Towards tensor renormalization group study of lattice QCD

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Tensor renormalization group (TRG)

Toward tensor renormalization group study of lattice QCD — *Atis Yosprakob*
14:45 - 15:05

Initial tensor construction and dependence for tensor renormalization group — *Katsumasa Nakayama*
15:05 - 15:25

Tensor renormalization group study of (1+1)-dimensional O(3) nonlinear sigma model w/ and w/o finite chemical potential
Yoshinobu Kuramashi

Grassmann Tensor Renormalization Group for two-flavor massive Schwinger model with a theta term — *Hayato Kanno*
16:35 - 16:55

Phase structure analysis of 2d CP(1) model with θ term by tensor network renormalization — *Hayato Aizawa*
16:55 - 17:15

Grassmann bond-weighted tensor renormalization group approach to 1+1D two-color QCD with staggered fermions at fi…
Ho Pai Kwok

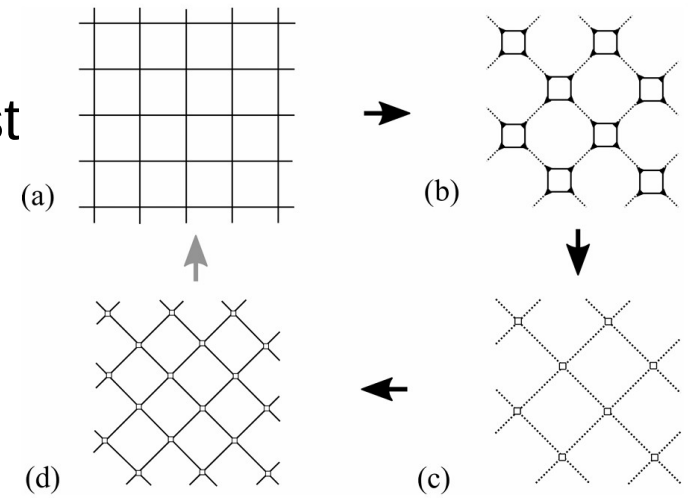
Entanglement entropy by tensor renormalization group approach — *Gota Tanaka*
14:35 - 14:55

Tensor renormalization group study of (1+1)-dimensional U(1) gauge-Higgs model at θ=π with Lüscher's admissibility c…
Shinichiro Akiyama

Spectroscopy by Tensor Renormalization Group Method — *Mrs Fathiyya Izzatun Az Zahra*
15:15 - 15:35

Tensor renormalization group (TRG)

- An alternative to Monte Carlo methods based on coarse graining
- No sampling = No sign problem
- Can access large volumes with log cost
- Can handle fermion/Grassmann numbers directly; Grassmann TRG



[Figures from Okunishi-Nishimo-Ueda; 2022]

Challenges

- TRG can be challenging when the local Hilbert space is large
- By that, I meant QCD
  - Multiple fermion flavors ==> dimension ~ exp(N_f)
  - Non-abelian gauge symmetry ==> Redundancy in the TN

I will talk about my works on these two directions.

Part I: Multi-layer construction for multi-flavor gauge theory

Based on [JHEP11(2023)187], with **Jun Nishimura** (KEK) and **Kouichi Okunishi** (Niigata U)

Multi-flavor gauge theory

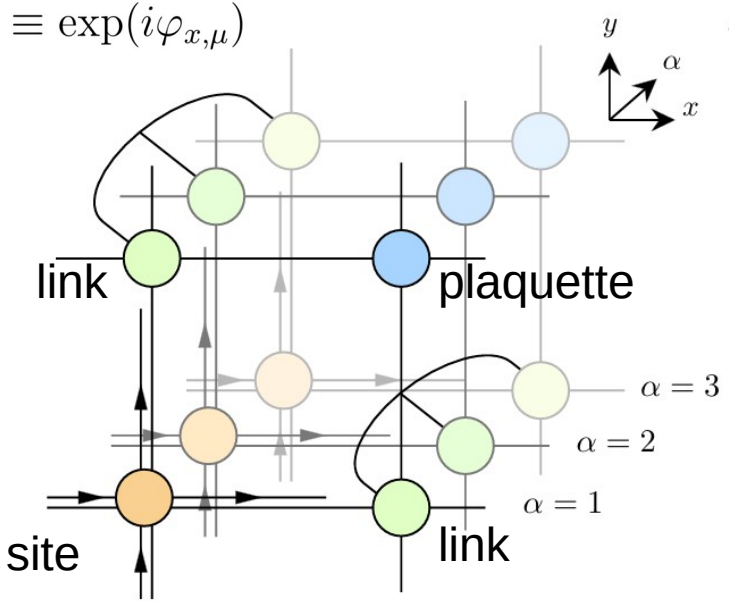
Key idea: Offload the information into several layers

$$Z = \int D\varphi \prod_{\alpha=1}^{N_\mathrm{f}} \left(D\varphi^{(\alpha)} D\psi^{(\alpha)} D\bar{\psi}^{(\alpha)} \right) \delta(\varphi^{(\alpha)} - \varphi) e^{-\sum_\alpha S^{(\alpha)}}$$

local action for each flavor

$$S^{(\alpha)} = \frac{1}{N_\mathrm{f}} S_\mathrm{gauge}[\varphi^{(\alpha)}] + \sum_{x \in \Lambda_2} \bar{\psi}_x^{(\alpha)} \not{D}^{(\alpha)} \psi_x^{(\alpha)}$$

$$U_{x,\mu} = \exp(iaA_{x,\mu}) \equiv \exp(i\varphi_{x,\mu})$$



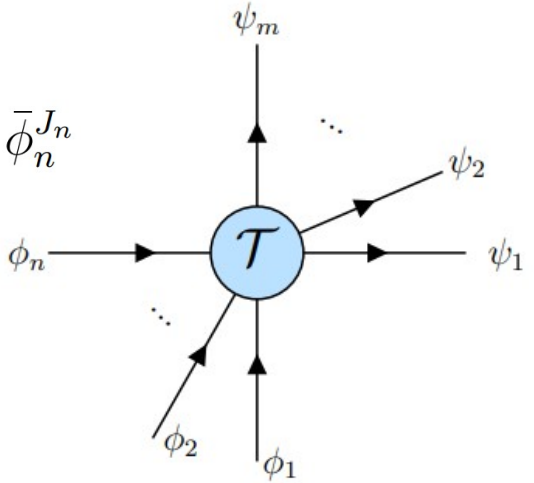
Fermions are handled directly with Grassmann tensors.

Quick intro: Grassmann tensors

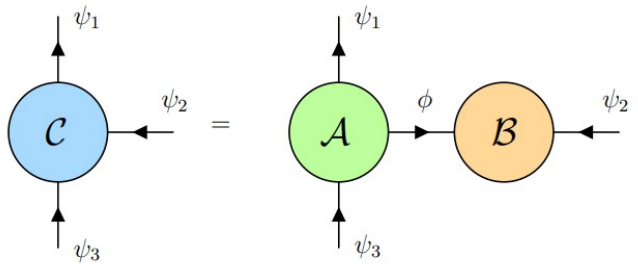
$$\mathcal{T}_{\psi_1 \cdots \psi_m \bar{\phi}_1 \cdots \bar{\phi}_n} = \sum_{I_1,\cdots,I_m,J_1,\cdots,J_n} T_{I_1 \cdots I_m J_1 \cdots J_n} \psi_1^{I_1} \cdots \psi_m^{I_m} \bar{\phi}_1^{J_1} \cdots \bar{\phi}_n^{J_n}$$

Coefficient tensor

Multi-component Grassmann numbers



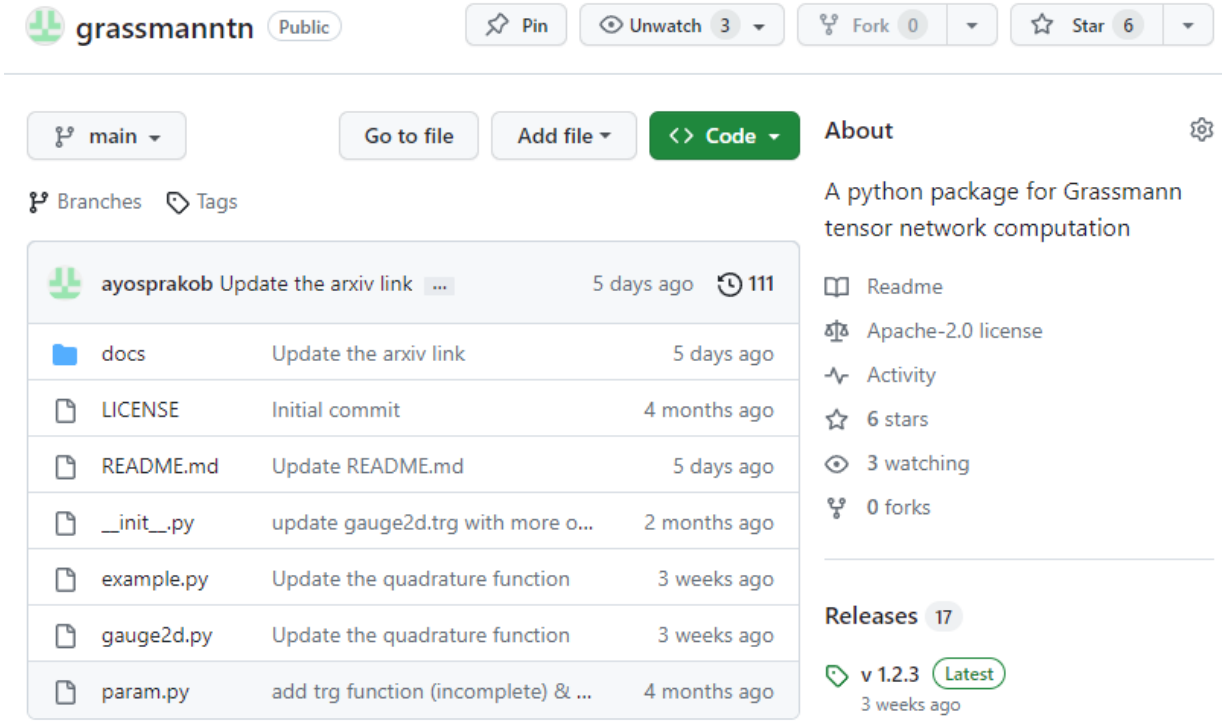
Grassmann tensor contraction



$$C_{IJK} = \sum A_{ILK} B_{JL} s_{JKL}$$

$$s_{JKL} = \sigma_L \times (-)^{p(L)(p(J)+p(K))+p(J)p(K)}$$

GrassmannTN:
a python package for Grassmann TRG/DMRG



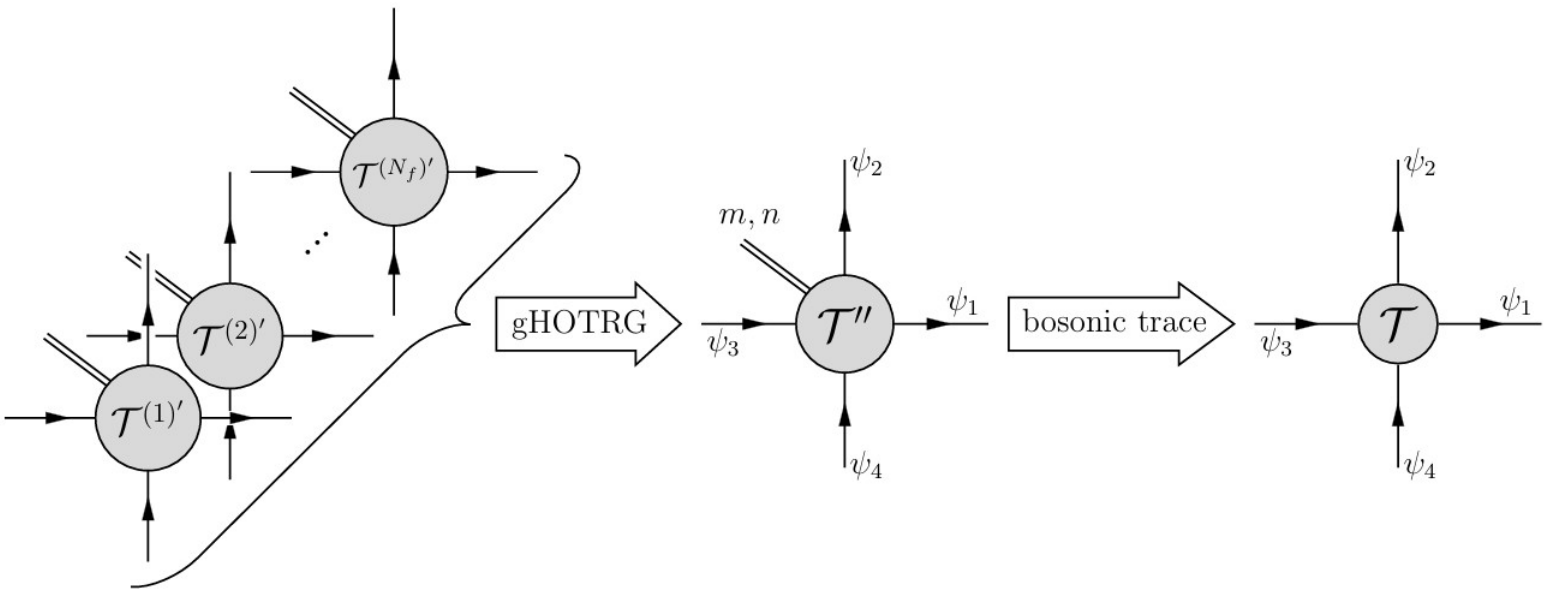
Features:
Grassmann contractions,
Tensor reshapes,
SVD/EigVD,
dense/sparse conversions,
Grassmann arithmatic,
Berezin integrals, etc.

complete tutorial for
1+1D Schwinger model (TRG)

https://github.com/ayosprakob/grassmanntn

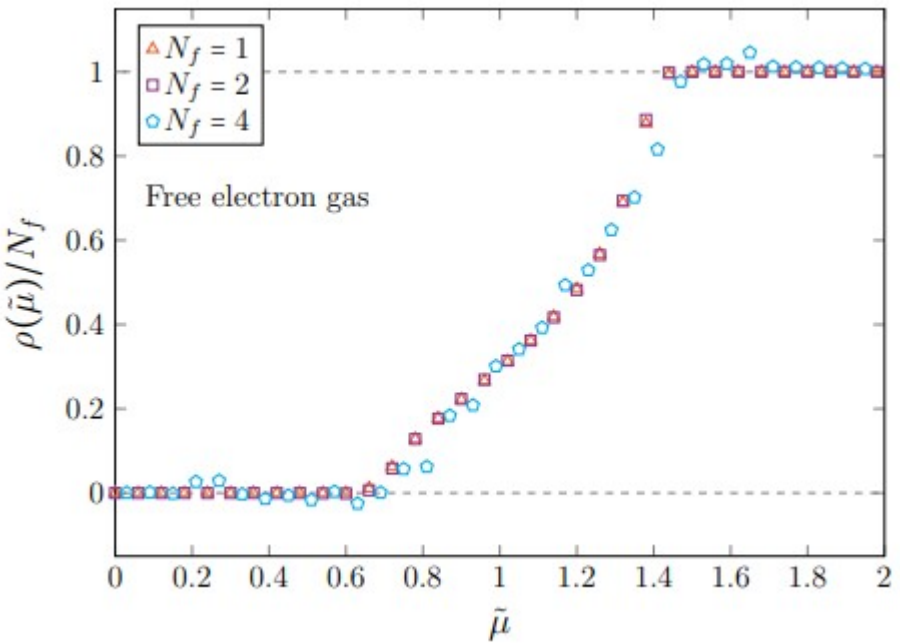
Multi-flavor gauge theory

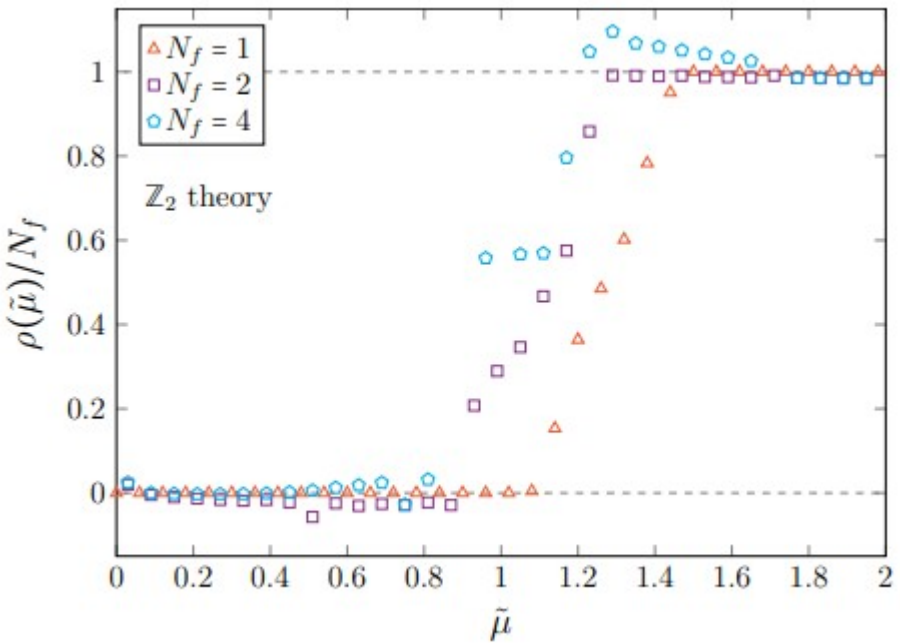
Multiple flavors are then combined via a TRG algorithm like an extra dimension



Finite density and Silver Blaze

Silver Blaze is reproduced up to $N_f = 4$





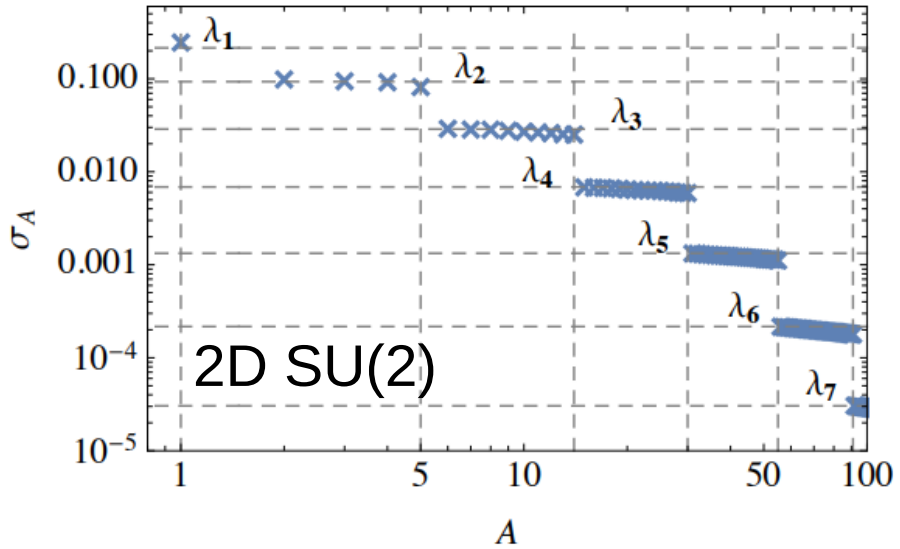
This is typically difficult in Monte Carlo due to the sign problem!

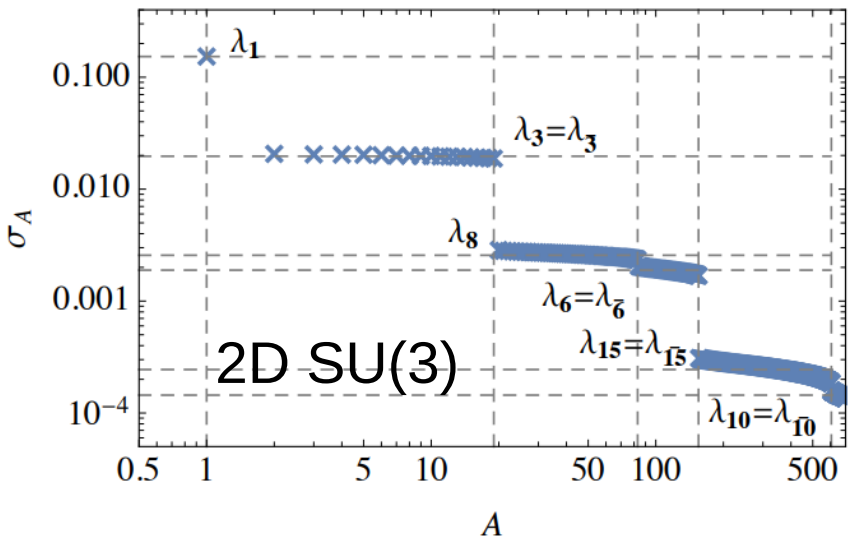
Part II:
Armillary sphere
Non-abelian gauge theory in higher dimensions

Based on [arXiv:2311.02541] (Formulation)
and [arXiv:2406.16763] (Numerical) with **Kouichi Okunishi** (Niigata U)

Why is non-abelian tensor network difficult?

Internal symmetry (from matrix indices) makes the tensor badly degenerated.





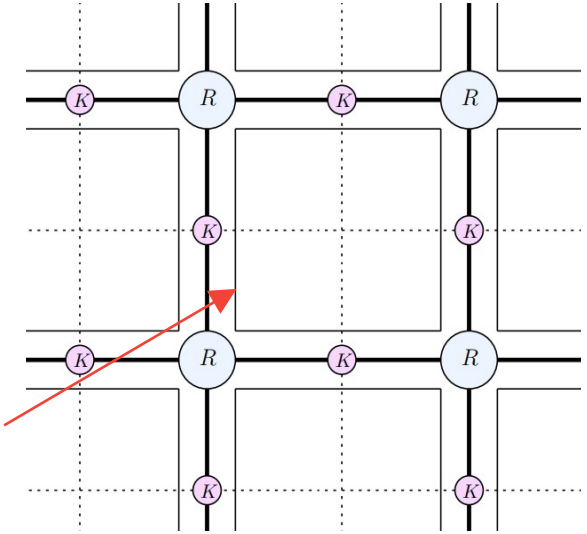
Figures from [Fukuma-Matsumoto; 2021]

Character expansion

- Lesson from 1+1D: the (matrix) index loops can be traced out if we use character expansion

[Hirasawa, Matsumoto, Nishimura, **A.Y.**; 2021]

- Degeneracy is completely eliminated

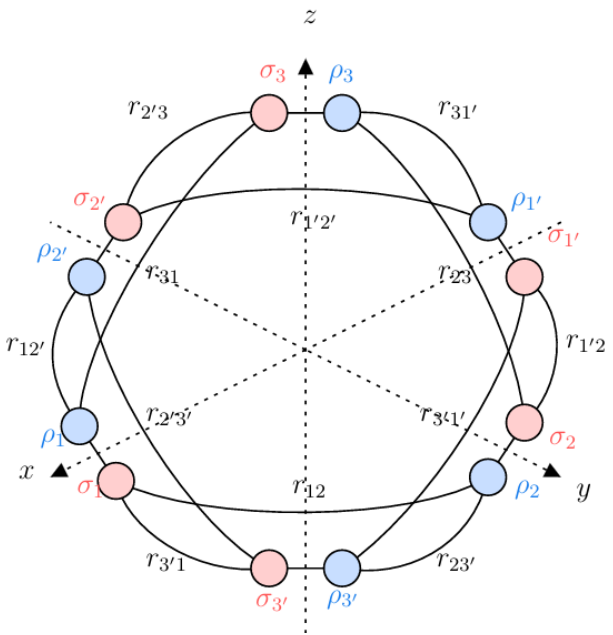


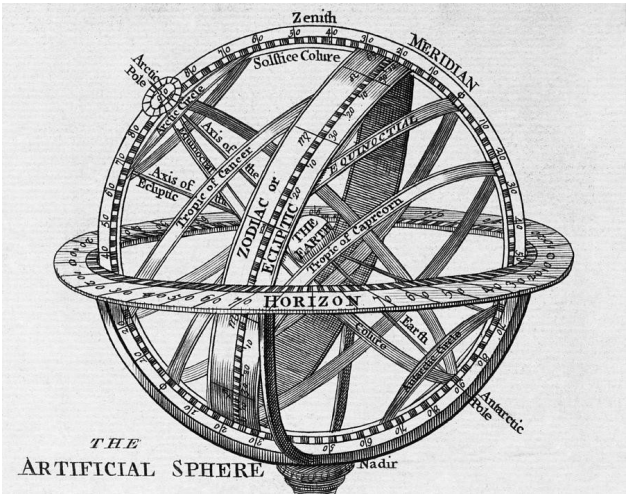
the source of singular value degeneracy

Can we do the same thing for any dimension?

The armillary sphere

Yes! There is a similar closed network in any dimension
Which we call the armillary sphere



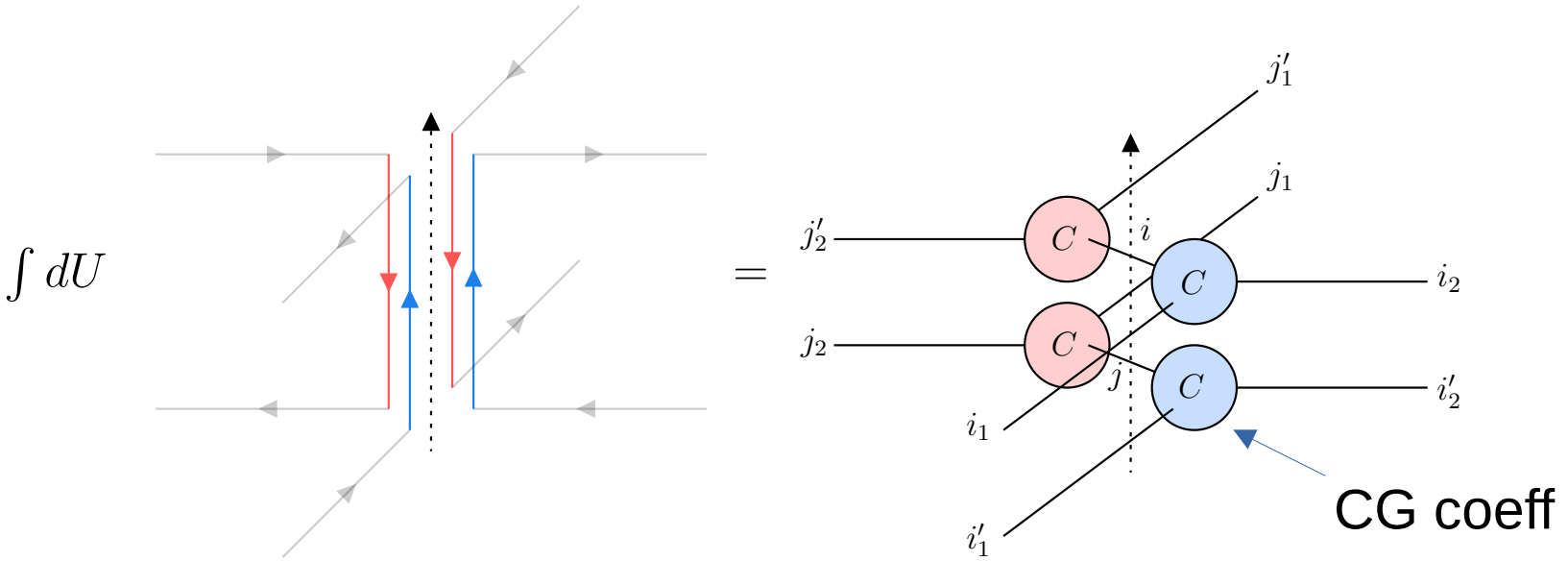


armillary sphere
= intersecting circles

This was first noticed by [Oeckl & Pfeiffer;2001] in the context of the spin foam model.

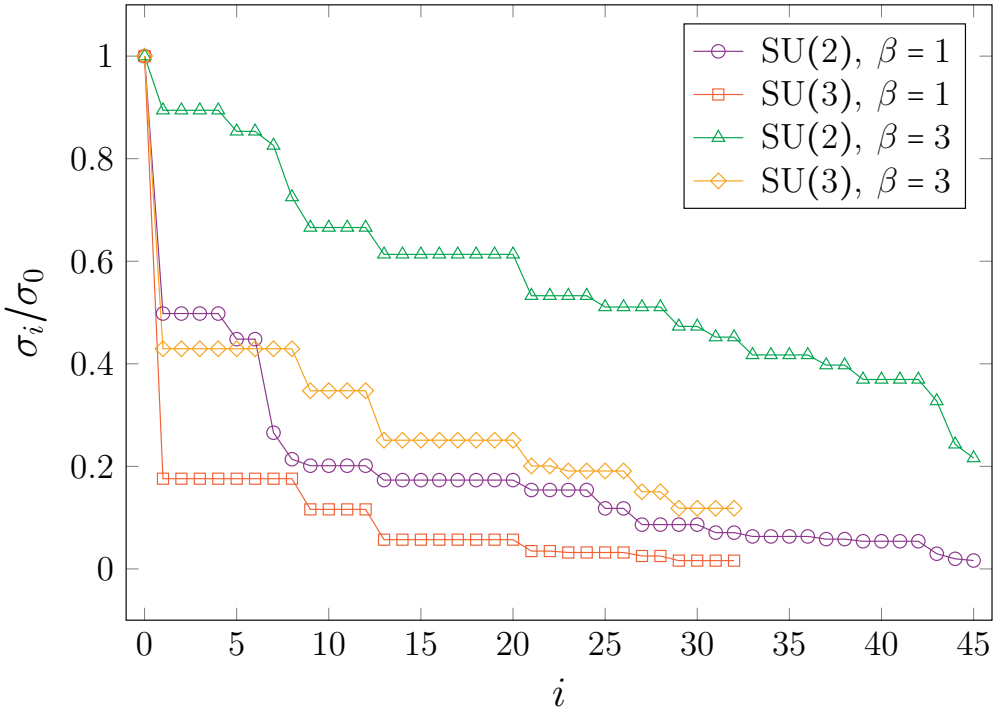
The armillary sphere

This structure can be obtained directly using character expansion



This formulation works for any gauge-invariant action and in any dimensions!

Result: singular value spectrum

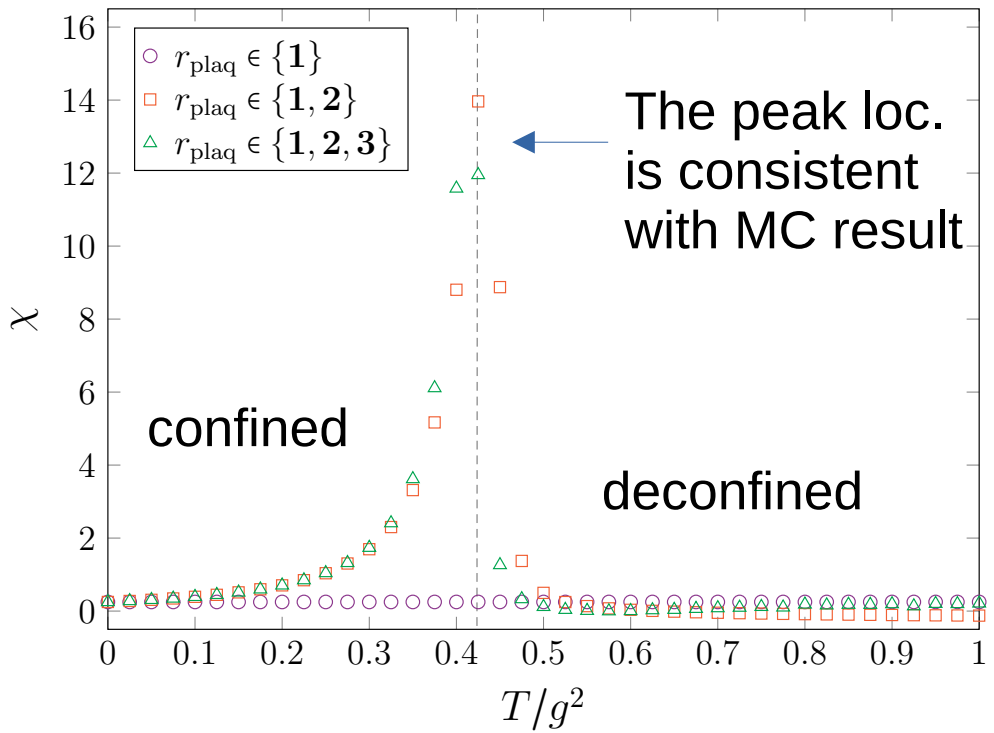


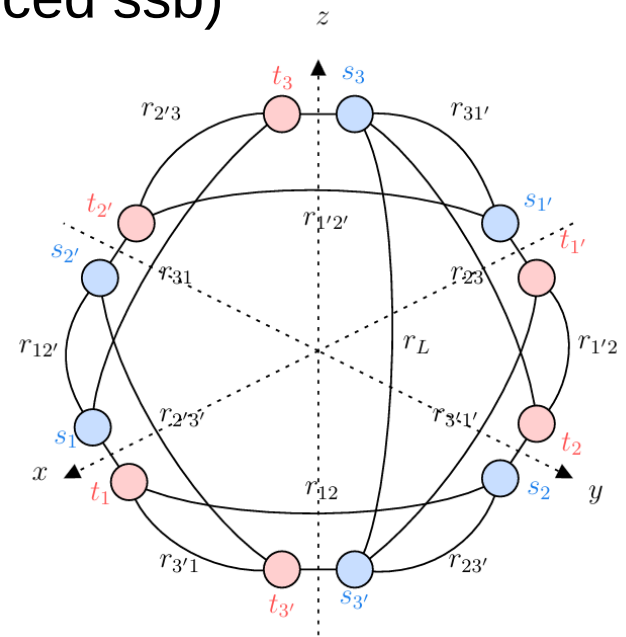
Singular value spectrum of the initial tensor do not have large degeneracy

Result: deconfinement @ finite temperature

TRG; V = 1x1024^2; D_{cut} = 64

Polyakov loop susceptibility (with induced ssb)





Armillary sphere with polyakov loop

Summary

- TRG is a promising methods for studying lattice theories
- We address 2 challenging aspects toward lattice QCD
  - Multiple fermion flavors can be handled with Grassmann Tensors with multi-layer construction
  - Degeneracy in non-abelian tensor network can be eliminated with the armillary sphere technique

Future prospects

- Armillary sphere with matter fields?
  - Can be done in many ways---need more investigation.
- Eliminate degeneracy in other schemes?
  (i.e., without using char. expansion?)
- Go to 3+1 dimensions
  - Theta vacuum and other interesting topics
- Etc.