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Finite-temperature critical point of heavy-quark QCD on large lattices

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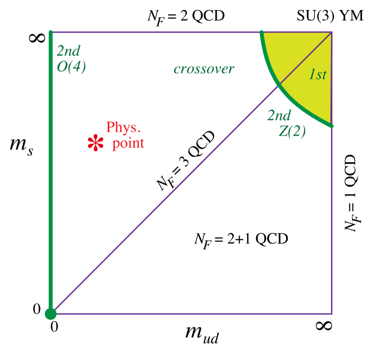
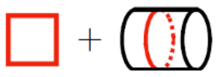
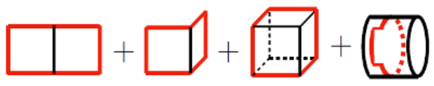
0. QCD in the heavy-quark region



Properties around the physical point may be affected by nearby **C**ritical **P**oints.
CP in the light-quark side turned out to be more distant ⇒ **CP in the heavy-quark side**
Binder cumulant analysis for precise CP ⇒ **large lattices required for FSS** [1]
⇒ **H**opping **P**arameter **E**xpansion (heavy-quark expansion) to simulate large lattices:

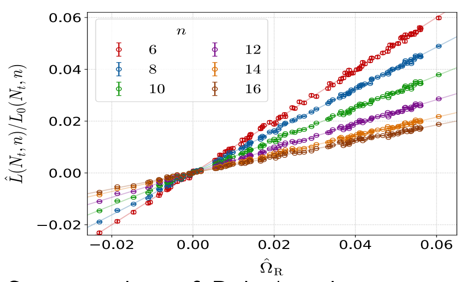
$S_{\rm LO} \sim$  pHB+OR *à la* pure YM applicable

$S_{\rm NLO} \sim$  incorporated by reweighting

⇒ **total cost ≈ qQCD** [1]

1. Effective incorporation of high-order terms of HPE

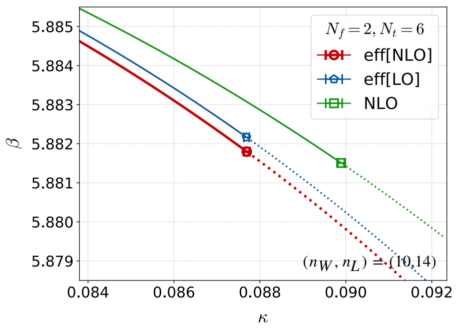
Convergence study of HPE ⇒ NLO sufficient down to CP at *Nt*=4, but higher orders needed at *Nt*≥6 [2].
Incorporate high-orders using strong linear correlation among different order terms of HPE:



Scatter plot of Polyakov-loop type operators of HPE at *Nt*=6 [3].

eff.[LO] method [2]
Incorporate NLO and higher orders (up to (n_W, n_L)th) by shifting the couplings in $S_{\rm LO}$.

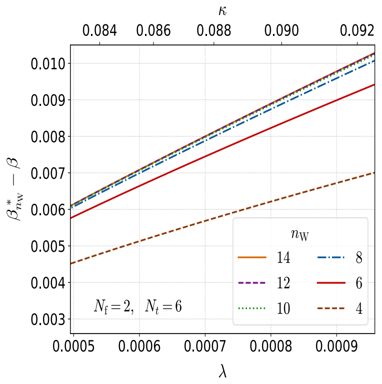
eff.[NLO] method [3]
Incorporate NNLO and higher orders by shifting the couplings in $S_{\rm NLO}$.
Superior to eff.[LO] ⇐ correlation stronger with smaller order-differences.



Test by the phase diagram at *Nt*=6 [3]:

- Transition line and CP with eff.[LO/NLO] shift from NLO
 ⇒ **NNLO and highers important at *Nt*≥6.**
- Dependence on the truncation order of HPE in eff.[LO/NLO] ⇒ **convergence of HPE** for $(n_W, n_L) \geq (10{,}14)$

We adopt eff.[LO/NLO] with $(n_W, n_L) = (10{,}14)$ at *Nt*=6.

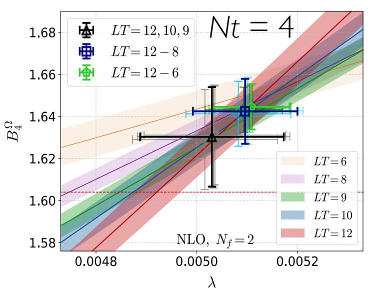


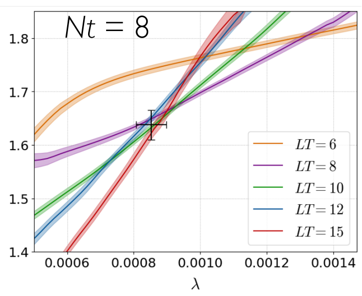
2. Simulations

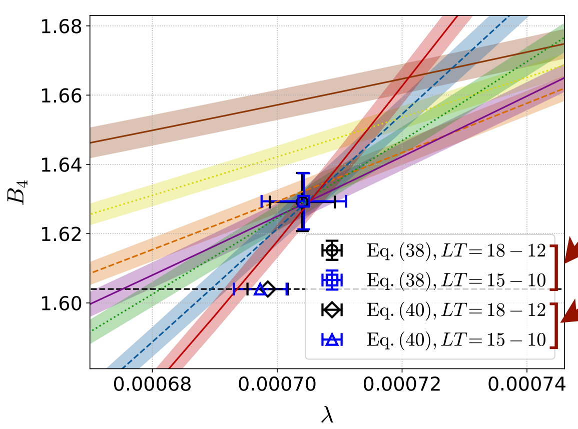
$S_g + S_{\rm LO} = -6N_{\rm site}\beta^*\hat{P} - \lambda N_s^3 {\rm Re}\hat{\Omega}$, $\beta^* = \beta + 16N_cN_f\kappa^4$, $\lambda = 2^{N_t+2}N_cN_f\kappa^{N_t}$, $\hat{\Omega}$ = Polyakov loop

Binder cumulant $B_4 = \dfrac{\langle {\rm Re}\hat{\Omega}^4\rangle_c + 3}{\langle {\rm Re}\hat{\Omega}^2\rangle_c^2}$ ⇐ multi-point reweighting to vary coupling parameters continuously.

- ☑ *Nt* = 4, *LT*=*Ns*/*Nt*=6-12, NLO ⇒ *LT* ≥ 9 required for FSS [1]
- ☑ *Nt* = 6, *LT*=*Ns*/*Nt*=6-18, eff.[LO/NLO] ⇒ *LT* ≥ 10 required [3]
- ☐ *Nt* = 8, *LT*=*Ns*/*Nt*=6-15, eff.[LO/NLO] ⇒ *LT* ≥ 10 required [4]







Results at *Nt*=6 [3]:

- Larger violation of FSS on finer lattice at *LT*≤9. ⇒ larger *LT* required.
- FSS fit $B_4 = b_4 + c(\lambda - \lambda_c)(LT)^{-1/\nu}$ works well with *LT* ≥ 10.
 ⇒ ν: consistent with 0.630 of Z(2), but b_4: 2σ discrepant from 1.604 of Z(2).
- **Fit with mixing of energy-like op.** $B_4 = (b_4 + c(\lambda - \lambda_c)(LT)^{-1/\nu})(1 + d(LT)^{Y})$
 works well by fixing b_4, ν, Y to Z(2) (*LT*≥10, $\chi^2/{\rm dof} \sim 0.5-1$). ⇐ Full 6 params fit unstable.

⇒ $\kappa_c = 0.08769(7)(^{+11}_{-0})$ for $N_{\rm f} = 2$ [cf. 0.0877(9) Cuteri+ ('21) fQCD, *Nt*=6, *LT*=4-7]

3. Toward the continuum limit of CP in physical units

Results of (β_c, κ_c) at *Nt*=4, 6, 8, combined with $m_{\rm PS}a$ at *T*=0 at the same (β_c, κ_c) estimated from previous studies, we find $m_{\rm PS}/T_c \approx 16.30(3),\ 18.04(4),\ 17.2(2)$ (*preliminary*) at *Nt*=4, 6, 8, respectively. ⇒ ***Nt*-dep. (*a*-dep.) looks mild.**

Publications:

1. A. Kiyohara, M. Kitazawa, S. Ejiri, K. Kanaya, Phys.Rev.D 104, 1144509 (2021)
2. N. Wakabayashi, S. Ejiri, K.Kanaya, M. Kitazawa, PTEP 2022, 033B05 (2022)
3. R. Ashikawa, M. Kitazawa, S. Ejiri, K. Kanaya, arXiv:2407.09156 (2024)
4. H. Sugawara, E. Ejiri, K. Kanaya, M. Kitazawa., *in preparation*