Leptonic decays of charmed mesons with Wilson quarks on $N_f = 2 + 1$ CLS ensembles

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Charmed decays

Leptonic decay constants of the D and $\mathrm{D_s}$ mesons

$$\mathrm{i} f_{\mathrm{D}} p_\mu = \langle 0 | A_\mu^{\mathrm{lc}} | \mathrm{D}(p) \rangle, \qquad \mathrm{i} f_{\mathrm{D_s}} p_\mu = \langle 0 | A_\mu^{\mathrm{sc}} | \mathrm{D_s}(p) \rangle$$

$\rightarrow$ constraints on the CKM matrix elements V_{cd} and V_{cs}

Leptonic vector and tensor decay constants of the D^* and D_s^* mesons

$$m_{\mathrm{D}^*_{(q)}} f_{\mathrm{D}^*_{(q)}} \epsilon_\mu^\lambda = \langle 0 | V_\mu^{q\mathrm{c}} | \mathrm{D}^*_{(q)}(p, \lambda) \rangle$$

$$\mathrm{i} f^{\mathrm{T}}_{\mathrm{D}^*_{(q)}} (\epsilon_\mu^\lambda p_\nu - \epsilon_\nu^\lambda p_\mu) = \langle 0 | T_{\mu\nu}^{q\mathrm{c}} | \mathrm{D}^*_{(q)}(p, \lambda) \rangle$$

$\rightarrow$ prediction of decay rates and comparison to QCD sum rules

Simulation setup

Simulation details

- partially quenched setup
- dynamical quarks: two degenerate light quarks and one strange quark
  $\rightarrow$ large number of ensembles at various pion masses/lattice spacings including two ensembles nearby the physical point
- charm quark: two charm quark masses per ensemble close to the physical charm quark mass

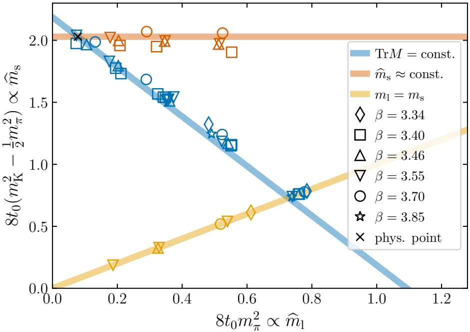
Extrapolation strategy

- rescale quantities with t_0 to obtain dimensionless quantities
- parametrize full quark mass dependence (light, strange, and charm) plus lattice spacing dependence
- perform simultaneous extrapolation to the continuum limit and the physical point
  $\rightarrow m_\pi = 134.8(3)$ MeV, $m_{\mathrm{K}} = 494.2(3)$ MeV, $m_{\bar{\mathrm{D}}} = \frac{2}{3} m_{\mathrm{D}} + \frac{1}{3} m_{\mathrm{D_s}} = 1899.4(3)$ MeV
- scale setting: $\sqrt{t_0} = 0.1449^{(7)}_{(9)}$fm [RQCD 23]

Details on the dynamical quarks

- utilize $2+1$f ensembles obtained within the CLS (Coordinated Lattice Simulations) effort
  - two degenerate light quarks and one strange quark
  - non-perturbatively improved Wilson action (clover)
  - tree-level improved Symanzik gauge action
- implementing open boundaries (at small lattice spacings) $\rightarrow$ avoid topological freezing

Extrapolation strategy: light/strange quark masses



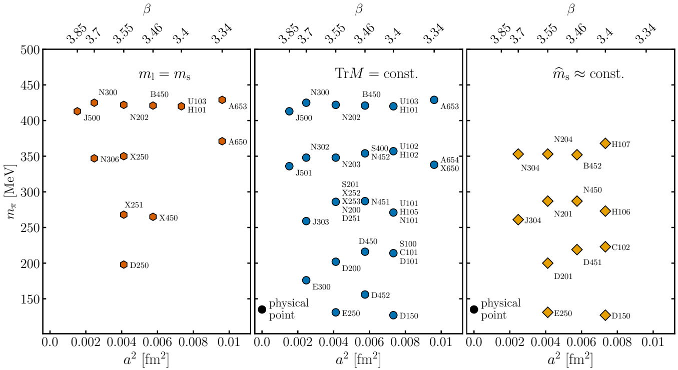
General simulation strategy

simulations along 3 different trajectories:

- $\mathrm{Tr}M = \mathrm{const.}$
  $\rightarrow$ sum of bare quark masses kept constant:
  $$\mathrm{Tr}M = 2m_l + m_s = \mathrm{const.}$$
- $\widehat{m}_s \approx \mathrm{const.}$
  $\rightarrow$ (approx.) constant strange quark mass
- $m_l = m_s$
  $\rightarrow$ equal light and strange quark mass

$\Rightarrow$ enables very controlled chiral extrapolation towards the physical point

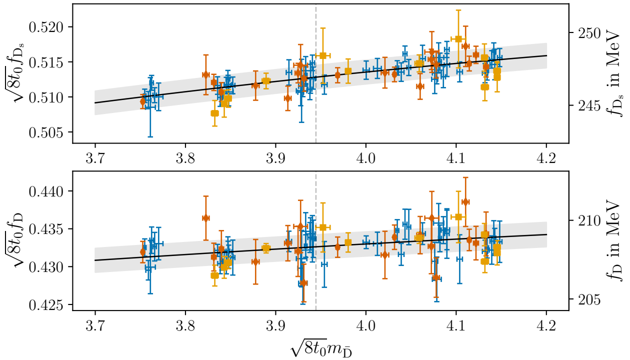
Ensemble overview



- 6 different lattice spacings ($a \approx 0.1 - 0.04$fm)
- 2 ensembles at the physical point
- dedicated ensembles to study possible finite volumes effects
- 49 ensembles in total with large statistics (1000-20000 MDUs)

RQCD ALPHA Collaboration

Fixing the charm quark mass



Charm quark mass parametrization via $m_{\bar{D}}$ (← and η_c^{con} for cross-checking)

- two charm quark masses per ensemble
- global fit allows to test for curvature → small higher order term is resolved $\sim m_{\bar{D}}^2$
- dashed line indicates physical point
- data points shifted using the fit to the physical m_π, m_K, lattice spacing effects are removed

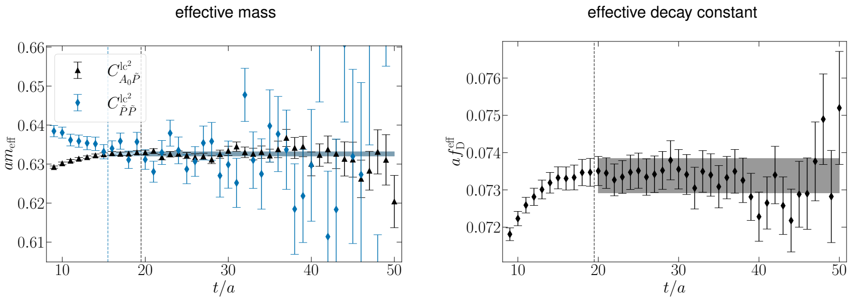
Extraction of the bare quantities

Bare decay constants obtained from spectral decomposition

$$C^{qc}_{A_0\tilde{P}}(t) = A^{qc}_{A_0\tilde{P}}\, \mathrm{e}^{-m_{\mathrm{D}_{(q)}} t}\,,\; C^{qc}_{\tilde{P}\tilde{P}}(t) = A^{qc}_{\tilde{P}\tilde{P}}\, \mathrm{e}^{-m_{\mathrm{D}_{(q)}} t} \quad \Rightarrow \quad f_{\mathrm{D}_{(q)}} = \frac{\sqrt{2}A^{qc}_{A_0\tilde{P}}}{\sqrt{A^{qc}_{\tilde{P}\tilde{P}} m_{\mathrm{D}_{(q)}}}}\,.$$

- point-to-all propagators with Gaussian smearing on APE-smoothed links $\rightarrow$ 3-37 sources per configuration
- matrix elements extracted with simultaneous one-state fits.
- effective decay constant: $f^{\mathrm{eff}}_{\mathrm{D}_{(q)}}(t) = \dfrac{\sqrt{2}C^{qc}_{A_0\tilde{P}}(t)}{\sqrt{C^{qc}_{\tilde{P}\tilde{P}}(t)m_{\mathrm{D}_{(q)}}\exp(-m_{\mathrm{D}_{(q)}}t)}}$

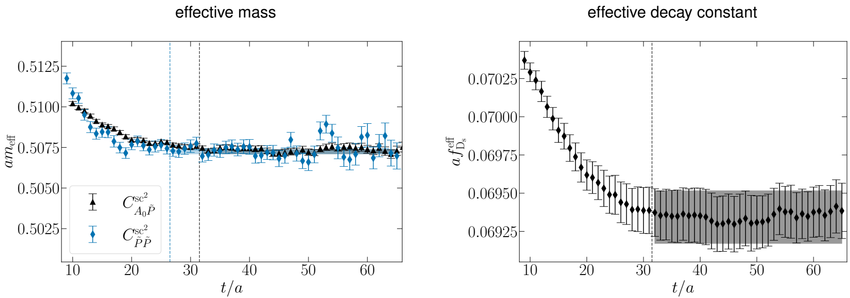
Extraction of the bare quantities



f_{D} for E250: $a \approx 0.064$ fm, $m_\pi \approx 130$ MeV, periodic boundary conditions

- groundstate dominance is explored by means of a two-state fit
  $\rightarrow$ time slices where contribution of excited state is negligible determines fit range
- vertical dashed lines indicate determined starting time slice for the ground state fit
- boundary effects wrt open b.c. are determined $\rightarrow$ only use data in bulk region

Extraction of the bare quantities



$f_{\mathrm{D_s}}$ for E300: $a \approx 0.05$ fm, $m_\pi \approx 174$ MeV, open boundary conditions

- groundstate dominance is explored by means of a two-state fit
  $\rightarrow$ time slices where contribution of excited state is negligible determines fit range
- vertical dashed lines indicate determined starting time slice for the ground state fit
- boundary effects wrt open b.c. are determined $\rightarrow$ only use data in bulk region

Renormalization and improvement

→ following [Bhattacharya et al., 2005]

Pseudoscalar decay constants

$$f^{\mathrm{R}}_{\mathrm{D}_{(q)}} = Z_{\mathrm{A}} \left[1 + a \left(b_{\mathrm{A}} m_{q\mathrm{c}} + \bar{b}_{\mathrm{A}} \mathrm{Tr} M\right)\right] \left(f^{(0)}_{\mathrm{D}_{(q)}} + c_{\mathrm{A}} f^{(1)}_{\mathrm{D}_{(q)}}\right) + \mathrm{O}(a^2)$$

Vector and tensor decay constants

$$f^{\mathrm{R}}_{\mathrm{D}^*_{(q)}} = Z_{\mathrm{V}} \left[1 + a \left(b_{\mathrm{V}} m_{q\mathrm{c}} + \bar{b}_{\mathrm{V}} \mathrm{Tr} M\right)\right] \left(f^{(0)}_{\mathrm{D}^*_{(q)}} + c_{\mathrm{V}} f^{(1)}_{\mathrm{D}^*_{(q)}}\right) + \mathrm{O}(a^2)$$

$$f^{\mathrm{T,R}}_{\mathrm{D}^*_{(q)}} = Z_{\mathrm{T}} \left[1 + a \left(b_{\mathrm{T}} m_{q\mathrm{c}} + \bar{b}_{\mathrm{T}} \mathrm{Tr} M\right)\right] \left(f^{\mathrm{T},(0)}_{\mathrm{D}^*_{(q)}} + c_{\mathrm{T}} f^{\mathrm{T},(1)}_{\mathrm{D}^*_{(q)}}\right) + \mathrm{O}(a^2)$$

Non-perturbative determinations of renormalization and improvement constants

- $Z_{\mathrm{A}}, b_{\mathrm{A}}, c_{\mathrm{A}}$: [1502.04999], [1604.05827], [1607.07090], [1808.09236]
- κ_{crit}: [2211.03744]
- $Z_{\mathrm{V}}, b_{\mathrm{V}}, c_{\mathrm{V}}$: [1805.07401], [1811.08209], [2010.09539]
- $Z_{\mathrm{T}}, b_{\mathrm{T}}, c_{\mathrm{T}}$: [1910.06759], [2012.06284], [2305.04717]

Chiral and continuum extrapolation

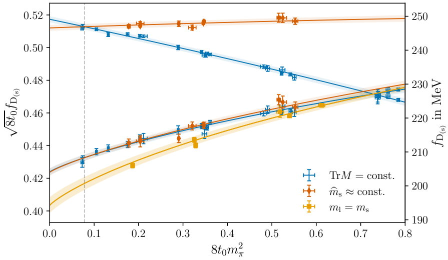
Combined chiral and continuum fit

- all quantities rescaled by t_0 to make them dimensionless
- quark masses are parametrized by
  $\bar{\mathbb{M}}^2 \sim 2m_{\mathrm{K}}^2 + m_\pi^2 \sim (2m_{\mathrm{l}} + m_{\mathrm{s}})\,, \quad \delta\mathbb{M}^2 \sim m_{\mathrm{K}}^2 - m_\pi^2 \sim (m_{\mathrm{s}} - m_{\mathrm{l}})\,, \quad \mathbb{M}_{\bar{\mathrm{D}}} \sim m_{\mathrm{c}}$
- general ansatz inspired by SU(3) chiral perturbation theory

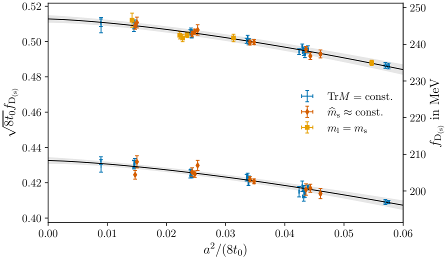
$$\sqrt{8t_0} f_{\mathrm{D_s}} = f_0 + c_1\,\bar{\mathbb{M}}^2 + \frac{2}{3} c_2\,\delta\mathbb{M}^2 + c_3\,(4\mu_{\mathrm{K}} + \frac{4}{3}\mu_\eta) + c_4\,\mathbb{M}_{\bar{\mathrm{D}}} + \ldots$$
$$\sqrt{8t_0} f_{\mathrm{D}} = f_0 + c_1\,\bar{\mathbb{M}}^2 - \frac{1}{3} c_2\,\delta\mathbb{M}^2 + c_3\,(3\mu_\pi + 2\mu_{\mathrm{K}} + \frac{1}{3}\mu_\eta) + c_4\,\mathbb{M}_{\bar{\mathrm{D}}} + \ldots$$

  $\rightarrow$ includes chiral logs $\mu_X = 8t_0 m_X^2 \log(8t_0 m_X^2)$
- vary fit model including $\leftarrow \chi^2/dof = 1.09 - 0.92$
  $\rightarrow$ quadratic, cubic and quartic terms in a $\leftarrow$ mass-dependent and independent terms
  $\rightarrow$ various higher order terms and combinations in light, strange, and charm quark masses
- perform AIC average using correlated fits

Chiral and continuum extrapolation



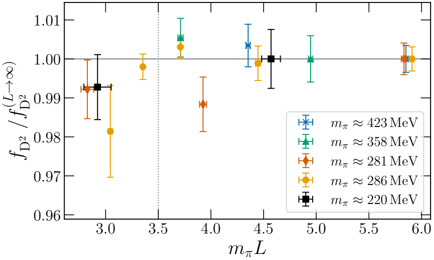
chiral extrapolation

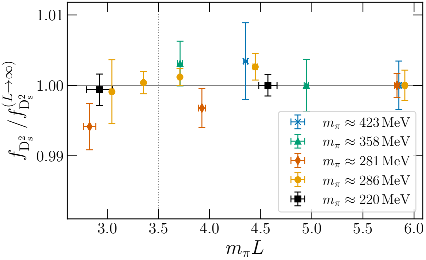


continuum extrapolation

- plots correspond to best fit $\chi^2/dof = 0.92$
- data points are projected according to the fit to continuum limit/physical point
- this fit includes higher order terms: $\sim m_D^2$, $\sim a^3$
- all three chiral trajectories are utilized

Systematics: Finite volume effects



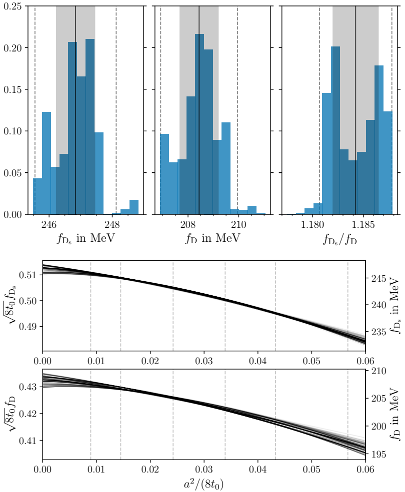


Dedicated ensembles with small/large volumes

$\rightarrow$ no significant finite volume effects for $m_\pi L > 3.5$

$\rightarrow$ exclude ensembles with $m_\pi L < 3.5$ and/or $L < 2.3$ fm

Systematics: Model and lattice spacing dependence



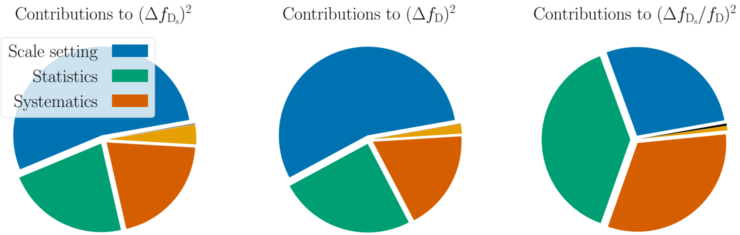
Model average: AIC weights

- histogram of fit results weighted by AIC weight
- systematic uncertainty from model average
- ratio computed from extrapolated f_D and f_{D_s}

Model average: a-dependence

- opacity $\sim$ weight
- pure a^2 fits have a small weight

Results and error budget

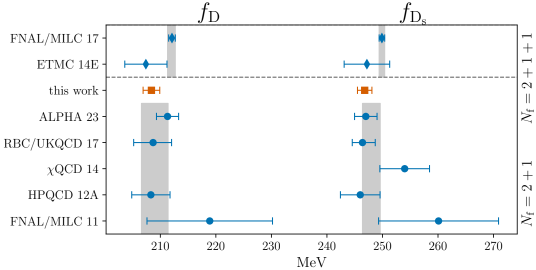


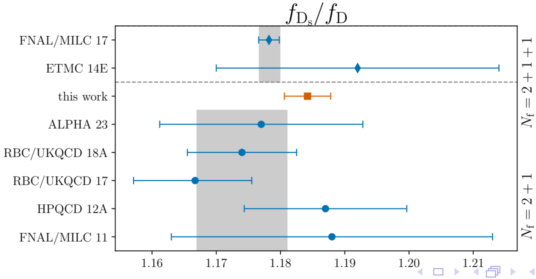
$f_{D_s} = 246.8(1.3)$ MeV $f_D = 208.4(1.5)$ MeV $f_{D_s}/f_D = 1.1842(36)$

$\rightarrow$ uncertainty of decay constants limited by scale setting

$\rightarrow$ balanced statistical and systematic uncertainties

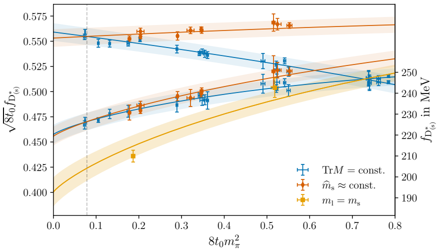
Comparison to other determinations

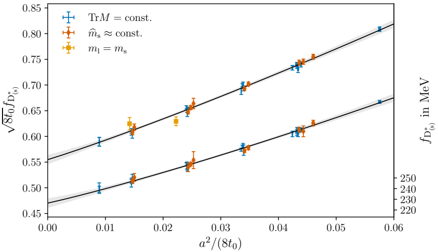




Outlook: Vector and tensor decay constants

→ work in progress





Very first attempt of an chiral and continuum extrapolation

→ $\chi^2/dof = 1.3$, 32 ensembles, 15 parameters

→ starting to investigate fit models and systematics

Conclusions

Leptonic decay constants of the D and $\mathrm{D_s}$ mesons

- determined f_D and $f_{\mathrm{D_s}}$ in partially quenched $2+1$ flavor QCD with sub-percent precision
- large number of ensembles constraints chiral and continuum extrapolation tightly
- bare decay constants at $\sim$ 0.5% uncertainty
- uncertainty limited by scale setting
- note: for higher precision also isospin breaking effects need to be computed

$\rightarrow$ find more details in our publication JHEP07(2024)090, [2405.04506]

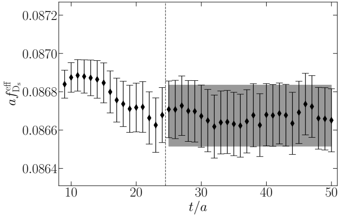
Leptonic vector and tensor decay constants of the D^* and D_s^* mesons

- work in progress: first attempt of a chiral and continuum limit
- final uncertainty will be limited by scale setting and $Z_\mathrm{T}(\mu)$.

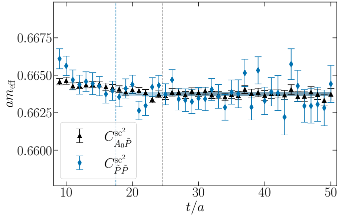
Backup Slides

Backup Slides

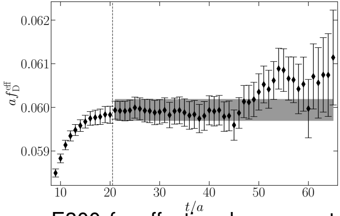
Extraction of the bare quantities



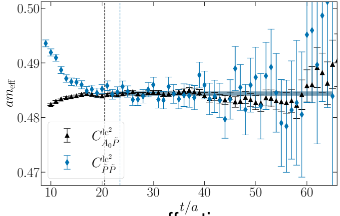
E250 f_{D_s}: effective decay constant



effective mass



E300 f_{D}: effective decay constant



effective mass

Fit form of the best fit

Quark mass dependence parametrized by $\bar{\mathbb{M}}^2$, $\delta\mathbb{M}^2$, and $\bar{\mathbb{M}}_{\mathrm{H}}$

$$\bar{\mathbb{M}}^2 \equiv 8t_0\frac{1}{3}\left(2m_{\mathrm{K}}^2 + m_\pi^2\right) \propto (2m_{\mathrm{l}} + m_{\mathrm{s}})\,, \quad \delta\mathbb{M}^2 \equiv 8t_0 2\left(m_{\mathrm{K}}^2 - m_\pi^2\right) \propto (m_{\mathrm{s}} - m_{\mathrm{l}})\,,$$

$$\bar{\mathbb{M}}_{\mathrm{H}} \equiv m_{\bar{\mathrm{D}}} = \frac{2}{3}m_{\mathrm{D}} + \frac{1}{3}m_{\mathrm{D_s}} \propto m_{\mathrm{c}}\,, \quad \mathbb{a}^2 \equiv \frac{a^2}{8t_0}$$

Fit form

$$\begin{aligned}\sqrt{8t_0}f_{\mathrm{D_s}}(m_\pi, m_{\mathrm{K}}, m_{\mathrm{H}}, a) = & f_0 + c_1\,\bar{\mathbb{M}}^2 + \frac{2}{3}c_2\,\delta\mathbb{M}^2 + c_3\,(4\mu_{\mathrm{K}} + \tfrac{4}{3}\mu_\eta) + c_4\,\bar{\mathbb{M}}_{\mathrm{H}} \\ & + c_5\,\bar{\mathbb{M}}_{\mathrm{H}}^2 + c_6\,\delta\mathbb{M}^2\bar{\mathbb{M}}_{\mathrm{H}} + c_8\,\bar{\mathbb{M}}^2\bar{\mathbb{M}}_{\mathrm{H}} + c_9\,\mathbb{a}^2 + c_{10}\,\mathbb{a}^2\bar{\mathbb{M}}_{\mathrm{H}} \\ & + c_{11}\,\delta\mathbb{M}^2\bar{\mathbb{M}}_{\mathrm{H}}^2 + c_{13}\,a^3 + c_{14}\,a^3\delta\mathbb{M}^2\,, \\ \sqrt{8t_0}f_{\mathrm{D}}(m_\pi, m_{\mathrm{K}}, m_{\mathrm{H}}, a) = & f_0 + c_1\,\bar{\mathbb{M}}^2 - \frac{1}{3}c_2\,\delta\mathbb{M}^2 + c_3\,(3\mu_\pi + 2\mu_{\mathrm{K}} + \tfrac{1}{3}\mu_\eta) + c_4\,\bar{\mathbb{M}}_{\mathrm{H}} \\ & + c_5\,\bar{\mathbb{M}}_{\mathrm{H}}^2 + c_7\,\delta\mathbb{M}^2\bar{\mathbb{M}}_{\mathrm{H}} + c_8\,\bar{\mathbb{M}}^2\bar{\mathbb{M}}_{\mathrm{H}} + c_9\,\mathbb{a}^2 + c_{10}\,\mathbb{a}^2\bar{\mathbb{M}}_{\mathrm{H}} \\ & + c_{12}\,\delta\mathbb{M}^2\bar{\mathbb{M}}_{\mathrm{H}}^2 + c_{13}\,\mathbb{a}^3 + c_{14}\,\mathbb{a}^3\delta\mathbb{M}^2\,.\end{aligned}$$

Finite volume effects m_π, m_K, m_D, $m_{\mathrm{D_s}}$