Towards the β function of SU(2) with adjoint matter using Pauli–Villars fields

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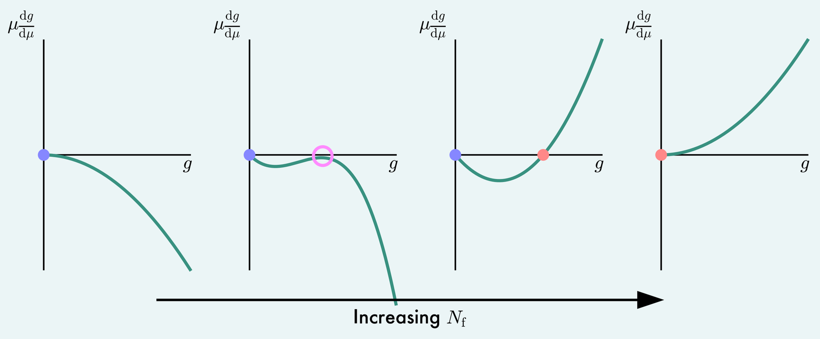
Context

For a non-Abelian theory with a given gauge group, for small numbers of massless flavours N_f the behaviour is expected to be QCD-like, while for very large N_f asymptotic freedom is lost. In between these extremes, there is expected to be an intermediary region where the theory is conformal: the *conformal window*. Theories near the lower edge of this window are interesting from a number of perspectives, not least because they may allow a light scalar Higgs state to arise as a pseudo-Nambu–Goldstone boson of a broken symmetry of a new strongly-coupled dynamics, with an anomalous dimension sufficiently large to account for experimental constraints.



The SU(2) theory with 2 adjoint Dirac flavours has been studied extensively on the lattice; spectroscopic results and studies of the step scaling beta function are consistent with it being well inside the conformal window. Its anomalous dimension has been found (at finite lattice spacing) to be 0.304(4) [1]. The theory with 1 adjoint flavour has proven more challenging to study: it shows signs of chiral symmetry breaking, but is not consistent with chiral perturbation theory, and can be fitted with a hyperscaling Ansatz. At large lattice spacing it shows a large anomalous dimension O(1), decreasing to 0.170(7) in the continuum limit [1].

In this work we aim to use the gradient flow beta function to observe the fixed point position and anomalous dimension in the 2-flavour theory, and whether a fixed point or any remnant thereof is visible in the 1-flavour theory.

Implementation

We generate ensembles using the Wilson gauge action and Wilson fermion action.

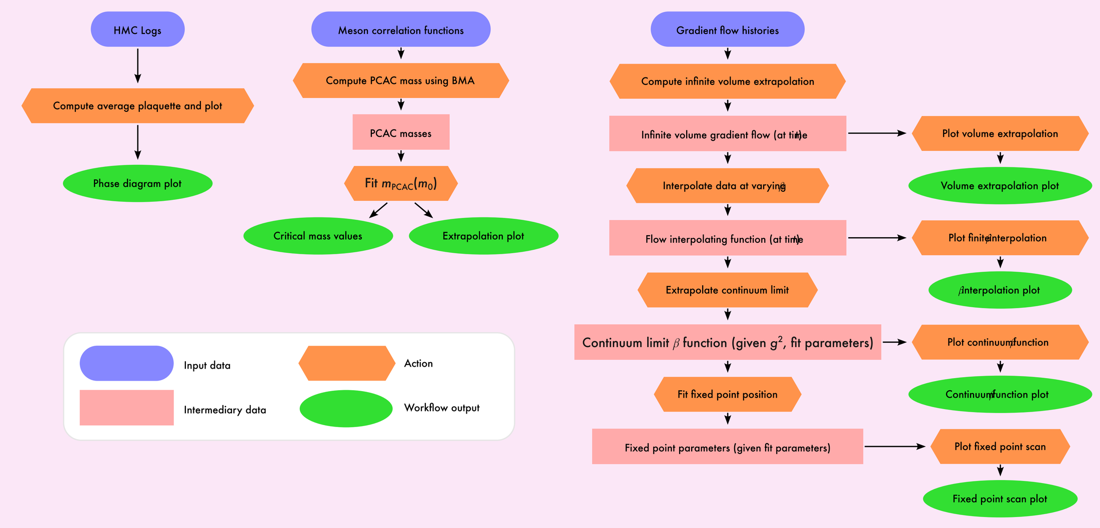
$$S_{\mathrm{G}} = \beta_0 \sum_p \mathrm{Tr}\left[1 - \frac{1}{2}U(p)\right] \qquad S_{\mathrm{F}} = \sum_{\alpha=1}^{N_{\mathrm{f}}} \overline{\psi}_\alpha(x)\left(i\not{D} - m\right)\psi_\alpha(x)$$

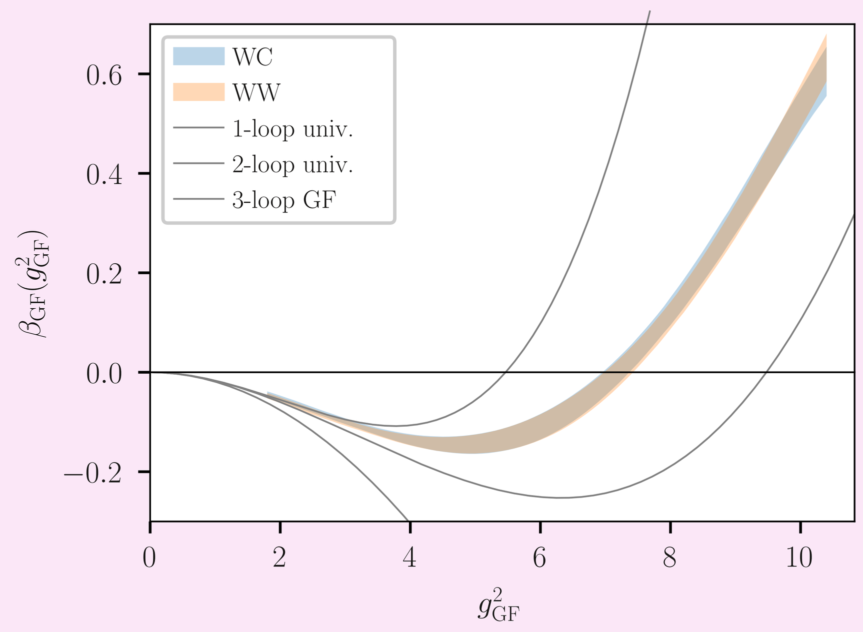
To this, we add N_{PV} species of Pauli–Villars field with mass m_{PV}, to widen the range of β_0 that can be probed [2].

The Hybrid Monte Carlo algorithm is used for the two-flavour theory, while the RHMC is used for the one-flavour theory. Ensembles are generated using the HiRep [3] code, both on CPUs and A100 GPUs. GNU Parallel is used to manage large swarms of small jobs. Meson correlation functions and gradient flow histories are computed using HiRep on CPUs.

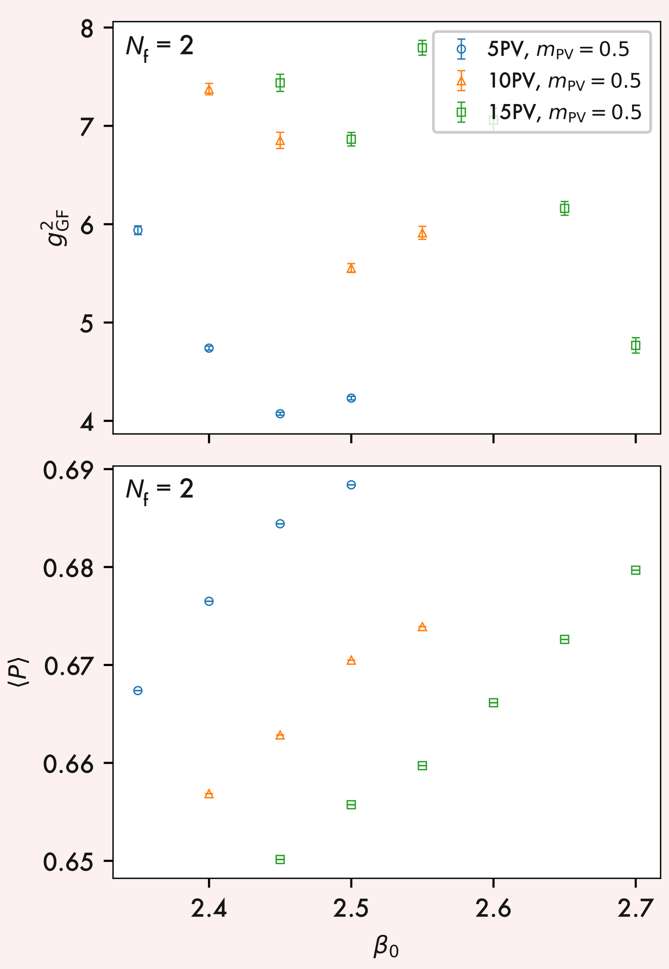
Analysis workflow

The data analysis pipeline is written using Snakemake, connecting individual tools each making use of pyerrors. This allows simple rules (e.g. how to transform one or more input data files to an output containing one or more statistical quantities) to be easily composed, independent steps to be parallelised, and redundant computations to be skipped.
To generate every plot on this poster, excluding the one in this box, starting from the raw output files transferred from HPC, requires a single command (twice: once per N_f), which launches 344 + 81 job steps taking 3 minutes on a 6-core laptop. All required packages are installed automatically; no setup is required beyond installing Snakemake.





This workflow was tested by adapting it to work with the open data released by Hasenfratz and Peterson [4]. This reproduces their finding of a conformal fixed point in the beta function of the SU(3) theory with 12 fundamental Dirac flavours.. Reproducing every figure of the corresponding paper deriving from the released data requires a single command, which launches 4227 job steps taking 19 minutes on a 6-core laptop.



Spanning of parameter space

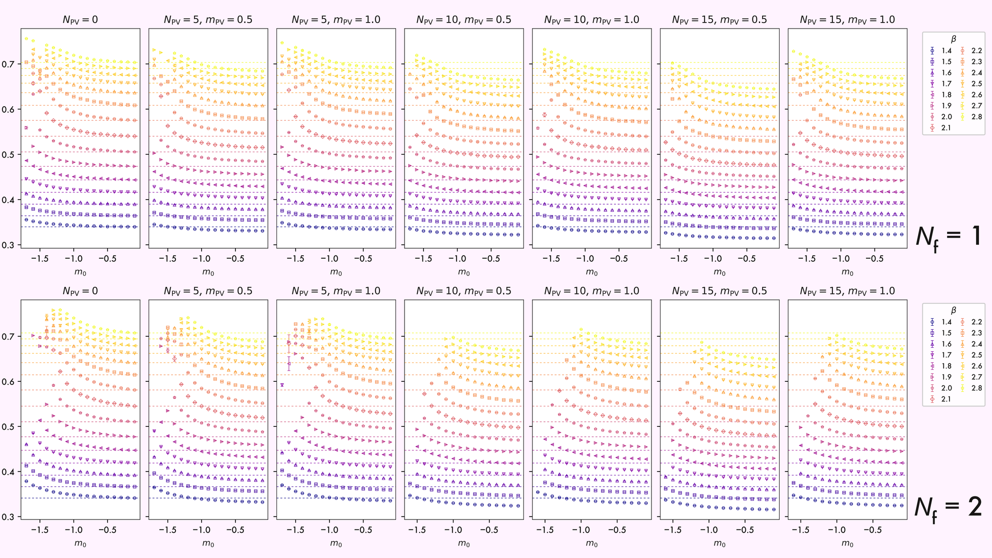
Studies of SU(3) with fundamental matter suggest that we should observe an increase in the plaquette value as more Pauli–Villars fields are added. We do not observe this; instead, we are able to reach lower plaquette values without hitting the bulk phase, as seen in the $N_f = 2$ data to the left.

It is unclear whether this difference is due to a physical difference such as the different fermion representation, or an algorithmic one such as the use of the Wilson fermion action.

Looking at the range of values of the running coupling spanned, we see that adding more Pauli–Villars fields does appear to enable this; however, the data for $N_{PV} = 15$ are currently insufficient to give a reliable infinite-volume extrapolation, so the full range of accessible couplings is not yet known.

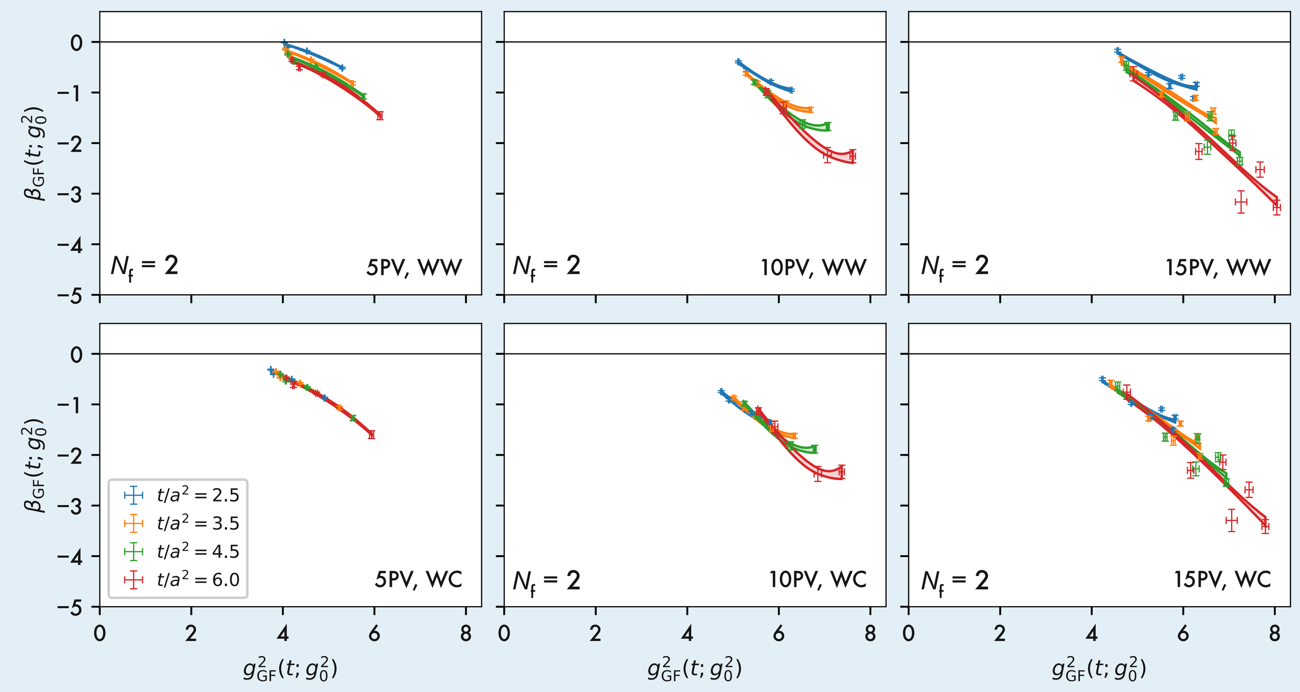
Phase diagram

To understand the phase structure of the theory and where to target computations to avoid the bulk phase, we study the average plaquette on an 8^4 lattice, with three values of $N_{PV} = 5, 10, 15$ and two values for $m_{PV} = 0.5, 1.0$. In each case we scan $\beta_0 \in [1.4, 2.8]$, $m_0 \in [-1.4, -0.1]$. For comparison, we plot the heaviest mass in the theory with no Pauli–Villars fields for each value of β considered as a dashed horizontal line.
For $N_f = 2$ we then focus on the cases $N_{PV} = 5, 10, 15$, $m_{PV} = 0.5$, while for $N_f = 1$ we focus on the cases $N_{PV} = 15$, $m_{PV} = 0.5$.



Interpolating $N_f = 2$ at finite running coupling

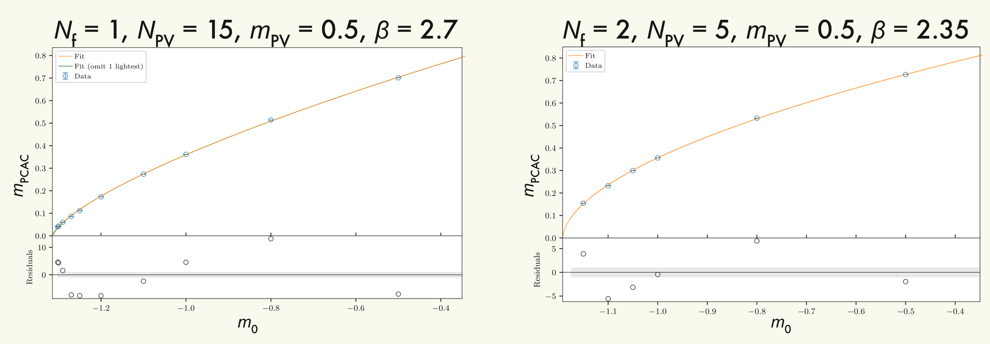
The range of β_0 considered in production runs gives a relatively small range of running couplings for $N_{PV} = 5, 10$. For $N_{PV} = 15$, there are currently insufficient statistics to give a reasonable interpolation, but the data appear to still be far from the fixed point.



Critical mass tuning

As the Wilson fermion action introduces an additive renormalisation to the fermion mass, we must tune the bare fermion mass to identify the chiral point at which to produce ensembles to study. We generate test ensembles on a 32×16^3 lattice, and compute the PCAC mass as a renormalised fermion mass, which we then fit using the Ansatz

$$m_{\mathrm{PCAC}} = B(m_0 - m_0^{\mathrm{cr}})^C$$



Towards the beta function

The running coupling at finite volume and lattice spacing can be computed as a function of the gradient flow time t as

$$g^2_{\mathrm{GF}}(t; L, g_0^2) = \langle t^2 E(t) \rangle$$

where E can be computed via a plaquette or via a clover operator. The β function can then be computed as:

$$\beta(t; L, g_0^2) = t\frac{\mathrm{d}}{\mathrm{d}t} g^2_{\mathrm{GF}}(t; L, g_0^2)$$

where we may fit subsets of the volumes considered and compute the fit results' mean weighted using a modified Akaike Information Criterion.

Each of these can be extrapolated to the infinite-volume limit using a linear fit form in the reciprocal of the lattice volume:

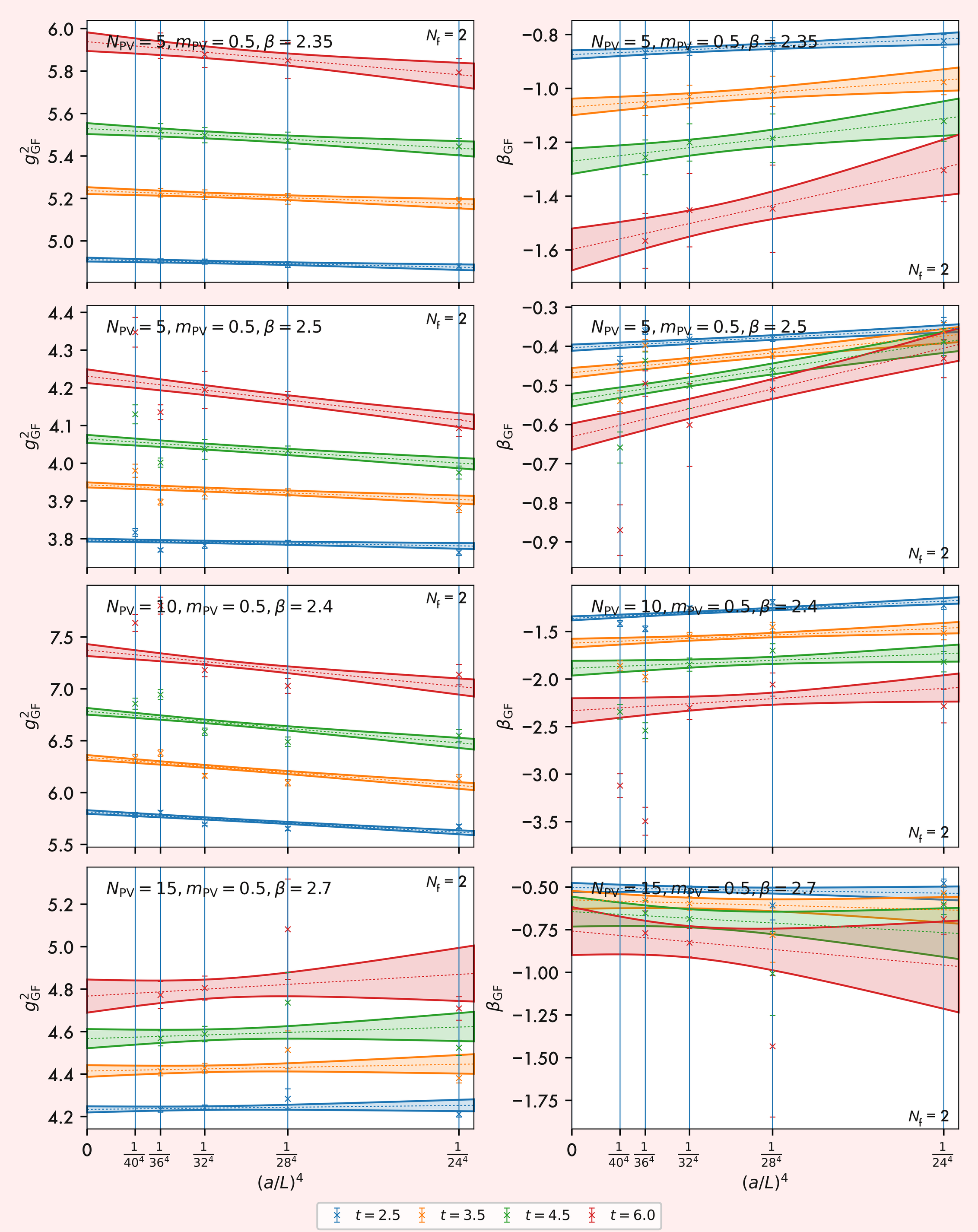
$$g^2_{\mathrm{GF}}(t; L, g_0^2) = g^2_{\mathrm{GF}}(t; g_0^2) + C_{g^2} L^{-4}$$
$$\beta(t; L, g_0^2) = \beta(t; g_0^2) + C_\beta L^{-4}$$

These may then be interpolated against each other to obtain the β function at arbitrary running coupling.

$$\beta^{\mathrm{int}}(t, g^2_{\mathrm{GF}}) = g^4_{\mathrm{GF}} \sum_{n=0}^{N-1} p_n g^{2i}_{\mathrm{GF}}$$

with N fixed to be the lowest value that gives a reasonable fit. This allows the β function to be scanned across a range of running coupling, once the continuum limit is taken via extrapolating a linear Ansatz

$$\beta^{\mathrm{int}}(t, g^2_{\mathrm{GF}}) = \beta^{\mathrm{cont}}(g^2_{\mathrm{GF}}) + \frac{C_{\mathrm{cont}}}{t}$$



Infinite volume extrapolation

Based on preliminary data, with ensemble generation still ongoing, for the two-flavour theory, we observe that in some cases there is a significant shift in the β function at large volumes, which may be due to severe finite volume effects at smaller volumes, or insufficient statistics in the larger, more expensive ensembles. $N_f = 1$ data are not yet available for sufficiently many volumes to perform an extrapolation.

Conclusions

We are progressing towards a study of the gradient flow running coupling for SU(2) with 1 and 2 flavours of adjoint Dirac fermion, using Pauli–Villars fields to expand the range of coupling we can study. This expansion is more limited than observed in SU(3) candidate BSM models. In the two-flavour case, we are currently far from the fixed point, and would need to go to significantly stronger coupling to observe it. In the one-flavour case, more data at large volumes are needed to reach the infinite volume limit before further conclusions may be drawn. These ensembles are currently thermalising.

References and Resources

[1] Bennett, E. et al. (2024). SU(2) gauge theory with one and two adjoint fermions towards the continuum limit. In preparation.
[2] Pica, C. et al. (2024). *HiRep*. https://github.com/claudiopica/HiRep
[3] Hasenfratz, A. et al. (2021). *Taming lattice artifacts with Pauli–Villars fields.* Phys.Rev.D 104 (2021) 7, 074509. arXiv:2109.02790
[4] Peterson, C., & Hasenfratz, A. (2024). *Twelve flavor SU(3) gradient flow data for the continuous beta-function* [Data set]. Zenodo. https://doi.org/10.5281/zenodo.10719052

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