Unfreezing topology with Nested Sampling

in the 2d quenched Schwinger model

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Nested sampling (John Skilling, 2004)

Bayesian evidence integral:

$$Z = \int \mathscr{L}(\theta)\pi(\theta)d\theta$$

$\pi(\theta)$: prior distribution

$\mathscr{L}(\theta)$: likelihood

Z: evidence

Nested sampling (John Skilling, 2004)

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Transform to 1-dim. integral:

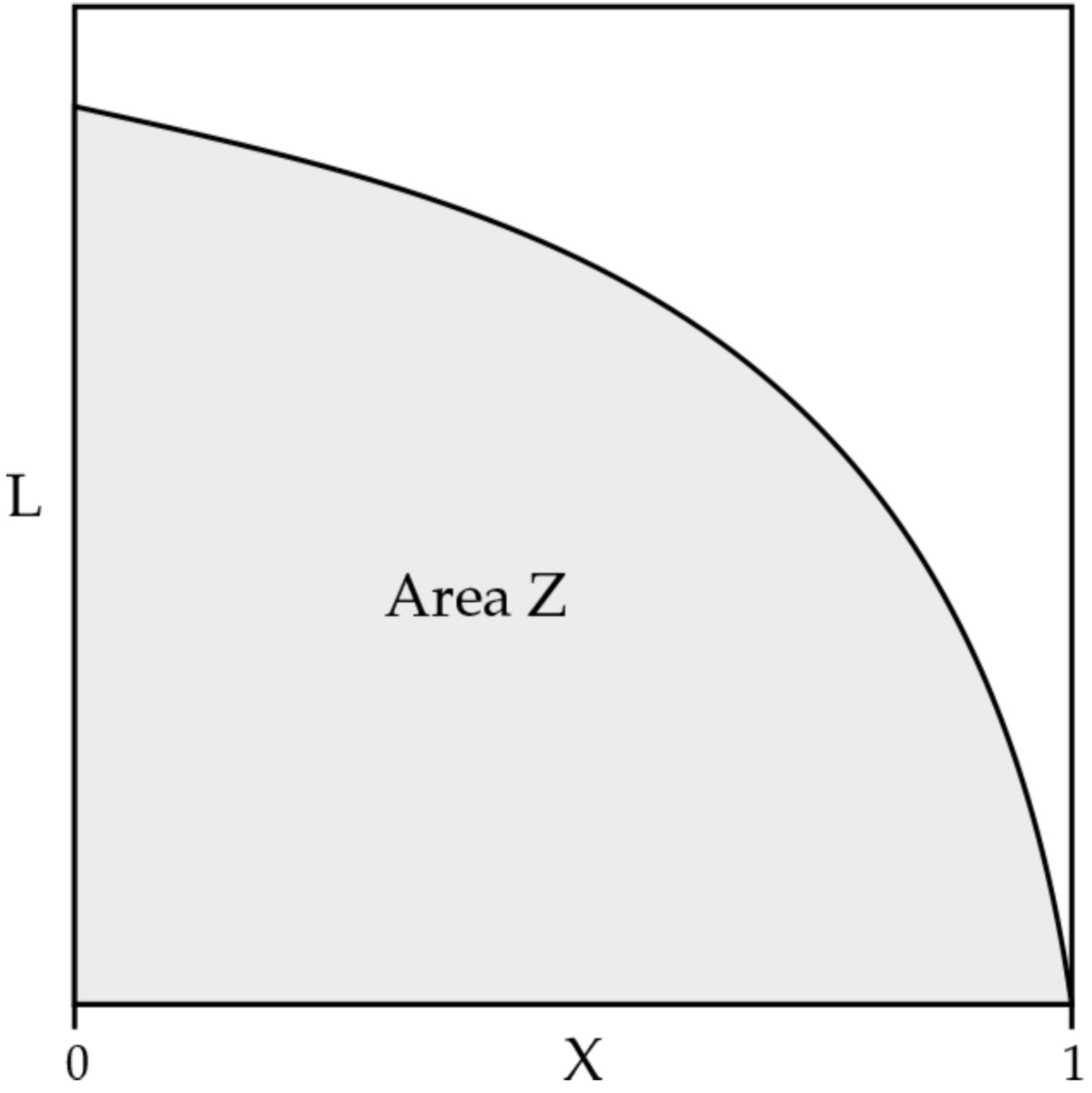
$$dX = \pi(\theta)d\theta$$

$$X(\lambda) = \int_{\mathscr{L}(\theta)>\lambda} \pi(\theta)d\theta$$

such that:

$$Z = \int_0^1 \mathscr{L}(X)dX$$

Nested sampling (John Skilling, 2004)



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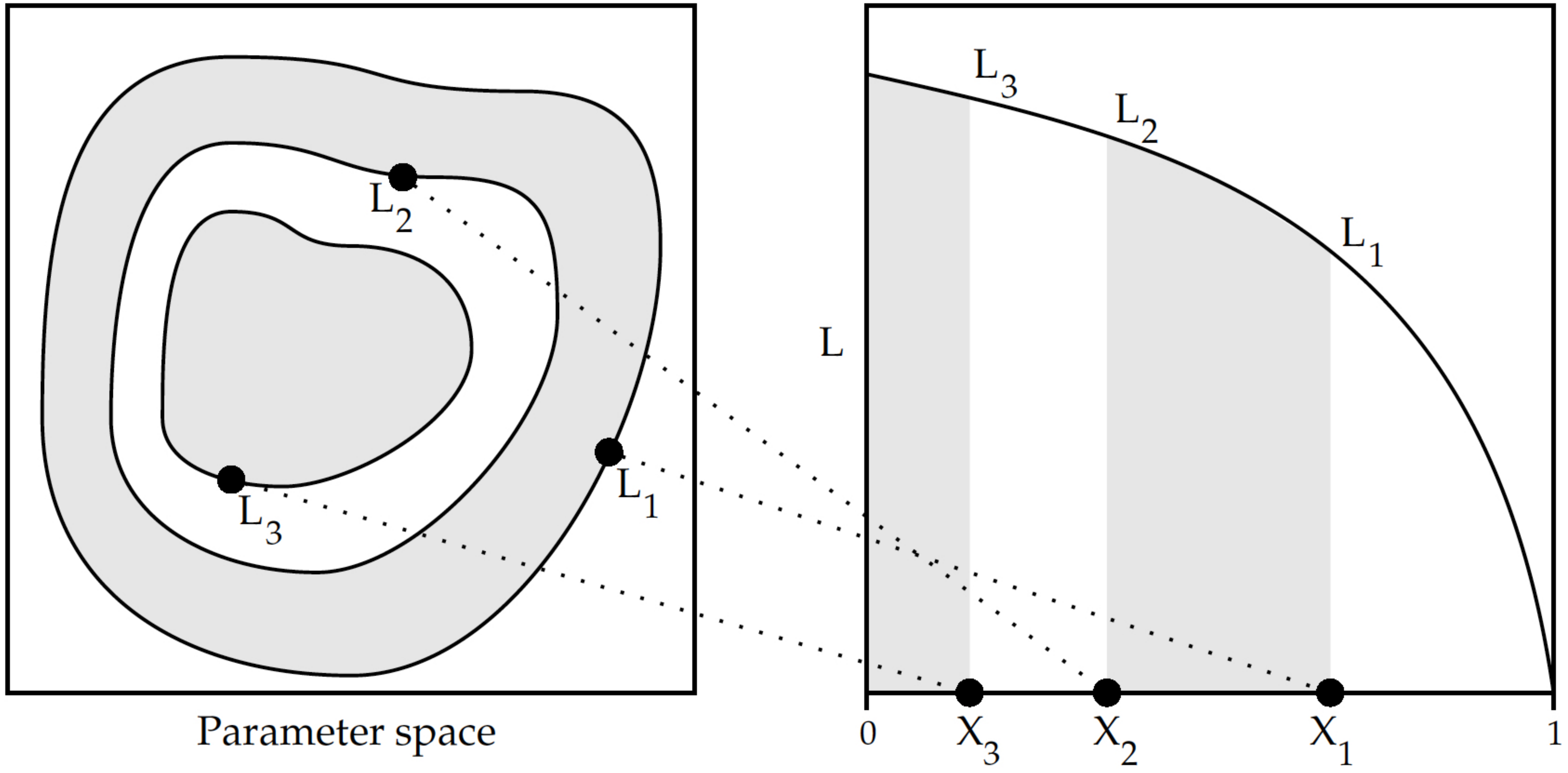
Draw samples uniformly from $\pi(\theta)d\theta$:



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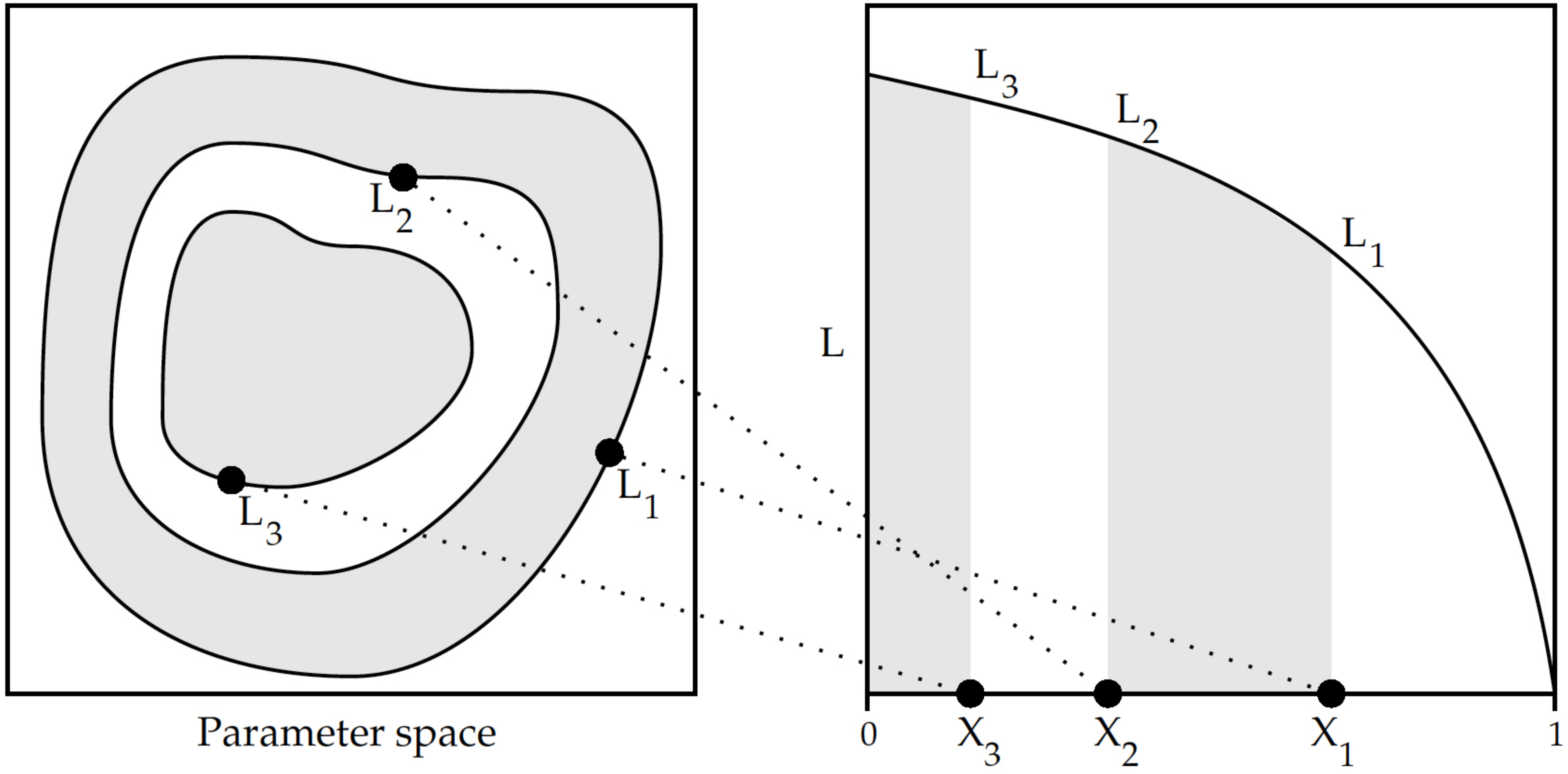


L_i's can be calculated.

X_i's are unknown, but:

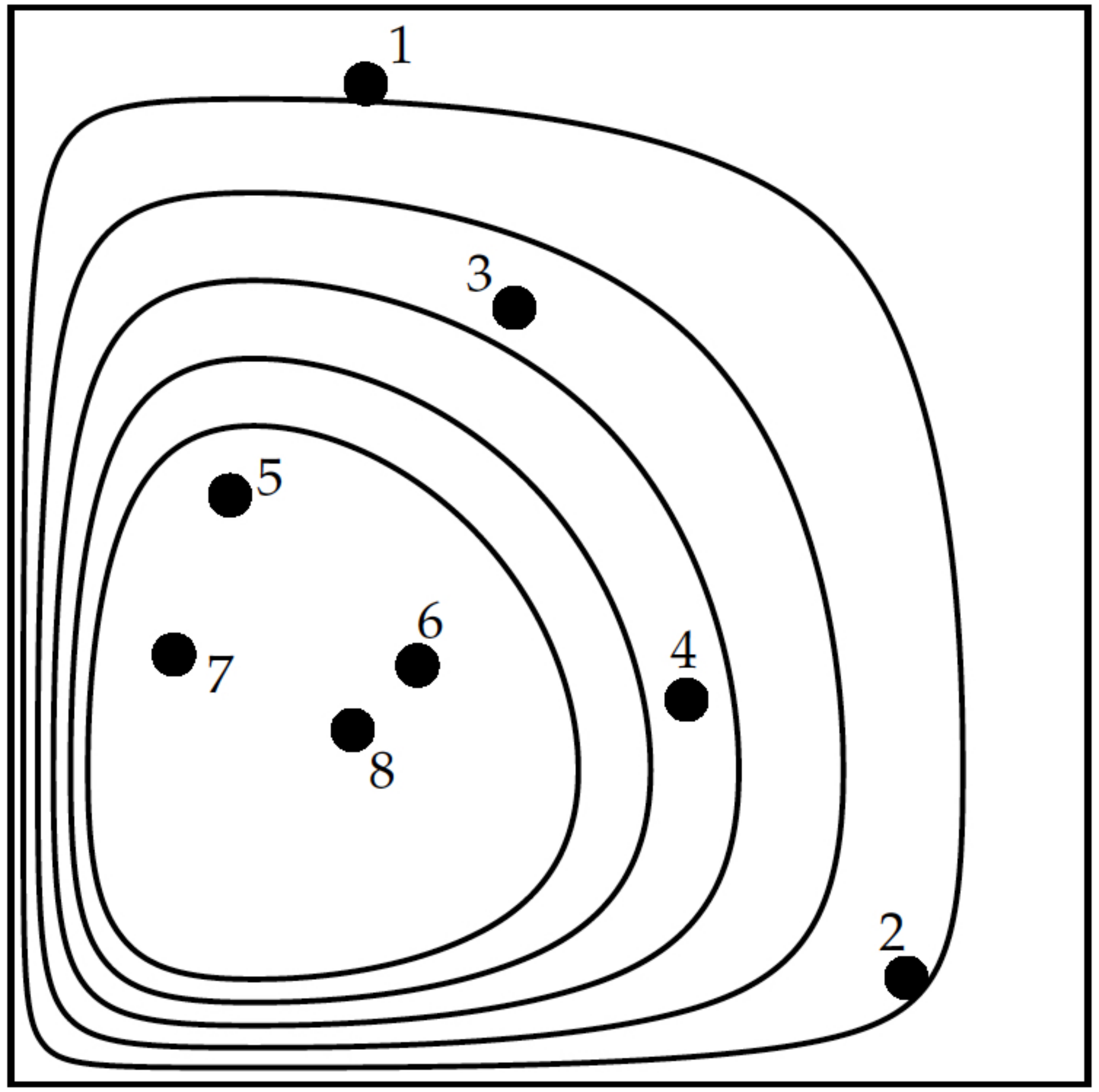
$$X_0 = 1, \quad X_i = t_i X_{i-1}$$

$$\Pr(t_i) = N\, t_i^{N-1} \quad \text{in } (0,1)$$

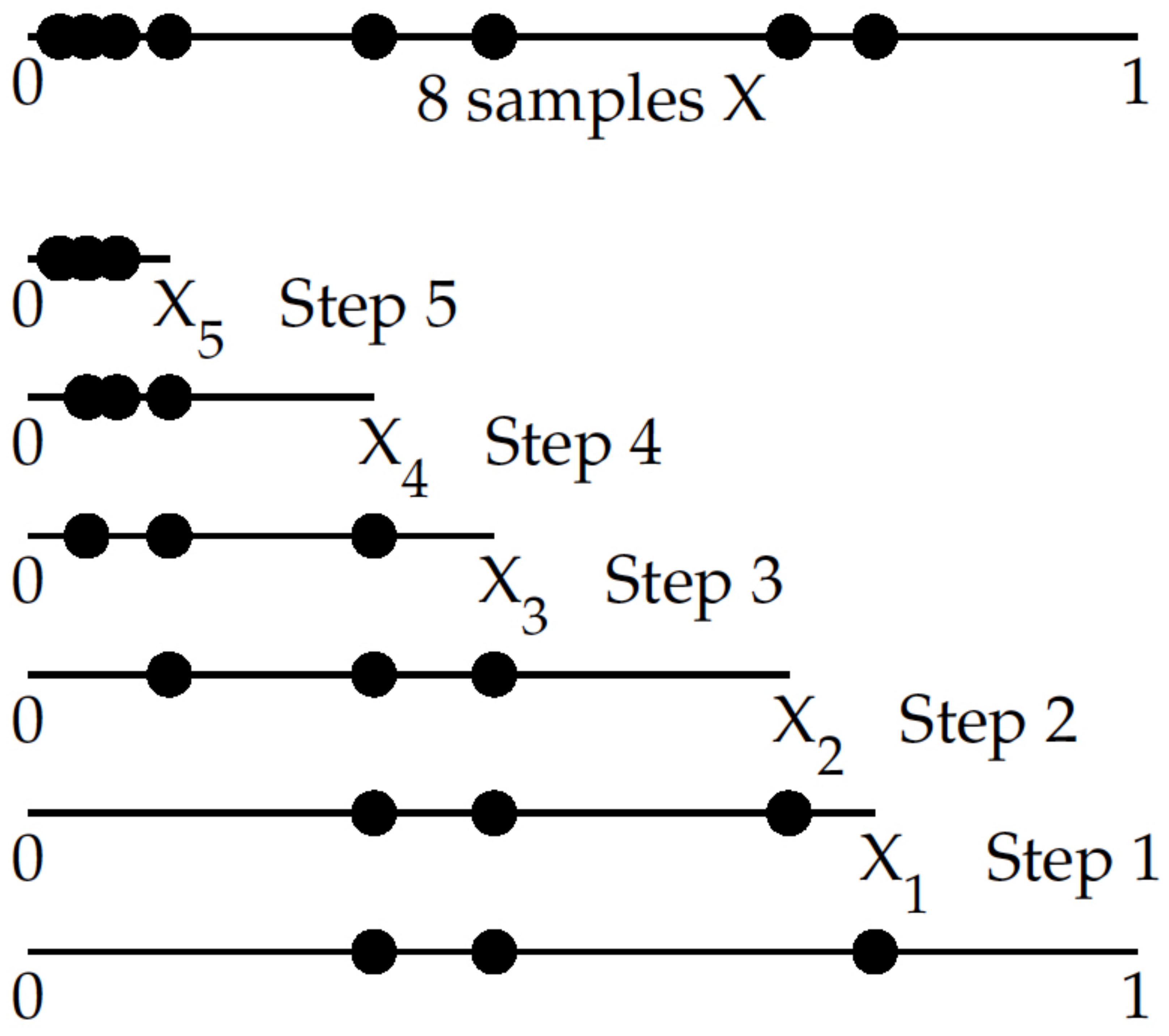
with

$$\langle \ln t \rangle = -1/N$$

Nested sampling (John Skilling, 2004)

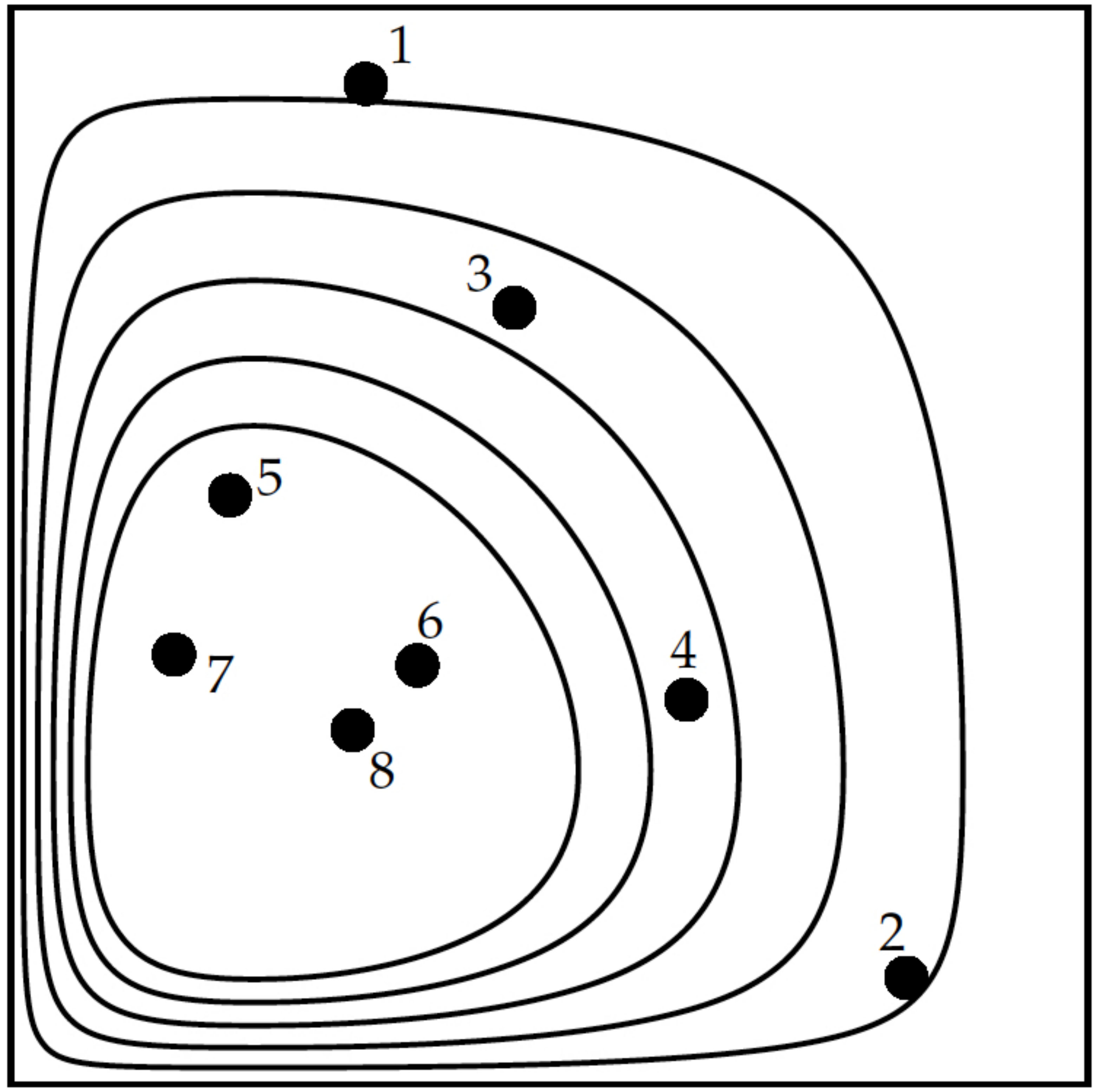


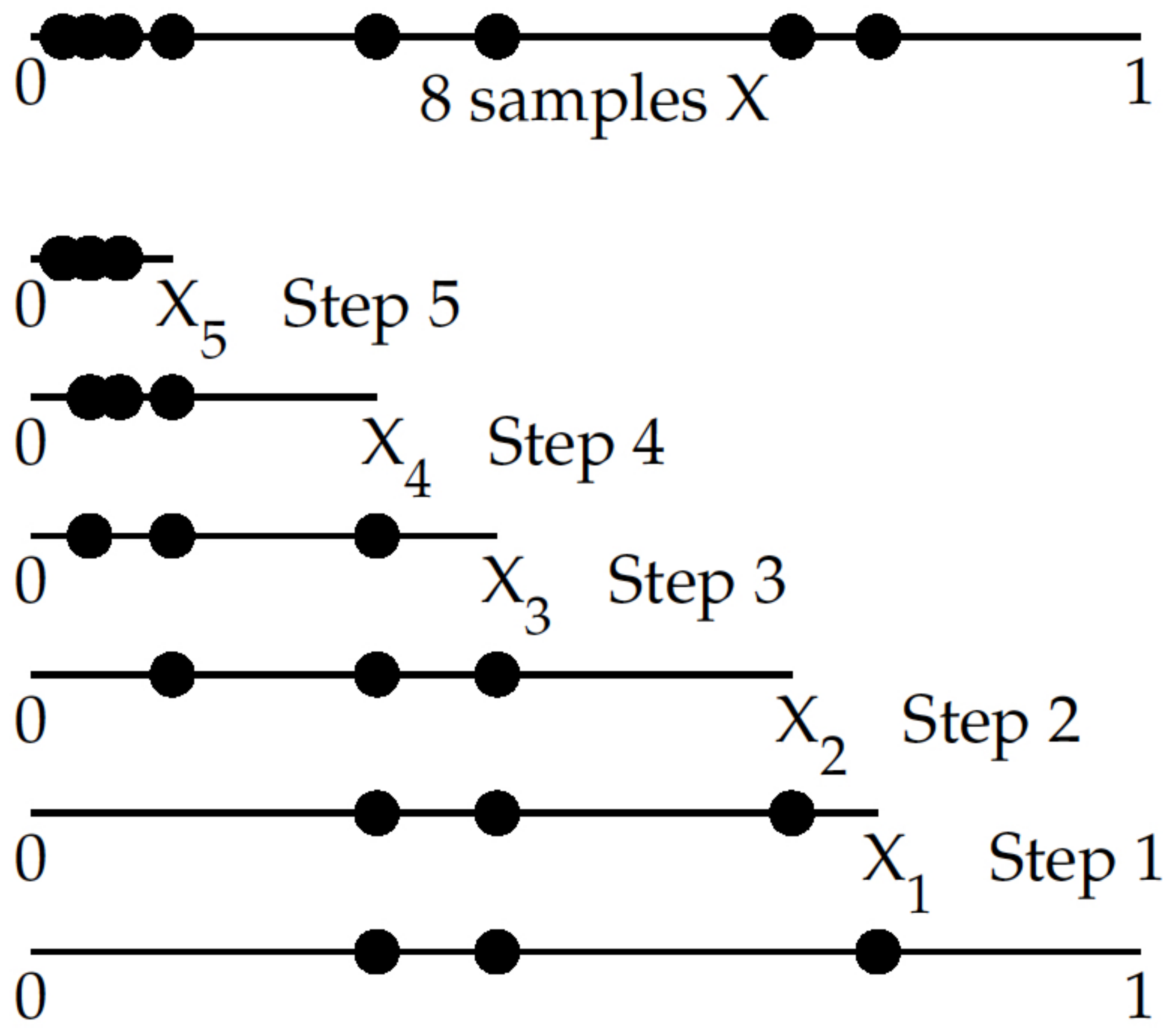
Parameter space



Enclosed prior mass X

Nested sampling (John Skilling, 2004)





Result from simulation: $\Rightarrow$ ordered list of $\{X_i, L_i\}$

Application to QFT

$$Z(\beta) = \int L^\beta \, dX \qquad \text{with } L = \exp(-S)$$

Nested sampling also yields density of states $\rho(S) = -\dfrac{dX}{d \ln L}$

and hence:

$$Z(\beta) = \int e^{-\beta S} \rho(S) \, dS$$

a posteriori for any β !

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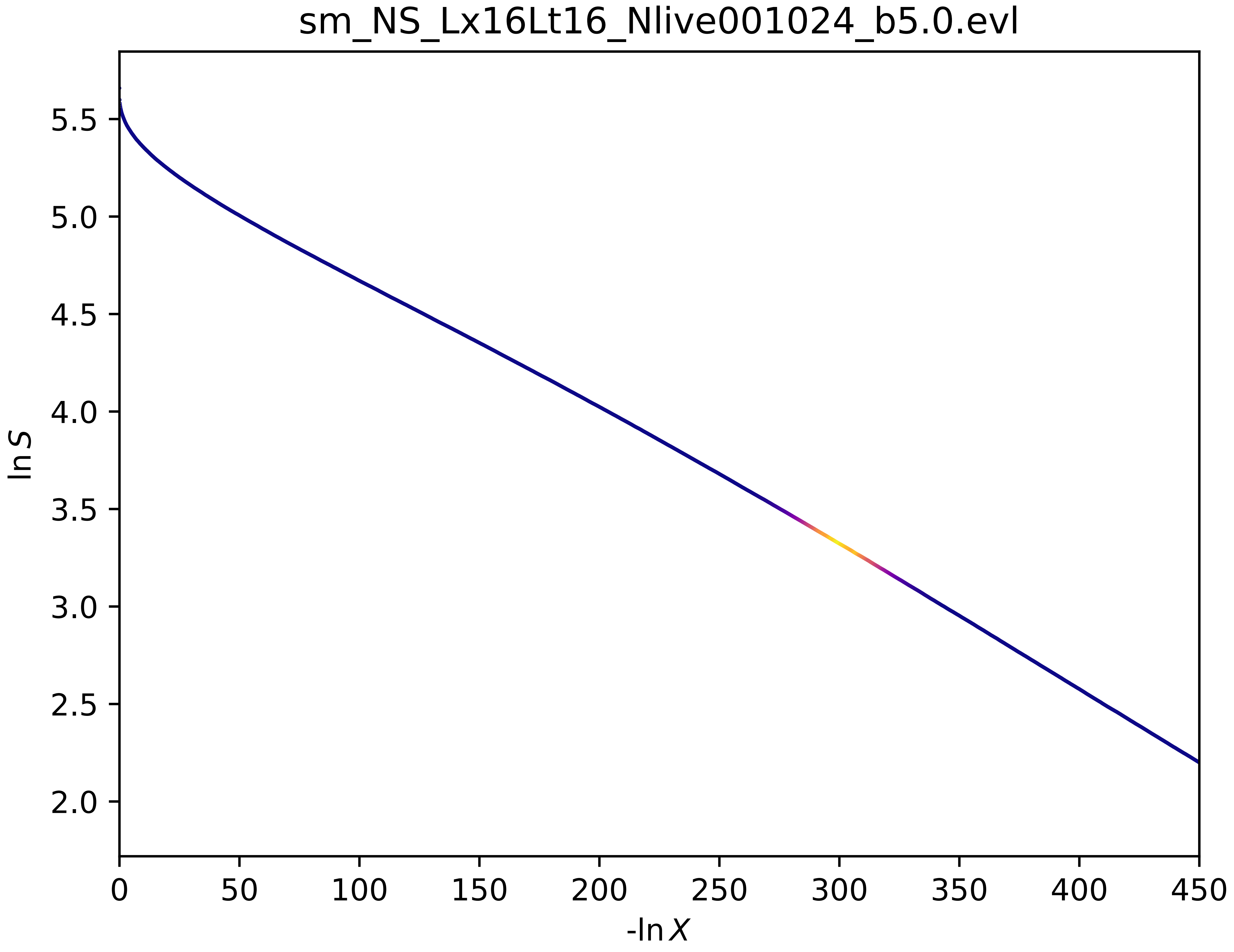
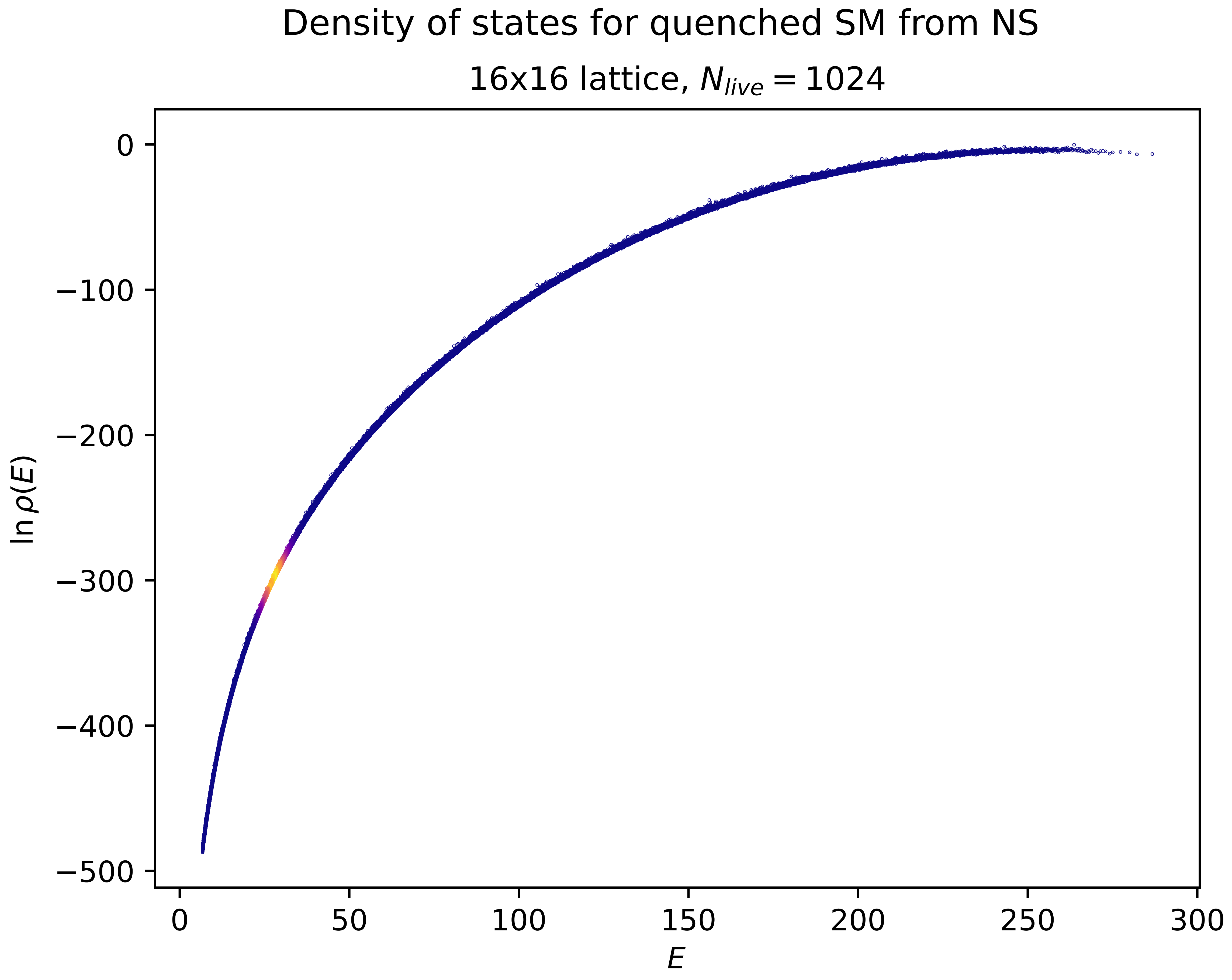
Example application: 2d quenched Schwinger model

Likelihood L vs. prior volume X and density of states

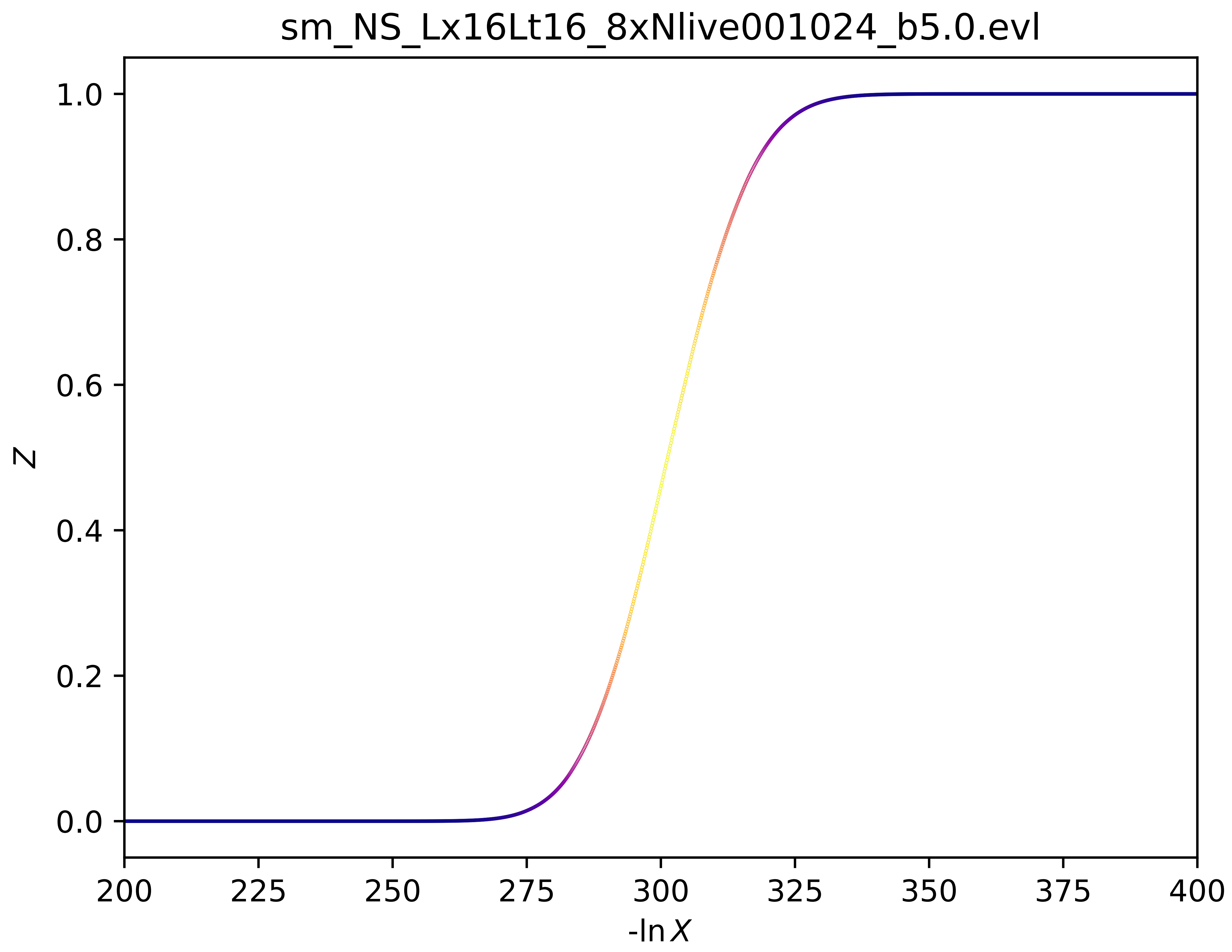
…or better $-\ln L = S$ and $\ln S$ vs $-\ln X$:

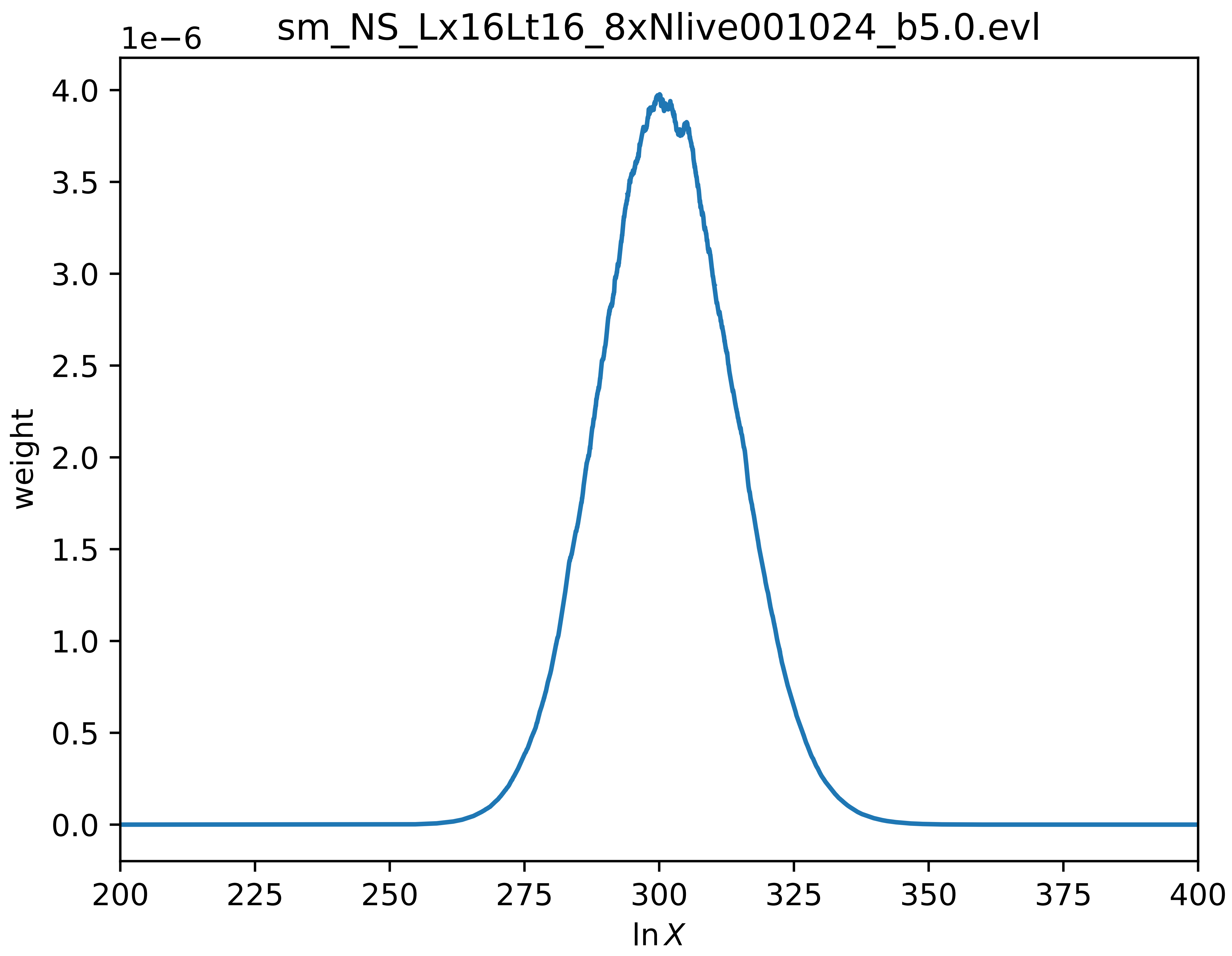


…or $\rho(E) = dX/dE$ and $\ln \rho(E) = -X\, d\ln X / d\ln L$:

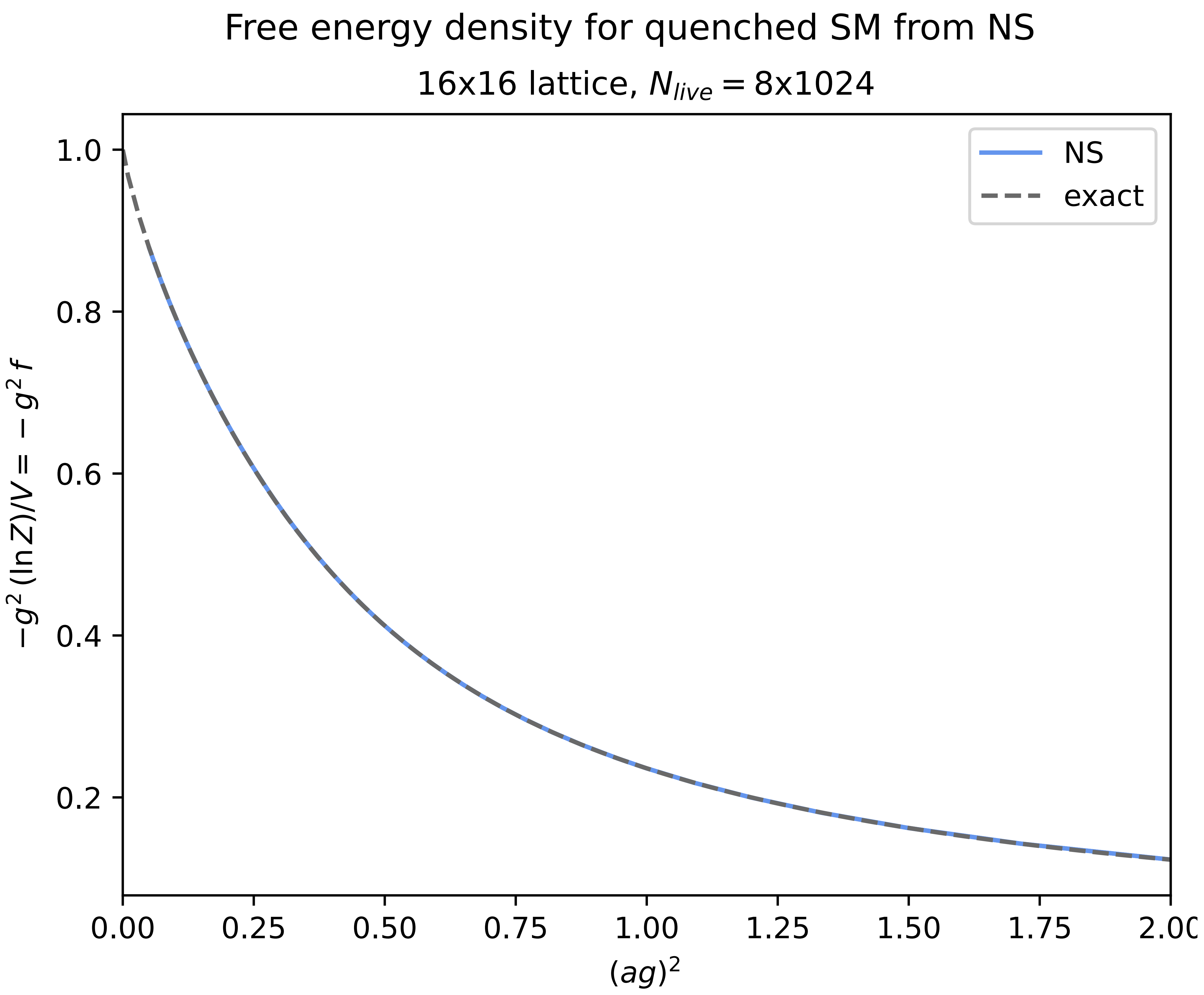


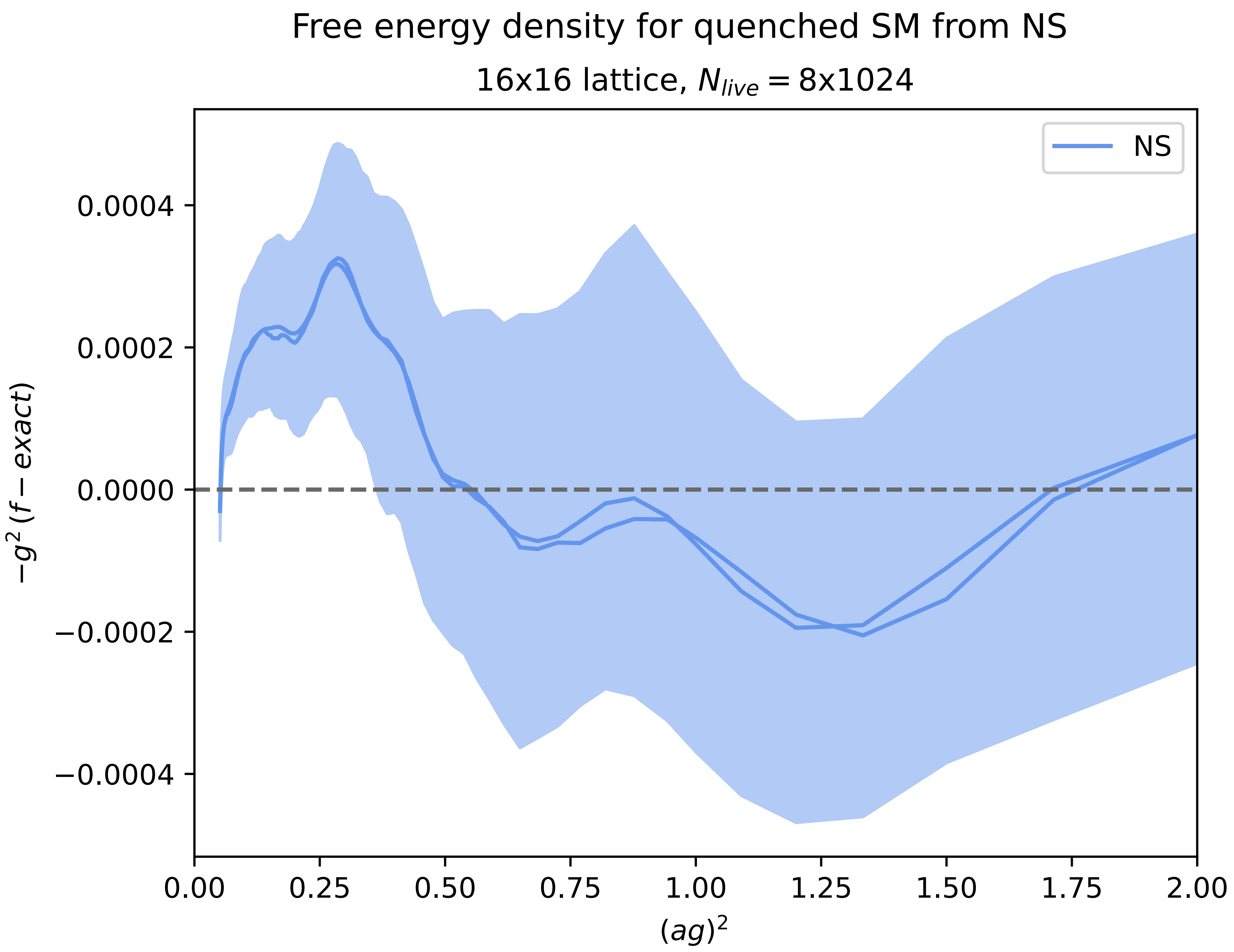
Partition function Z with weights at $\beta = 5.0$



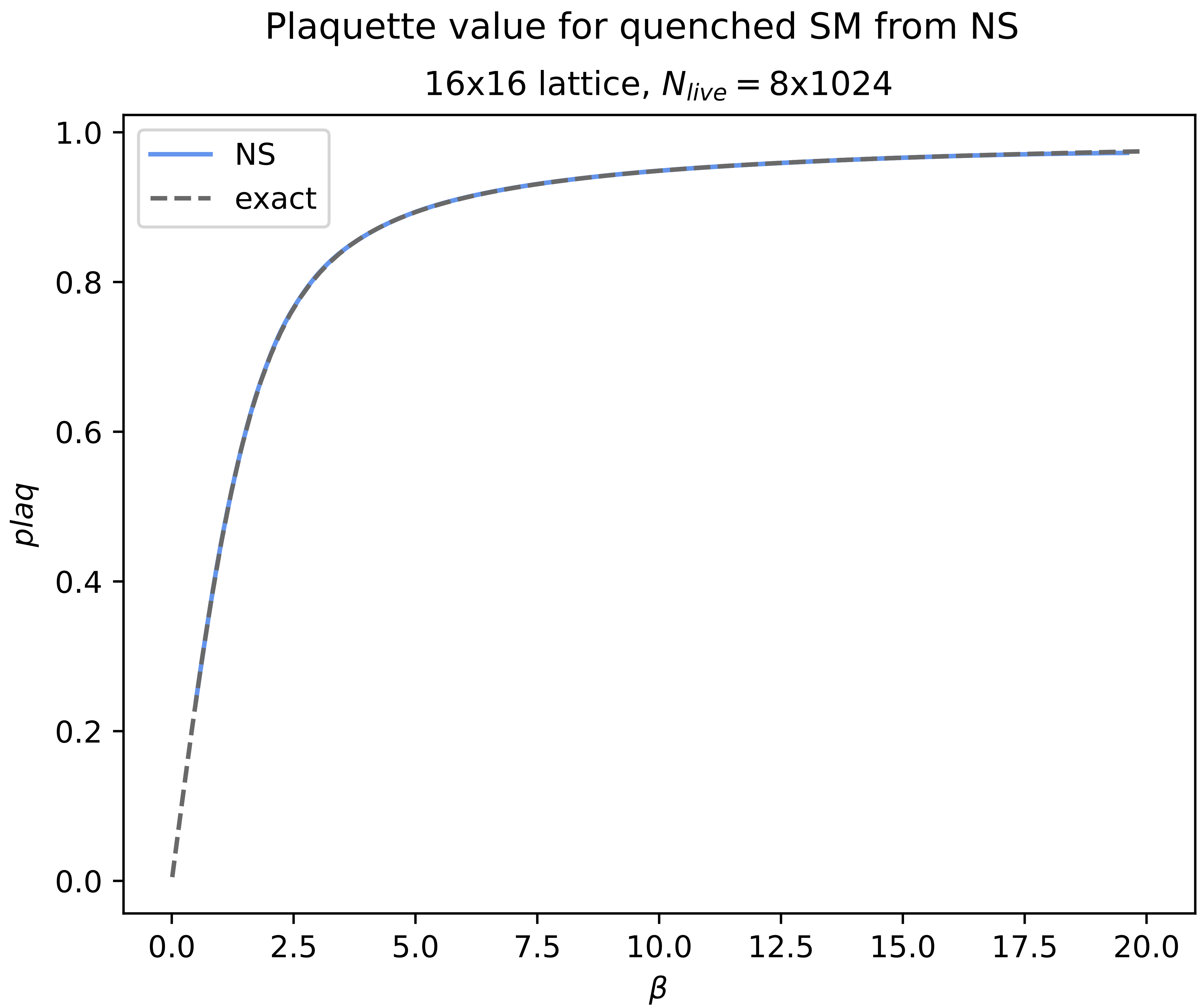


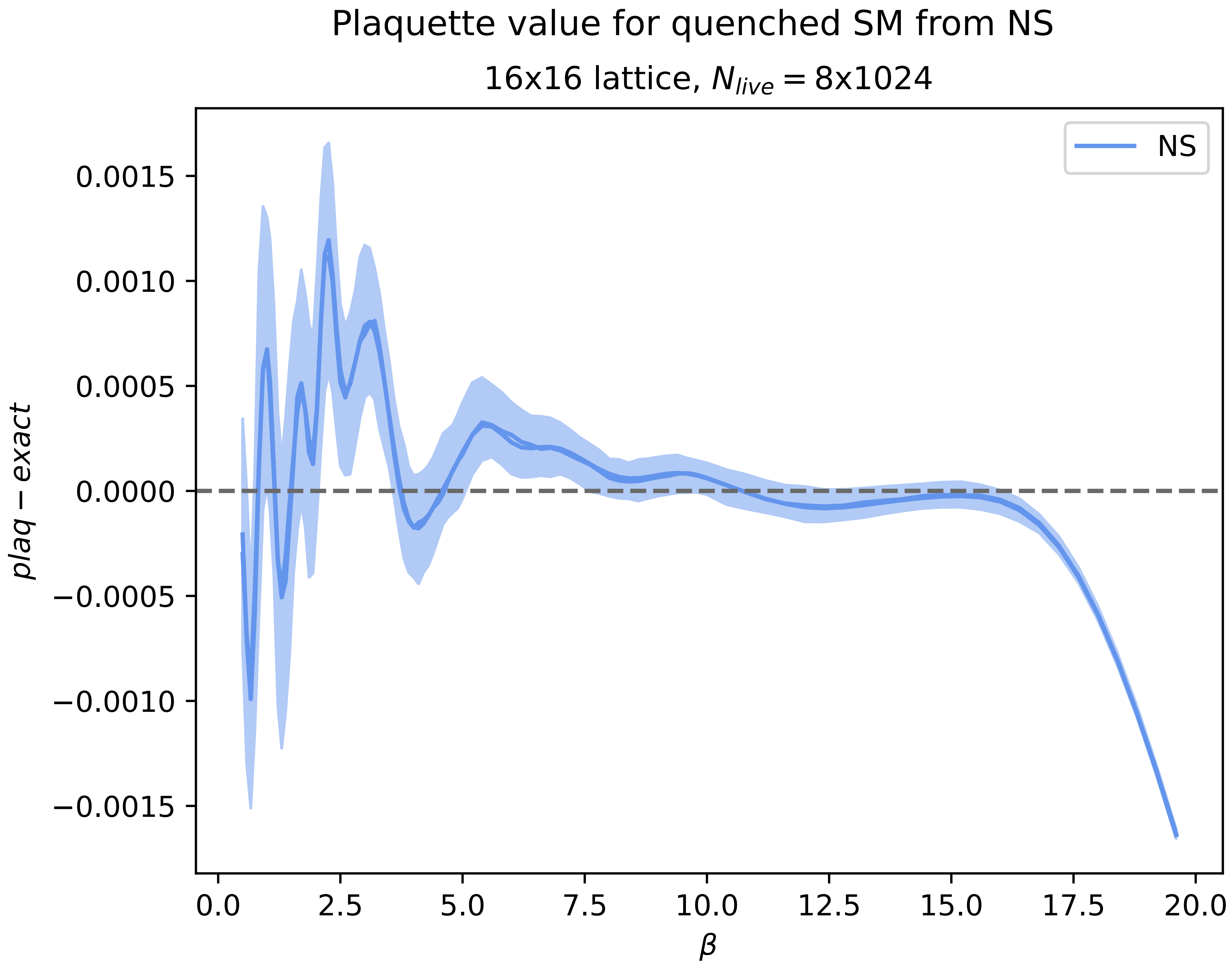
Free energy density, continuum limit



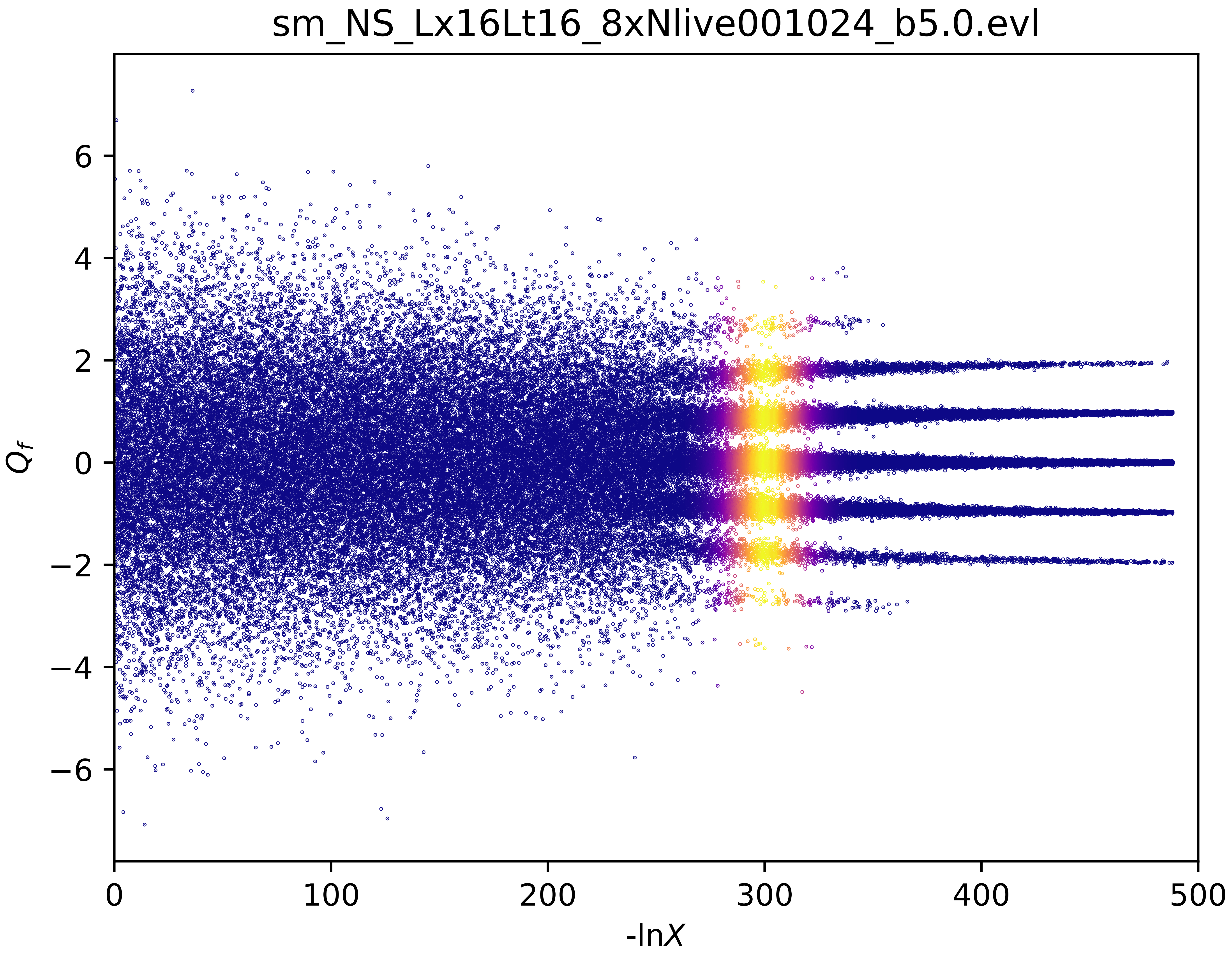


Plaquette value vs β

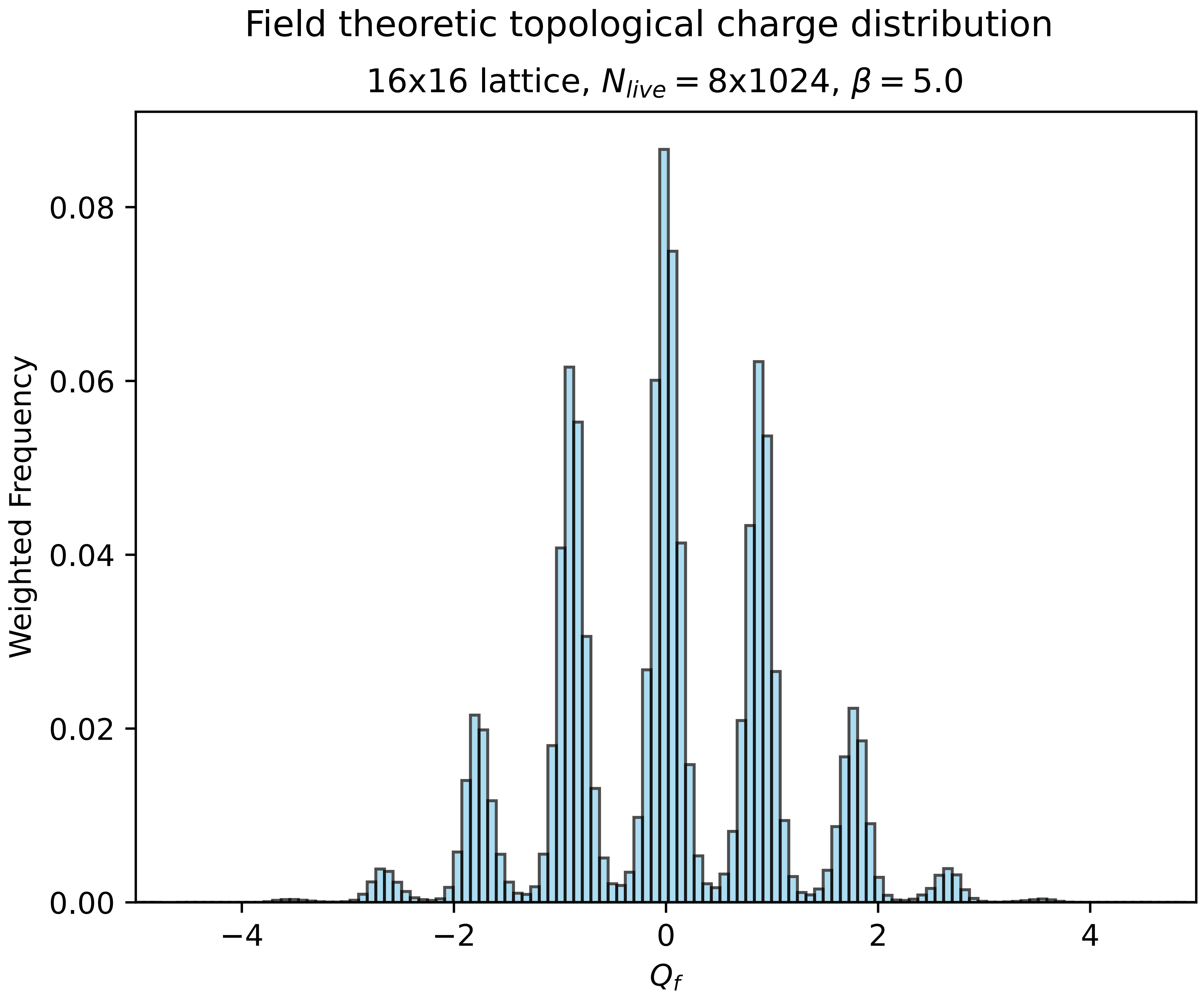


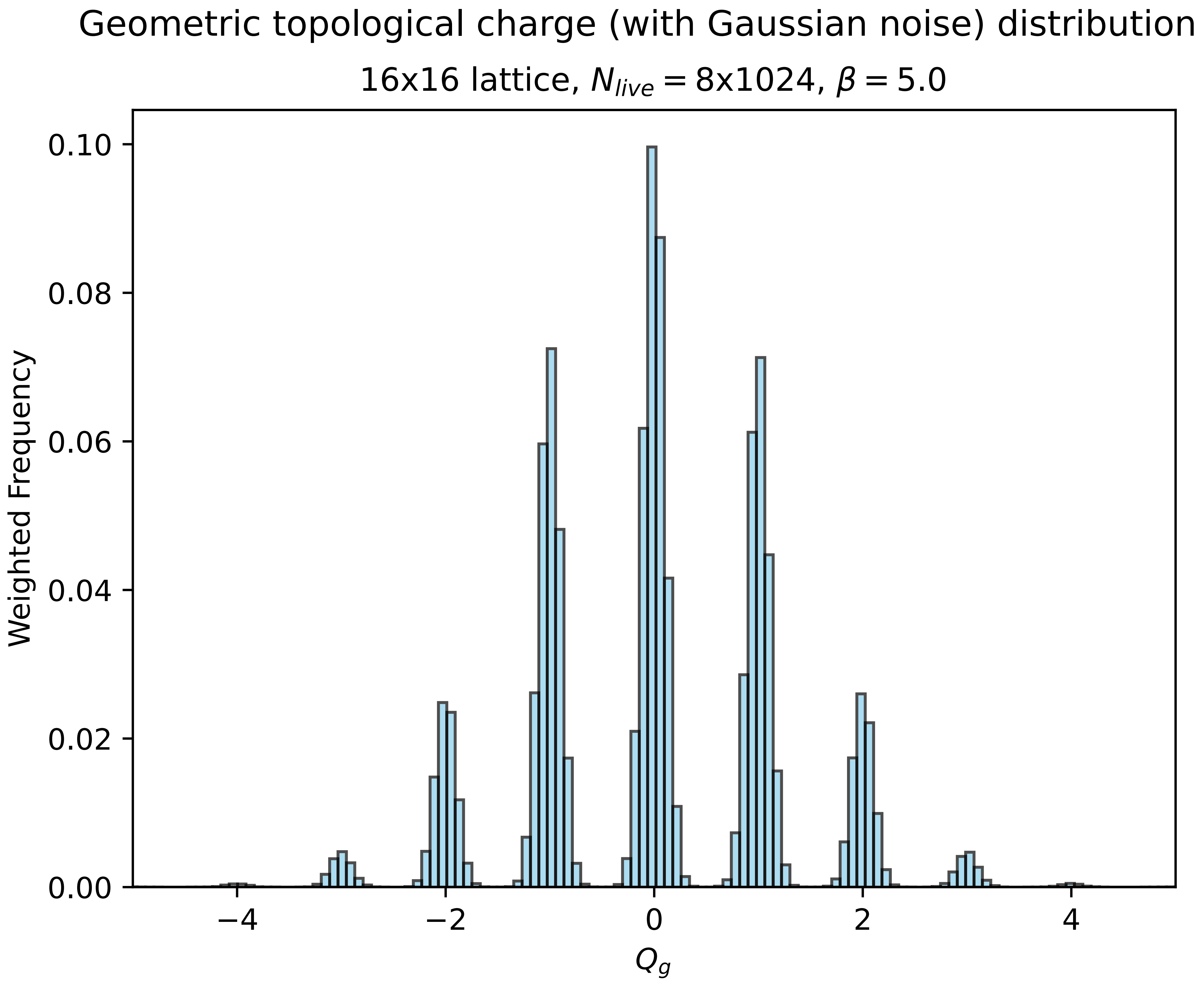


Topological charge with weights at $\beta = 5.0$

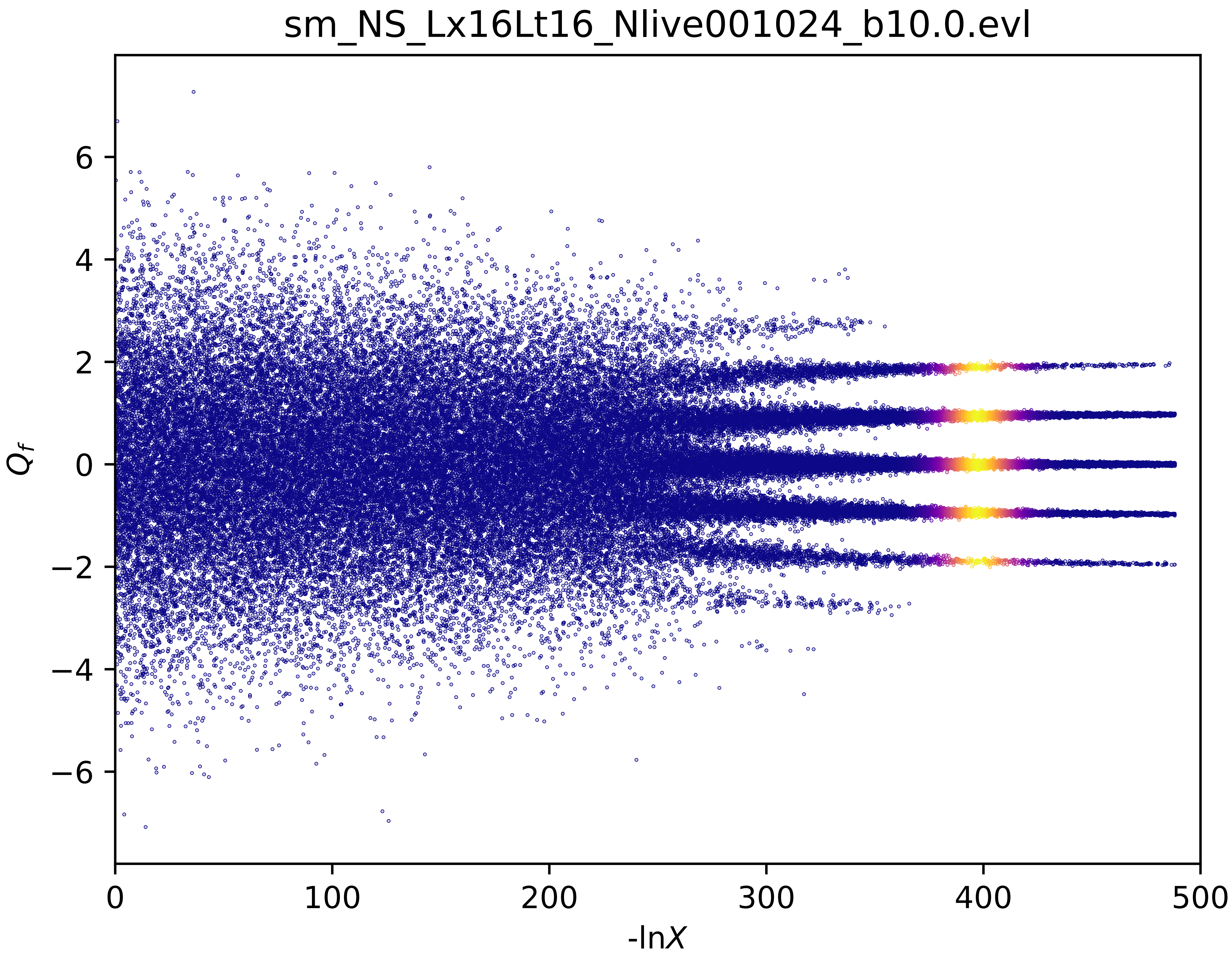


Topological charge distribution





Topological charge with weights at $\beta = 10.0$



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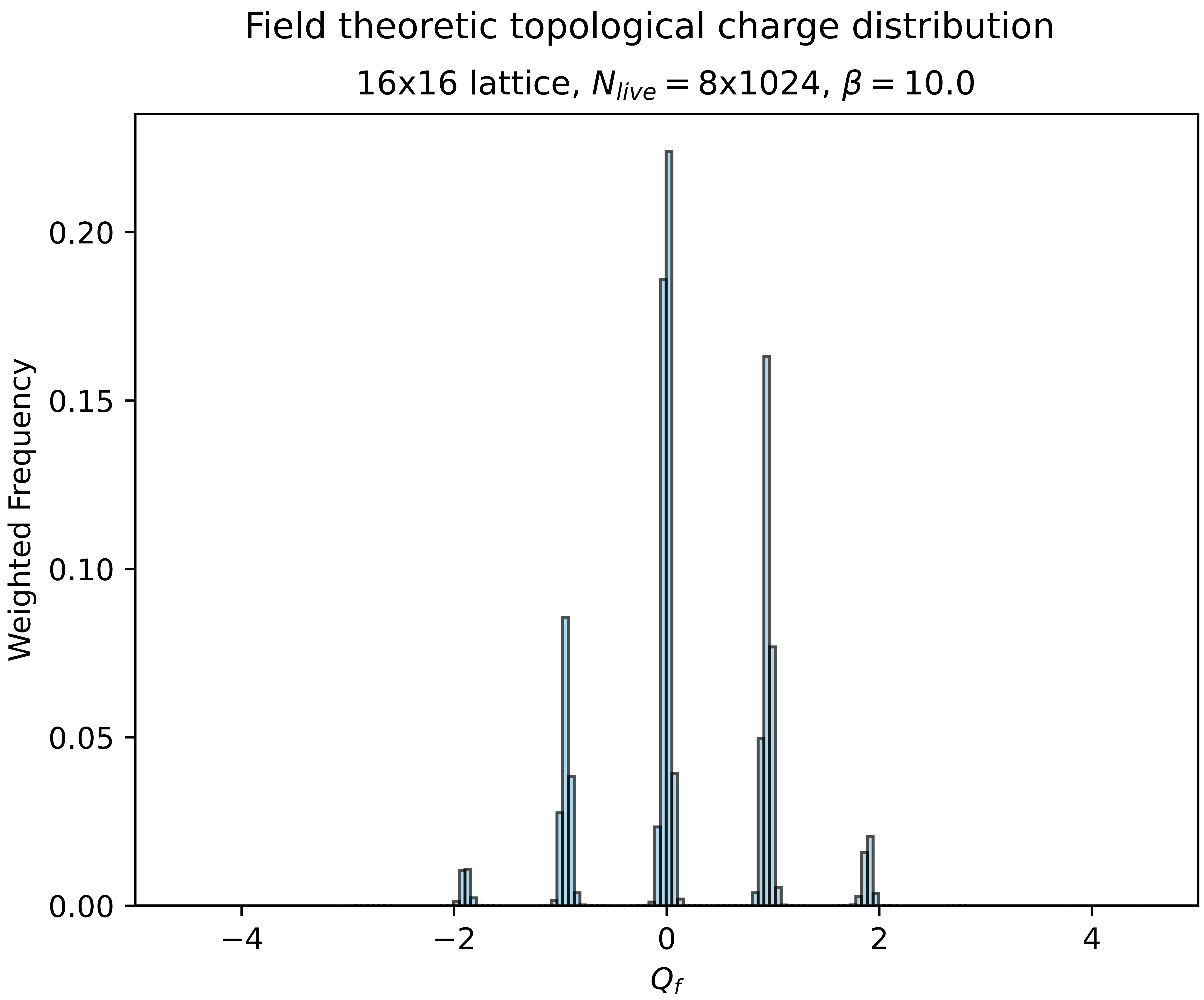
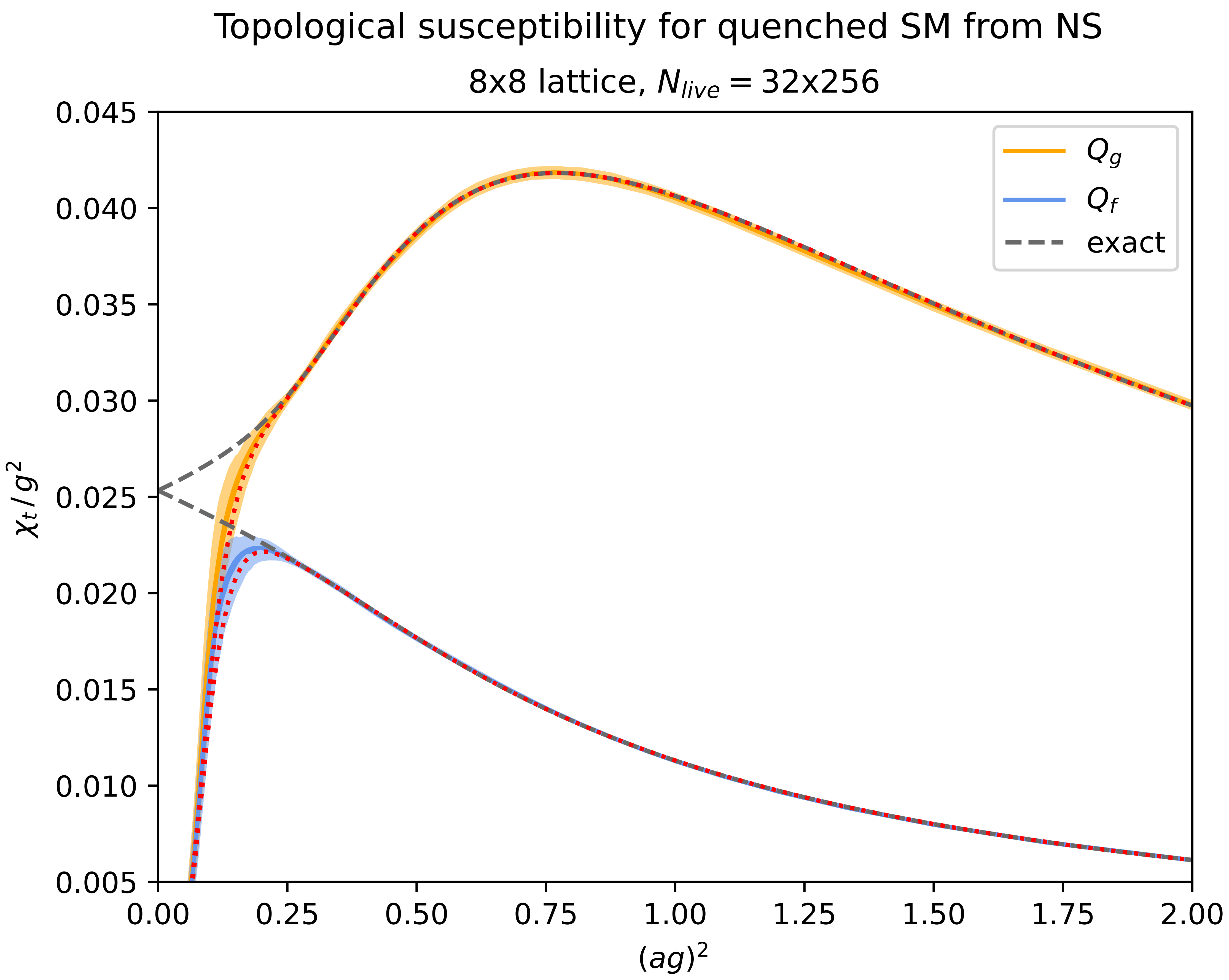
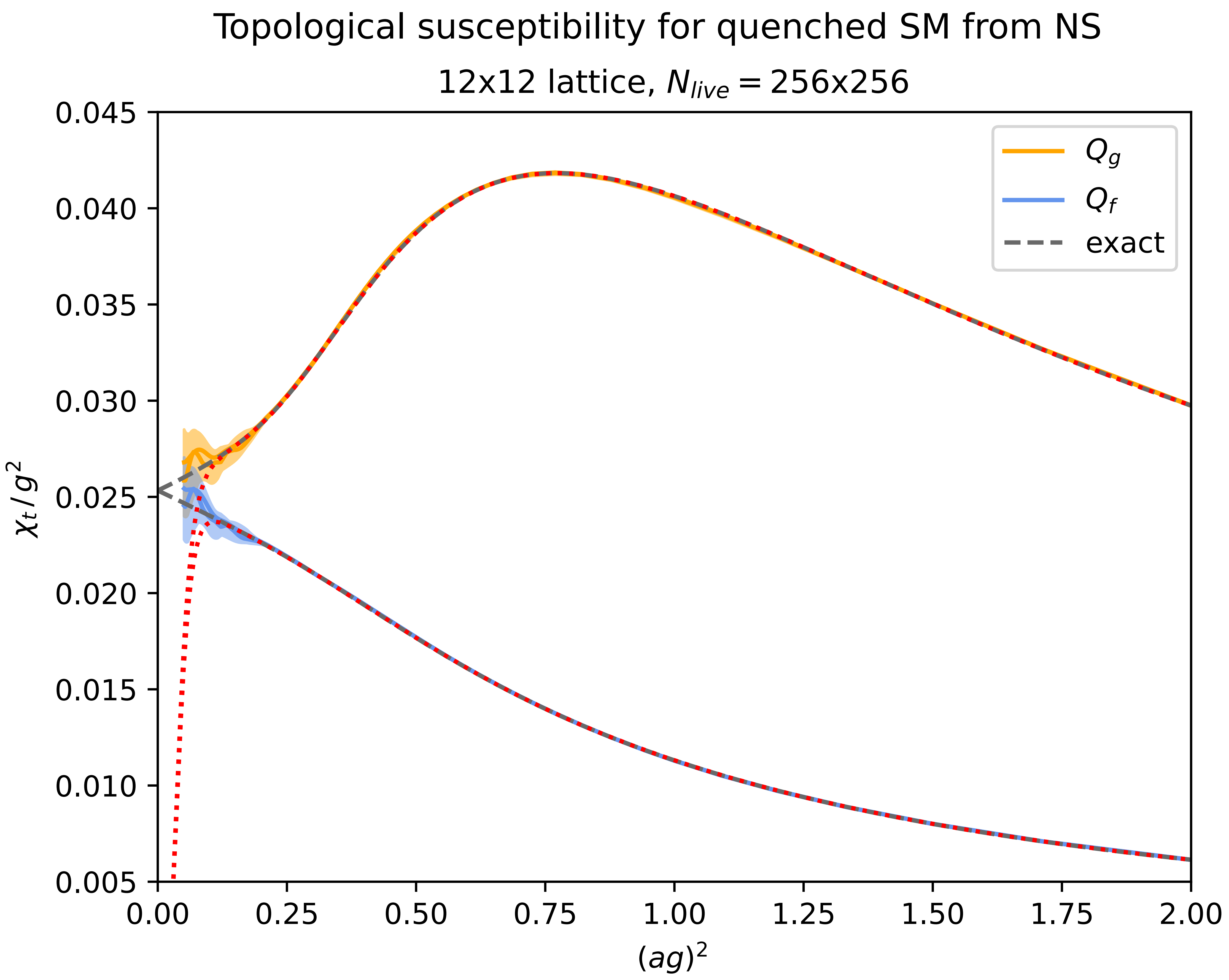


Topological susceptibility continuum limit





Conclusion & outlook

- Nested sampling has the potential to unfreeze topology
- Nested sampling parallelizes trivially
- Scaling with V needs to be investigated
- Inclusion of fermions
- Application to 1st order phase transitions

Thanks!

SIGN25

Bern, 20-24 January 2025
Workshop on the sign problem in QCD and beyond

http://sign25.itp.unibe.ch/

Hope to welcome you soon in Switzerland!