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Real-time dynamics from convex geometry

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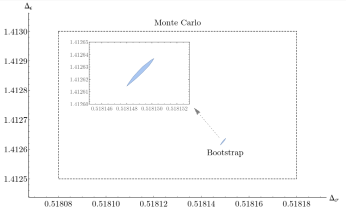
Convex geometry in quantum physics

$\langle \mathcal{O}^\dagger \mathcal{O} \rangle \geq 0$ "...the rest is commentary."

Consider vectors $E_i \equiv \langle \mathcal{O}_i \rangle$. With a basis of N operators, this defines a space $\approx \mathbb{R}^N$. The *allowed* space is a convex subset.

Convex spaces can be efficiently explored

CFT Bootstrap



From `arXiv:1603.04436`

[Han-Hartnoll-Kruthoff 2004.10212]
[Berenstein-Hulsey 2108.08757]
[**SL** 2111.13007]
[**SL** 2211.08874]

Spectral densities, in QM or QFT:

$$\rho(\omega) \geq 0$$

The spectral density function

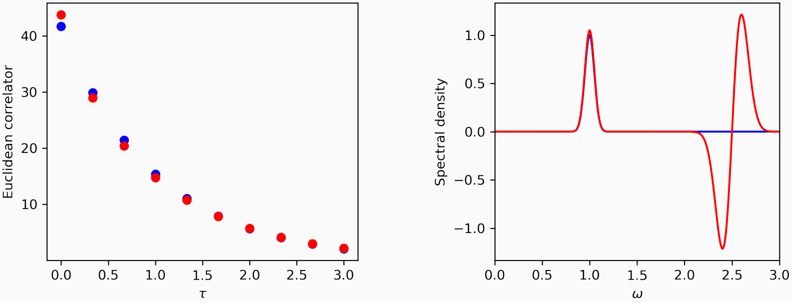
$G^{(R)}(t) = \mathrm{Im}\,\langle \mathcal{O}(t)\mathcal{O}(0)\rangle$ and $G^{(E)}(\tau) = \langle \mathcal{O}(-i\tau)\mathcal{O}(0)\rangle$

$$G^{(R)}(t) = -\int_0^\infty d\omega\,\rho(\omega)\sin\omega t$$

$$G^{(E)}(\tau) = \int d\omega\,\rho(\omega)\frac{\cosh\omega\left(\frac{\beta}{2}-\tau\right)}{\sinh\frac{\beta\omega}{2}}$$

$G^{(\cdot)(t)} \leftrightarrow \rho(\omega)$ is linear

The ill-posed inverse problem



But we *have not imposed* $\rho(\omega) \geq 0$.

Asking the right question

An experimentalist *cannot* measure $\rho(\omega)$.

Only various integrals.

An experimentalist *cannot* measure $G^{(R)}(t)$.

Time-energy uncertainty!

It makes much more sense to ask about:

$$\tilde{G}^{(R)}(t;\sigma) = \int dt'\, G^{(R)}(t') e^{-\frac{(t-t')^2}{2\sigma^2}}$$

In general: $\int \rho(\omega)\mathcal{K}(\omega)$ for smooth, rapidly decaying kernel $\mathcal{K}$.

Transport coefficients are often defined in the infinite time limit.

This is not probed by any experiment!

Example: hydrodynamic modelling of heavy-ion collisions

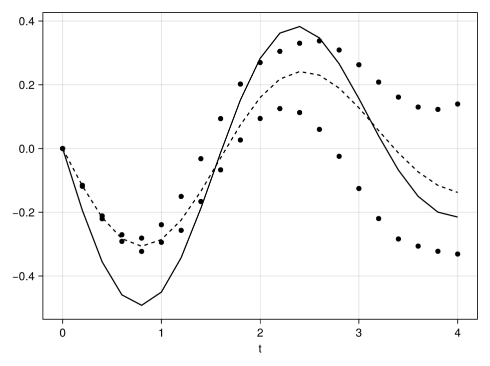
Positive AC I: Continuation from exact Euclidean data

minimize $\int \mathcal{K}\rho$

subject to $\int K_i \rho = C_i$

and $\rho \geq 0$

Solved via interior-point method.



Solid: real-time correlator.
Dashed: smeared correlator.
Points: bounded smeared correlator.
$N_\tau = 20$, $\sigma = 0.5$, $\beta = 0.5$,
$H = \frac{1}{2}p^2 + \frac{1}{2}x^2 + \frac{1}{2}x^4$

The frequentist blob

Measured correlators: $C_i \approx G^{(E)}(\tau_i)$

Error vector: $v_i = C_i - \int \rho(\omega) K(\tau_i, \omega)$

where $K(\tau, \omega) = \frac{\cosh \omega\left(\frac{\beta}{2} - \tau\right)}{\sinh \frac{\beta\omega}{2}}$

Via (statistical) bootstrap, we can get the distribution of $v^T M v$ for any procedure for constructing M. This defines a "confidence region" of ρ:

$$\{\rho(\omega) \,|\, v^T M v < \epsilon_{99\%}\}$$

This region is convex. Intersection with another convex region ($\rho \geq 0$) is also convex.

So you think you know what a Lagrangian is...

$$L(x, \lambda) = f(x) - \lambda g(x)$$

The primal:

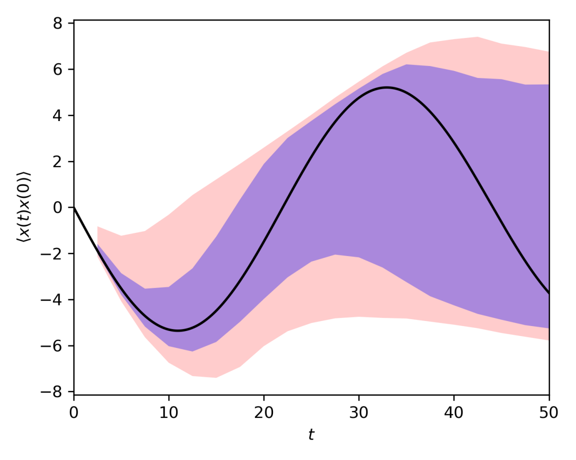
$$\begin{array}{r} \text{minimize } f(x) \\ \text{subject to } g(x) \geq 0 \end{array} \implies p^* = \min_x \max_{\lambda \geq 0} L(x, \lambda)$$

The dual:

$$d^* = \max_{\lambda \geq 0} \min_x L(x, \lambda) \implies \begin{array}{rl} \text{maximize} & \min_x L(x, \lambda) \\ \text{subject to} & \lambda \geq 0 \end{array}$$

Weak duality: $d^* \leq p^*$. Strong duality: $d^* = p^*$.

Positive AC II: The anharmonic oscillator



$\beta = 20$, $\omega^2 = 0.01$, $\lambda = 0.001$.

From 10^2 and 10^3 samples. Smeared with $\sigma = 5.0$.

99% confidence region.

Closing thoughts

- Heavy-ion collisions happen pretty quickly!
- Bounds of this form are "optimal" provided no other constraints are known.
- Is Gaussian smearing optimal/necessary?
- More than one operator can be included in the analysis, enabling access to "off-diagonal" correlators, and suggesting the use of Schwinger-Dyson relations.

This is a rapidly developing field. **Stay tuned...**