Semiconductor quantum simulator for lattice gauge theories

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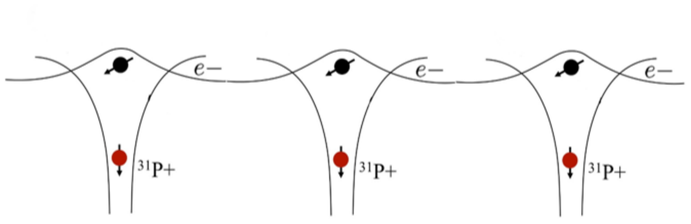
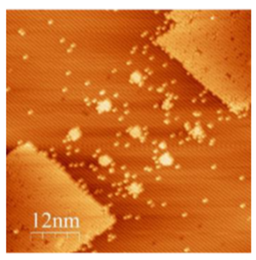
1. Motivation: The Dopant Platform

- **Why Quantum Simulation:** Classical computation of lattice gauge theories (LGTs) becomes increasingly time and resource-intensive for real-time dynamics and finite density systems. Quantum simulation is a promising approach to take advantage of the entanglement and superposition properties of qubits to overcome this problem.
- **Challenges with a digital approach:** High encoding cost for fermionic d.o.f in higher dimensions. High encoding and gate cost for large spin (bosonic) d.o.f.
- **Solution:** Arrays of donor atoms in a silicon host gives controllable fermionic d.o.f without encoding and large nuclear spin (bosonic) d.o.f. This enables using analog simulation techniques for Hamiltonians involving interactions between these d.o.f.





Wang et al, Nature Communications(2022)

- Parameters like chemical potential (μ) at the fermionic sites, tunneling (J) between them, hyperfine coupling (g) between electron and nuclear spins and external magnetic fields (h^x,h^z) are fairly tunable allowing periodic driving to engineer Hamiltonians.

$$H = \sum_n \left[J_n(t)\left(c_n^\dagger c_{n+1} + \text{h.c.}\right) + \mu_n c_n^\dagger c_n + g_n I_n^z c_n^\dagger c_n + h^z I_n^z + h^x(t) I_n^x \right]$$

2. Z_2 LGT in 1+1D

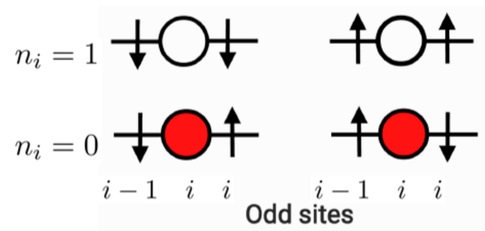
- **First step:** Theory in 1+1D with (small) spin-1/2 and staggered fermions.

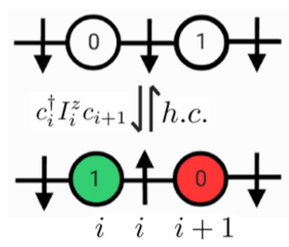
$$H_{Z_2} = \sum_n \left[J c_n^\dagger I_n^z c_{n+1} + h.c. + (-1)^n m_f c_n^\dagger c_n + f I_n^x \right]$$

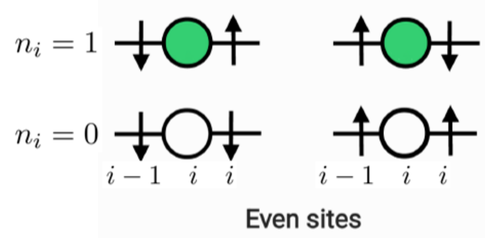
- **Gauss Law:** Physical states must obey Gauss' Law.

$$G_i \left|\psi_{\text{phys}}\right\rangle = g \left|\psi_{\text{phys}}\right\rangle, \quad [G_i, H_{Z_2}] = 0$$

$$G_i = I_i^x I_{i-1}^x e^{-i\pi\left(c_i^\dagger c_i - \frac{1-(-1)^i}{2}\right)}, \quad g = 1$$







- Given the fermionic occupation states and a single nuclear spin state, the other nuclear spins are fixed by Gauss Law.
- The gauge link associated with the hopping term updates the nuclear spin in a manner that preserves Gauss Law.

3. Floquet Engineering: Floquet-Magnus Expansion

- When parameters in the Hamiltonian are driven at high frequencies, the effective Hamiltonian is given by an inverse frequency expansion. For a small time period $T=2\pi/\Omega$:

$$U(T,0) = \mathcal{T} e^{-i\int_0^T H(t)dt} = e^{-iH_F T}$$

$$H_F = \sum_{n=0}^{\infty} H_F^{(n)}$$

$$H_F^{(0)} = \frac{1}{2\pi}\int_0^{2\pi} dt H(t)$$

$$H_F^{(1)} = \frac{1}{2!i\Omega}\frac{1}{2\pi}\int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \left[H(t_1), H(t_2)\right]$$

$$H_F^{(2)} = \frac{1}{3!i^2\Omega^2}\frac{1}{2\pi}\int_0^{2\pi} dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \left(\left[H(t_1), \left[H(t_2), H(t_3)\right]\right] + (t_1 \leftrightarrow t_3)\right)$$

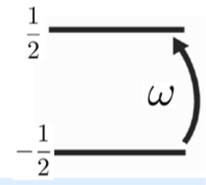
- Carefully choose the time dependence of parameters to ensure the required terms are dominant and non-vanishing. Ensure unwanted terms are sub-dominant or zero.

4. Floquet Engineering: Rotating Frame

- When Hamiltonian terms are of the same order as the drive frequency, resummation of infinite terms is required at each order in the inverse frequency expansion.
- This process can be simplified by first transitioning to the rotating frame (interaction picture) with respect to those terms.

$$\left.\begin{aligned} H(t) &= H_0 + V(t) \\ U(t) &= e^{-i\int_0^t V(t)dt} \end{aligned}\right\} \quad H^{rot}(t) = U^\dagger(t) H_0 U(t)$$

- A Floquet expansion can now be performed effectively.
- Example: Rabi Oscillations in 2-level system



$$H_{\text{lab}} = \frac{\omega}{2}\sigma_z + \Omega\cos(\omega t)\sigma_x$$

$$H^{rot} = \frac{\Omega}{2}(\sigma_x + \cos(2\omega t)\sigma_x + \sin(2\omega t)\sigma_y) \longrightarrow H_F^{(0)} = \frac{\Omega}{2}\sigma_x$$

5. Floq. Eng. Z_2: Fast Step

- Rotating frame w.r.t. chemical potential term:

$$H \to \sum_n \left[J_n(t) e^{i(\mu_n - \mu_{n+1})t} c_n^\dagger c_{n+1} + h.c. + g_n I_n^z c_n^\dagger c_n + h^z I_n^z + h^x(t) I_n^x \right]$$

- Choose $J_n(t) = J_0 + 2j_n(t)\cos\left((\mu_{n+1} - \mu_n)t\right)$ to get

$$H_F^{(0)} = \sum_n \left[j_n(t) c_n^\dagger c_{n+1} + h.c. + g_n I_n^z c_n^\dagger c_n + h^z I_n^z + h^x(t) I_n^x \right]$$

$$H_F^{(1)} \sim \mathcal{O}\left(J_0^2/\Omega_0\right) + \cdots$$

- j and h^x are slowly varying and can be treated as constants in this fast step.
- This step replaces a large J(t) (non-zero mean) with a smaller j(t) (which can average to 0). This gives more flexibility and helps with convergence in the slower step that follows.

6. Slow Step

- Rotating frame w.r.t. both hyperfine interaction and nuclear Zeeman terms and choosing time-dependence

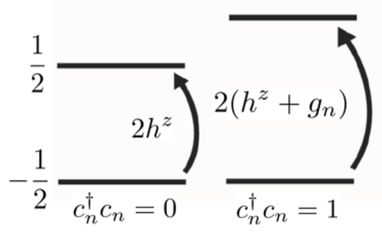
$$j_n(t) = A\sin(g_n t)\cos(g_{n+1} t), \quad (g_n \neq g_{n+1})$$

$$h^x(t) = R\left(\cos\left(2\left(h^z + g_n\right)t\right) + \cos\left(2h^z t\right)\right)$$

gives the Floquet Hamiltonian

$$H_F^{(0)} = \sum_n \left[\frac{iA}{4} c_n^\dagger I_n^z c_{n+1} + h.c. + \frac{R}{2} I_n^x \right]$$

$$H_F^{(1)} \sim \mathcal{O}\left(\frac{A^2}{\Omega}, \frac{AR}{\Omega}, \frac{R^2}{\Omega}\right)$$



- It is necessary for the hyperfine coupling at adjacent sites to be different for the scheme to distinguish the Gauss law preserving term $c_i^\dagger I_i^z c_{i+1}$ from $c_i^\dagger I_{i+1}^z c_{i+1}$, a term that violates it.
- The I^z terms (Zeeman and hyperfine terms) are resonantly driven by a transverse fields inducing "Rabi" oscillations to give the electric field term I^x.

7. Ongoing Work

- Obtain the staggered fermion mass term.
- Perform numerics to estimate the effect of higher order Floquet contributions.
- Test the feasibility of obtaining non-trivial dynamics with experimental parameters numerically in preparation for hardware implementation.

8. Future Directions

- Implementation of higher dimensional LGTs with the primary challenge being implementing the non-trivial plaquette terms.
- Implementation of LGTs with larger spins.

References

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2. Ali Rad, Alexander Schuckert, Eleanor Crane, Gautam Nambiar, Fan Fei, Jonathan Wyrick, Richard M. Silver, Mohammad Hafezi, Zohreh Davoudi, Michael J. Gullans. Analog Quantum Simulator of a Quantum Field Theory with Fermion-Spin Systems in Silicon. arXiv:2407.03419 [quant-ph]

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