Smeared R-ratio in isoQCD with Low Mode Averaging

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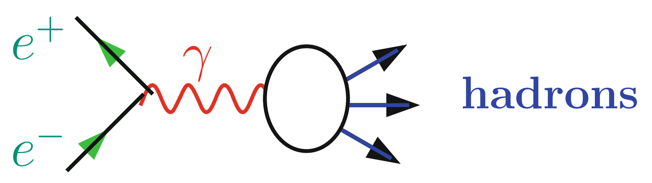
Introduction

Low Mode Averaging (LMA) is a technique to increase the quality of the signal-to-noise ratio in the long time separation of Euclidean correlators. It is highly beneficial in computing the vector-vector light connected two-point correlators and derived physical quantities, e.g. $a_\mu^{\rm HVP}$, in the ETMC mixed action lattice setup [1]. Here, we focus on preliminary results for the smeared R-ratio [2] using the Hansen-Lupo-Tantalo (HLT) spectral-density reconstruction method [3], now with Gaussian kernels of width down to $\sigma \sim 250$ MeV. This is enough to appreciate the ρ resonance around 770 MeV.

R-ratio via HLT method (mini review)

$$R(\omega) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



$R(\omega)$ is a distribution and has to be probed by using smearing kernels. We extract the smeared R-ratio

$$R_\sigma(E) = \int_0^\infty d\omega\, G_\sigma(E-\omega) R(\omega)\,, \text{ with normalized Gaussian kernels,}$$

$$G_\sigma(E-\omega) = \exp\left(-(E-\omega)^2/2\sigma^2\right)/\sqrt{2\pi\sigma^2}\,.$$

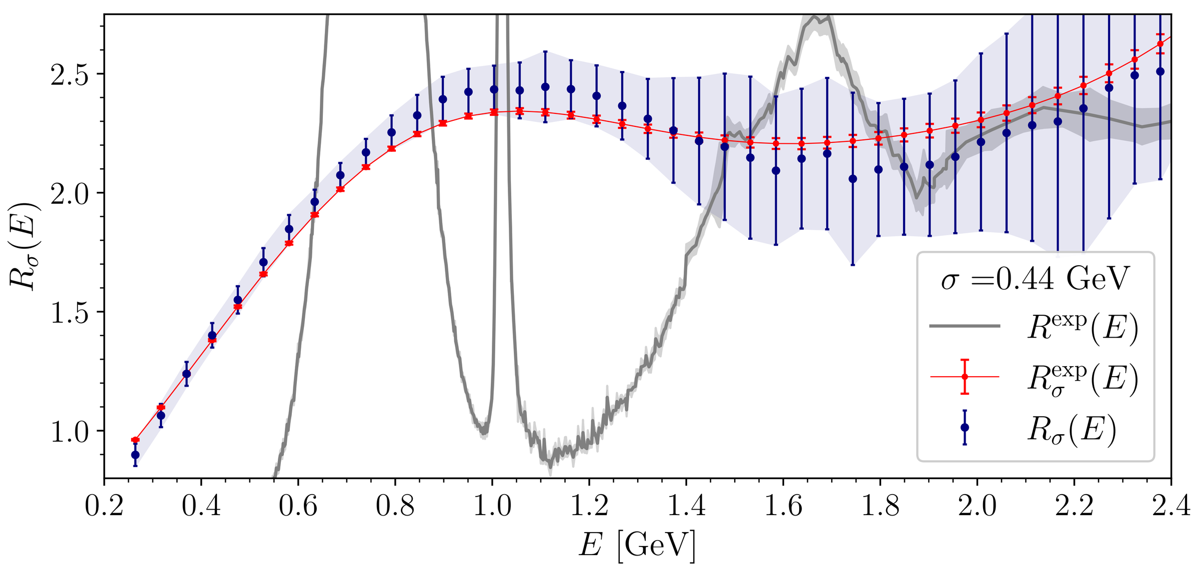


Figure 1: The first-principle lattice results of 2023 [2] compared to the smeared experimental R-ratio from KNT19 compilation [4]. Errors increase as σ decreases.

Two-point Euclidean correlator of the quark electromagnetic current $C(t)$ is connected to the R-ratio by

$$C(t) = -\frac{1}{3}\sum_{i=1}^{3}\int d^3x \mathrm{T}\,\langle 0|\, J_i(x) J_i(0)\,|0\rangle = \frac{1}{12\pi^2}\int_{2m_\pi}^\infty d\omega\, e^{-\omega t}\,\omega^2 R(\omega)\,.$$

In [3] the smearing kernels are approximated as $K(\omega;\mathbf{g}) = \sum_{\tau=1}^{T/a} g_\tau e^{-a\omega\tau}$

with an accuracy measured by

$$A_{\rm n}[\mathbf{g}] = \int_{E_0}^\infty d\omega\, w_{\rm n}(\omega)\left|K(\omega;\mathbf{g}) - \frac{12\pi^2 G_\sigma(E-\omega)}{\omega^2}\right|^2,$$

that, for weight-functions $w_n > 0$, correspond to a class of weighted L_2-norms in functional space. The $\mathbf{g}$ coefficients result from minimizing

$$W[\lambda,\mathbf{g}] = (1-\lambda)A_{\rm n}[\mathbf{g}] + \lambda B[\mathbf{g}], \quad \text{where } B[\mathbf{g}] = \sum_{\tau_{1,2}=1}^{T/a} g_{\tau_1} g_{\tau_2} \mathrm{Cov}_{\tau_1\tau_2}\,,$$

λ is a parameter varied/optimized in stability analysis, see [2] Supp. Mat.

Low Mode Averaging

$$D_r = D_W + ir\mu\gamma_5\,,\quad Q_r \equiv \gamma_5 D_r = Q_W + ir\mu\,,\quad S_r = (D_r)^{-1} = Q_r^{-1}\gamma_5\,,$$

where $r = \pm 1$ and μ are Wilson and twisted mass parameters.
The two-point functions read

$$C_{rr'}^{AB}(t) = \frac{1}{L^3}\mathrm{Tr}\left[\sum_{\vec{x},\vec{y}} S_r(\vec{x},t_0;\vec{y},t_0+t)\,\Gamma_A S_{r'}(\vec{y},t_0+t;\vec{x},t_0)\,\Gamma_B\right].$$

Following [5–7], we split the propagator as:

$$S_r(x,y) = |P_{\rm IR}Q_r^{-1}P_{\rm IR}\eta(x)\rangle\langle\eta(y)|\gamma_5 + |P_{\rm UV}Q_r^{-1}P_{\rm UV}\eta(x)\rangle\langle\eta(y)|\gamma_5$$
$$= \sum_{j=1}^{K}\frac{|v_j(x)\rangle\langle v_j(y)|\gamma_5}{\lambda_j + ir\mu} + \frac{1}{N}\sum_\eta |\tilde{\phi}_r^\eta(x)\rangle\langle\eta(y)|\gamma_5\Big|_{N\gg 1}\,,$$

K and N count the lowest modes and the stochastic sources respectively.

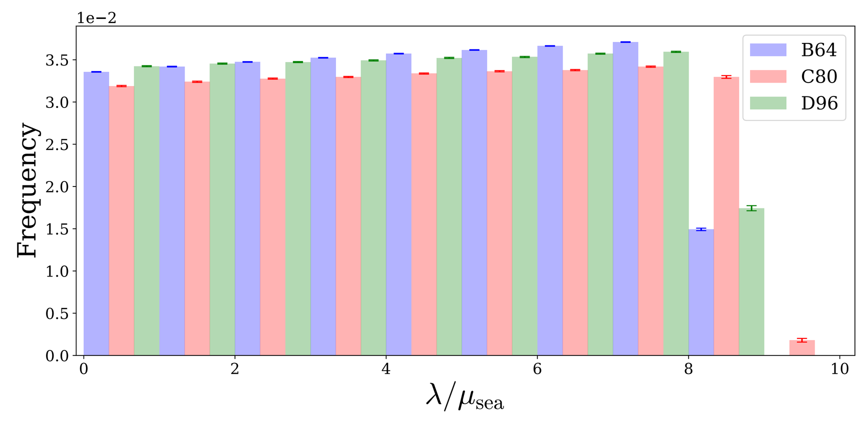


Figure 2: The frequency histogram of the eigenvalues. As the frequency of $\lambda_{\rm min} \propto L^3 T$, we scale K roughly in this way while varying a: typically K is such that $\lambda_{\rm max}/\mu_{ud} \sim 8$.

Ensemble	$(L/a)^3 \times T/a$	a[fm]	#confs.	#eigv.	#sources
cB211.072.64	$64^3 \times 128$	0.07951(4)	790	400	1024
cC211.060.80	$80^3 \times 160$	0.06816(8)	550	530	960
cD211.054.96	$96^3 \times 192$	0.05688(6)	400	530	960

Table 1: ETMC ensembles [8] used for two-point vector correlators in isoQCD below.

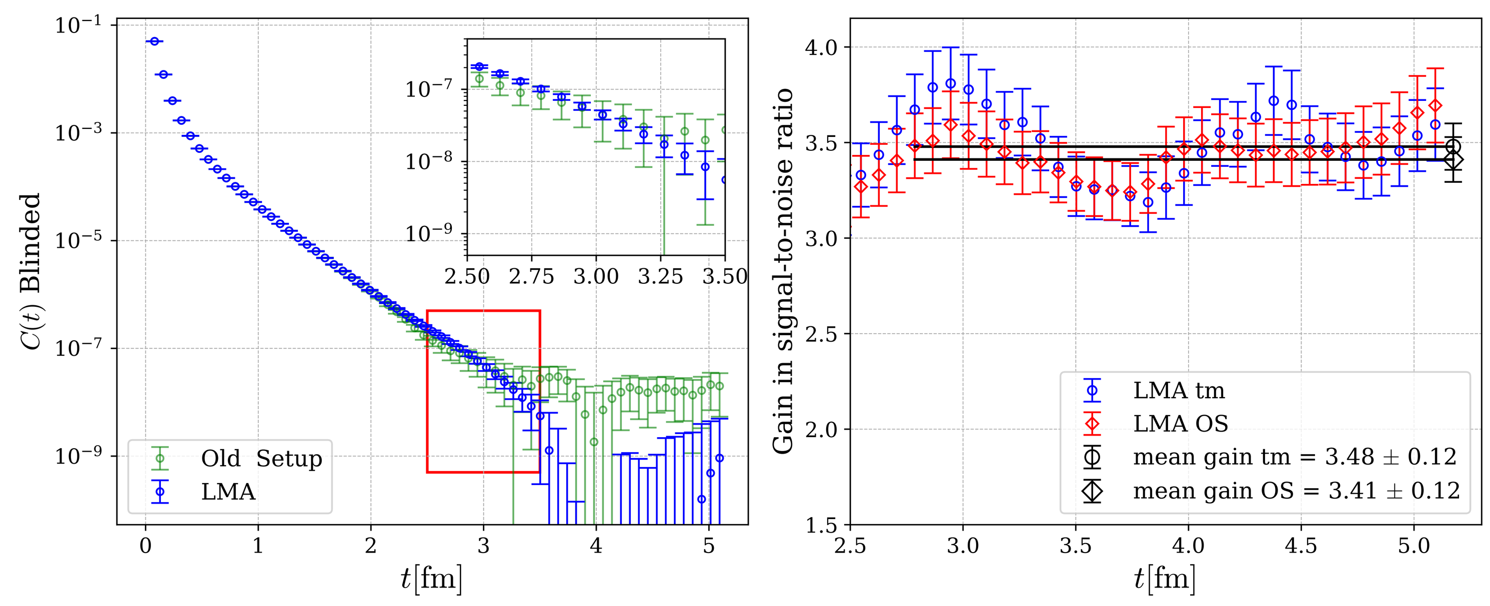


Figure 3: Left: vector-vector light connected two-point correlator on B64 ensemble at $L = 5.1$ fm using old [9] and new LMA setup. Right: the gain in the signal-to-noise ratio.

References

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R-ratio via HLT + LMA *Preliminary*

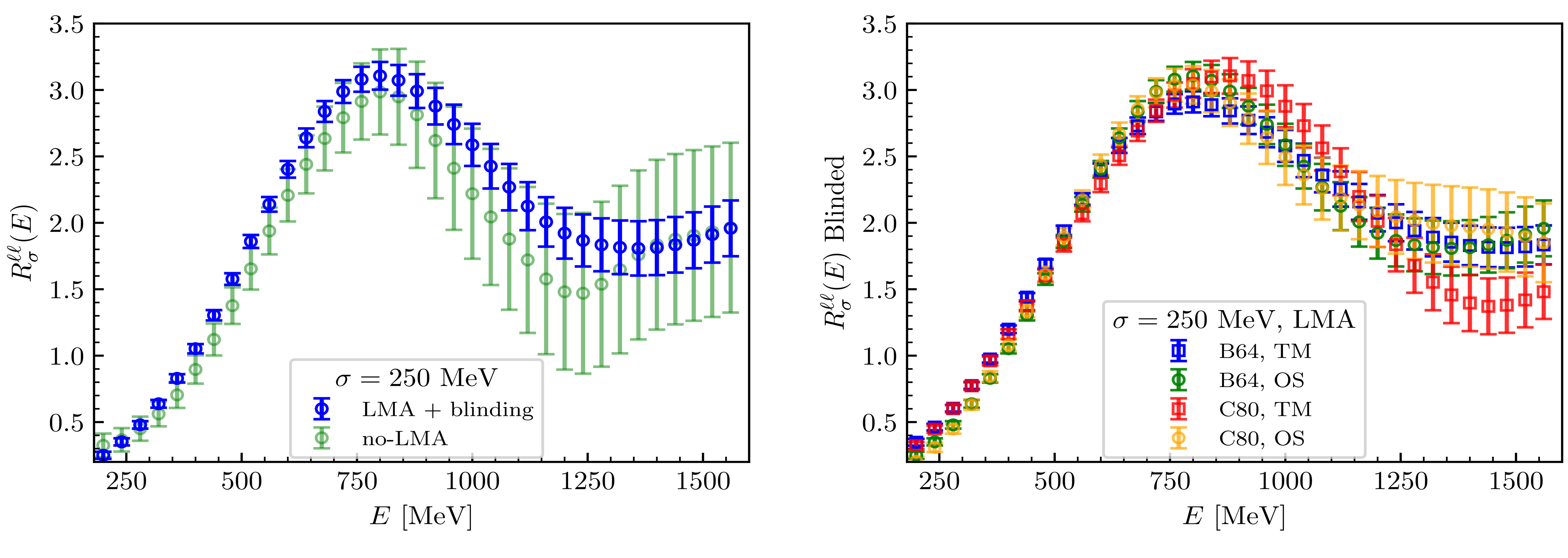

Figure 4: Smeared R-ratio with Gaussian kernels of width $\sigma = 250$ MeV and central energy up to 1.6 GeV.