α_s in (2+1+1)-Flavor QCD from the Static Energy

Viljami Leino
Helmholtz Institute Mainz, JGU Mainz

*For the TUMQCD collaboration
A. Bazavov, N. Brambilla, A. S. Kronfeld, J .Mayer-Steudte,
P. Petreczky, S. Steinbeißer, A. Vairo and J. H. Weber

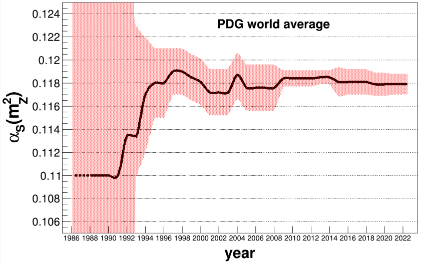
Lattice 2024
1st of August 2024
Liverpool, UK

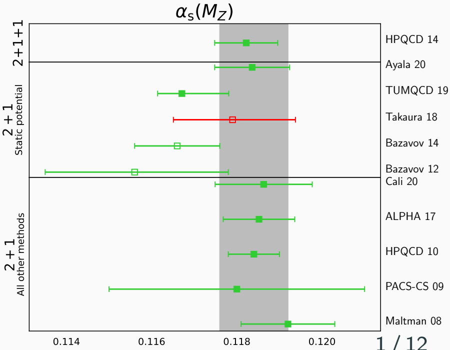
Motivation

- α_s:Fundamental parameter of SM
- Lattice dominates the global average and error
- TUMQCD measures α_s from the static energy

N_f	
0 :	2010, 2024
2+1 :	2012, 2014, 2019
2+1+1 :	This talk (preliminary)

- Very few 2+1+1 extractions on the lattice





Upper figure: D. d'Enterria *et.al.* Snowmass21
Lower: comparison of selected results to FLAG21 average

Static energy

- Static energy $E_0(r)$ between a static quark and antiquark
- Determined from the large-time behavior of Wilson loops

$$E(r) = -\lim_{T\to\infty} \frac{\ln\langle \mathrm{Tr}(W_{r\times T})\rangle}{T}, \qquad W_{r\times T} = P\left\{ \exp\left(i \oint_{r\times T} dz_\mu \, gA_\mu \right) \right\}$$

- Perturbatively known to $\mathrm{N^3LL}$ [1]:

$$E_0(r) = \Lambda_\mathrm{s} - \frac{C_\mathrm{F}\alpha_\mathrm{s}}{r}\left(1 + \#\alpha_\mathrm{s} + \#\alpha_\mathrm{s}^2 + \#\alpha_\mathrm{s}^3 + \#\alpha_\mathrm{s}^3 \ln\alpha_\mathrm{s} + \#\alpha_\mathrm{s}^4 \ln^2\alpha_\mathrm{s} + \#\alpha_\mathrm{s}^4 \ln\alpha_\mathrm{s} \ldots\right)$$

- The ultra-soft scale μ_us gives rise to $\ln\alpha_\mathrm{s}(1/r)$ term
- Scheme independent: can directly compare lattice and $\overline{\mathrm{MS}}$

[1] For review of perturbative results, see: X. Tormo Mod. Phys. Lett. A28 (2013)

Renormalon

- *Dimensional regularization*: requires a renormalon subtraction
- *Lattice regularization*: diverges as $1/a$ towards continuum limit
- Both regularization problems can be absorbed into a constant term
- Two approaches to reduce renormalon effects

Integrated force

- Static force has no renormalon

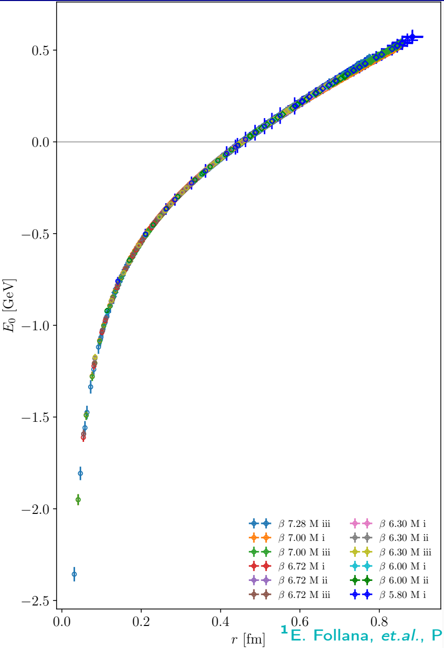
$$E_0(r) = \int_{r^*}^{r} \mathrm{d}r' F(r') + \mathrm{const}$$

- Now has integration constantt
  See e.g. Bazavov *et.al.*PRD90 (2014)

Minimal Renormalon Subtraction

- Relate the factorial growth of perturbative series to a power correction
- For more see: Komijami JHEP62 (2017), TUMQCD PRD97 (2018), Kronfeld JHEP12 (2023)

Static energy on the lattice



- Static energy in 2+1+1 QCD
- Measured for scale setting in: TUMQCD, PRD107 (2023)
- 2+1+1 HISQ[1] MILC[2] ensembles
- Three different light quark masses; physical strange and charm
- Data ranging from $r \approx 0.03$–0.09fm
- Coulomb gauge Wilson line correlators allow off-axis directions
- Massive charm: $1/m_c \sim 0.15 fm$
- This talk: $\Lambda_{\overline{\text{MS}}}$ from the small distance behavior

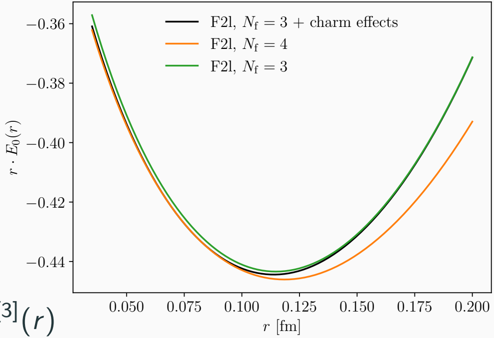
[1] E. Follana, *et.al.*, PRD75 (2007); [2] A. Bazavov, *et.al.*, PRD98 7 (2018)

Charm quark mass effects in perturbation theory

- Effects due to finite mass of a heavy quark give correction $\delta V_{\mathrm{m}}^{(N_f)}(r)$

$$E_{0,m}^{(N_f)}(r) = \int_{r^*}^{r} \mathrm{d}r' F^{(N_f)}(r') + \delta V_{\mathrm{m}}^{(N_f)}(r) + \mathrm{const}$$

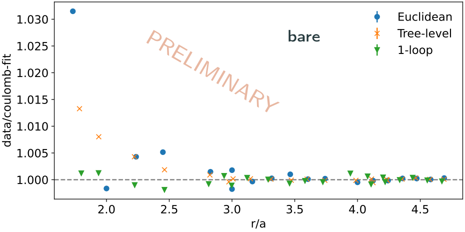
$$E_{0,m}^{(N_f)} \to E_0^{(N_f)} \quad \text{for } r \gg 1/m$$
$$E_{0,m}^{(N_f)} \to E_0^{(N_f+1)} \quad \text{for } r \ll 1/m$$

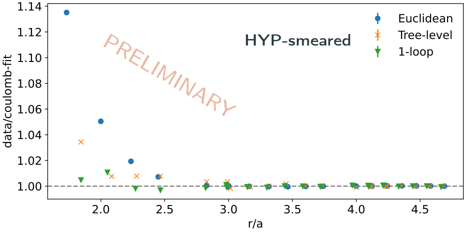


- Finite mass corrections known up to two loops[1]
  $\delta V_{\mathrm{m}}^{(N_f)}(r) = \delta V_{\mathrm{m}}^{(N_f),[2]}(r) + \delta V_{\mathrm{m}}^{(N_f),[3]}(r)$

[1] D. Eiras, J. Soto, PRD61 (2000); M. Melles PRD62 (2000); A. H. Hoang hep-ph/0008102 (2000)

Discretization effects





- Lattice breaks rotational symmetry
- $E_0(r, a)$ is discrete and direction dependent
- Ongoing effort to calculate the 1-loop improvement
- Use HPsrc and HiPPy programs to numerically calculate the diagrams
- Promising results, not finalized
  $\Rightarrow$ Use tree-level and inflate errors on $r/a \leq \sqrt{8}$ by 0.1% for this talk

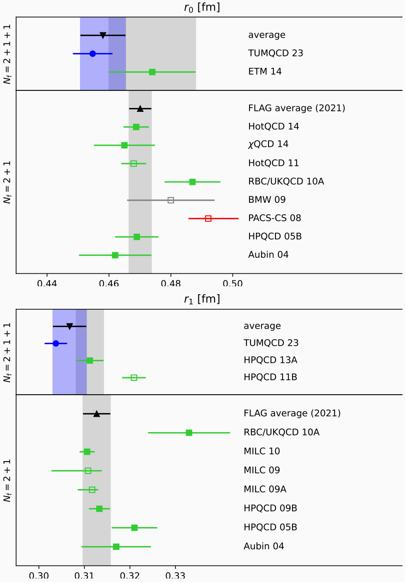
G. v. Hippel, V.L, S. Steinbeißer, in preparation TUM-EFT 171/22

Lattice scales

- Scales r_i measured recently:
  TUMQCD, PRD107 (2023)

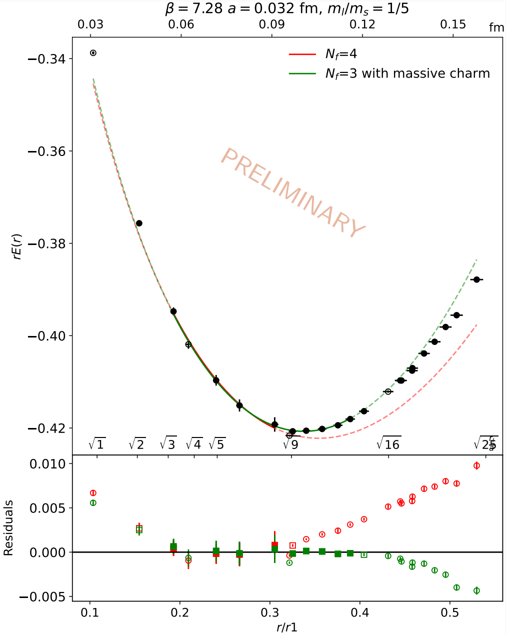
$$r_i^2 F(r_i) = \begin{cases} 1.65, & i = 0\ ^{1} \quad r_0 = 0.4547(64)\,\mathrm{fm} \\ 1.0\ , & i = 1\ ^{2}\ , \quad r_1 = 0.3037(25)\,\mathrm{fm} \\ 0.5\ , & i = 2\ ^{3} \quad r_2 = 0.1313(41)\,\mathrm{fm} \end{cases}$$

- $r_2 \sim 1/m_c$ affected by charm mass
- r_0/r_1 agrees well between 2+1 and 2+1+1
- Discrepancies between 2+1 and 2+1+1 r_1 more likely based on physical observable
- We use f_{p4s}



[1] R. Sommer, NPB411 (1994); [2] C. Bernard, *et.al.*, PRD62 (2000); [3] A. Bazavov, *et.al.*, PRD97 (2018)

Fitting lambda



- Start fits from $r/a = \sqrt{3}$
- From TUMQCD2019
  PT works up to $\sim 0.13fm$
- Charm effects noticeable already at $r > 0.1fm$
- Charm effects:
  limit to 2-loop accuracy
- Drop on-axis points due to large discretization effects
- Model average (AIC) over valid fit ranges
- Correlated fits,
  blocked jackknife

← Example: Finest ensemble, 2-loops no us-resum., MRS

Finest lattice results for $r_1\Lambda^{(3)}$

- Previous TUMQCD results of $r_1\Lambda^{(3)}$
  2014 : 0.495^{+28}_{-18}, 2019: $0.494(9)^{\mathrm{stat}}(5)^{\mathrm{lat}}(^{+21}_{-3})^{\mathrm{soft}}(6)^{\mathrm{us}}$

  ↖ 3-loop with us-resummation results with full error analysis

- With 2-loop tree-level TUMQCD19 has mean value of 0.502
  Likewise 3-loop without us-resummation is slightly lower

- We get preliminary results (statistical&model selection errors only):

N_f	loops	renormalon	$r_1\Lambda^{(3)}$
3+1	2	MRS	0.513(46)
3+1	2	Force	0.514(46)
4	2	MRS	0.502(29)
4	2	Force	0.502(29)

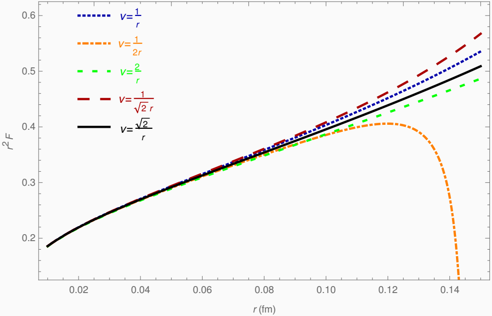
N_f	loops	renormalon	$r_1\Lambda^{(3)}$
3+1	3	MRS	0.485(47)
4	3	MRS	0.485(27)

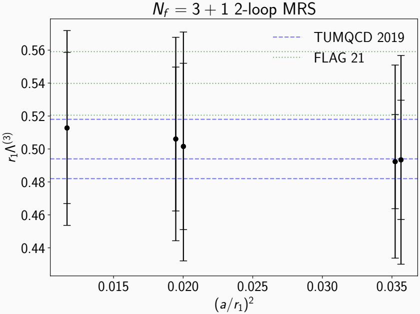
↖ 3-loop potential, 2-loop charm
no us-resummation

- No major differences between integrated force and MRS methods

- Everything agrees within errors

Note about systematics





- To study soft-scale dependence vary it by factor of $\sqrt{2}$ (or 2)
- Continuum limit yet to be performed, relatively stable at finer lattices
- US-scale variation needs to be studied, also higher order loop effects

Left figure from A. Bazavov, *et.al.*, PRD100 (2019)

Physical Units

- Converting earlier table with $r_1 = 0.3037(25)$fm based on f_{p4s} scale :

N_f	loops	renormalon	$\Lambda^{(3)}$ MeV	$\alpha_s(M_Z, N_f = 5)$
3+1	2	MRS	333(30)	0.1179(20)
4	2	MRS	326(19)	0.1175(13)

- *Preliminary numbers with partial errors and no-continuum limit!*
- TUMQCD19: $\Lambda^{(3)} = 314.0^{+15.5}_{-8.0}$MeV and $\alpha_s(M_Z) = 0.11660^{+0.00110}_{-0.00056}$
- Previous TUMQCD19 with $r_1 = 0.3106(17)$fm[1] based on f_π
- Upcoming determination of 2+1 $r_1 \approx 0.3040(25)$fm[2] based on f_K
- TUMQCD19 new r_1: $\Lambda^{(3)} = 321^{+18}_{-12}$MeV and $\alpha_s(M_Z) = 0.1171^{+00120}_{-0.00080}$
- FLAG21: $\Lambda^{(3)} = 338(12)$MeV and $\alpha_s(M_Z) = 0.1184(8)$
- PDG20 non-lattice: $\alpha_s(M_Z) = 0.1176(11)$

[1] A. Bazavov *et.al.* PoSLATTICE2010; [2] Private communication (P. Petreczky, J.H. Weber)

Conclusions

- Preliminary results for α_s measured from static energy $E_0(r)$ with 2+1+1 flavors
- Massive charm effects are clearly visible
- Preliminary results within error of previous results and literature
- Scale setting quantities can make a big difference
- Still missing from analysis:
  - One loop improvement
  - Full soft and ultrasoft scale variations
  - Continuum limit

Conclusions

- Preliminary results for α_s measured from static energy $E_0(r)$ with 2+1+1 flavors
- Massive charm effects are clearly visible
- Preliminary results within error of previous results and literature
- Scale setting quantities can make a big difference
- Still missing from analysis:
  - One loop improvement
  - Full soft and ultrasoft scale variations
  - Continuum limit

Thank you for your attention!