High statistical computation of the Landau gauge ghost-gluon vertex

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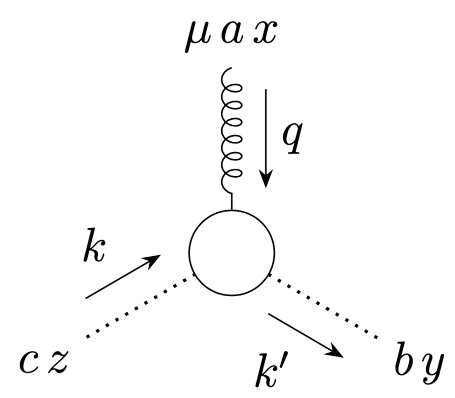
Abstract: The lattice computation of the one-particle irreducible ghost-gluon Green function in the Landau gauge is revisited with a set of large gauge ensembles. The large statistical ensembles enable a precision determination of this Green function over a wide range of momenta, accessing its IR and UV properties with a control on the lattice effects.

Motivation and Definitions

In QFT the one-particle irreducible Green's functions (1PI) summarize the dynamics of the theory. For QCD and for two-point functions a good understanding of the non-perturbative features of the 1PI has been achieved for the full range of momenta. Efforts are now on the evaluation of 1PI functions with more than two external legs as is the case of

► The Ghost-Gluon 1PI



whose vertice reads

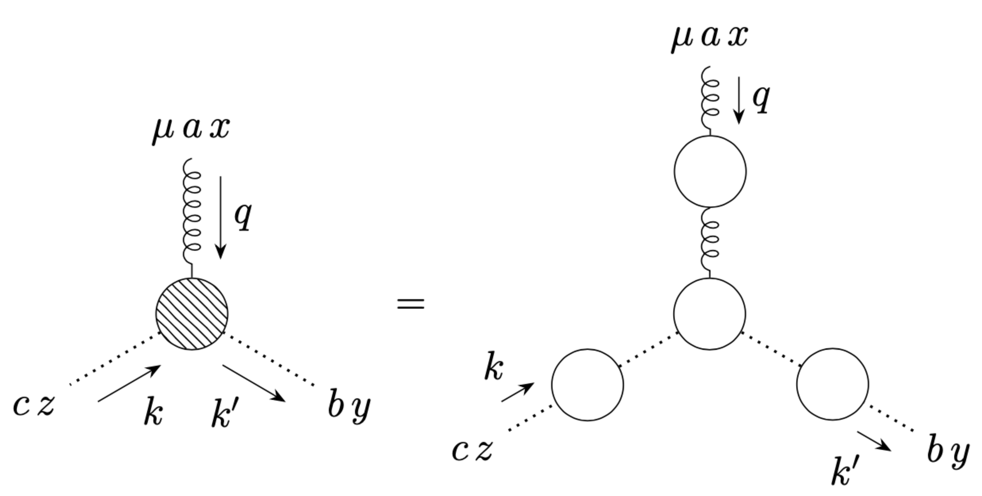
$$\Gamma^{\mu} = i\,g\,f_{abc}\left(k'_{\mu}\,H_1(k^2,\,k'^2,\,q^2) + q_{\mu}\,H_2(k^2,\,k'^2,\,q^2)\right)$$

► g is the strong coupling constant;
► H_i are Lorentz scalar form factors.

Γ^{μ} is important for the QCD dynamics, determining, for example, the ghost propagator.

The Lattice Green Function

On a lattice simulation only the full Green functions $\mathcal{G}$ can be computed and, therefore, to access the ghost-gluon 1PI the function to be calculated is



In the Landau gauge, due to the orthogonality of gluon propagator, the contribution of the form factor H_2 vanishes and only H_1 can be accessed.

► The tree level lattice 1PI

$$\Gamma^{Lat}_{\mu} = i\,g\,f_{abc}\,k'_{\mu}\,\cos\left(\frac{a\,k_{\mu}}{2}\right)$$

with

$$k_{\mu} = \frac{2\,\pi}{a\,N_{\mu}}\,n_{\nu}\ , \qquad n_{\mu} = 0, \ldots, N_{\mu}/2\ ,$$

a being the lattice spacing N_{μ} the number of lattice points in direction μ and $k'_{\mu} = (2/a)\,\sin(a\,k_{\mu}/2)$ being the improved momenta as it reproduces the naive lattice momenta up to corrections of $\mathcal{O}(a^2)$. The determination of $\mathcal{G}$ requires the computation of

the gluon [1,2] and ghost propagators [2,3].

See also [4] for further details on the definitions used in the computation.

The form factor H_1 is obtained computing the Lorentz-color contraction

$$H_1 = \frac{\Gamma_{Lat}\,\mathcal{G}}{\Gamma_{(i)}\,D_{gl}\ \ D_{gh}\,D_{gh}\,\Gamma_{(i)}}$$

where we have omitted the Lorentz and color indices as well as the momenta. In the above expression D_{gl} stands for the gluon propagator, D_{gh} for the ghost propagator and the vertex $\Gamma_{(i)}$ represents either Γ_{Lat} of the continuum vertex Γ_{Cont} (that differs of Γ_{Lat} by ignoring the cos-term in the definition of Γ_{Lat}).

The results reported here use the following ensembles

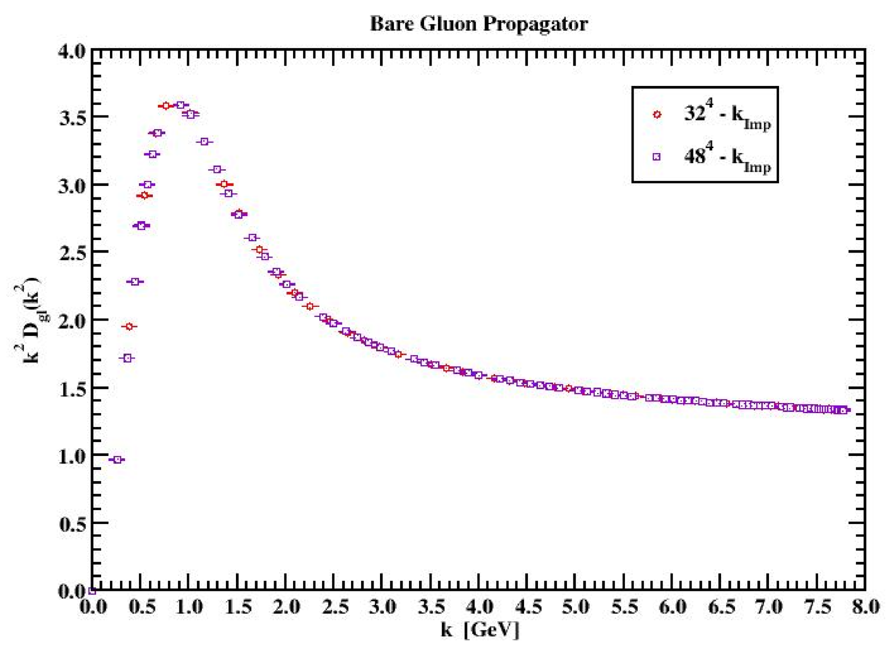
β	L	#
6.0	32	3000
6.0	48	2000

that were generated for studying the four gluon vertex [5]. The conversion to physical units uses as lattice spacing $1/a = 1.943$ GeV (see [2] and references therein). Statistical errors were computed with the bootstrap method using a confidence level of 67.5%.

Two and Three Point Functions

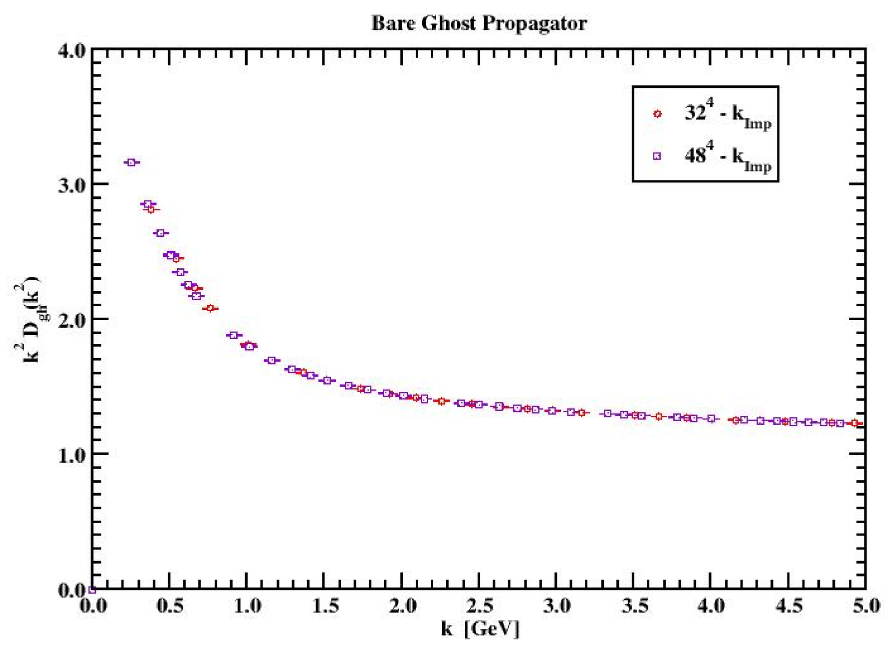
In the Landau gauge, the gluon propagator is given by

$$D^{ab}_{\mu\nu}(k) = \delta^{ab}\left(\delta_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2}\right)D_{gl}(k^2)$$

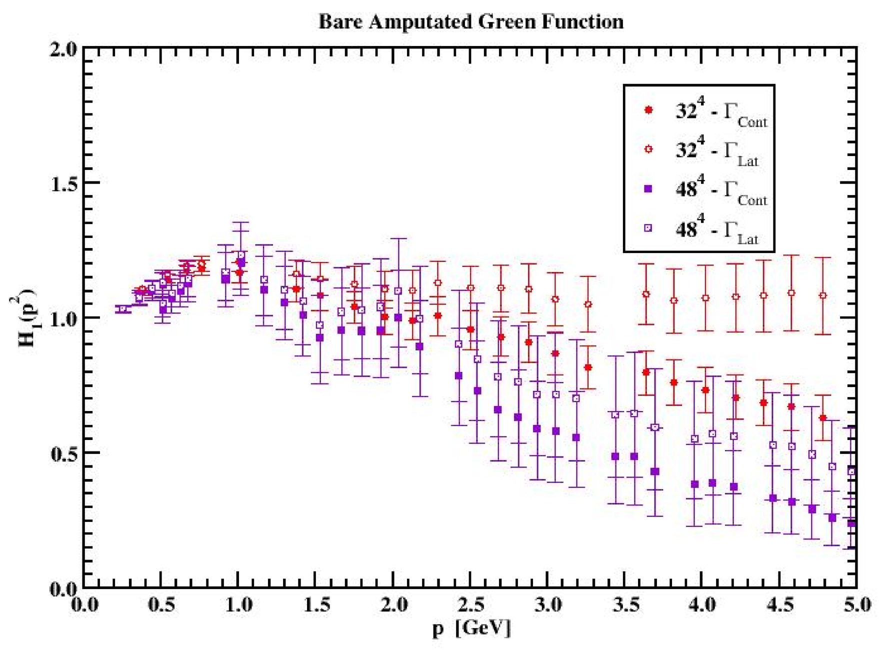


while the ghost propagator is

$$D^{ab}(k) = \delta^{ab}\,D_{gh}(k^2)$$



In the soft gluon limit defined by $q = 0$ the three point function form factor is



Summary and Conclusions

► Good agreement over a large range of momenta and specially at IR
► Good agreement with previous lattice calculations despite their large statistical errors for the SU(3) case [6,7,8]
► Lattice spacing effects for $k \gtrsim 3$ GeV that need to be understood
► H_1 seems to be flat and the data suggests that it decreases slightly at low momenta
► Increase the statistical precision of the calculation

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