Scattering phase shift in Gross-Neveu model

Muhammad Asaduzzaman*, Zheyue Hang*, Goksu Can Toga, Simon Catterall**, Yannick Meurice***

*University of Iowa, Iowa City, IA 52242, USA

**Syracuse University, Syracuse, NY 13210, USA

Abstract

We compute scattering phase shift due to interaction between different flavors of fermions in a (1+1) D lattice toy model- Gross-Neveu model. The model shares some similar characteristics with QCD

– Asymptotically free relativistic field theory.

– Dynamical mass generation.

Real time evolution of the model with quantum computer has been investigated in [1]. There has been recent investigations of scattering with quantum computers for different lattice models [2-5]. Here, we explore further on finding the phase shifts due to the interaction introduced between different flavors of fermions on the same lattice site.

Introduction

Lattice Hamiltonian [1] for *L*-spatial sites and *N*-flavors of fermions in terms of reduced staggered fields χ_n^f

$$H^{(N)} = \sum_{n=1}^{L}\left[i\sum_{f=1}^{N} \chi_n^{\dagger f}\left[\chi_{n+1}^f - \chi_{n-1}^f\right] + m(-1)^n \chi_n^{\dagger f}\chi_n^f + G^2\left(\sum_{f=1}^{N}\chi_n^{\dagger f}\chi_n^f\right)^2\right]$$

where, *f* is the flavor index and *n* is the lattice site index. Qubitization of the model is performed using Jordan-Wigner transformation

$$H^{(N)} = \sum_{n=1}^{L-1}\left[\sum_{f=1}^{N}\left(-X_n^f Y_{n+1}^f + Y_n^f X_{n-1}^f + (-1)^n m(1-Z_n^f) + \frac{G^2}{2}\sum_{g,g>f}(I-Z_n^f)(I-Z_n^g)\right)\right].$$

Now, time-evolution of a state can be performed using some local-rotation and entangling operations

$$\mathcal{U}(t) = \exp(-i\delta t\, X_{fL+n}Y_{fL+n+1})\exp(i\delta t\, Y_{fL+n}X_{fL+n-1})\exp(-iG^2\delta t\, Z_{fL+n}Z_{gL+n})$$

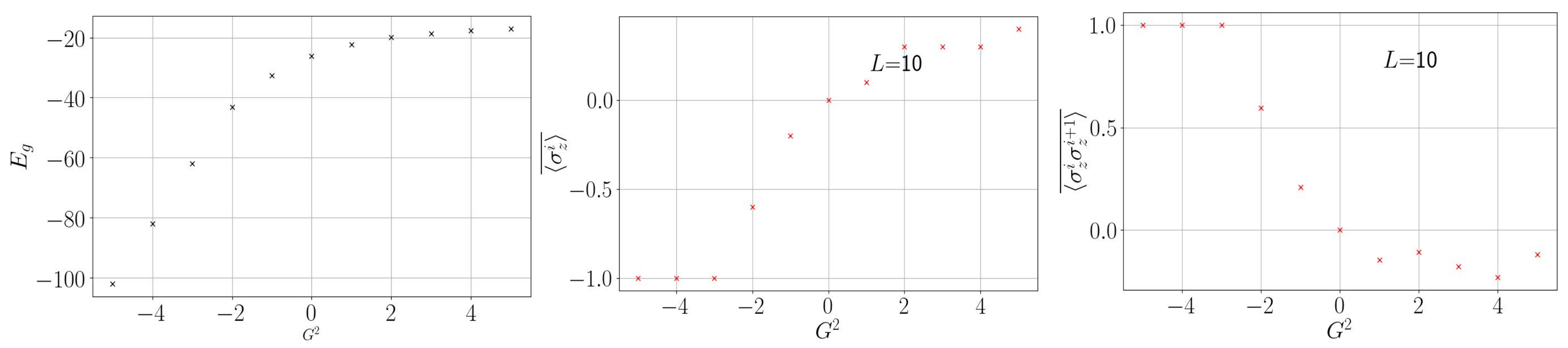


Fig 1. Gap(left), average Magnetization (middle) and average same-site correlation of the magnetization of two flavors of fermions (right).

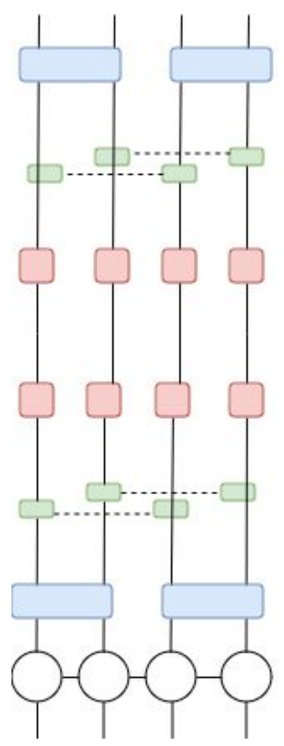


Fig 2: TEBD circuit structure for the time evolution of the MPS.

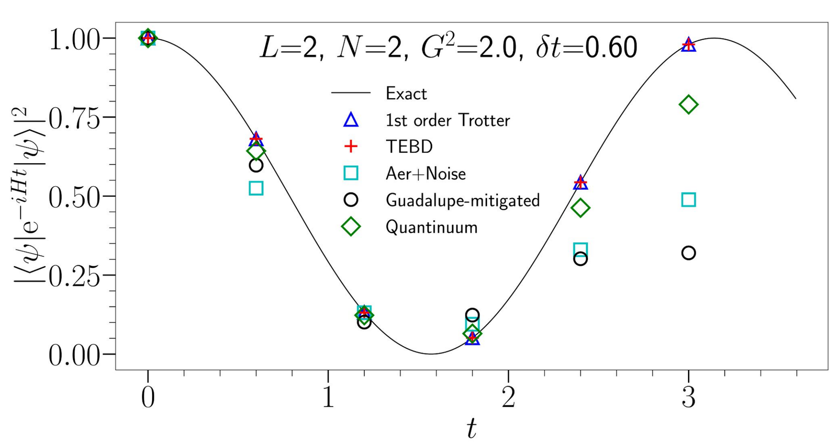
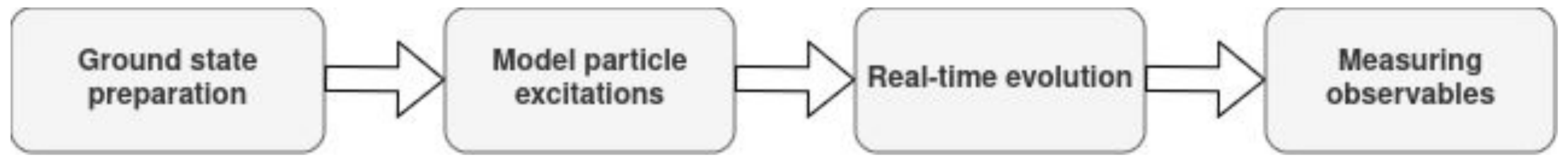


Fig 3. Real time evolution of a state with IBM's Guadalupe and Quantinuum's H1-device with 4000 and 300 shots respectively.

Computing Time Delay of Wavepacket



1. Prepare the ground state: $|\Omega\rangle$
2. Create two wavepackets in the position space separated by a finite distance *N/2*

$$\psi_w(x^{(1)}, x^{(2)}) = \frac{1}{N}\left(e^{ik^{(1)}x^{(1)} - (x^{(1)}-N/4)^2/(N/8)^2} e^{-ik^{(2)}x^{(2)} - (x^{(2)}-3N/4)^2/(N/8)^2}\right)$$

3. We time evolve the resulting excitation to obtain the evolved state $|\psi_f\rangle$.
4. Find projections on the (slowest) left and the right moving momentum eigenstates.

$$|k_\pm^{(f)}\rangle = \frac{1}{\sqrt{N}}\sum_{j=0}^{N-1}\exp\left(\pm i\frac{2\pi}{N}j^{(f)}\right)|j^{(f)}\rangle \qquad P_\pm^{(f)} = \langle k_\pm^{(f)}|\psi_f\rangle$$

5. Compute rescaled probability with and without interaction

$$R_-^{(G^2)} = \frac{P_-^{(1)}}{P_-^{(1)} + P_+^{(1)}}$$

6. Compute time delay:

$$T_W(G^2, k) = t(R_-^{(0)} = 0.5) - t(R_-^{(G^2)} = 0.5)$$

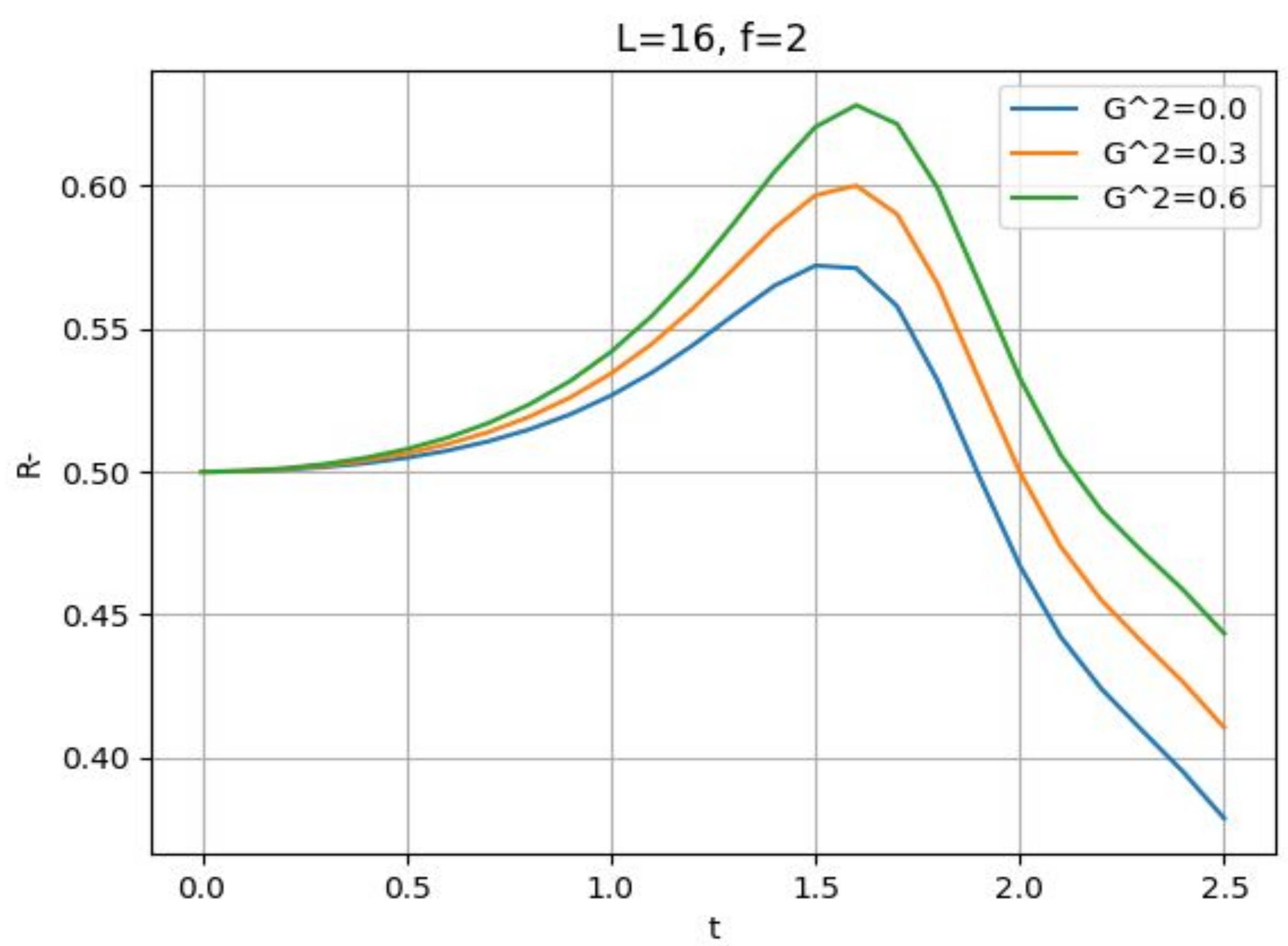


Fig 4. Phase delay due to interaction can be computed from the scaled return probability. The phase delay can be computed from the delay at $R_- \sim 0.5$.

Results

Parameters:

Matrix product state (MPS) based Time Evolving Block Decimation (TEBD) algorithm used for the simulation.

1. lattice site: 16
2. no of flavor of fermions: 2
3. chosen momenta: $k = \frac{2\pi}{N}$

Results:

Computed time delay after scattering due to interaction

$$T_W(0.3, \pi/8) = 0.1039 \qquad T_W(0.6, \pi/8) = 0.2343$$

Future Prospects

Using the information of the time delay due to interaction, the scattering phase shift can be estimated.

$$2\delta(k_f) = 2\delta(k_0) + \int_{k_0}^{k_f} dk T_W(k)(\partial E/\partial k)$$

Additional steps to estimate time delay using quantum simulation?

— applying efficient state preparation algorithm for the ground state and the wave packet creation, QETU algorithm [6].

— perform inverse Fourier transform for the measurements in the momentum basis [7].

— investigate the impact of different sources of noise in the time-delay calculation: coherent and incoherent. Demonstrate importance of the error mitigation in solving scattering problems with real time evolution.

Contact

Muhammad Asaduzzaman (Asad): masaduzzaman@uiowa.edu

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