Proton decay matrix elements on PACS configurations

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Program for Promoting Researches on the Supercomputer Fugaku
Large-scale lattice QCD simulation and development of AI technology

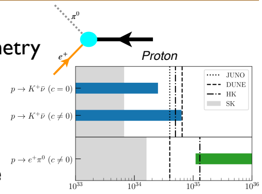
Proton decay -Smoking Gun of New Physics

Proton decay

Bayron number violation for matter-antimatter asymmetry

Current bounds [1]: $\tau(p \to e^+\pi_0) \geq 1.6 \times 10^{34}$ yrs .
$\tau(p \to \nu K^+) \geq 5.9 \times 10^{33}$ yrs .

Next experiments: Hyper-K, DUNE and JUNO.



Proton decay matrix elements on the lattice

Proton is a QCD object → determined by non-perturbative quark dynamics
bridge between GUTs and Experiments

Lattice status

PACS collaboration[2]: Nf=2+1 Wilson-clover physical point ensembles
Single lattice cutoff ($a^{-1} \sim 2.3$ [GeV])

RBC/UKQCD [3]: Nf=2+1 MDWF physical point ensembles
Two (coarse) lattice cutoff ($a^{-1} \sim 1.0, 1.4$ [GeV])
Direct ME calculation ≃ Indirect LEC + ChPT calculation

Next task for PACS

Remove syst. errs (Finite volume, discretization)→ **≲10% error in future**

Constrains on GUTs or SUSY-GUTs

Constraining GUTs and Relevant form factor

GUTs effective Lagrangian Charge-conjugated $\{\bar{q}, \bar{l}\}^C = \{q, l\}^T C$ * l: fermion or sfermion

$$\mathscr{L}_{\rm eff} = \sum_I^{\dim O_I \geq 5} C_I O_I + {\rm h.c.}, \quad O_I = \epsilon^{abc}\left(\bar{q}^{aC} P_{\chi_I} q^b\right)\left(\bar{l}^C P_{\chi_I'} q^c\right)$$

Chirality projector $P_{\chi^{(\prime)}} = \frac{1 \pm \gamma_5}{2}$

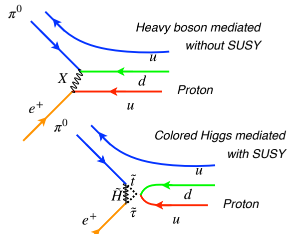
Wilson coefficients $C_I = \tilde{c}_I / \Lambda^2_{\rm GUT}$ for non-SUSY and $\tilde{c}_I / M_{H_c}$ for SUSY

A dimensionless $O(1)$ coupling at nuclear scale $\tilde{c}_I$

Baryon → meson + anti-lepton

$$\langle \pi^0, e^+ | p \rangle_{\rm BSM} = \sum_i C^i(\mu) \cdot \langle \pi^0, e^+ | O^i(\mu) | p \rangle_{\rm SM}$$

$$\langle \pi^0, e^+ | (ud)(eu) | p \rangle = \overline{v_e^c} \langle \pi^0 | (ud) u | p \rangle$$



Two Form factors: W_0, W_1

$$\langle \pi^0 | (ud)_\Gamma u_L | p \rangle = P_L \left[W_0 - \frac{i q_\alpha \gamma^\alpha}{m_p} W_1 \right] u_p$$

$$\to \langle \pi^0, e^+ | (ud)_\Gamma (eu)_L | p \rangle = W_0 \cdot (v_e, u_p)_L + \frac{m_e}{m_p} W_1 \cdot (v_e, u_p)_R$$

Partial width: τ "Constrain $C^i(\mu)$ with τ from Expt. and W_0^i from lattice"

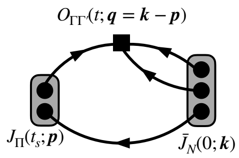
e.g.

$$\frac{1}{\tau}(p \to \pi^0 + e^+) = \frac{m_p}{32\pi}\left[1 - \left(\frac{m_\pi}{m_p}\right)^2\right]^2 \left| \sum_i C^i W_0^i (p \to \pi^0) \right|^2$$

$$\to \tau / {\rm Br}(p \to \pi \bar{l}) \sim 1.4 \times 10^{33} {\rm yrs.} \cdot \left(\frac{\Lambda_{\rm GUT}}{10^{15}\ {\rm GeV}}\right)^4 \cdot \frac{1}{|\tilde{c}_I|^2}$$

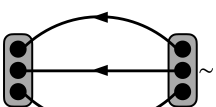
Proton decay matrix element on PACS configurations

Bare matrix elements (Direct)

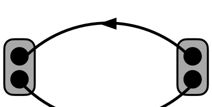


Operator: $O_{\Gamma\Gamma'} = \epsilon^{abc}(\bar{u}^{Ta}\Gamma d^b)\Gamma' s^c$

$$\sim \frac{\sqrt{Z_\Pi Z_N}}{4 E_\Pi E_N} e^{-E_\Pi(t_s - t) - E_N t} \cdot P_{\Gamma'} \left[W_0^{\Gamma\Gamma'} - \frac{i q_\alpha \gamma^\alpha}{m_p} W_1^{\Gamma\Gamma'} \right] u_N(k)$$



$$\sim Z_N(\boldsymbol{k}) \frac{E_N + m_N}{E_N} e^{-E_N t}$$



$$\sim \frac{Z_\Pi(\boldsymbol{p})}{E_\Pi} e^{-E_\Pi t}$$

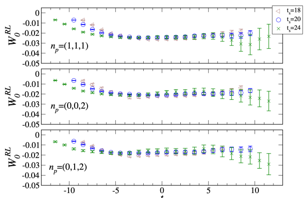
Construct a ratio of correlation functions →Read off the form factor from plateau

Nf=2+1 PACS ensemble of 64^4 ($m_\pi L = 3.8, a^{-1} \sim 2.3$[GeV])

- Iwasaki gauge $\beta = 1.82$
- Stout smeared Wilson-clover fermion
- ud and s quarks are on **the physical point**

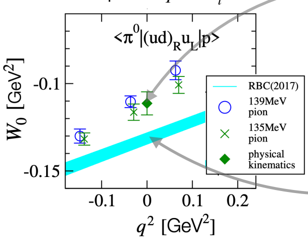
e.g. Form factor W_0 for pion final state $\langle \pi^0 | (ud)_R u_L | p \rangle$

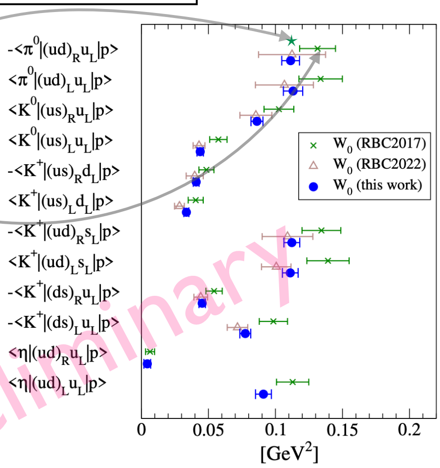
- Meson momentum: $\boldsymbol{p} = \frac{2\pi}{L}\boldsymbol{n}$
- Excited-state contamination is negligible



Renormalized matrix elements

On-shell lepton: $-q^2 = m_l^2 = 0$



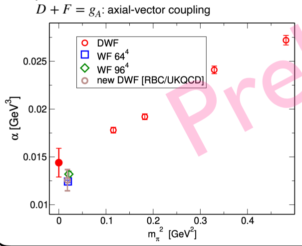


* Indirect: LO in ChPT Claudson, Wise, Hall, 1982

$W_0[\langle \pi^0 | (ud)_R u_L | \rangle] \simeq \alpha / \sqrt{2} f (1 + D + F)$

with f :pion decay constant

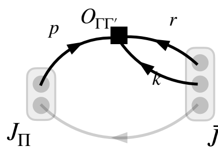
$D + F = g_A$: axial-vector coupling



- PACS with the statistical error only
- Consistent with new RBC/UKQCD DWF results [3]
- No chiral limit surprise!
  ↔ Matrin & Staving (skim chiral bag)

Non-perturbative renormalization (NPR)

Compute with RI intermediate scheme, and pertubatively match into $\overline{\rm MS}$(2GeV)



$S = 1,\ P = \gamma_5,\ V = \gamma_\mu,\ A = \gamma_\mu\gamma_5,\ T = \sigma_{\mu\nu} = \frac{1}{2}\{\gamma_\mu, \gamma_\nu\},\ \tilde{T} = \gamma_5 \sigma_{\mu\nu}$

Classify based on 1. Parity ($\mathscr{P}$) & 2. Exact SU(2) symmetries ($\mathscr{S}$)

→ Only consider mixing of $X, Y = \{SS, PP, AA\}$

	$\mathscr{S}^-$			$\mathscr{S}^+$	
$\mathscr{P}^-$	SS	PP	AA	VV	TT
$\mathscr{P}^+$	SP	PS	−AV	−VA	$\tilde{T}$T

$(C\Gamma)^T = -C\Gamma$ for S, P, A ($\mathscr{S}^-$)
$(C\Gamma)^T = +C\Gamma$ for V, T ($\mathscr{S}^+$)

- **NPR process** ★ Only $O_{LL} = (\bar{u}^T P_L d) \cdot P_L s$ and $O_{RL} = (\bar{u}^T P_R d) \cdot P_L s$ are independent

1. Projected vertex function $M^{A,B}(p^2) = \Lambda^{A,abc}_{\alpha\beta\gamma\delta} \times P^{B,abc}_{\beta\alpha\delta\gamma}$, $A, B = \{SS, PP, AA\}$
2. The renormalization condition $\tilde{Z}^{BC}_{\rm ND} M^{CA}|_{\mu^2 = q^2} = \delta^{BA}$
3. Parity to Chiral $\tilde{Z}^{\rm chiral}_{\rm ND} = \mathscr{T} \tilde{Z}^{\rm Parity}_{\rm ND} \mathscr{T}^{-1}$
4. Multiply Z_q: $Z^{BC}_{\rm ND} = (Z^{\rm SF}_{V(A)} \Lambda^{\rm SMOM_{(\gamma_\mu)}}_{V(A)})^{3/2} \tilde{Z}^{BC}_{\rm ND}$
5. Perturbative matching $Z^{XX}_{\overline{\rm MS}}(2{\rm GeV}) = C \times \tilde{Z}^{XX}_{\rm ND}$, $X = \{LL, RL\}$

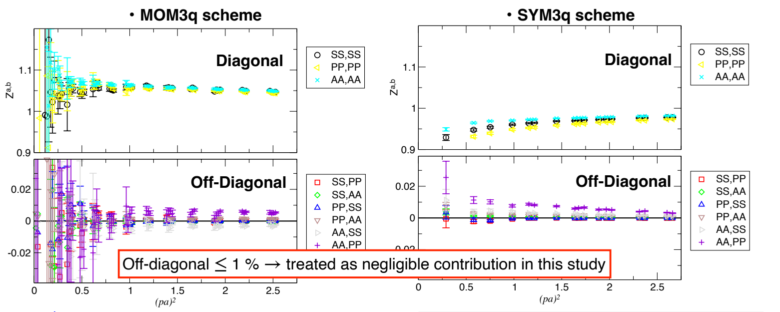
- **Scheme dependence**

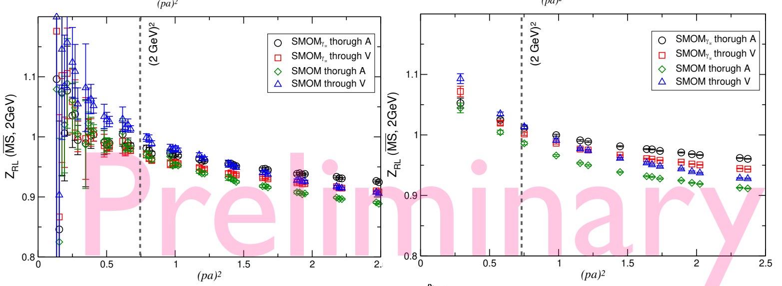
Two intermediate scale

MOM3q:$q^2 = p^2 = k^2 = r^2, q = p = k = r$ "NLO matching"

SYM3q : $q^2 = p^2 = k^2 = r^2,\ q = p + k + r = 0$ "NNLO matching"

Syst. error?





Fitting ansatz[4]:

$$f_{\rm Global}(\mu_0 \geq 1{\rm GeV}) = \frac{c_{-1}}{(\Lambda^{-1}_{\rm IR}\mu_0)^2} + Z + \sum_{k>0}^{k_{\rm max}} c_k (a\mu_0)^{2k}$$

$$f_{\rm IR-trunc.}(\mu_0 \geq 2{\rm GeV}) = Z + \sum_{k>0}^{k_{\rm max}} c_k (a\mu_0)^{2k}$$

→Syst. Error for the fitting ansatz
+ Perturbative truncation
+ selection of the model

Negligible scheme dependence

→ $Z_{RL} = 1.016(5)_{\rm stat}(29)_{\rm scheme}(26)_{\rm model}(6)_{\rm fit} \to 1.016(5)_{\rm stat}(40)_{\rm syst}$: SYM3q
$Z_{RL} = 1.002(30)_{\rm stat}(39)_{\rm scheme}(16)_{\rm model}(16)_{\rm fit} \to 1.002(30)_{\rm stat}(45)_{\rm syst}$: MOM3q

cf) 0.98(7) from PoS[2]

Summary

- **NOTE: all results are PRELIMINARY!**
- Proton decay matrix elements at the physical point
  - **PACS Wilson-clover, fine lattice: $a \lesssim 0.1$ fm**; RBS/UKQCD DWF, coarse lattice
  - Physical point calculation = Systematic error for the chiral extrapolation is eliminated
- Non-perturbative renormalization utilizing SMOM scheme
  - SYM3q scheme[3] = NNLO matching is available: Gracey (2012)→ error ≲ 4% from NPR
  - **Sophisticated fitting treatment[4] applied: Reduce error as 7% → 4%**
- Consistent results for both W_0 and LEC α with RBC/UKQCD DWF[3]
- **Continuum limit on PACS10 configurations in future!**

References & Acknowledgements

[1] J. Elis, L. Evans, N. Nagata *et al.*, *Eur. Phys. J C* **80**, 332 (2020).
[2] Y. Aoki *et al.*, PoS LATTICE2019, 141 (2020).
[3] J.S Yoo *et al.*, Phys. Rev. D **105**, 074501 (2022).
[4] R. Tsuji *et al.*, Phys. Rev. D **106**, 094505 (2022).

Used Computers
- Supercomputer-K, Oakforest PACS, Supercomputer Fugaku, Wisteria/BDEC-01

Programs
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