Strangeness-Neutral Line in Dense and Magnetized QCD at Imaginary Chemical Potential

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Abstract

We simulate QCD with 3 quark flavours for the case of an external magnetic field and imaginary chemical potential in the temperature range of the crossover. This poster clarifies how to match experimental conditions, i.e bringing the system into strangeness neutrality as well as predicting the new simulation parameters for runs with increasing imaginary chemical potential by comparing different approaches.

Introduction

Using the method of lattice QCD dense strongly interacting systems (e.g see [1]) [a] as well as such systems in external magnetic fields (e.g see [2]), which has a critical endpoint (CEP) in the $T-B-$plane around $B = [4-9]\,\mathrm{GeV}^2$ [3], have been well studied in the past. Physical systems such as magnetars or as in non central heavy ion collisions (HIC) however are both dense and have large magnetic fields. We want to study such systems to find a possible first order connecting line between the CEP in the $T-B-$ and the possible CEP in the $T-\mu-$ Plane.

To match experimental conditions in e.g HIC we musst fulfill strangeness neutrality and isospin asymmetry, i.e

$$\langle n_S \rangle = 0 \quad \text{and} \quad \langle n_Q \rangle = 0.4 \langle n_B \rangle \tag{1}$$

where $\langle n_i \rangle$ represents the expectation value of the strange (S), charge (Q) or baryon (B) number. The input variables of the simulation, i.e $\hat{\mu}_{B,sim} = \frac{\mu_B}{T}$ introduces a strangeness to our physical system, which should be canceled by $\hat{\mu}_{S,sim}$. $\hat{\mu}_{Q,sim}$ has to be introduced due to the fact that we also simulate using an external magnetic field. We will discuss the routine of extrapolating $\hat{\mu}_{Q,sim}$ and $\hat{\mu}_{S,sim}$ for larger $\hat{\mu}_B$ from strangeness neutral points.

Strangeness Neutrality

Eq. (1) should be fullfilled, therefore the susceptibility χ_S should be 0. In general we define the quark number susceptibilities in eq. (2)

$$\chi^{u,d,s}_{i,j,k} = \frac{T N_t^{4-i-j-k}}{V} \frac{\partial^{i+j+k} \log Z}{(\partial \mu_u)^i (\partial \mu_d)^j (\partial \mu_s)^k}, \tag{2}$$

in which $N_t^{4-i-j-k}$ makes the result dimensionless. By using

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q \qquad \mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q \qquad \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S, \tag{3}$$

we can calculate χ^S_1. The result of the simulation at $\hat{\mu}_B = 3\frac{i\pi}{8}$ where $\hat{\mu}_S$ and $\hat{\mu}_Q$ were determinant by a linear extrapolation is shown in figure 1[b].

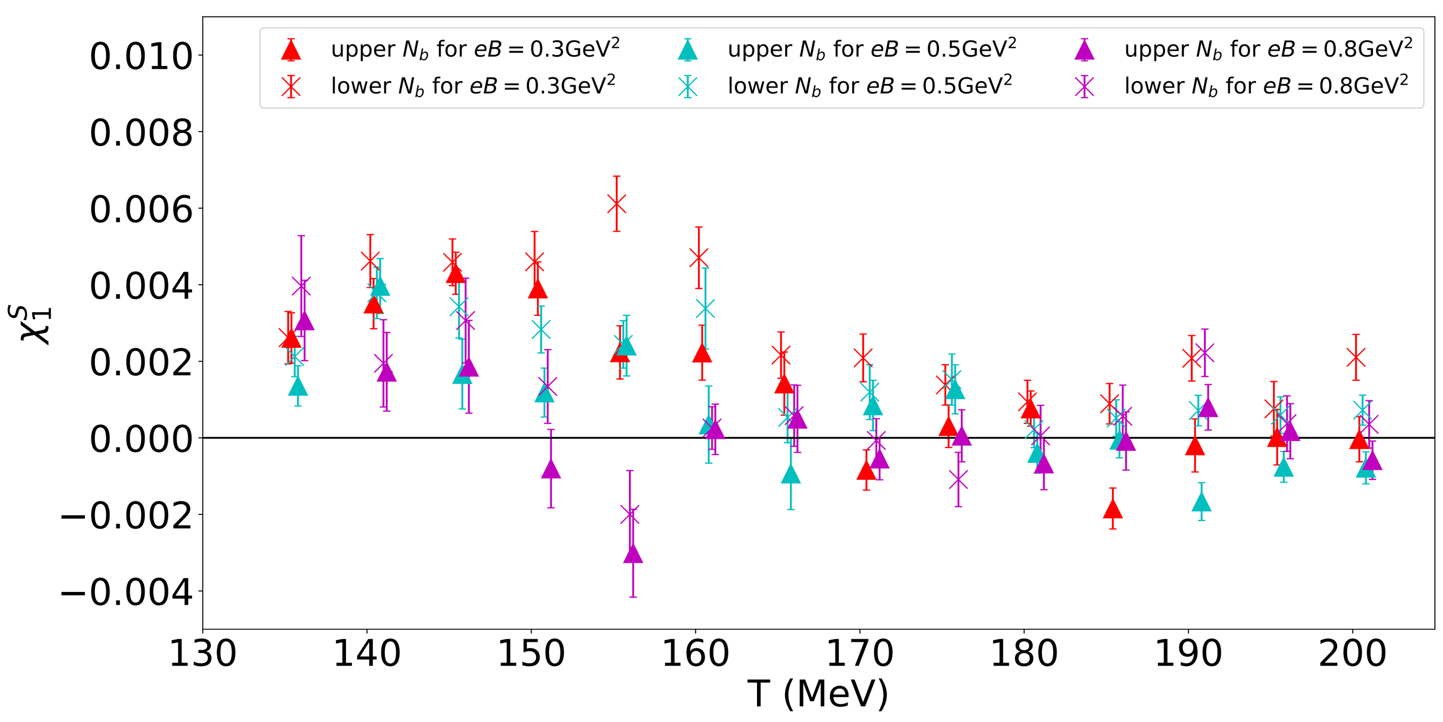

Figure 1. Values of χ^S_1 from the measurements without correction for $\hat{\mu}_B = 3\frac{i\pi}{8}$.

We bring the measurements into strangeness neutrality by using a Taylor expansion up to 2nd order*. With this we can calculate our observables in strangeness neutrality. For the χ^B_1 we plot the difference from the naive and the strangeness neutral case in figure (2)

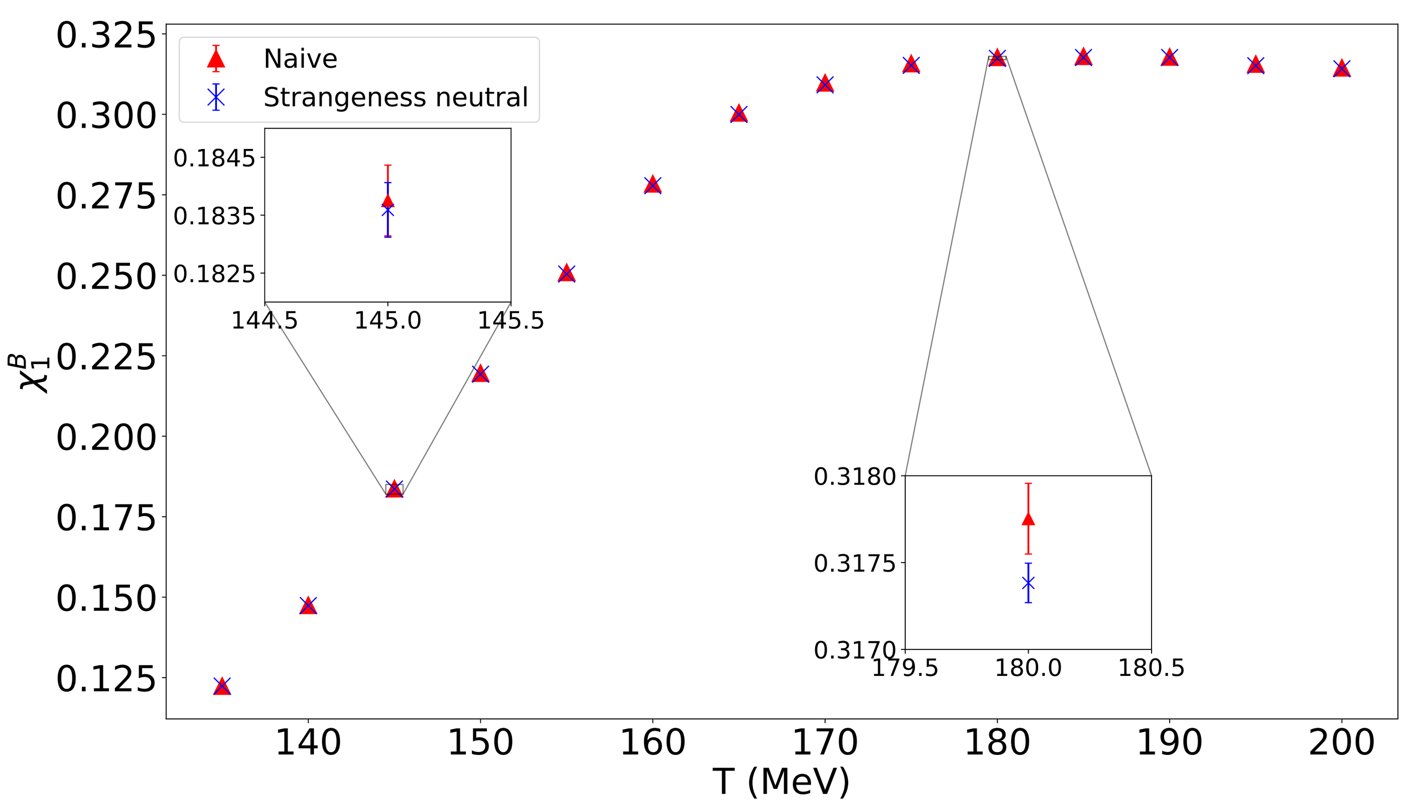

Figure 2. χ^B_1 for the naive and strangeness neutral measurement at $0.5GeV^2$ with $\hat{\mu}_B = 3\frac{i\pi}{8}$

[a]**Note**: Direct simulations at real chemical potential are hampered by the sign problem

[b]**Note**: Due to the quantization condition for magnetic fields on the lattice, we can only simulate with integer N_b and later need to interpolate to the N_b for $0.3\,\mathrm{GeV}^2$, $0.5\,\mathrm{GeV}^2$ or $0.8\,\mathrm{GeV}^2$ respectively

Extrapolate New Simulation Parameters

We want to predict how $\hat{\mu}_Q$ and $\hat{\mu}_S$ behave w.r.t $\hat{\mu}_B$ by using a Taylor series as ansatz (see eq.4) ($X \in \{Q, S\}$)

$$\hat{\mu}_X(\hat{\mu}_B) = x_1\hat{\mu}_B + x_3\hat{\mu}_B^3 + x_5\hat{\mu}_B^5 + \mathcal{O}(\hat{\mu}_B^7) \tag{4}$$

$$\Leftrightarrow \frac{d\hat{\mu}_X}{d\hat{\mu}_B}(\hat{\mu}_B) = x_1 + 3_3\hat{\mu}_B^2 + 5x_5\hat{\mu}_B^4 + \mathcal{O}(\hat{\mu}_B^5) \tag{5}$$

A schematic depiction of this can be seen in figure (3).

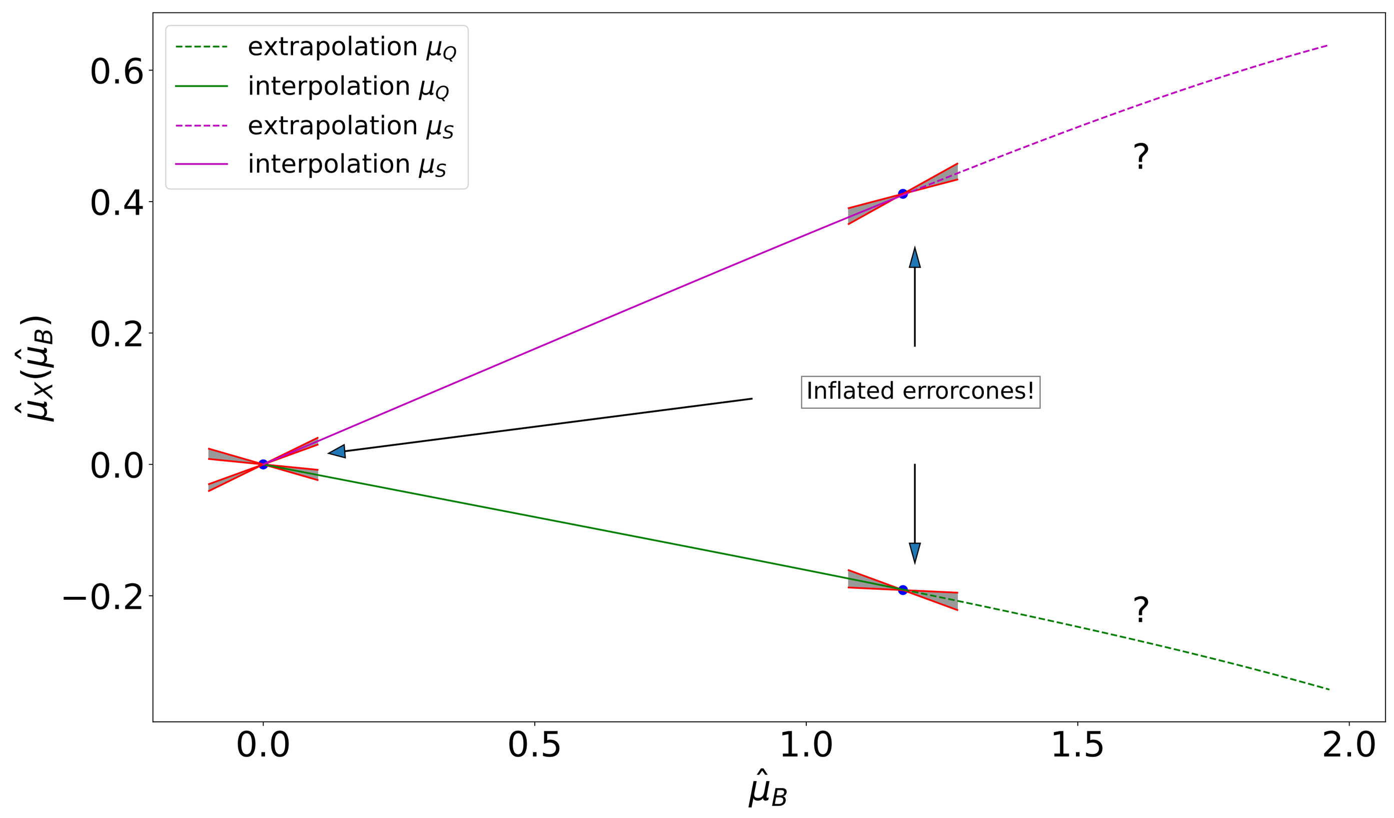

Figure 3. Schematic depiction of the ansatz in eq. (4)

Using this ansatz we follow the following ideas:

1. Solving eq (4) and (5) for x_3 and x_5, where x_1 is the value $\frac{d\hat{\mu}_X}{d\hat{\mu}_B}(\hat{\mu}_B = 0)$
2. Fit the ansatz of eq. (4) where x_5 is set to 0 and solve for x_1 and x_3. This idea can also be expanded with a spline interpolation for the temperatures to smooth the $\hat{\mu}_X(T)$ dependence despite usage of integer N_b.

The prediction for the next $\hat{\mu}_B = 4\frac{i\pi}{8}$ runs for the different ideas can be seen in fig. (4) for $\hat{\mu}_Q(T)$ and in fig. (5) for $\hat{\mu}_S(T)$ for the runs around $eB = 0.5\,\mathrm{GeV}^2$

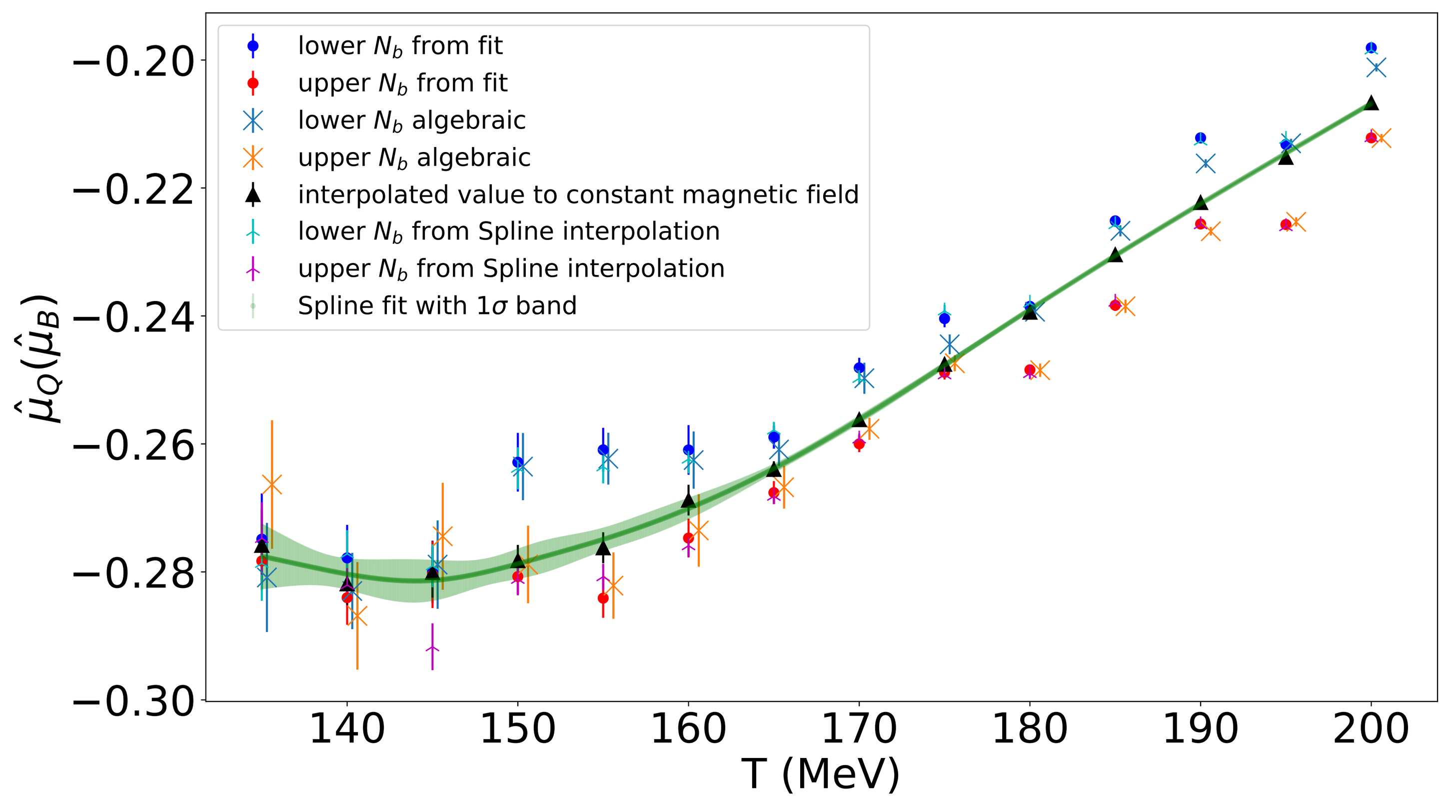

Figure 4. Extrapolation for the temperature range for $\hat{\mu}_Q(\hat{\mu}_B = 4\frac{i\pi}{8})$

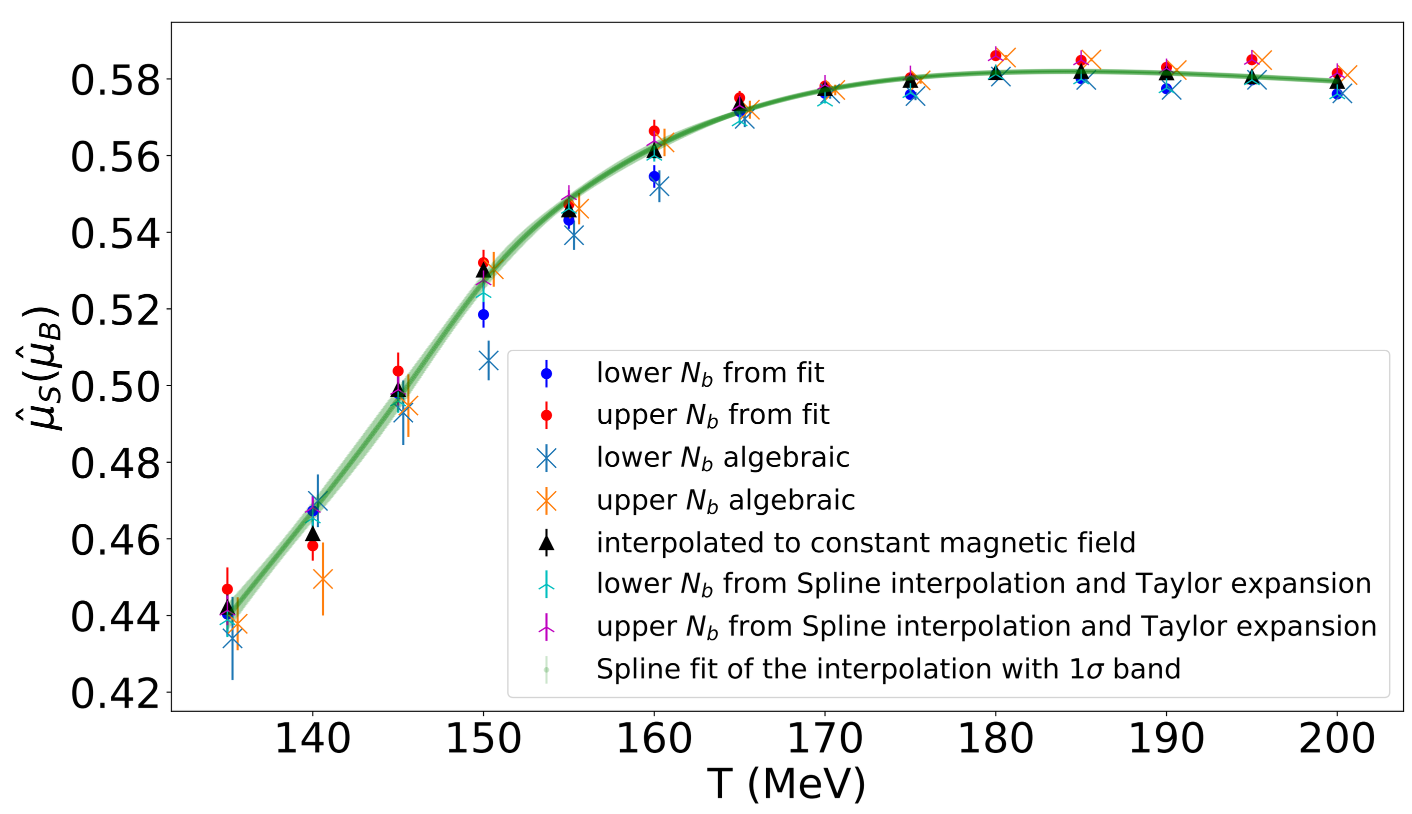

Figure 5. Extrapolation for the temperature range for $\hat{\mu}_S(\hat{\mu}_B = 4\frac{i\pi}{8})$

References

[1] Borsányi et. al. [Phys.Rev.Lett. 2020] arXiv: 2002.02821

[2] G. Endrődi et. al. [JHEP 2011] arXiv:1111.4956

[3] D'Elia et. al. [Phys. Rev. D 105, 2022] arXiv:2211.12166

* For details and physical interpretation see the talk by Dean Valois from Tue. 30.07 14:15 in session TR5.