Partially connected contributions to baryon masses in QCD+QED

S. Rosso[1]

[1]Consejo Superior de Investigaciones Científicas, Humboldt-Universität zu Berlin, Instituto de Física de Cantabria

Abstract Full QCD+QED simulations allow to evaluate **isospin breaking corrections** to hadron masses. With the openQ*D code, we are able to perform these simulations employing **C-periodic boundary conditions**, implemented through a doubling of the physical lattice along one spatial direction. The use of these boundary conditions introduces **non-zero Wick contractions between two quark or two antiquark fields**, that, in the case of the computation of **baryon masses**, lead to **partially connected additional contributions** that we expect to vanish in the infinite volume limit. These contributions are challenging because they involve an **all-to-all propagator** connecting one point in the physical lattice and one in the mirror lattice. We present a way to compute these corrections to the Ω^- baryon mass using a combination of **point and stochastic source inversions**. This work is part of the program of the **RC* collaboration**.

1. Motivation

In order to shed light on possible violations of the Standard Model lattice QCD simulations aim to reach **percent or sub-percent precision level**. To attain such precision, isospin breaking effects must be accounted for in the measurements of numerous hadronic observables, including meson and **baryon masses**. These effects do not come only from the **difference in mass of the light quarks** but also from the **difference in their electric charges**, for this reason the coupling of QCD to electrodynamics (QED) has to be investigated.

2. C-periodic boundary conditions

All the fields of the theory get **charge conjugated** when they cross the boundaries of the lattice, making it possible to have a total **non zero electric charge**.

$$Q = \int_0^L dx^3 \rho(x) = \int_0^L dx^3 \nabla \cdot E(x) = 0 \qquad (1)$$

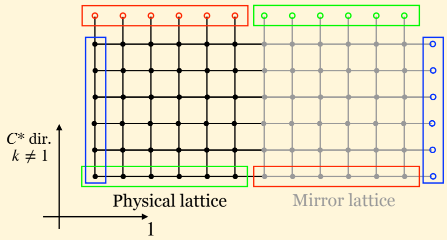
With periodic boundary conditions

$$\downarrow$$

$$Q = \int_0^L dx^3 \rho(x) = \int_0^L dx^3 \nabla \cdot E(x) \neq 0 \qquad (2)$$

With C- periodic boundary conditions

The boundary conditions are implemented in the code with an **orbifold construction**[1]



3. Modifications of Wick contractions

taking into account the boundary conditions we get **additional non-zero contributions** to quark Wick contractions[2]

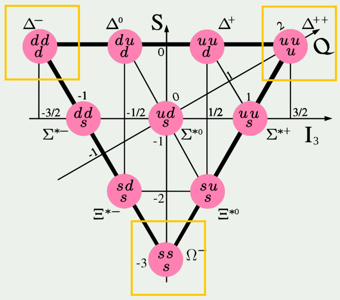
$$\langle q_a^A(x)\, \overline{q}_b^B(y)\rangle = D^{-1}(x,y)_{ab}^{AB} \qquad (3)$$

$$\langle q_a^A(x) q_b^{B,T}(y)\rangle = -\,D^{-1}(x, y+L\hat{1})_{ad}^{AB} C_{db} \qquad (4)$$

$$\langle \overline{q}_a^{A,T}(x)\overline{q}_b^B(y)\rangle = C_{ad} D^{-1}(x+L\hat{1}, y)_{db}^{AB} \qquad (5)$$

4. Correlators for decuplet vertices

Baryons correlators get **partially connected contributions** due to the modification of Wick contractions[3]. Focusing on **decuplet vertices**:



Interpolating operators :

$$v^{m;d}(x) = \sum_{\substack{abc\\ABC}} W_{abc;ABC}^{d;m} \psi_c^C(x)\psi_a^A(x)\psi_b^B(x) \qquad (6)$$

$$\overline{v}^{m;d}(x) = \sum_{\substack{abc\\ABC}} \overline{W}_{abc;ABC}^{d;m} \overline{\psi}_b^B(x)\overline{\psi}_a^A(x)\overline{\psi}_c^C(x) \qquad (7)$$

Where:

$$W_{abc;ABC}^{d;m} = \epsilon^{ABC}[P_{dc}^{ml}\Gamma_{ab}^l + P_{db}^{ml}\Gamma_{ac}^l + P_{da}^{ml}\Gamma_{cb}^l]$$
$$\overline{W}_{abc;ABC}^{d;m} = \epsilon^{ABC}[P_{cd}^{ml}\Gamma_{ab}^l + P_{bd}^{ml}\Gamma_{ac}^l + P_{ad}^{ml}\Gamma_{cb}^l] \qquad (8)$$

$$P^{ml} = [\delta^{ml} Id_{4\times4} - \frac{1}{3}\gamma^m\gamma^l]$$
$$\Gamma^l = C\gamma^l \qquad (9)$$

Two-point correlation function:

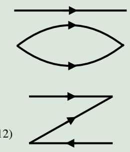
$$C(x_0) = \sum_{\substack{dd'\\m}} \sum_{\mathbf{x}} P_{dd'}^+\, v^{m;d}(x)\overline{v}^{m;d'}(0) \qquad (10)$$

$$P^+ = \frac{1+\gamma^0}{2} \qquad (11)$$

6. Types of contributions

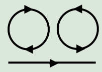
Connected: 3 quark propagators connecting x and 0

$$C(x_0) = \sum_{\substack{d'd\\m}} \sum_{\substack{\alpha bc\\\alpha' b'c'}} \sum_{\mathbf{x}} \sum_{\substack{ABC\\A'B'C'}} \Big[\overline{W}_{a'b'c';A'B'C'}^{d';m} P_{dd'}^+ W_{abc;ABC}^{d;m} \psi_a^A(x)\overline{\psi}_{a'}^{A'}(0)\, \psi_b^B(x)\overline{\psi}_{b'}^{B'}(0)\, \psi_c^C(x)\overline{\psi}_{c'}^{C'}(0)\Big] \qquad (12)$$

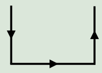


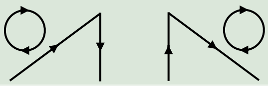
Partially connected: 1 quark propagator connecting x and 0

$$C(x_0) = -\sum_{\substack{d'd\\m}} \sum_{\substack{\alpha bc\\\alpha' b'c'}} \sum_{\mathbf{x}} \sum_{\substack{ABC\\A'B'C'}} \Big[\overline{W}_{a'b'c';A'B'C'}^{d';m} P_{dd'}^+ W_{abc;ABC}^{d;m} \overline{\psi}_{a'}^{A'}(0)\overline{\psi}_{b'}^{B'}(0)\, \psi_c^C(x)\overline{\psi}_{c'}^{C'}(0)\, \psi_b^B(x)\psi_a^A(x)\Big] \qquad (13)$$



$$= -\sum_{\substack{d'd\\m}} \sum_{\substack{\alpha abc\\\alpha' a'b'c'}} \sum_{\mathbf{x}} \sum_{\substack{ABC\\A'B'C'}} \Big[\overline{W}_{a'b'c';A'B'C'}^{d';m} P_{dd'}^+ W_{abc;ABC}^{d;m} C_{a'\alpha'} D^{-1}(L\hat{1},0)_{\alpha'b'}^{A'B'} D^{-1}(x,0)_{cc'}^{CC'} D^{-1}(x, x+L\hat{1})_{b\alpha}^{BA} C_{\alpha a}\Big] \qquad (14)$$





7. Strategy for computing partially connected contributions

Point sources can be used to compute the inverse of the Dirac operator at a fixed source point, while stochastic sources can be used to obtain the all-to-all propagator.

Point sources

$$\eta(z)_{Vv}^{(A\alpha)} = \delta_{VA}\delta_{v\alpha}\delta_{0,z} \qquad (15) \qquad \xi(z)_{A'\alpha'}^{(B'b')} = D^{-1}(z;z')_{\alpha'v}^{A'V}\eta(z')_{Vv}^{(B'b')} \qquad (16)$$

Stochastic sources

Using the identity:

$$\frac{1}{N_s}\sum_n \chi(x)_a^{(n)\dagger A}\chi(y)_b^{(n)B} = \delta_{AB}\delta_{ab}\delta_{xy} \qquad (17)$$

$$D^{-1}(x; x+L\hat{1})_{b\alpha}^{BA} =$$

$$\frac{1}{N_s}\sum_n D^{-1}(x,z)_{be}^{BE}\chi^{(n)}(z)_e^E \ \ \chi^{\dagger(n)}(x+L\hat{1})_\alpha^A = \frac{1}{N_s}\sum_n [D^{-1}\chi^{(n)}]_b^B(x)\ \ \chi^{\dagger(n)}(x+L\hat{1})_\alpha^A \qquad (18)$$

Defining:

$$T_{\alpha'b'c',\alpha bc}^{A'B'C',ABC} = \sum_{\substack{d'd\\m}} \overline{W}_{a'b'c';A'B'C'}^{d';m} P_{dd'}^+ W_{abc;ABC}^{d;m} C_{a'\alpha'} C_{\alpha a} \qquad (19)$$

$$C(x_0) = -\sum_{\substack{abc\\a'b'c'}} \sum_{\mathbf{x}} \sum_{\substack{ABC\\A'B'C'}} T_{a'b'c',abc}^{A'B'C',ABC} \xi(L\hat{1})_{A'a'}^{(B'b')} \xi(x)_{Cc}^{(C'c')} \frac{1}{N_s}\sum_n \Big[D^{-1}\chi^{(n)}\Big](x)_b^B\ \ \chi^{\dagger(n)}(x+L\hat{1})_a^A \qquad (20)$$

8. Implementation

The code is parallelized dividing the global lattice in local sublattices. Due to the presence in equation (20) of the stochastic source defined in the **mirror lattice**, communication between different MPI processes is needed. To reduce the amount of memory needed, the sum in the spatial coordinates of the sink point is the first sum to be performed.

The code is currently under testing, using check programs to verify **gauge invariance** and **translational invariance**

Conventions In every equation capital indices are colour indices, indices in [a,d] are Dirac indices, in [l,m] spin indices and in [x,z] space-time indices

References

[1] Campos et al., **openQ*D code: a versatile tool for QCD+QED simulations**, The European Physical Journal C 80, 195 (2020)

[2] Lucini et al., **Charged hadrons in local finite-volume QED+QCD with C* boundary conditions**, J. High Energ. Phys. 2016, 76 (2016).

[3] Bushnaq et al., **First results on QCD+QED with C* boundary conditions**, J. High Energ. Phys., 2023, 12 (2023)