Form factors in semileptonic decay of D meson

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Outline

- Motivations
- Formulae and Calculation details
- Preliminary results

Motivations

- Research of charm quark physics. Meson masses are at the typical non-perturbative energy region.
- A transition $c \to s$, determining $|V_{cs}|$ with experimental data (decay rate or branch ratio), and helping to test unitarity of CKM matrix and searching for new physics beyond SM.

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & \boxed{|V_{cs}|} & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & \boxed{0.97349 \pm 0.00016} & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}$$

(PDG, review of particle physics, DOI:10.1093/ptep/ptac097)

- Combining with experiment,

Experimental data SM parameter non-perturbative non-perturbative

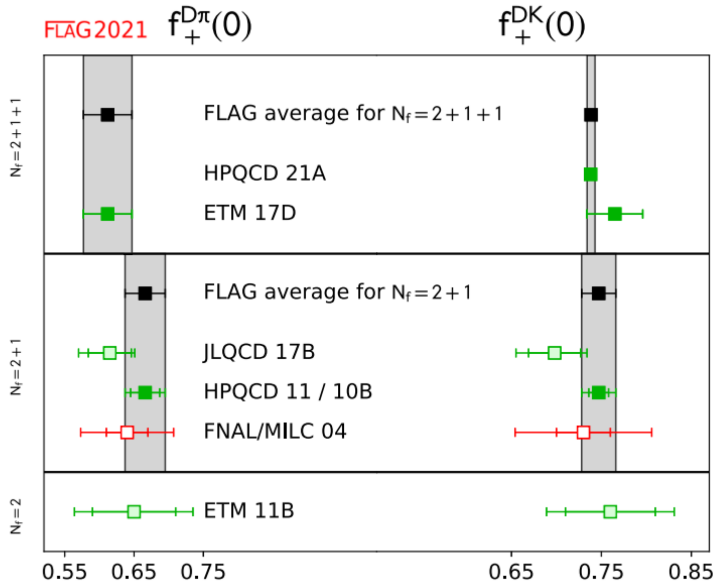
$$\boxed{\frac{d\Gamma(D \to P\ell\nu)}{dq^2}} = \frac{G_F^2 \boxed{|V_{cx}|^2}}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_D^2} \times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) m_D^2 (E_P^2 - m_P^2) \boxed{|f_+(q^2)|^2} + \frac{3m_\ell^2}{8q^2} (m_D^2 - m_P^2)^2 \boxed{|f_0(q^2)|^2} \right]$$

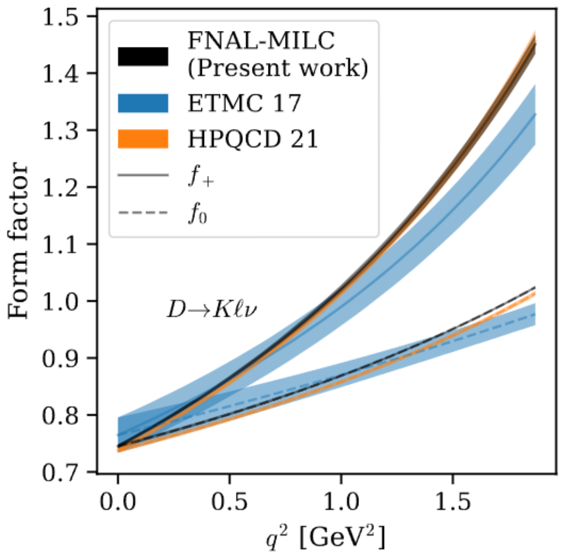
Motivations

- FLAG Review 2021 (Eur. Phys. J. C (2022) 82:869)
- Latest result

$N_f = 2+1+1$ Highly Improved Staggered Quark from Fermilab/MILC (2212.12648, PRD107.094516)

more precise ←





→ overlap fermions

a new cross check

Formulae of form factors

- **Vector** current

$$Z_V \langle P(p_P) | V^\mu | D(p_D) \rangle = \boldsymbol{f_+(q^2)} \left(p_D^\mu + p_P^\mu - \frac{M_D^2 - M_P^2}{q^2} q^\mu \right) + \boldsymbol{f_0(q^2)} \frac{M_D^2 - M_P^2}{q^2} q^\mu$$

- **Scalar** current

$$\langle P(p_P) | S | D(p_D) \rangle = \boldsymbol{f_0(q^2)} \frac{M_D^2 - M_P^2}{m_c - m_x}$$

- **Tensor** current (**beyond SM**)

$$Z_T \langle P(p_P) | T^{\mu\nu} | D(p_D) \rangle = \boldsymbol{f_T(q^2)} \frac{2}{M_D + M_P} [p_P^\mu p_D^\nu - p_P^\nu p_D^\mu]$$

The renormalization constants are from χQCD Collaboration, DOI:10.1103/PhysRevD.108.054506.

Formulism of correlation function

- 2-point correlation function (2pt),

$$C_2(t;\vec{p}) = \sum_{\vec{y}} \langle\Omega\big|O(t+t_0,\vec{y})O^\dagger(t_0,\vec{x}_0)\big|\Omega\rangle e^{-i\vec{p}(\vec{y}-\vec{x_0})}$$
$$\propto |\langle\Omega|O(0)|M,\vec{p}\rangle|^2 e^{-E_{\vec{p}}t}$$

- 3-point correlation function (3pt) ,

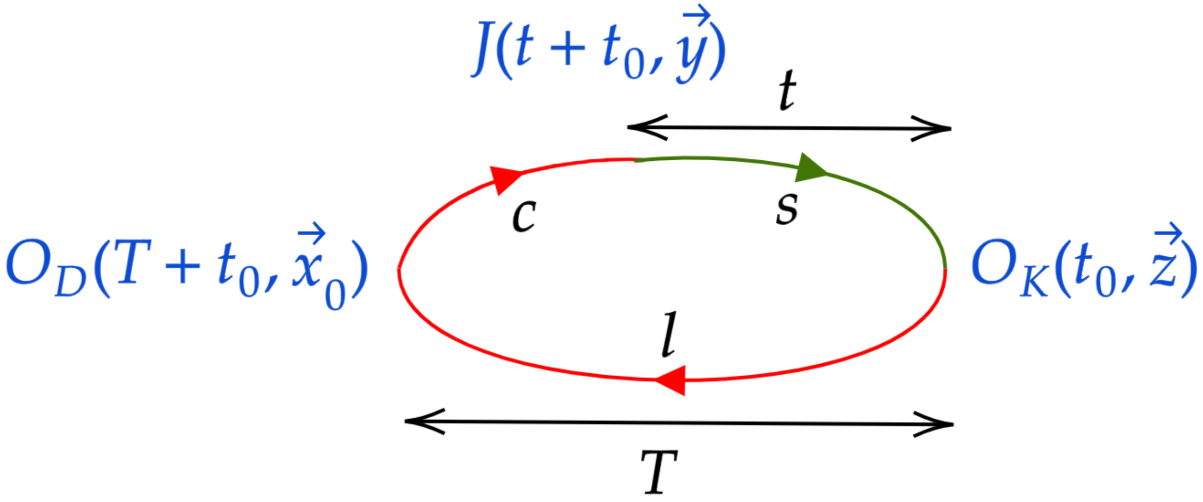
$$C_3(t,T;\overrightarrow{p_I},\overrightarrow{p_F}) = \sum_{\vec{z},\vec{y}} \langle\Omega\big|O_I(T+t_0,\overrightarrow{x_0})O_c(t+x_0,\vec{y})O_F^\dagger(t_0,\vec{z})\big|\Omega\rangle\, e^{i\overrightarrow{p_I}(\vec{y}-\overrightarrow{x_0})}e^{-i\overrightarrow{p_F}(\vec{z}-\overrightarrow{x_0})}$$
$$\propto \langle\Omega|O_I|n,\overrightarrow{p_I}\rangle\langle n,\overrightarrow{p_I}|O_c|m,\overrightarrow{p_F}\rangle\langle m,\overrightarrow{p_F}\big|O_F^\dagger\big|\Omega\rangle e^{-E_I t}e^{-E_F(T-t)}$$

Operators for mesons and currents we used,

$$O_I = \bar{\psi}_c\gamma^5\psi_l, O_F = \bar{\psi}_l\gamma^5\psi_s, O_C = \bar{\psi}_s\Gamma\psi_c, \Gamma = V^\mu, S, \sigma^{\mu\nu}$$

Calculation of correlation function

We set D meson at time $T + t_0$, and K meson at time t_0, so 3pt can be a function of time t between current and K meson.



And D meson is set be still while K meson is set to move, so we can change q^2.

Simulation details

- **Gauge configuration**, arXiv:1011.0892 RBC&UKQCD

$N_f = 2+1$ **domain wall** gauge action,

overlap fermions for valence quark,

to keep chiral symmetry.

$m_\pi^{sea} \cong 360\text{MeV}$

$1/a$ (GeV)	Label	am_l/am_s	Volume	$N_{conf} \times N_{src}$
2.383(9)	f004	0.004/0.03	$32^3 \times 64$	628×1
	f006	0.006/0.03	$32^3 \times 64$	42×16
	f008	0.008/0.03	$32^3 \times 64$	49×16

- **Parameters of caculation**

$T = 32$		$a\vec{p_I}$	$a\vec{p_F}$
am_l	0.0046, 0.0765, 0.0129, 0.024	(0, 0, 0)	(0, 0, 0) (0, 0, 1), (0, 1, 0), (1, 0, 0) (0, 1, 1), (0, 1, 1), ⋯, (1, −1, 0) (1, 1, 1), (1, 1, −1), ⋯, (−1, −1, 1)
am_s	0.037, 0.040, 0.043, 0.046, 0.049, 0.052		
am_c	0.45, 0.492, 0.50, 0.55		

Formulism of correlation function

- **Ratio**, 3pt divides 2pt,

$$\frac{C_3^{D\to K}(t,T;\overrightarrow{p_I},\overrightarrow{p_F})}{C_2^D(t;\overrightarrow{p_I})C_2^K(T-t;\overrightarrow{p_F})} \cong \frac{|\langle K,\overrightarrow{p_F}|O_c|D,\overrightarrow{p_I}\rangle|}{|\langle\Omega|O(0)|K,\vec{p}_F\rangle| \times |\langle\Omega|O(0)|D,\overrightarrow{p_I}\rangle|}$$

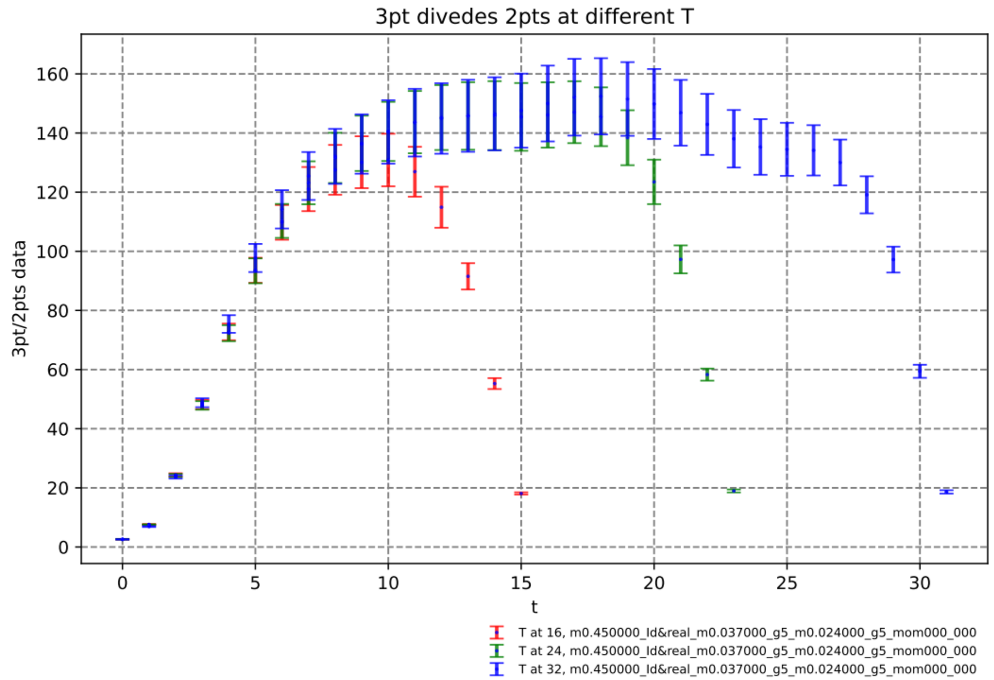
There should be a plateau when meson **ground states are dominant**.

- A sample of scalar current

at $T = 16,24,32$ and $a\overrightarrow{p_I} = (0,0,0)$

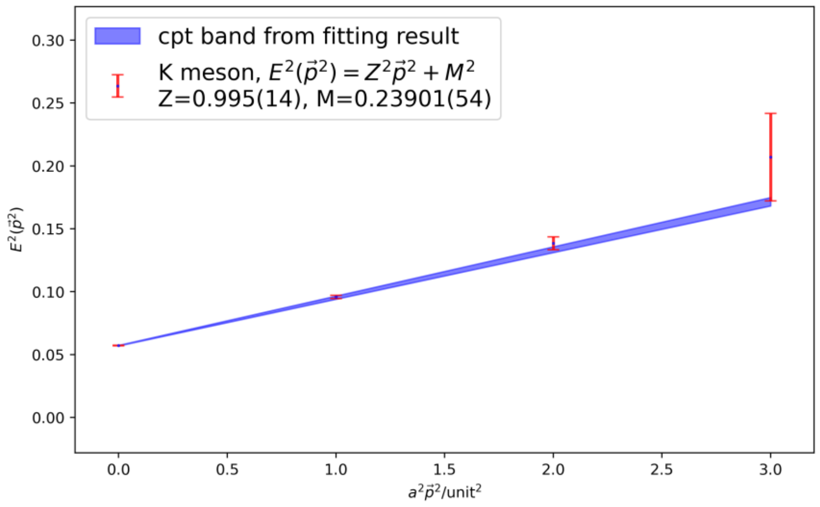
With $am_l = 0.024,\ am_s = 0.037,$

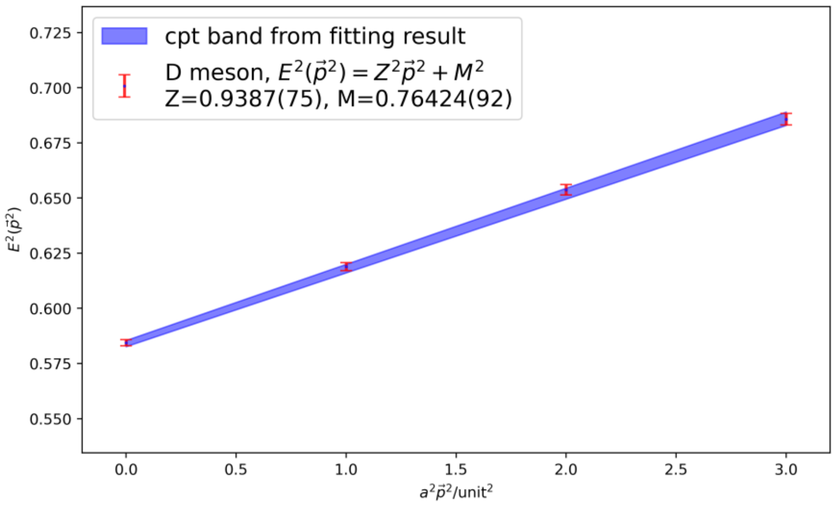
$am_s = 0.45.$



Dispersion relation

We checked the dispersion relation with energy of K meson and D meson at four different momenta and use a modified continuous dispersion relation as the fitting function $E^2(\vec{p}^2) = Z^2|\vec{p}|^2 + E^2(0)$





Unit is $2\pi/L$. Fitting results, $Z = 0.995\ (14)$ for K meson and $Z = 0.9387\ (75)$ for D meson. And continuous dispersion relation was used for a simple analysis.

Fitting of correlation functions

- Least χ^2 fitting.

$$\chi^2(\boldsymbol{p}) = \sum_k [f^k(\boldsymbol{x};\boldsymbol{p}) - y]_i^T C_{ij}^{-1} [f^k(\boldsymbol{x};\boldsymbol{p}) - y]_j$$

k is index of functions, and i, j are indexes of the i-th and j-th time points.

- Consider the correlation between 3pt and 2pt.
- Fitting functions are correlation functions with only ground state term with anti-period boundary condition,

$$C_3^{D\to K}(t;T) = \frac{Z_K Z_D O}{2E_K 2E_D}\left(e^{-E_D t} e^{-E_K(T-t)} + e^{-E_D(n_t-t)} e^{-E_K(n_t-T+t)}\right)$$

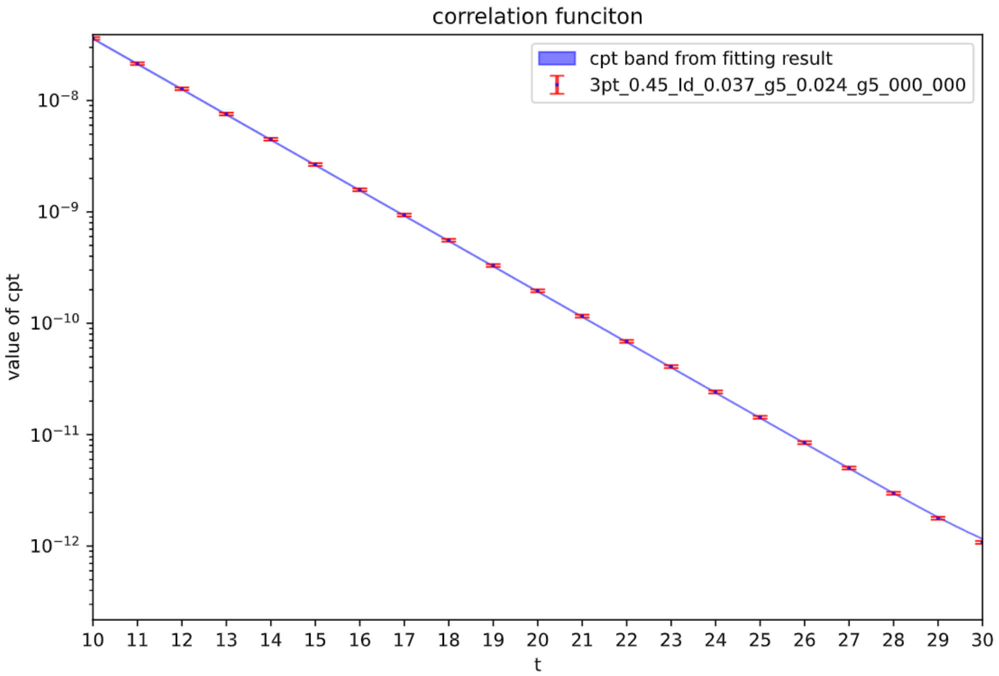
$$C_2^D(t) = \frac{Z_D^2}{2E_D}\left(e^{-E_D t} + e^{-E_D(n_t-t)}\right)$$

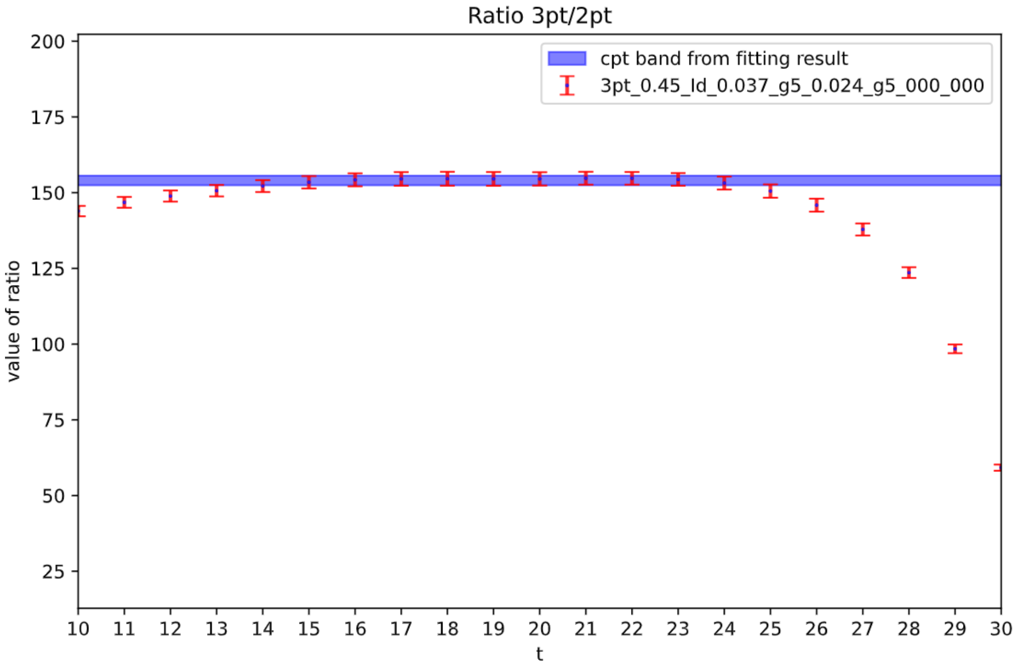
$$C_2^K(t) = \frac{Z_K^2}{2E_K}\left(e^{-E_K t} + e^{-E_K(n_t-t)}\right)$$

If we consider more current operators, we can take more matrix elements in count to do the joint fit.

Fitting correlation functions

There is a exemplary result of fitting with $am_c = 0.45$, $am_s = 0.037$ and $am_l = 0.024$ and $a\overrightarrow{p_F} = (0,0,0)$ and scalar current operator $\Gamma = \mathrm{Id}$.



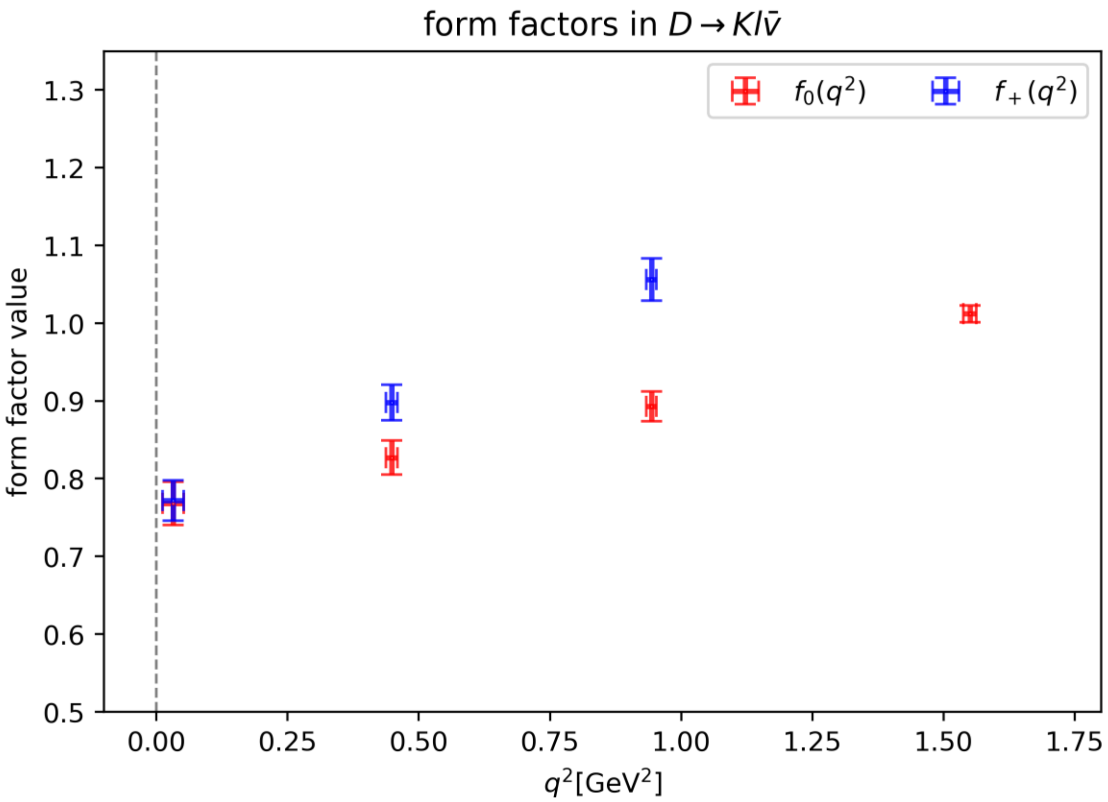


Finally $\chi^2/\mathrm{dof} = 1.2$ on fitting range $[16, 20]$, we can think that the fitting is good.

Form factors of f_0 and f_+

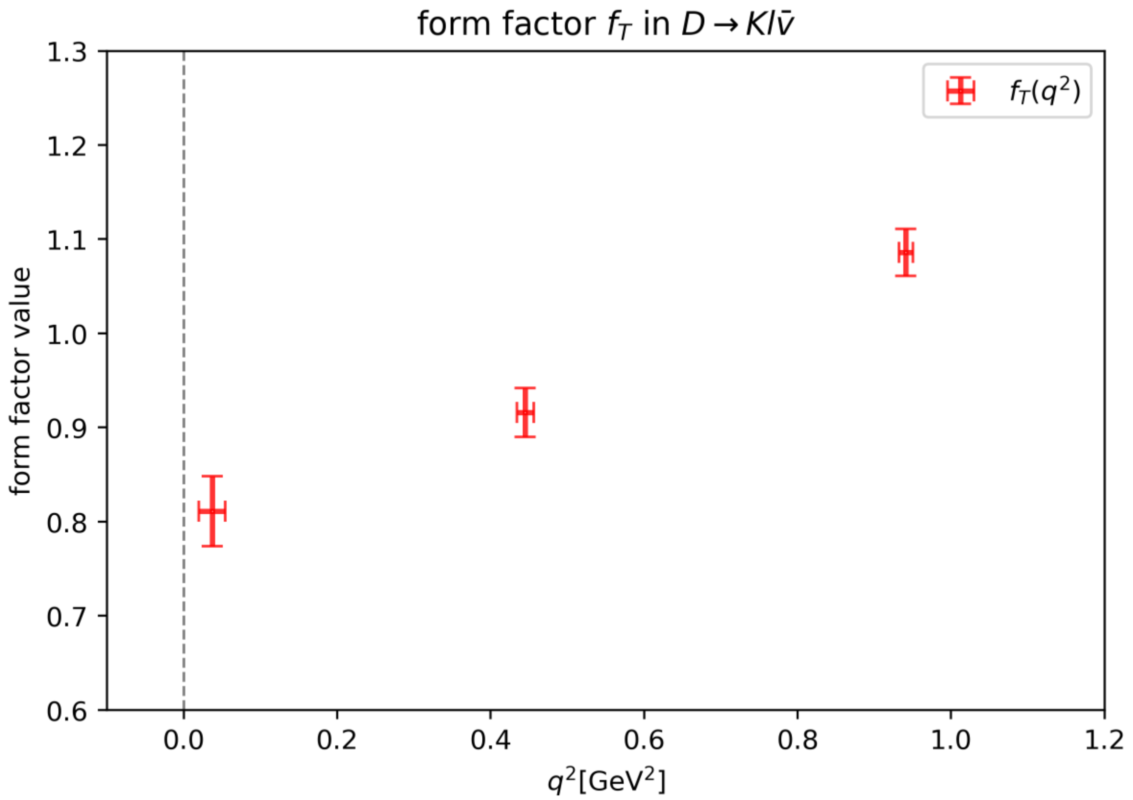
$f_0(0) = f_+(0)$

Non- singularity



Results of form factors of f_0 and f_+ with quark masses $am_c = 0.45$, $am_s = 0.037$ and $am_l = 0.024$.

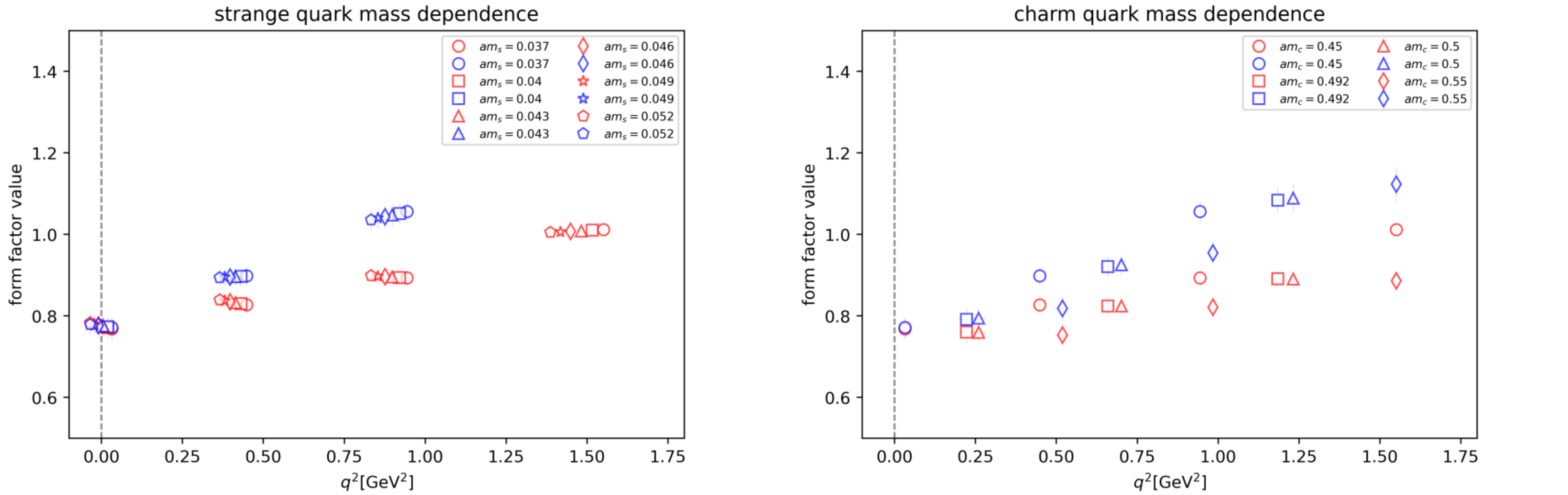
Form factors of f_T



Form factor $f_T(q^2)$ from tensor current with quark masses $am_c = 0.45$, $am_s = 0.037$ and $am_l = 0.024$. Those results have been multiplied by the renormalization constant.

Quark mass dependence

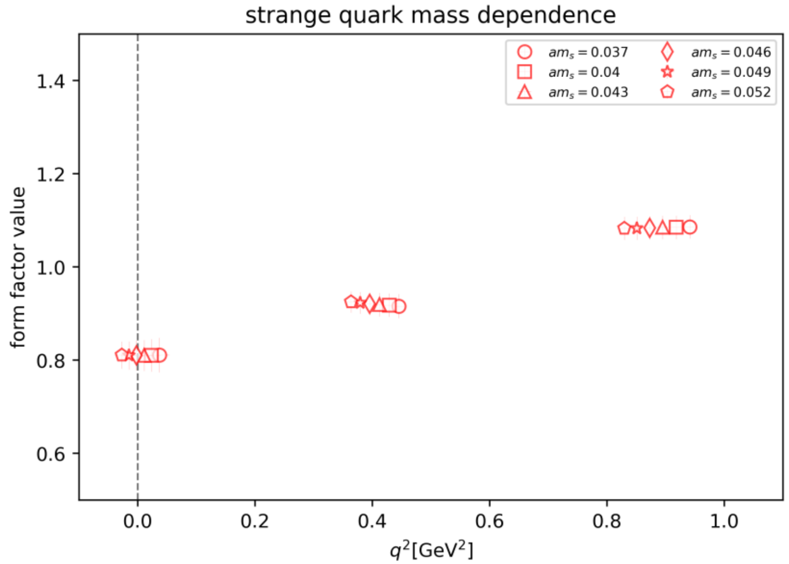
- Form factors of f_0 and f_+ change with masses of quarks,

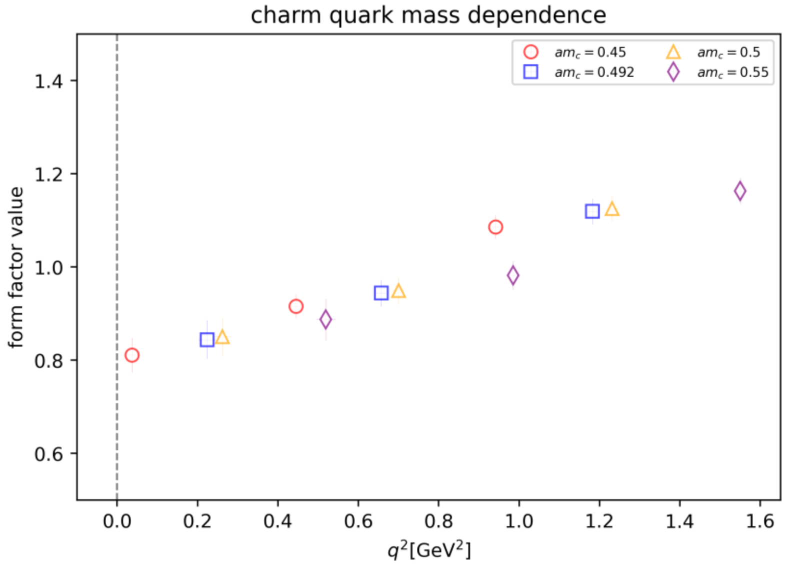


The left figure presents the results obtained by varying the strange quark mass, while the right figure shows the results obtained by varying the charm quark mass.

Quark mass dependence

- Form factor of f_T changes with masses of quarks,





The left figure presents the results obtained by varying the strange quark mass, while the right figure shows the results obtained by varying the charm quark mass.

Summary

- Preliminary results for 2-point correlation functions of D meson and K meson.
- Dispersion relation of mesons.
- Preliminary results for $D \rightarrow Kl\nu$ form factors on one lattice set with four different q^2.
- The dependence of the form factors on the masses of the quarks.

- Future work

Thank you for your attention

Extrapolation of form factors with respect to q^2.

Extrapolation/interpolation of results to the physical point.

More Statistics for decreasing statistical error.

More lattice sets for analysis of systematic error.

Back up

- About calculating 3pt

For computational cost it is most convenient to perform the calculation in the following way: two of the propagators, s and l, are generated from the same random wall source and the third quark propagator, the c, is an extended propagator (sequential propagator) using as a source the appropriate time slice of the light quark propagator.

- About renormalization constant

By $\overline{\mathrm{MS}}$ scheme at $2\mathrm{GeV}$, $Z_V = 1.0789(10)$, $Z_T = 1.157(11)$

- For Scalar current, it doesn't need a renormalization constant,

$Z_S \langle P(p_P)|S|D(p_D)\rangle \times Z_m(m_c - m_x) = f_0(q^2)(M_D^2 - M_P^2)$, for $Z_S Z_m = 1$

- Pion mass, $m_\pi^{\mathrm{sea}} \cong 360\mathrm{MeV}, m_\pi^{\mathrm{val}} \cong 504\mathrm{MeV}$

Back up

- How to extract V_{cs}?

First we need to calculate form factors at some q^2 points, then extend them to whole q^2 space with Z expansion technique, finally extract V_{cs} with experimental data like decay ray or branching fraction.

- How to test lepton flavour universality?

We can calculate the ratio of differential decay rate of different lepton in final state like electron and muon in frame of SM with our form factors, and the ratio is a function related to q^2, and it has been measured by BES experiment. But we have to note that we can compare those two result with ignoring long-distance QED corrections.