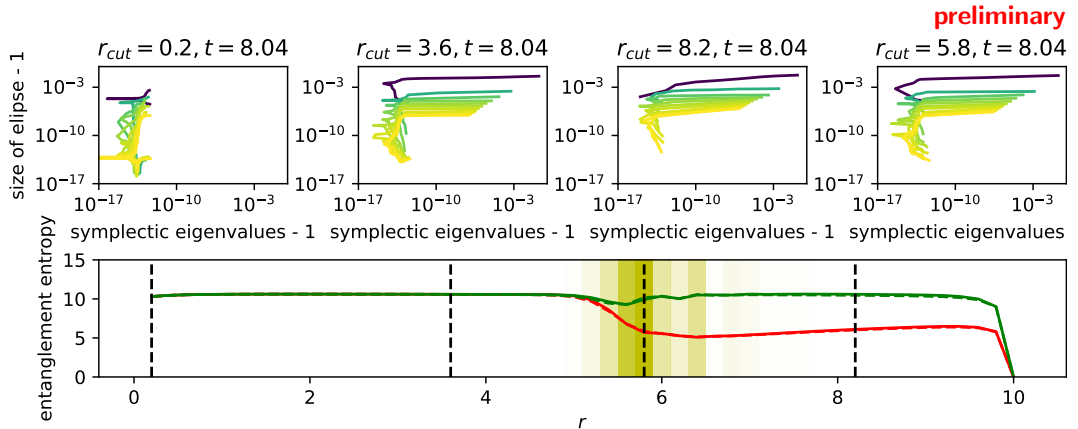
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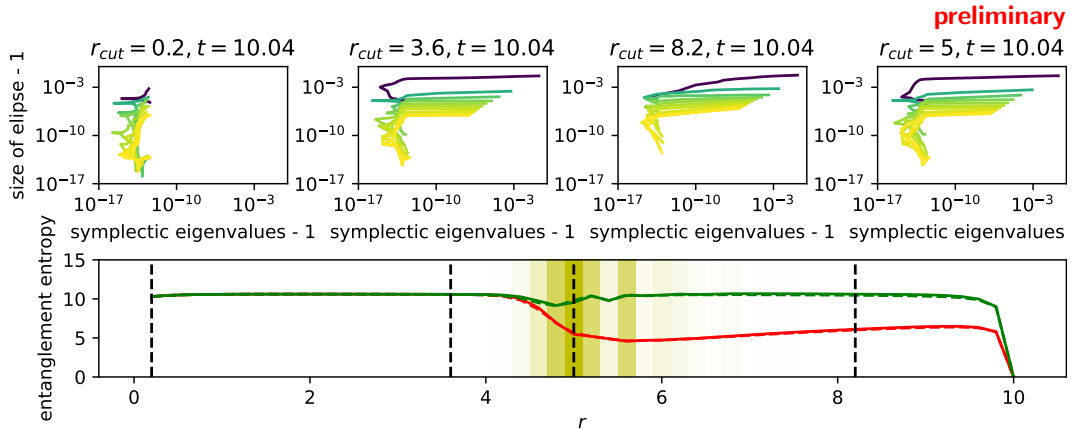
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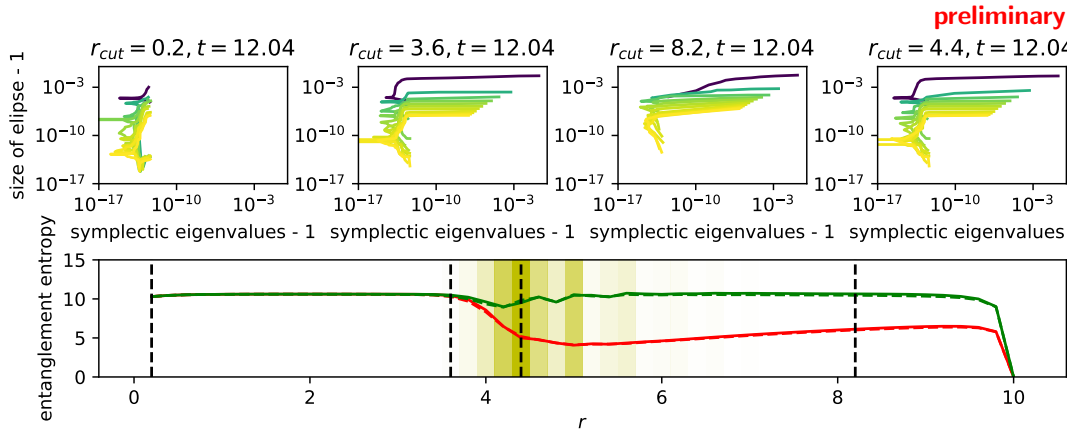
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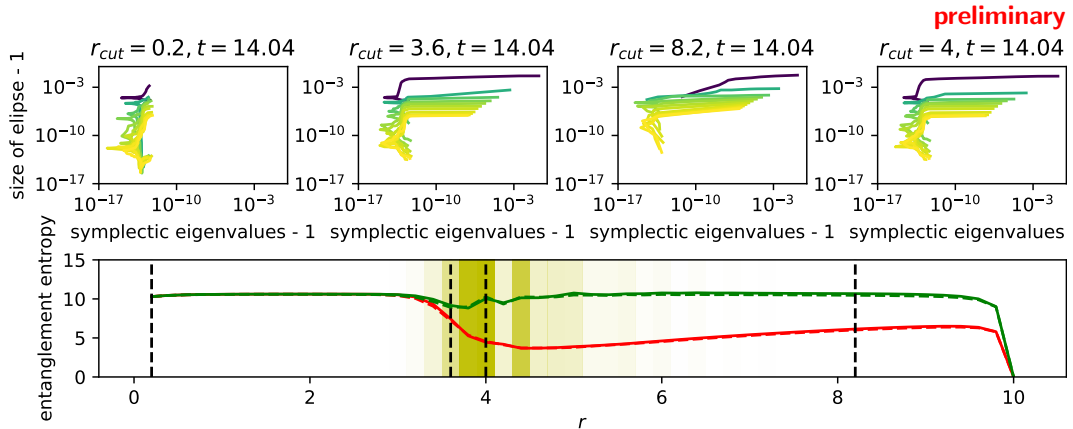
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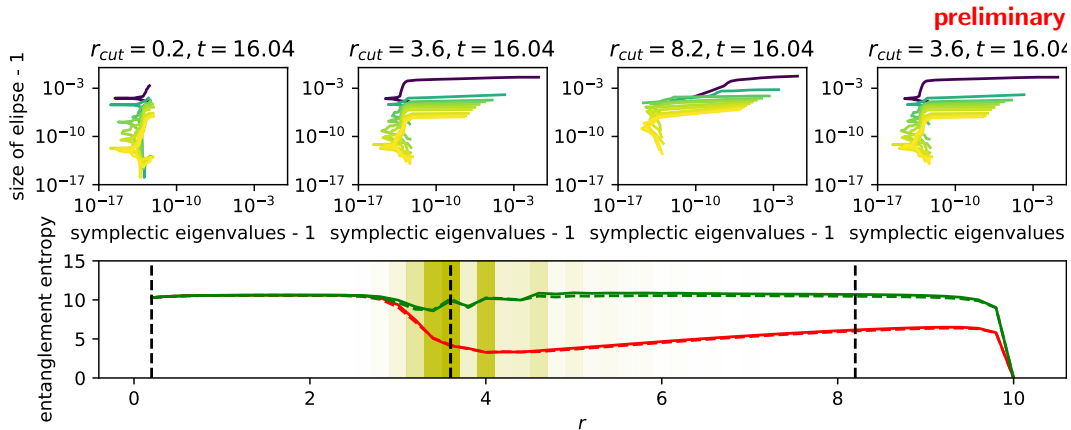
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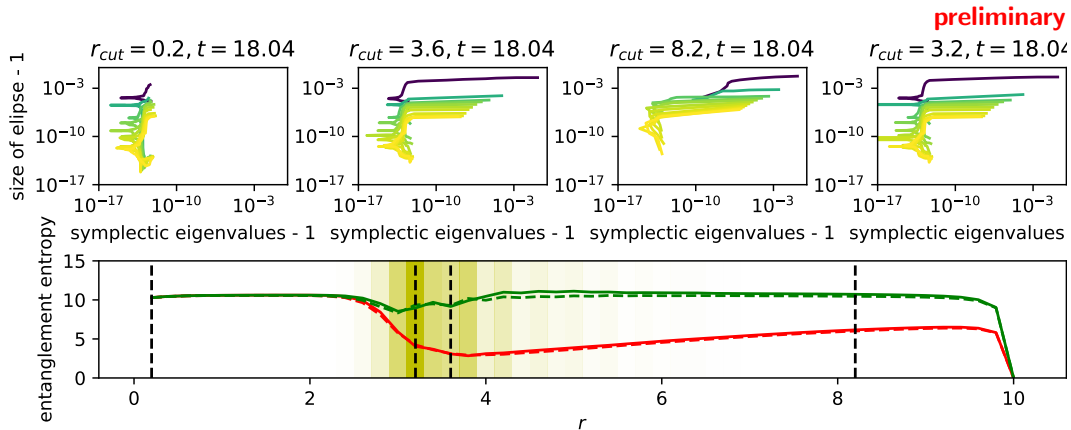
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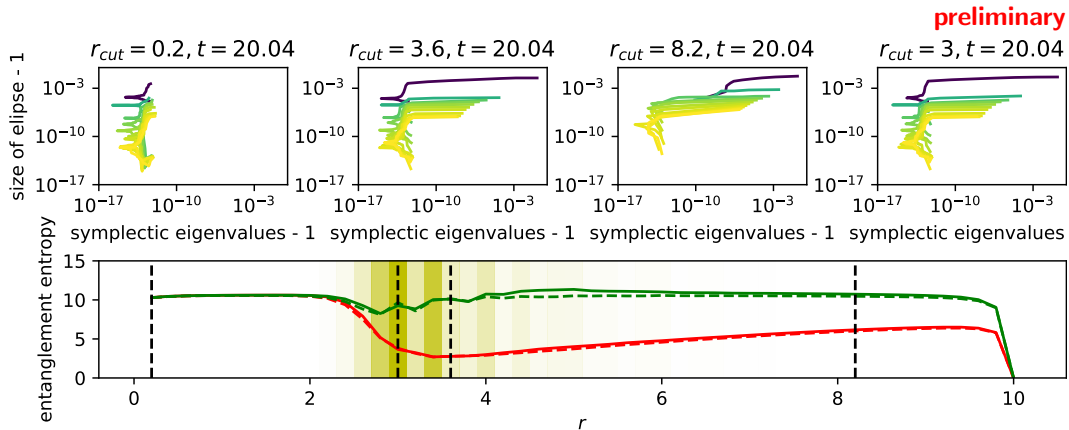
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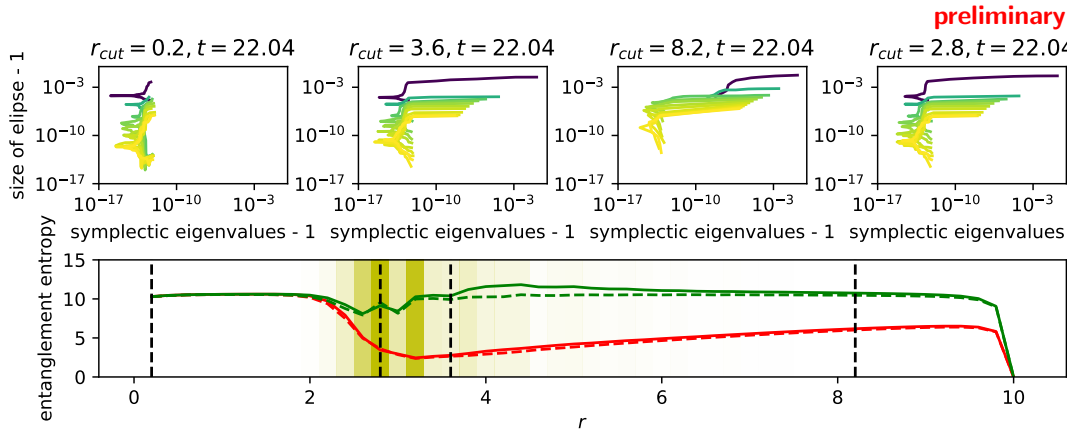
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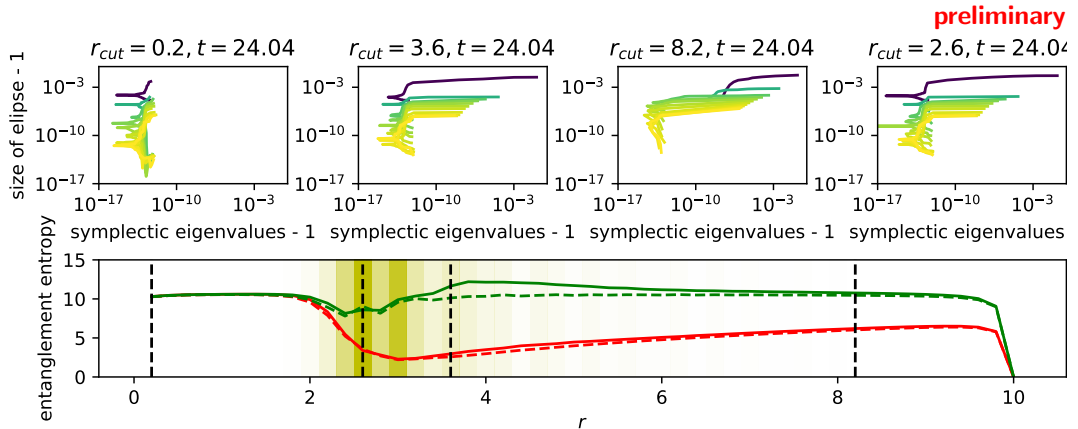
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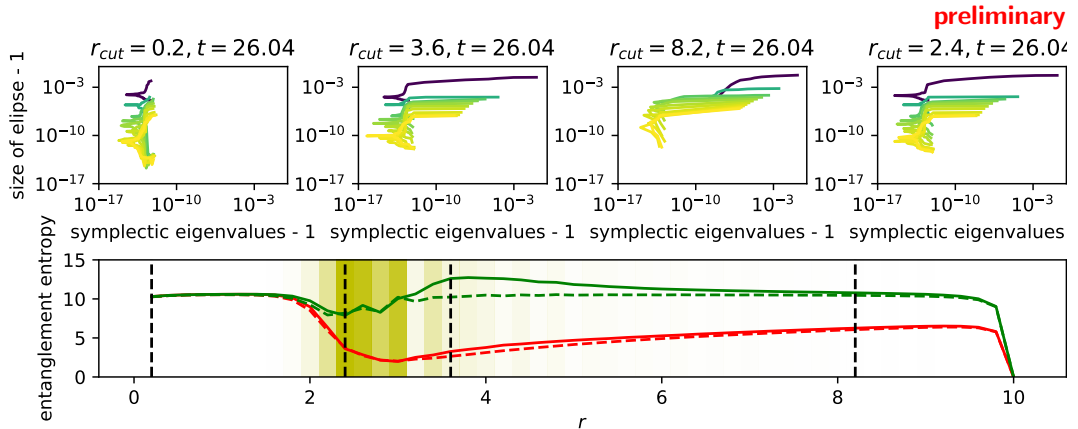
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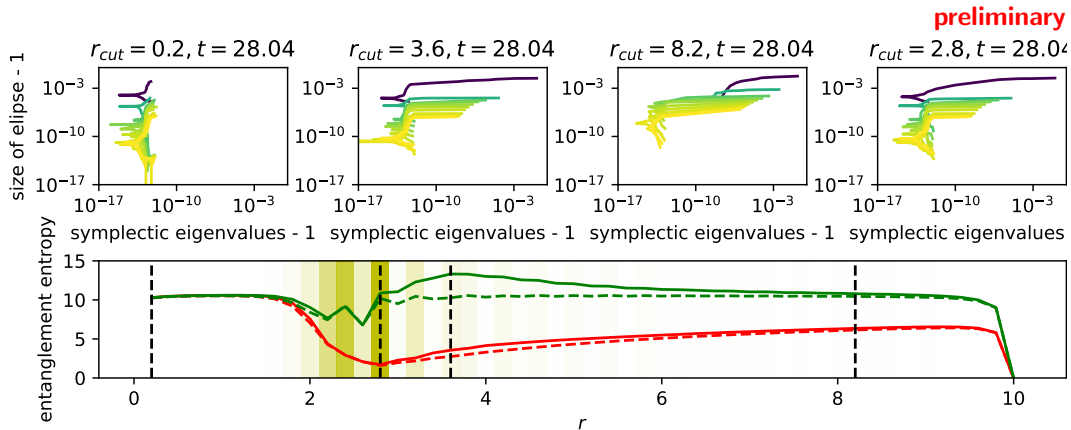
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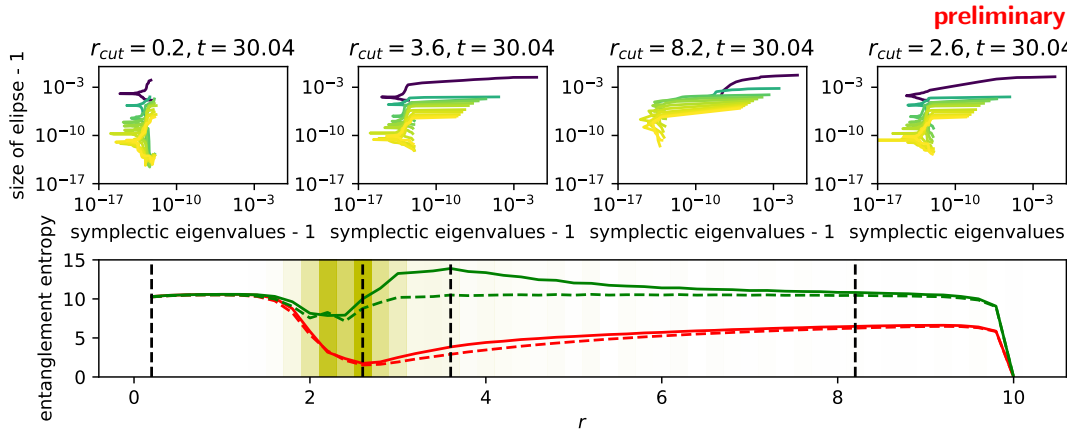
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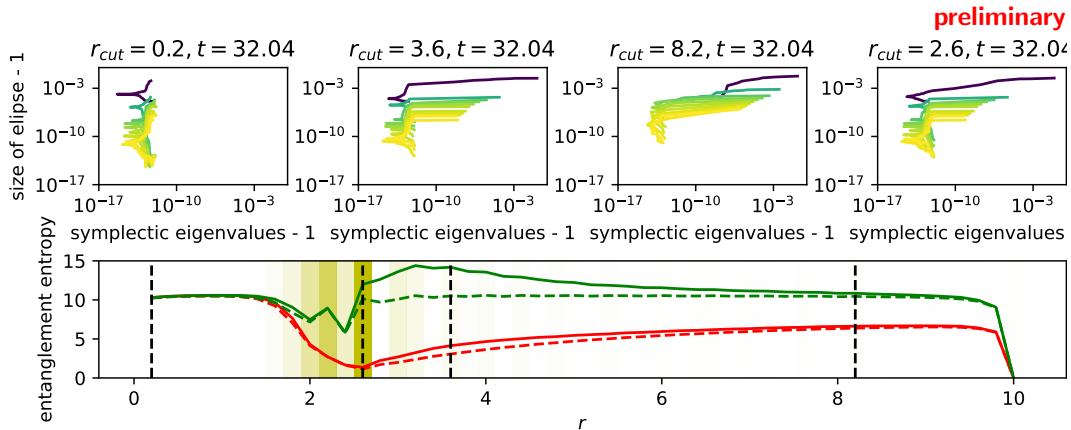
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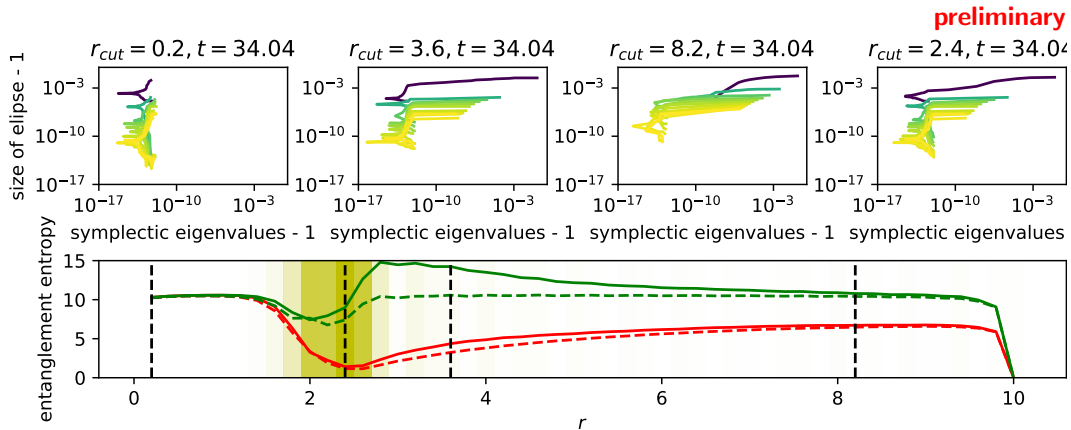
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Conclusion

- Simulation of the quenched gravitational collapse.
- Entropy is interesting observable in the quenched case.
- States are not purely thermal in the time range studied.
- Extraction of temperature challenging.

Outlook

- Systematic study of limits:
 - $dr \rightarrow 0$
 - $dt \rightarrow 0$ (seems harmless)
 - $I_{\max} \rightarrow 0$
- Better initial conditions (start at outermost point, slowly coupling in of the classical bump)
- Unquenching