The QED contributions to the short and intermediate windows of the hadronic vacuum polarization contribution to the muon g-2.

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Introduction

- The exciting recent results from the Fermilab Muon g-2 experiment for the Muon Anomalous Magnetic Moment (2104.03281) motivates reducing the error on lattice calculations of the hadronic contribution to a_μ^{LO}.
- The lattice QCD results from the BMW collaboration (2002.12347, 2407.10913) for a_μ^{LO} are in tension with the data driven estimates.
- We have reported lattice-QCD calculations of the light-quark connected contribution to window observables associated with the leading-order hadronic vacuum polarization contribution to the anomalous magnetic moment of the muon (2207.04765,2301.08274).

Here we present the connected QED contributions to window observables to further compare to data driven estimates.
BMW estimate that the connected QED contributions to a_μ^{LO} are ~ 0.2 %.

Formalism for computig $a_\mu^{HVP(LO)}$

Lattice-QCD calculations of the HVP are based on the Euclidean time vector-vector correlation

$$G_{ff'}(t) = Q_f Q_{f'} \sum_{\vec{x}} Z_V^2 \langle j_f^i(\vec{x},t) j_{f'}^i(0) \rangle \,. \qquad (1)$$

where f and f' are flavour indices, Q_f is the electric charge for that flavour in units of e, Z_V is the renormalisation factor for the lattice vector (electromagnetic) current.
The contribution to a_μ from a window (that isolates a region in time) is

$$a_\mu^w(t_1, \Delta t) = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, G_{ff'}(t)\; K_G^w(t)\,,$$

with a modified kernel,

$$K_G^w(t) \equiv K_G(t) W(t, t_i, \Delta t).$$

The short distance (SD) window is (T. Blum et al, arXiv:1801.07224)

$$W^{SD}(t;t_1) \equiv 1 - \Theta(t;t_1,\Delta)$$

with $t_1 = 0.4$ fm. This regime may be described by perturbation theory.
The intermediate window (W) is defined by

$$W(t;t_1,t_2) \equiv \Theta(t;t_1,\Delta) - \Theta(t;t_2,\Delta)$$

with $t_1 = 0.4$ fm and $t_2 = 1.0$ fm. This is a standard benchmark number for comparison of lattice and phenomenological numbers.
The Windows are defined (with $\Delta = 0.15$ fm) using

$$\Theta(t;t',\delta) \equiv \frac{1}{2} + \frac{1}{2}\tanh[\frac{t-t'}{\Delta}]$$

We estimate the QED contribution to the strange or light quarks via

$$\delta a_\mu = a_\mu[QCD+QED] - a_\mu[QCD]$$

We have **not** yet retuned the quark masses to include the QED contribution.

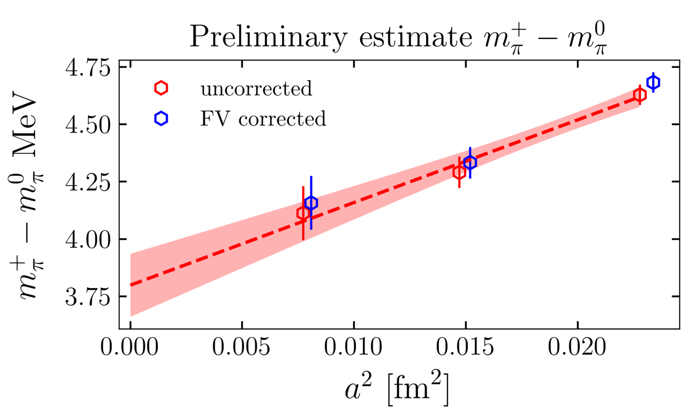
Connected quenched QED corrections

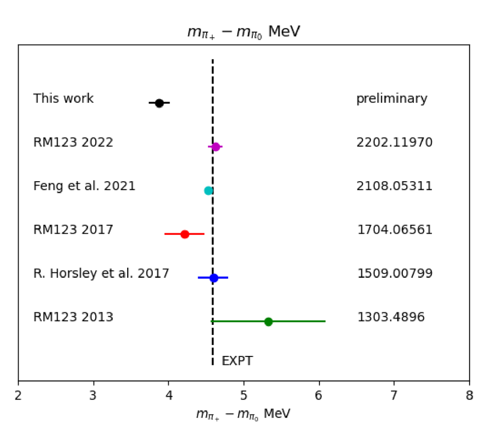
The basis of the calculation are MILC HISQ nf $= 2+1+1$ QCD gauge-field ensembles.

- We use the electro-quenched approximation and only compute connected correlators.
- The calculation used quenched QED fields fixed to the Feynman gauge with zero modes dealt with using the QED$_L$ prescription.
- We use the truncated solver method with 16 sloppy inversions and 1 precise inversion.

Ensemble	$L^3 \times T$	a[fm]	no. meas	masses
very coarse	$32^3 \times 48$	0.15	1844	m_u m_d $3/5/7m_l$ m_s
coarse	$48^3 \times 64$	0.12	967	$3/5/7m_l$ m_s
fine	$64^3 \times 96$	0.09	596	$3/5/7m_l$ m_s

Ensembles at physical pion masses, but because of noise increasing we use $3/5/7m_l$ valence quark masses (following BMW) and extrapolate to m_l for vector mesons. Lower statistics at physical point for pseudo-scalar mesons to test scheme dependence.





Connected strange quenched QED corrections

We have computed the SD and W contribution to the QED contribution to a_μ^{LO}.

$$\delta a_\mu^s = a_0(1 + a_1\alpha_s(\mu = \frac{2}{a})(\Lambda a)^2 + a_2(\Lambda a)^4)$$

with fit parameters a_0, a_1 and a_2. α_s in the V scheme and $\Lambda = 0.5$ GeV.

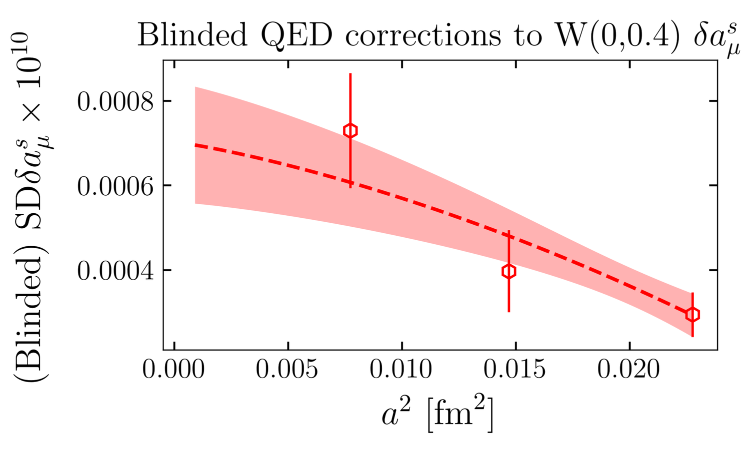


Fig. 3: χ^2/dof = 0.86

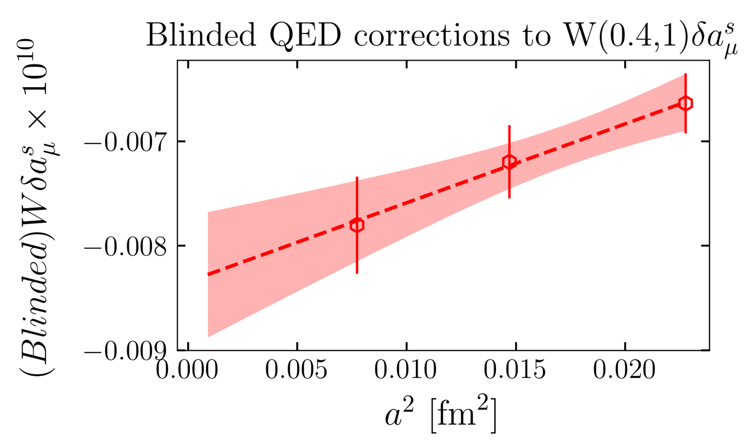


Fig. 4: χ^2/dof = 0.014

Connected light quenched QED corrections

We extrapolate the light quark data from quark masses $7m_l$, $5m_l$, and $3m_l$ to the physical light quark mass m_l.

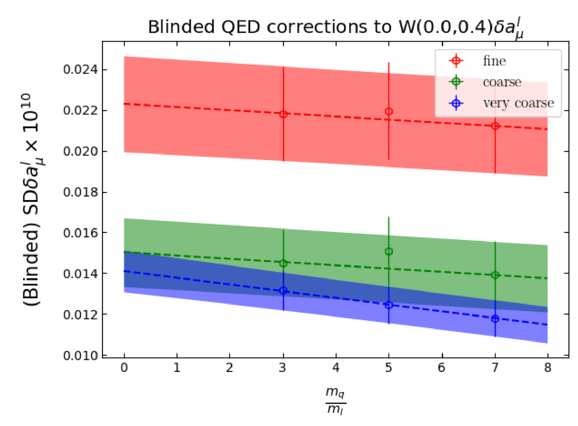


Fig. 5: Light short distance

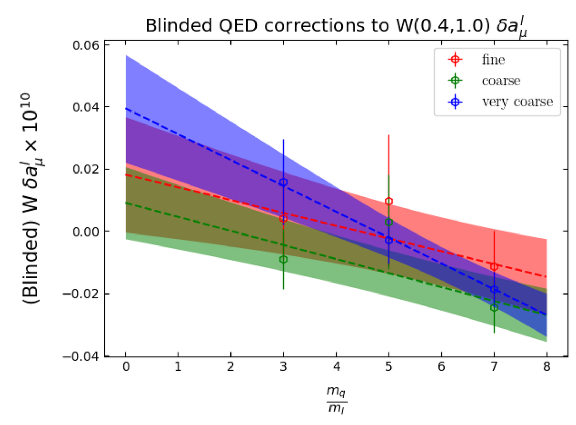


Fig. 6: Light intermediate window

For the short distance window we use the continuum limit model

$$\delta a_\mu^l = a_0(1 + a_1\alpha_s(\mu = \frac{2}{a})(\Lambda a)^2 + a_2(\Lambda a)^4)$$

with fit parameters a_0, a_1 and a_2. α_s in the V scheme, and $\Lambda = 1.2$ GeV.
For the W window we use

$$\delta a_\mu^l = a_0 + a_1\alpha_s(\mu = \frac{2}{a})(\Lambda a)^2 + a_2(\Lambda a)^4$$

because a_0 is small.

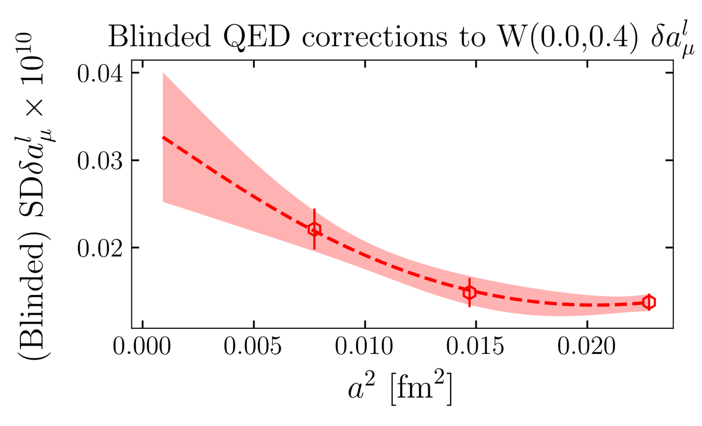


Fig. 7: χ^2/dof = 0.19

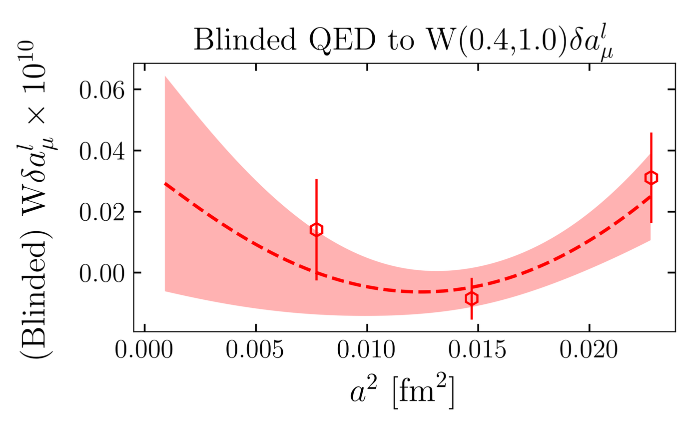


Fig. 8: χ^2/dof = 0.75

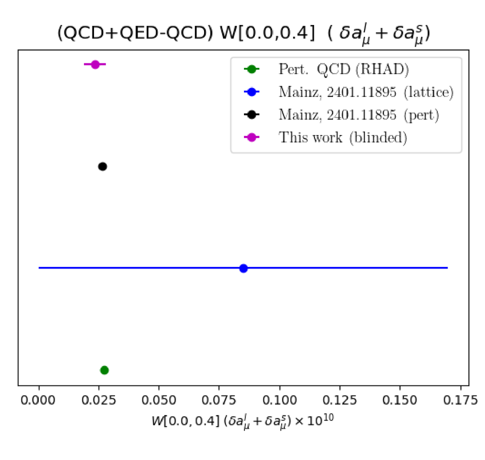


Fig. 9: Summary

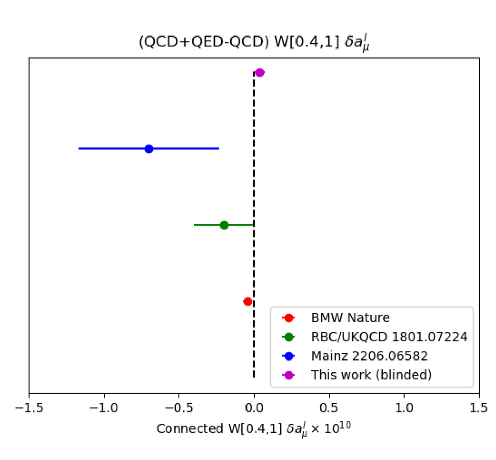


Fig. 10: Summary

Future work

- We next plan to compute the QED contributions to the disconnected diagrams.
- We are investigating using the perturbative approach (1706.05293) to estimate the QED corrections.
- We will then compute the remaining QED corrections.

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