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## One-loop Analysis for QCD Schrodinger functional

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The one-loop matching between the overline coupling and the MS coupling is done in two steps. First, we calculate the relation of the overline coupling to a renormalised lattice coupling. Combination with the known one-loop relation between the lattice and the MS couplings then yields the desired result.

\subsection{The basic calculation}

Expand  $\bar{g}(L)$  the running coupling in terms of  $g_0$  the overline coupling:

\begin{equation}

$$\overline{g}^2 = g_0^2 + p_1 g_0^4 + O(g_0^6),$$

\end{equation}

where the coefficient  $p_1$  depends on the number of flavors  $n_f$  the overline quark mass in lattice units  $m_0$  and the lattice size  $l \equiv \frac{L}{a}$ . For later convenience this dependence is split into the pure gauge part and the quark contribution:  $p_1 = p_1(n_f, z, l) = p_{1,0}(l) + n_f p_{1,1}(z, l)$ , (1) with:

$$z = \bar{m}L, \quad \bar{m} = \frac{1}{a} \ln(1 + m_0 a). \quad (2)$$

Note that expansion of  $\bar{m}$  w.r.t. small lattice spacing  $a$  gives the first order result  $\bar{m} = m_0$ .

Now expand  $g_0$  in terms of renormalised coupling  $g_{lat}(\mu)$  at the  $\mu$  scale gives:

$$g_0^2 = g_{lat}^2 + z_1 g_{lat}^4 + O(g_{lat}^6), \quad (3)$$

$$z_1 \equiv z_{1,0}(a\mu) + n_f z_{1,1}(a\mu) = 2b_0(n_f, 0) \ln(a\mu), \quad (4)$$

where  $b_0(n_f, 0)$  is known:

$$b_0(n_f, 0) = \frac{1}{4\pi^2} \left( 11 - \frac{2}{3} n_f \right) \quad (5)$$

Then we have the relation between renormalised coupling constants  $\bar{g}$  and  $g_{lat}$ :

$$\bar{g}^2 = g_{lat}^2 + (p_1 + z_1) g_{lat}^4 + O(g_{lat}^6). \quad (6)$$

The goal is to calculate the quark field contribution  $p_{1,1}$ . Using the standard Wilson plaquette action, we have:

\begin{equation}

$$p_{1,1}(z, l) = (k n_f)^{-1} \frac{\partial}{\partial \eta} \ln \det(D_5) |_{\eta=0},$$

\end{equation}

where:

\begin{align}

$$k = 12 l^2 \left( \sin\left(\frac{\pi}{3l^2}\right) + \sin\left(\frac{2\pi}{3l^2}\right) \right) = 12 (L/a)^2 \left[ \sin\left(\frac{1}{3}\pi(a/L)^2\right) + \sin\left(\frac{2}{3}\pi(a/L)^2\right) \right]$$

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$$D_5 = \gamma_5 (D + m_0)$$

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$$D = \sum_{\mu} \frac{\gamma_{\mu}}{2} (\nabla_{\mu} + \nabla_{\mu}^{-1}) - \frac{a}{2} \nabla_{\mu} \nabla_{\mu}$$

$$\begin{aligned}
& + \sum_{\mu\nu} c_{sw} \frac{ia}{4} \sigma_{\mu\nu} P_{\mu\nu} \\
& \backslash \\
& \sigma_{\mu\nu} = \frac{i}{2} [\gamma_{\mu}, \gamma_{\nu}] \\
& \end{aligned}$$

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