# Quark mass determination from overlap and clover fermion actions

**Yi-Bo Yang**

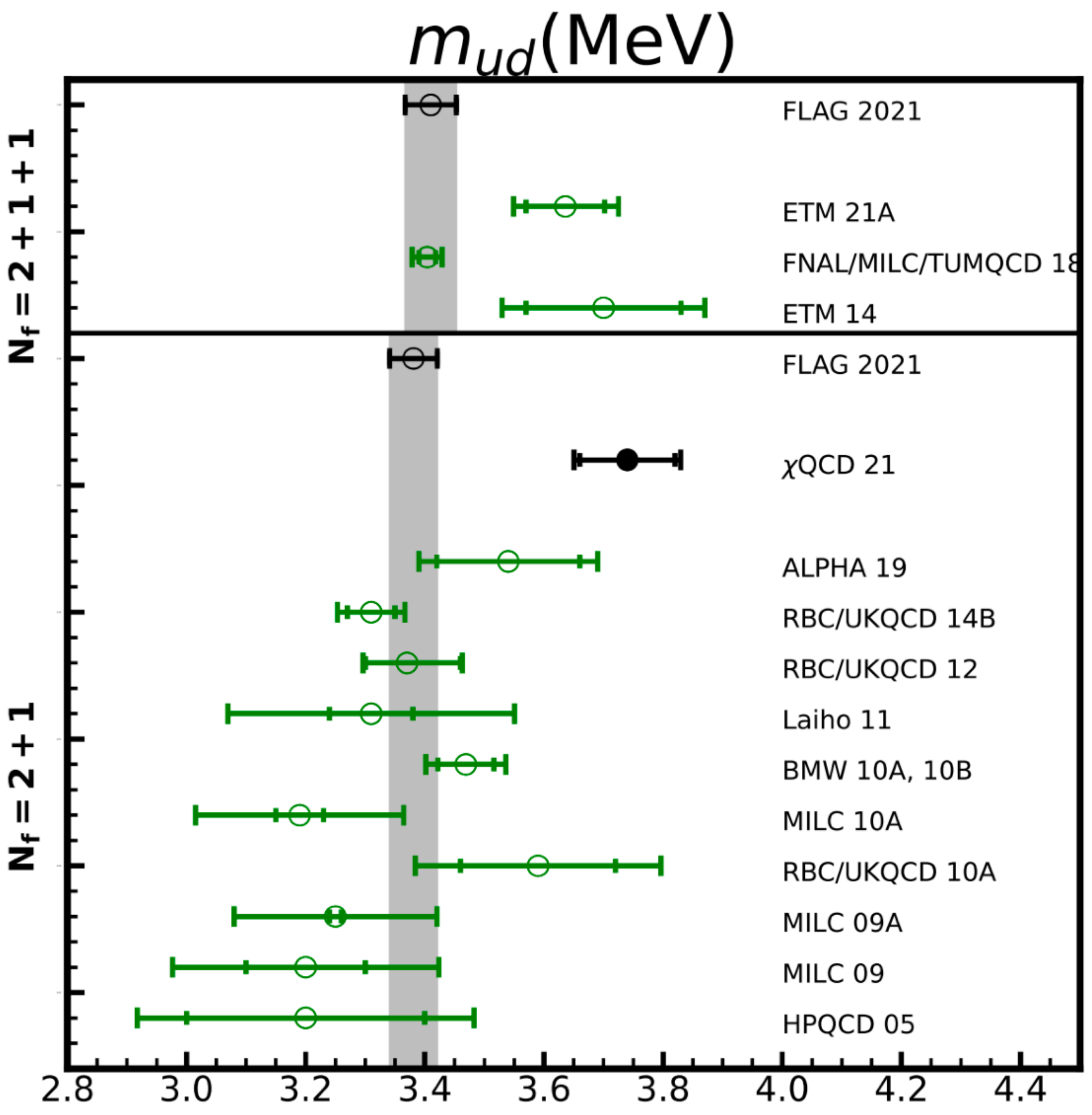
University of Chinese Academy of Sciences

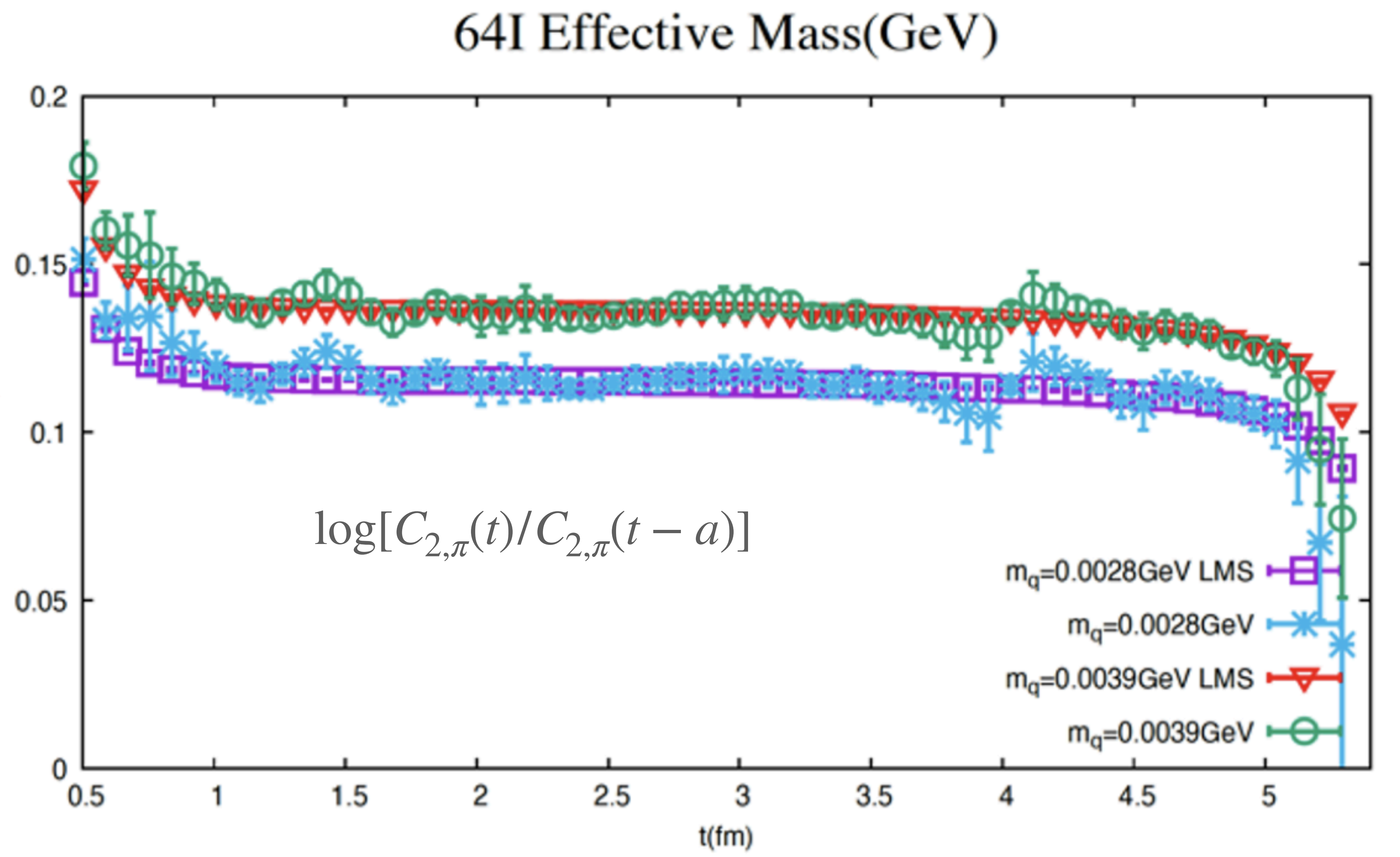
ICTP-AP International Centre for Theoretical Physics Asia-Pacific 国际理论物理中心-亚太地区

With Dian-Jun Zhao, Zhi-Cheng Hu, Bo-Lun Hu, Ji-Hao Wang, Hai-Yang Du, Geng Wang, Keh-Fei Liu and et.al.,

For $\chi$QCD and CLQCD

# Heavier light quark mass from overlap?



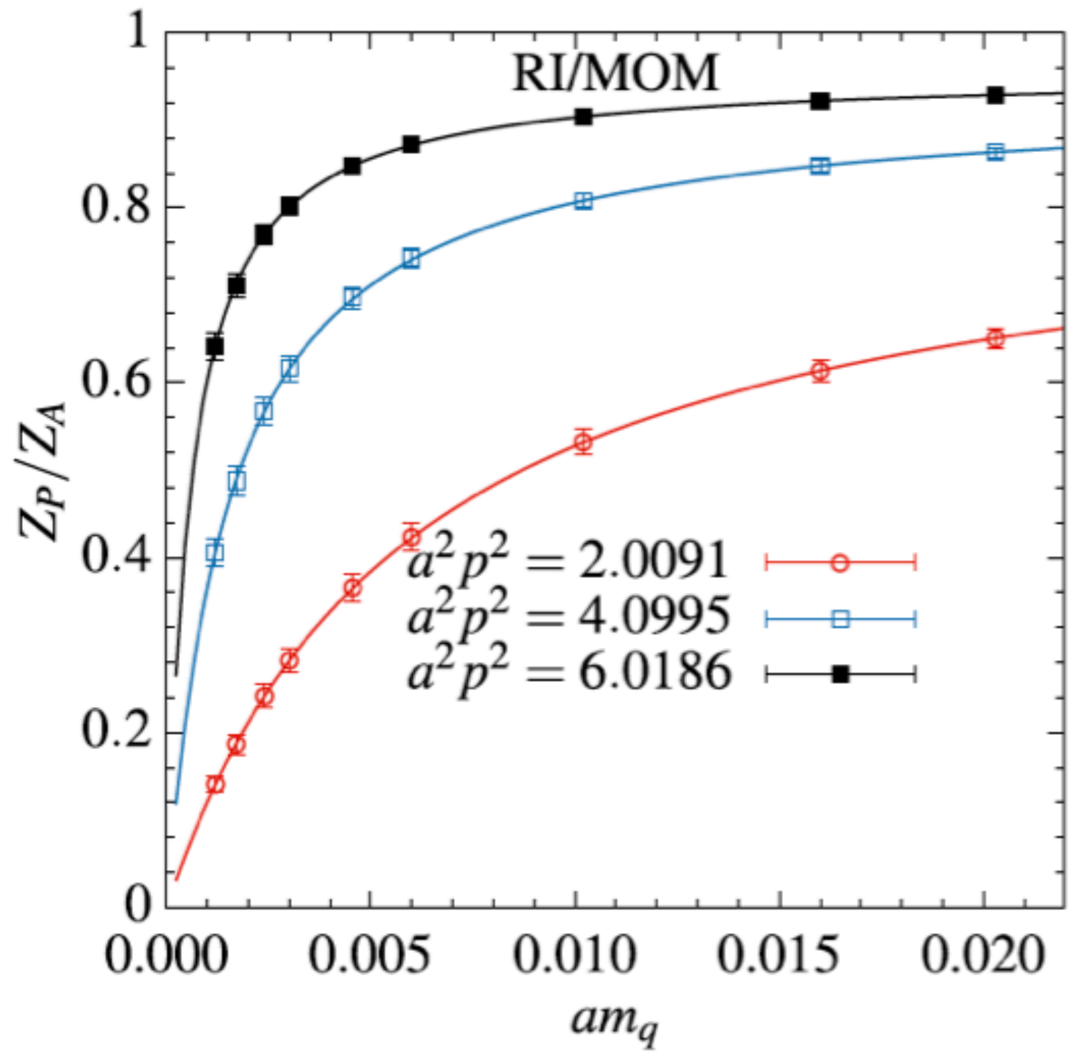



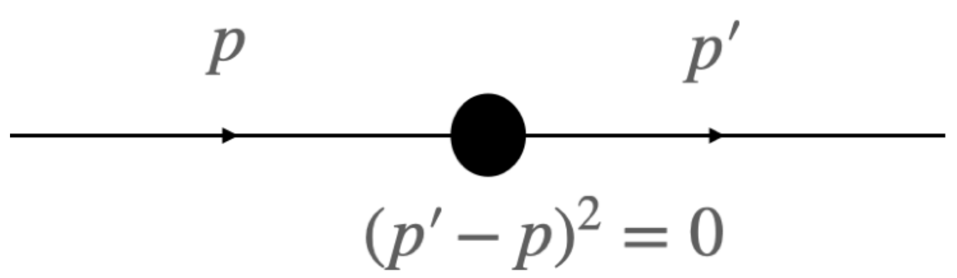

Dian-Jun Zhao, YBY, χQCD, *PoS* LATTICE2021 (2022) 198, 2201.04910

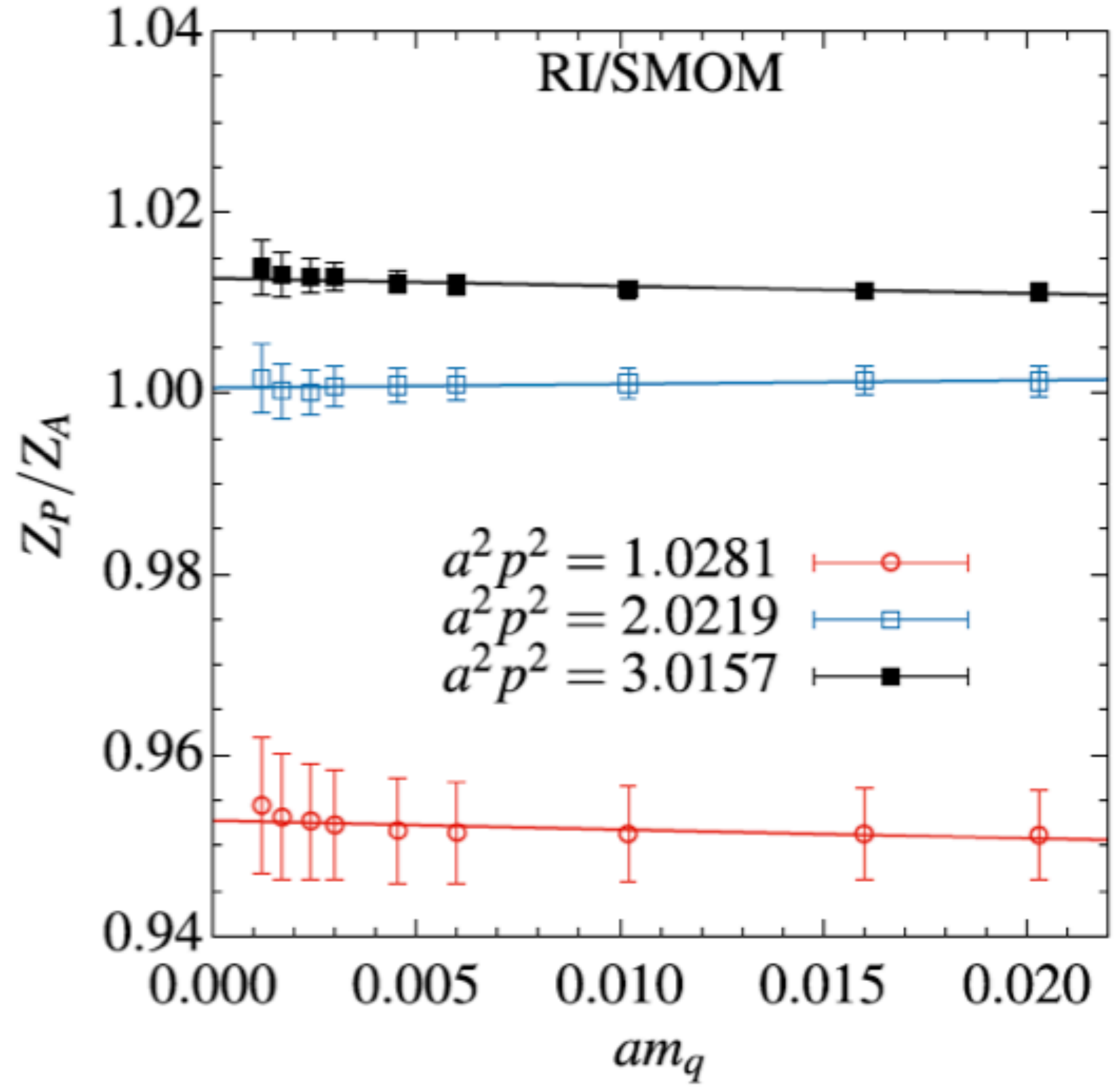
- Our previous light quark mass determination shows obvious deviation from the FLAG and also unitary DWF values.
- Using Overlap on DWF ensembles at $a = 0.114/0.084$ fm at physical point;
- Low mode substitution suppress the statistical uncertainty by two order of magnitude;
- Systematic uncertainty from the RI/MOM renormalization we used becomes critical.

$$
\begin{aligned}
C_2(t) = & \frac{1}{N_{\text{grid}}} \sum_{\vec{y}, \vec{x}_{i,j} \in \text{grid}} \langle S_1(\vec{y}, t; \vec{x}_i, 0) S_2^\dagger(\vec{y}, t; \vec{x}_j, 0) \\
& - S_{1,\text{L}}(\vec{y}, t; \vec{x}_i, 0) S_{2,\text{L}}^\dagger(\vec{y}, t; \vec{x}_j, 0) \rangle \\
& + \frac{1}{L^3 \times T} \sum_{\vec{y}, \vec{z}, t_0} \langle S_{1,\text{L}}(\vec{y}, t + t_0; \vec{z}, t_0) S_{2,\text{L}}^\dagger(\vec{y}, t + t_0; \vec{z}, t_0) \rangle \\
& \xrightarrow[0 \ll t \ll T]{} \frac{m_{\text{PS}}^3 f_{\text{PS}}^2}{2(m_{q_1} + m_{q_2})^2} (e^{-m_{\text{PS}} t a} + e^{-m_{\text{PS}} (T-t) a}),
\end{aligned}
$$

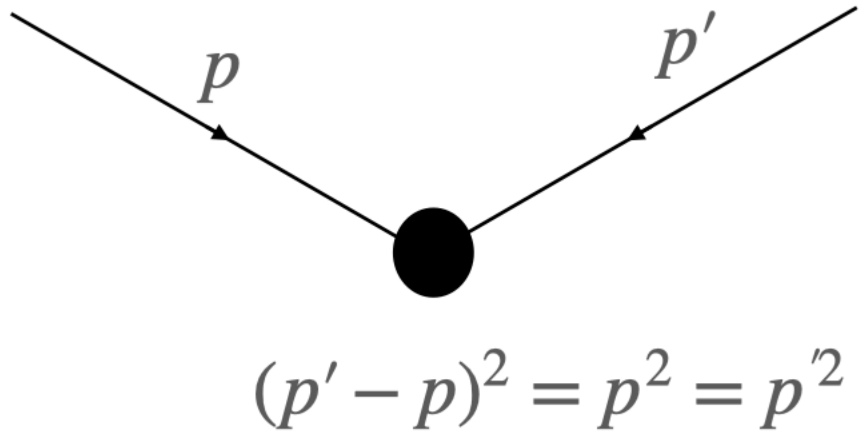
# RI/MOM renormalization



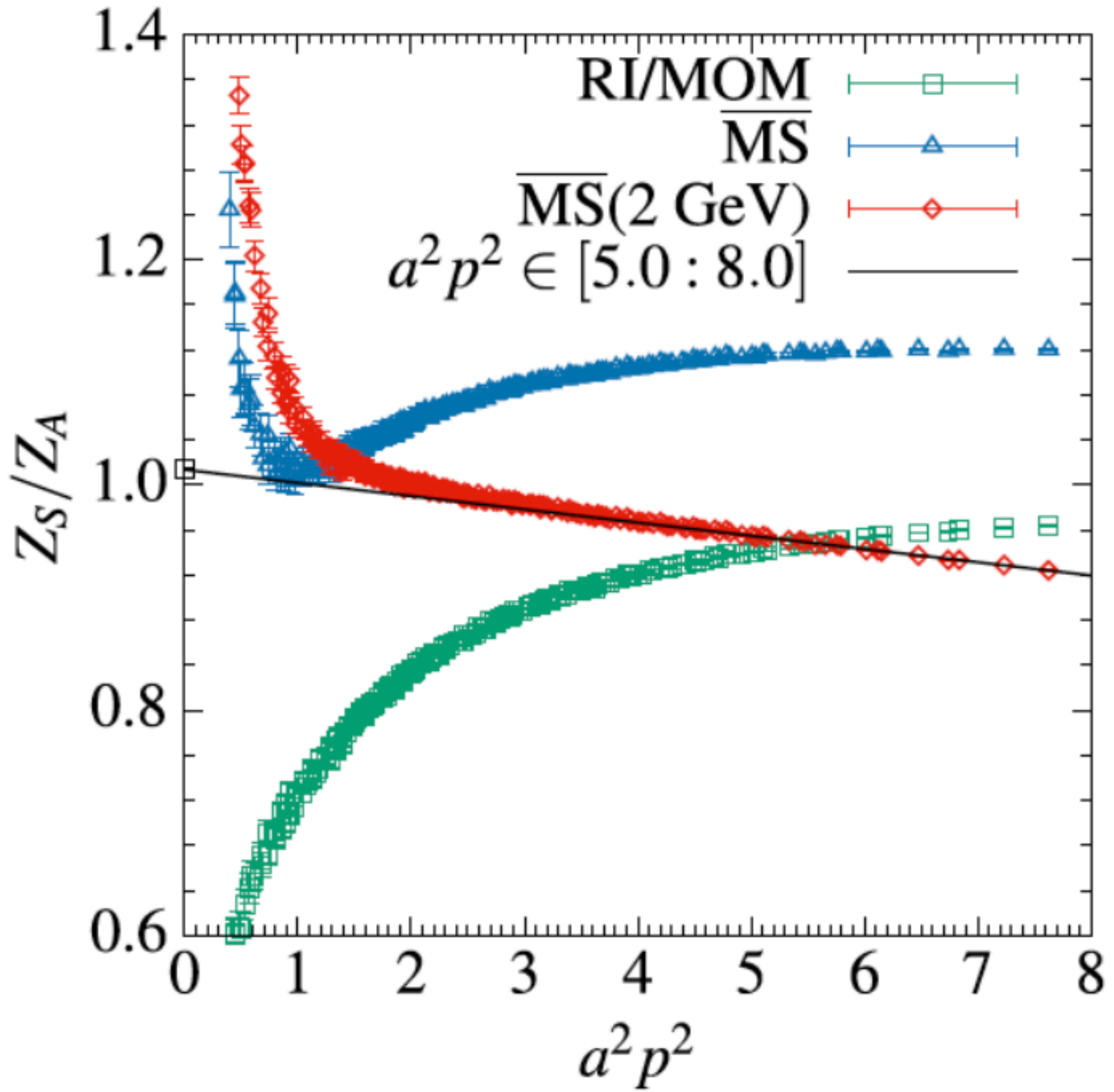



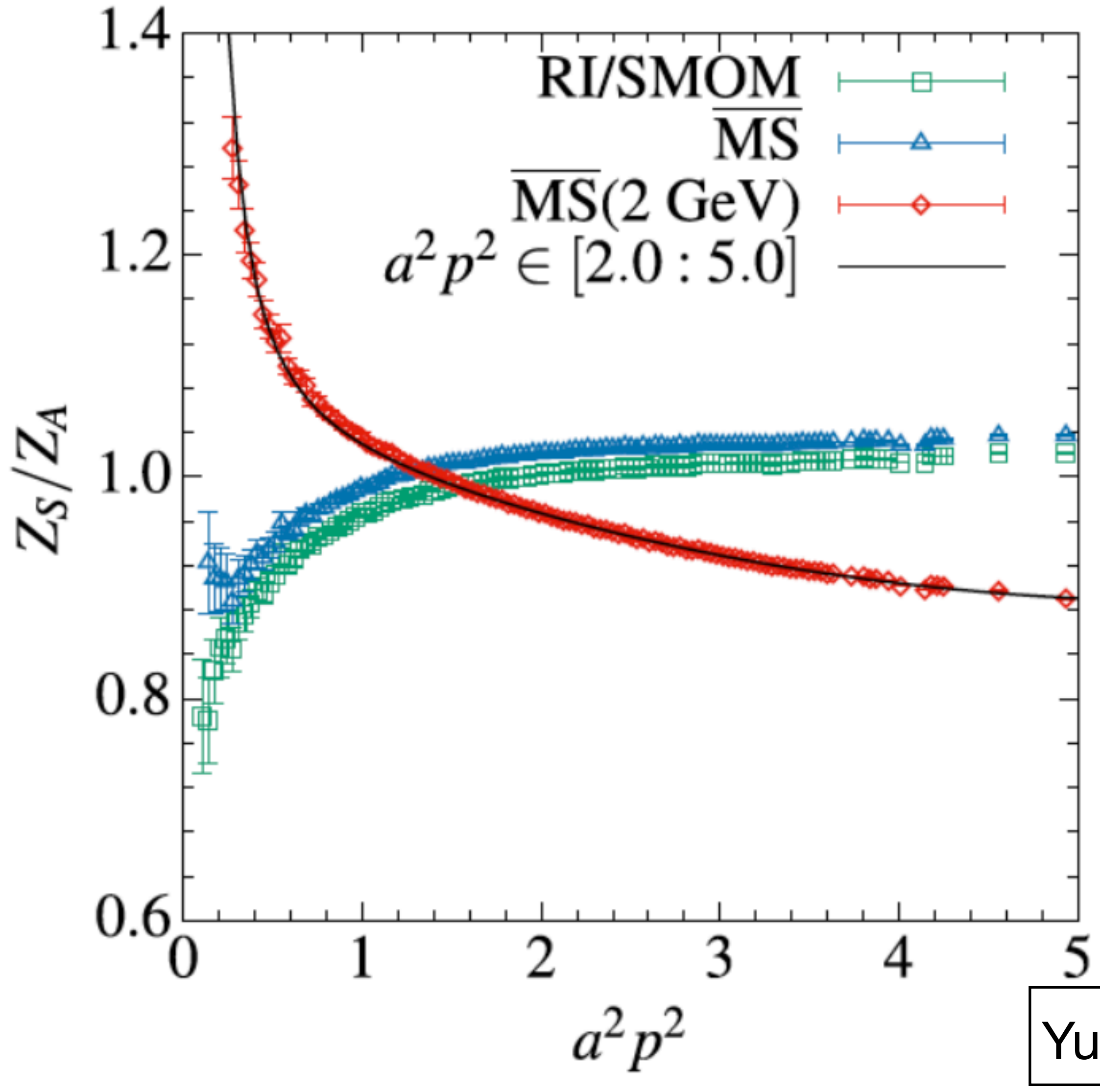

## MOM v.s. SMOM




- **MOM:**
  - **Non-trivial $m_q$ dependence;**
  - **Smaller $a^2p^2$ dependence;**
  - **Larger matching correction:** $\frac{Z_S^{\overline{\text{MS}}}}{Z_S^{\text{MOM}}}(4\text{GeV})$ $= 1 + 0.092 + 0.047 + 0.028 + \mathcal{O}(\alpha_s^4)$

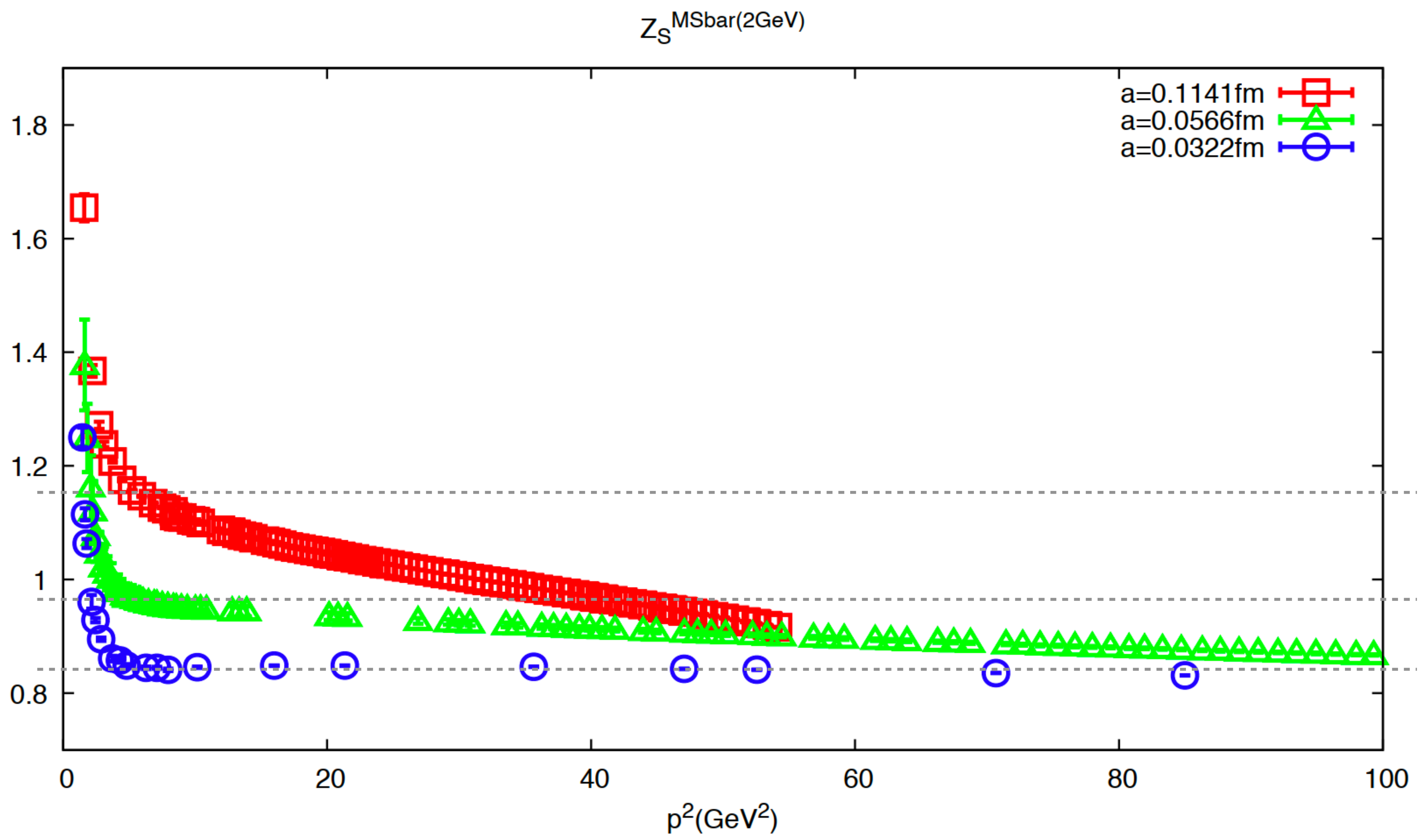
- SMOM:
  - Linear $m_q$ dependence;
  - Non-linear $a^2p^2$ dependence,
  - Smaller matching correction: $\frac{Z_S^{\overline{\text{MS}}}}{Z_S^{\text{SMOM}}}(4\text{GeV})$ $= 1 + 0.011 + 0.003 + 0.002 + \mathcal{O}(\alpha_s^4)$




Yujiang Bi, et.al, $\chi$QCD, PRD97(2018)094501, 1710.08678

# RI/MOM renormalization




Dian-Jun Zhao, et.al., $\chi$QCD, in preparation

## 2-step matching procedure

- $Z_S^{\overline{\rm MS},2\ \rm GeV}(1/a,p^2)$ with different 1/a and $p^2$ using 4-loop matching:

1. Curvatures at small $p^2$ are similar which suggest that they would come from the missing higher order matching;
2. $Z_S^{\overline{\rm MS},2\ \rm GeV}(1/a,p^2)$ has much smaller discretization error with same $p^2$ but smaller a, and less affected by the curve.

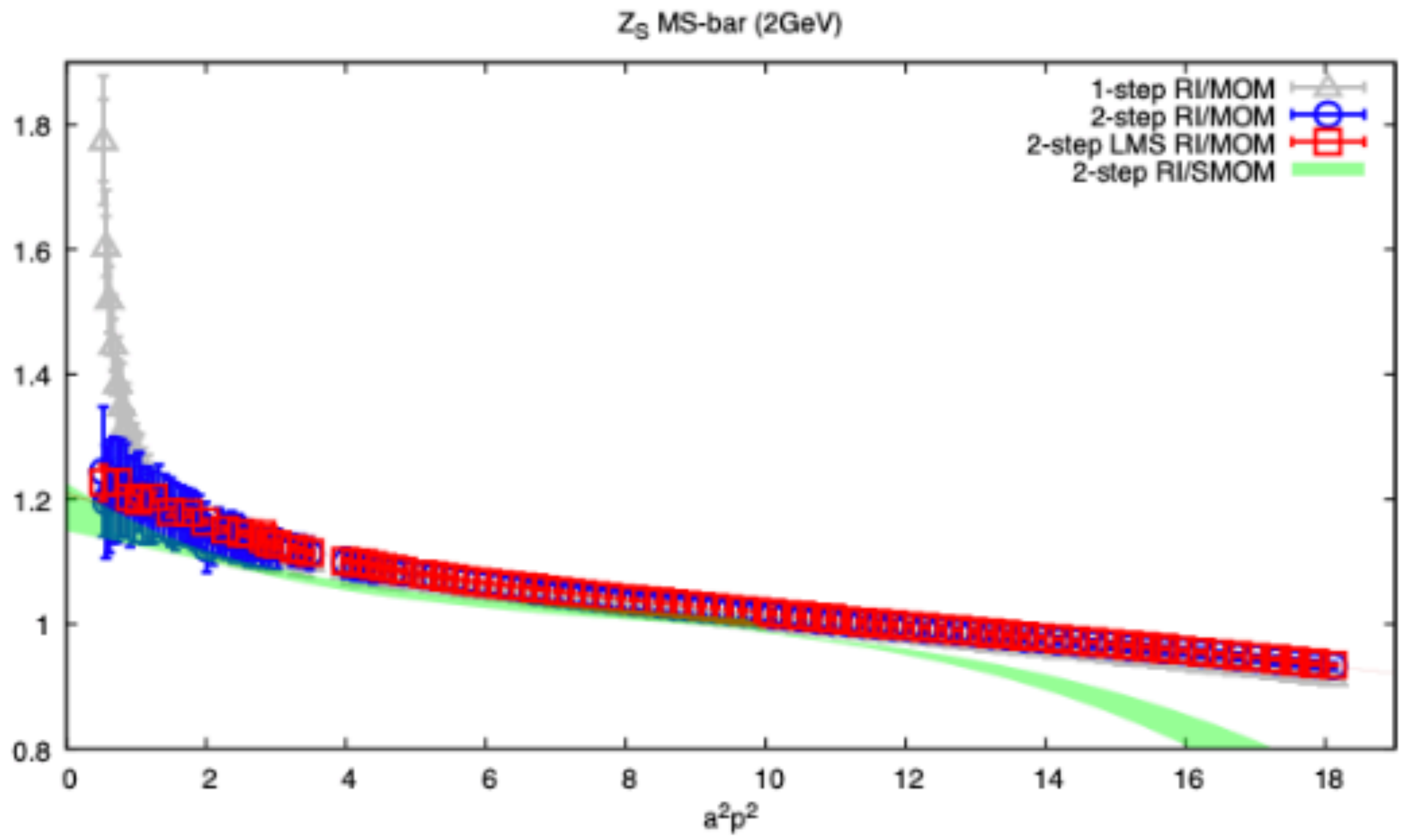
- The so-call 2-step matching separates the standard matching procedure $Z^{\overline{\rm MS}}(u_0;1/a,p^2)=C(\mu_0^2,p^2)Z^{\rm MOM}(1/a,p^2)$ into 2-steps:

1. Match the $Z^{\rm MOM}(1/a,p^2)$ to $Z^{\rm MOM}(1/a_0,p^2)$ with $a_0 \ll a$;
2. Convert $Z^{\rm MOM}(1/a_0,p^2)$ to the $\overline{\rm MS}$ scheme using the standard procedure.

$$Z_S^{\overline{\rm MS}}(\mu_0,1/a,p^2)=\frac{Z_S^{\overline{\rm MS}}(\mu_0,1/a,p^2)}{Z_S^{\overline{\rm MS}}(\mu_0,1/a_0,p^2)}Z_S^{\overline{\rm MS}}(\mu_0,1/a_0,p^2)$$
$$=\frac{Z_S^{\rm RI}(1/a,p^2)}{Z_S^{\rm RI}(1/a_0,p^2)}[Z_S^{\overline{\rm MS},1-\rm step}(\mu_0,1/a_0)+\mathcal{O}(a_0^2p^2)],$$

# RI/MOM renormalization

## Low mode substitution & 2-step matching




Dian-Jun Zhao, et.al., $\chi$QCD, in preparation

- The statistical uncertainty can be suppressed using the low mode substitution:

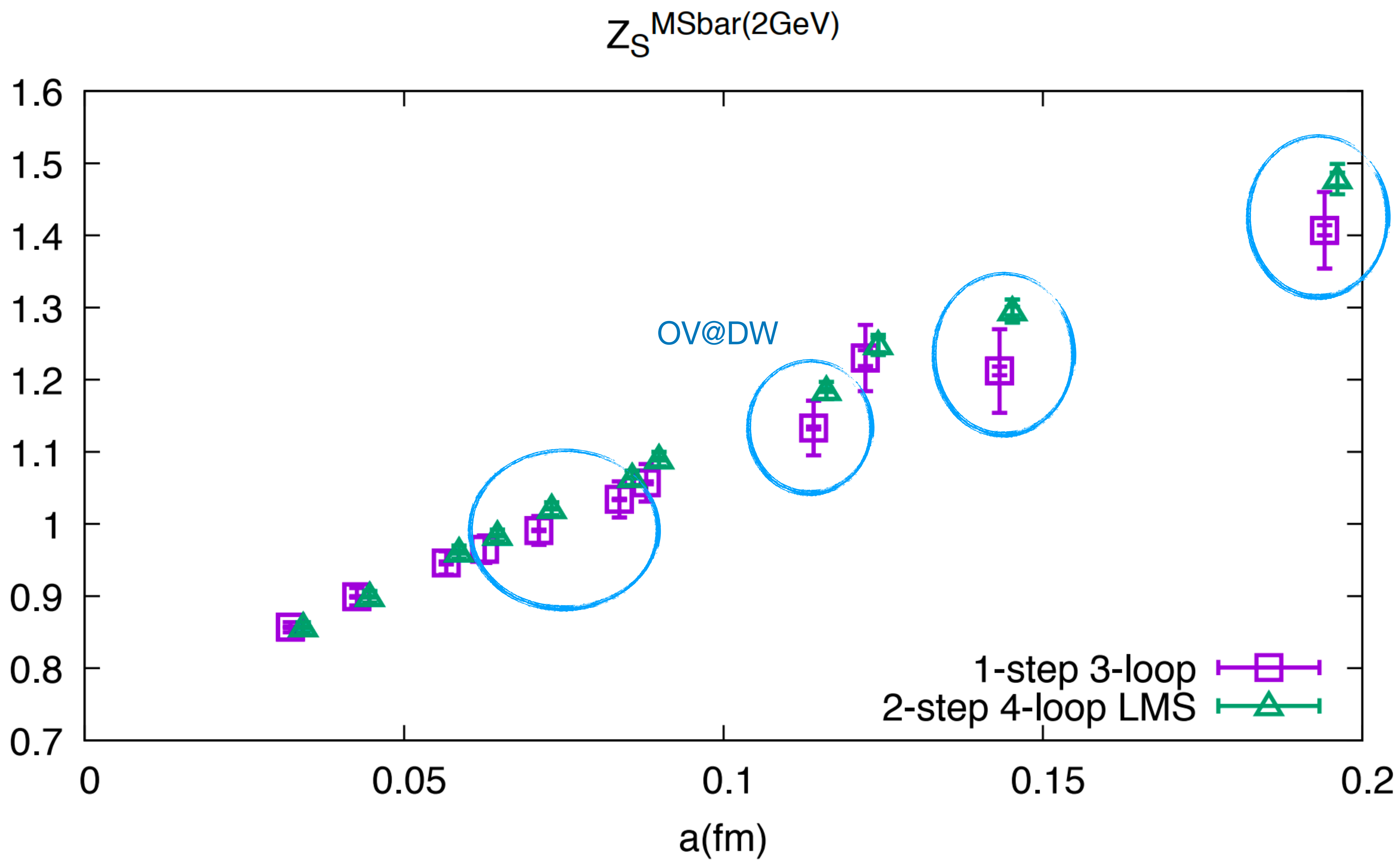
$$G_{\mathcal{O}}(p_1,p_2)=\sum_{x,y}e^{-i(p_1\cdot x-p_2\cdot y)}\langle S(x,0)\Gamma_{\mathcal{O}}S(0,y)$$
$$-S_L(x,0)\Gamma_{\mathcal{O}}S_L(0,y)$$
$$+\frac{1}{L^3\times T}\sum_z S_L(x+z,z)\Gamma_{\mathcal{O}}S_L(z,y+z)\rangle,$$
$$\langle S(p)\rangle=\sum_x e^{-ip\cdot x}\langle[S(x,0)-S_L(x,0)$$
$$+\frac{1}{L^3\times T}\sum_y S_L(x+y,y)]\rangle.$$

- The systematic uncertainty from the perturbative matching can be suppressed using the 2-step matching:

$$Z_S^{\overline{\mathrm{MS}}}(\mu_0,1/a,p^2)=\frac{Z_S^{\overline{\mathrm{MS}}}(\mu_0,1/a,p^2)}{Z_S^{\overline{\mathrm{MS}}}(\mu_0,1/a_0,p^2)}Z_S^{\overline{\mathrm{MS}}}(\mu_0,1/a_0,p^2)$$
$$=\frac{Z_S^{\mathrm{RI}}(1/a,p^2)}{Z_S^{\mathrm{RI}}(1/a_0,p^2)}[Z_S^{\overline{\mathrm{MS}},1-\mathrm{step}}(\mu_0,1/a_0)+\mathcal{O}(a_0^2p^2)],$$

# RI/MOM renormalization

## 1-step v.s. 2-step with LMS




Dian-Jun Zhao, et.al., $\chi$QCD, in preparation

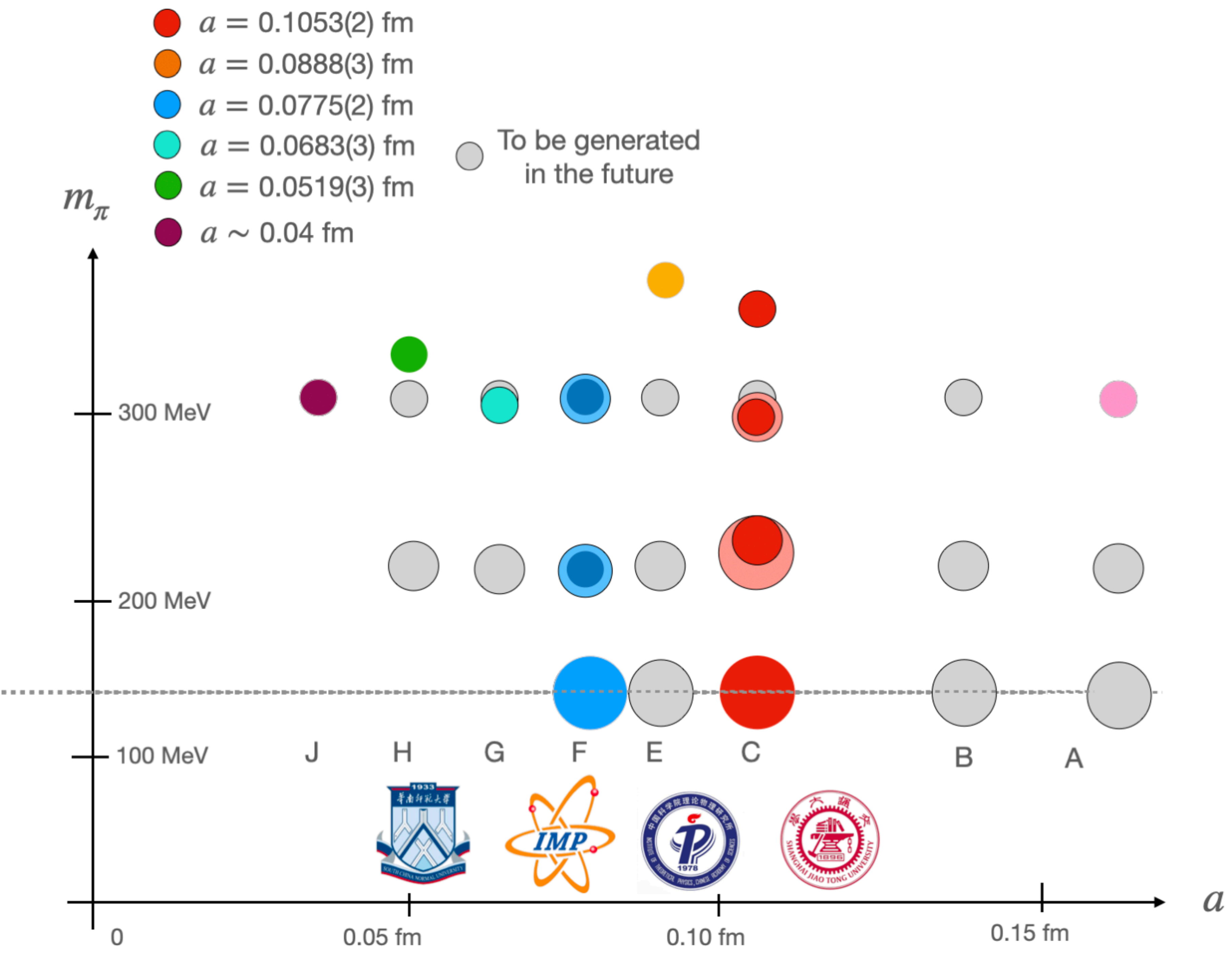
- 2-step eliminate most of the systematic uncertainty based on the renormalization calculation at $a = 0.032$ fm MILC ensemble;
- Total uncertainty of the renormalization can be a factor of 3 smaller;
- $Z_S^{\overline{\text{MS}},2\text{ GeV}}(1/a)$ of the overlap fermion is independent of the actions used in the configurations, at sub-percent level.

# RI/MOM for Clover fermion

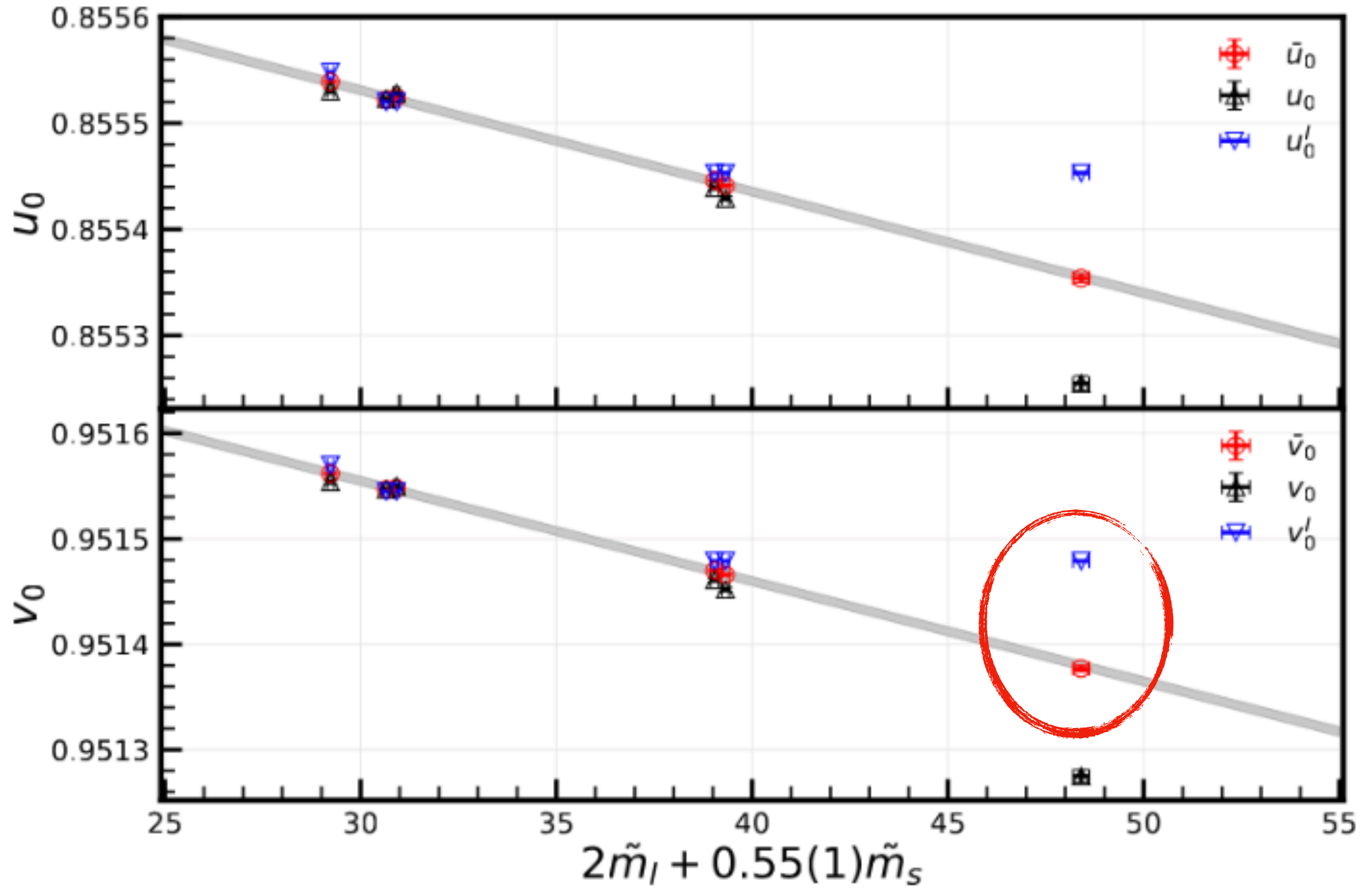
## CLQCD ensembles

$$S_g(g_0) = \frac{1}{N_c}\mathrm{Re}\sum_{x,\mu<\nu}\mathrm{Tr}\left[1 - 10/(g_0^2u_0^4)\left(\mathscr{P}^U_{\mu,\nu}(x) + \frac{1}{20u_0^2}\mathscr{R}^U_{\mu,\nu}(x)\right)\right]$$

$$S_q(m) = \sum_{x,\mu=1,\ldots,4,\eta=\pm}\bar\psi(x)\sum\frac{1+\eta\gamma_\mu}{2}V_{\eta\mu}(x)\psi(x+\eta\hat\mu a) + \sum_x\psi(x)\left[-(4+ma)\delta_{y,x} + \frac{1}{v_0^3}\sigma^{\mu\nu}g_0F^V_{\mu\nu}\right]\psi(x),$$




- Tadpole improved Symanzik gauge;
- Tadpole improved Clover fermion;
- Tadpole improvement requires fine-tuning of the tadpole factors $u_0$ and $u_l$;
- We tune those factors to the $0.001\ \%$ level, as the mistuning effect can be $\mathcal{O}(100)$ enhanced in the hadron and quark masses.



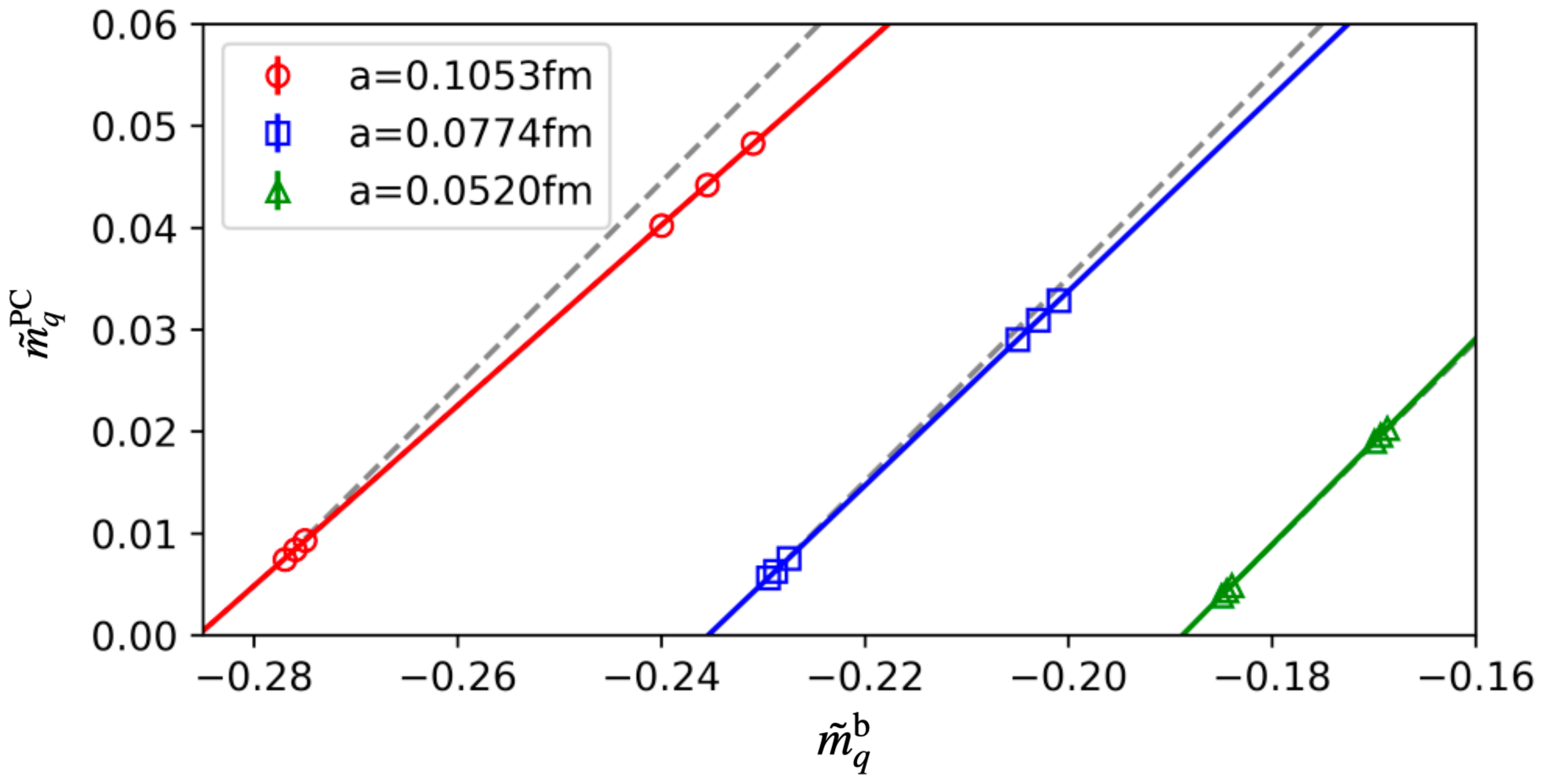

| | $\tilde m_{\mathrm{PS}}$ | $\tilde m_l^{\mathrm{PC}}$ | $\tilde f_{\mathrm{PS}}$ | $Z_{wp}$ |
|---|---|---|---|---|
| $1/(v_0^I)^3$=1.1609 | 0.1832(12) | 0.01191(13) | 0.0768(10) | 10.50(30) |
| $1/(\bar v_0)^3$=1.1613 | 0.1822(12) | 0.01178(13) | 0.0768(11) | 10.50(31) |
| difference | 0.0011(01) | 0.00013(01) | 0.0000(00) | 0.00(01) |

Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, 2310.00814

# RI/MOM for Clover fermion

## additive Chiral symmetry breaking

$$m_q^{\mathrm{PC}} = \frac{F^2}{2\Sigma} m_\pi^2 + \mathcal{O}(m_\pi^4) \sim \frac{m_{\pi^2}}{5\ \mathrm{GeV}}$$




Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, 2310.00814

- Due to the additive $\alpha_s/a$ correction, the dimensionless bare quark mass $\tilde{m}_q^{\mathrm{b}} = m_q^{\mathrm{b}} a$ is negative.
- The renormalized quark mass should be defined as $m_q^R = Z_m(m_q^{\mathrm{b}} - m_{\mathrm{crti}})$, where $m_{\mathrm{crti}}$ is defined as the $m_q^{\mathrm{b}}$ which vanishes the pion mass.
- One can avoid this difficulty by defining the quark mass through PCAC relation:

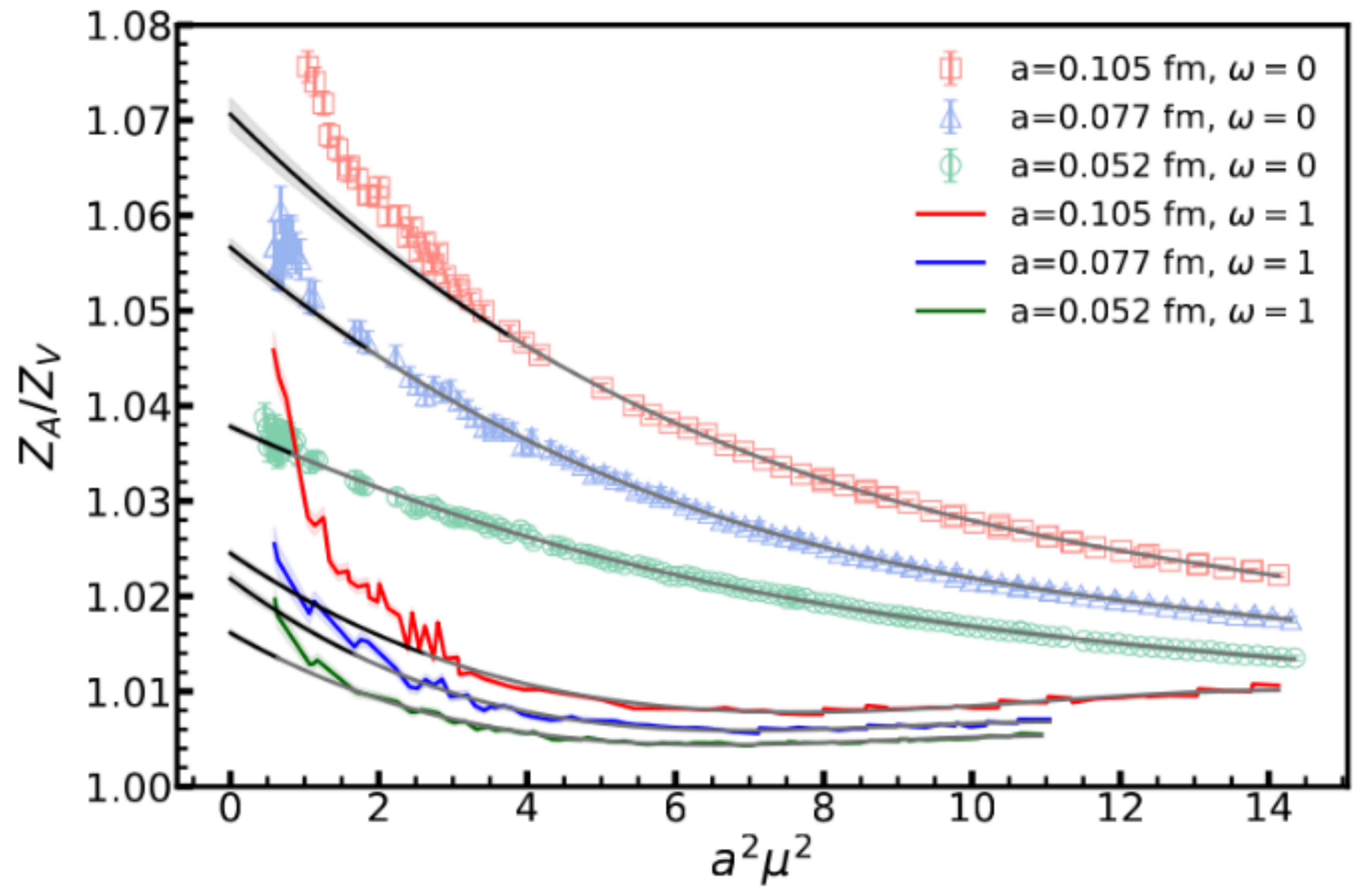
$$\langle 0 | \partial_4 A_4 | \mathrm{PS} \rangle = (m_q^{\mathrm{PC}} + m_{\bar{q}}^{\mathrm{PC}}) \langle 0 | P | \mathrm{PS} \rangle$$

T. Ishikawa, et.al., JLQCD, Phys.Rev.D78 (2008) 011502

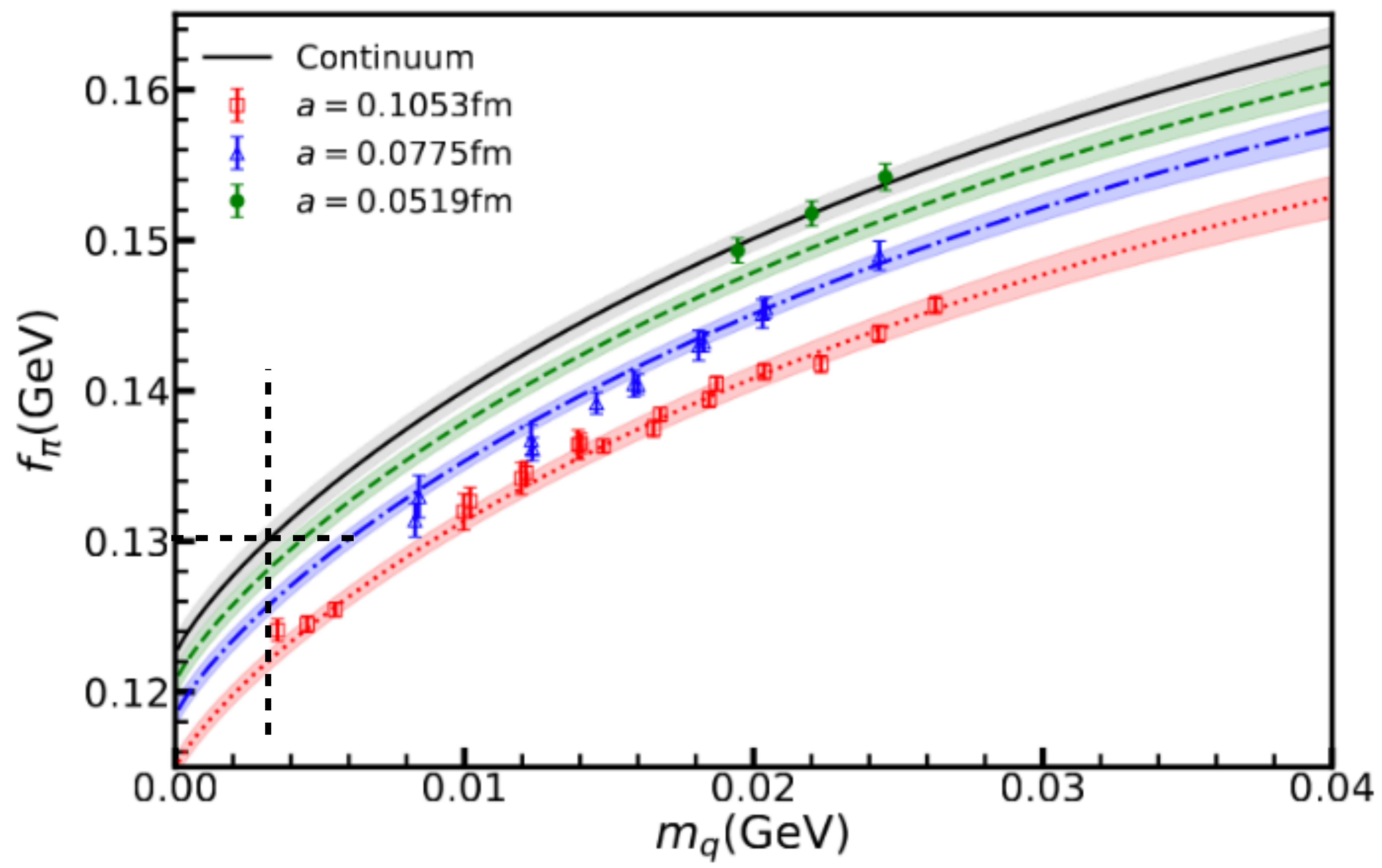
- And then $m_q^{\mathrm{PC}}$ is always positive and can be renormalized as $m_q^R = Z_P/Z_A m_q^{\mathrm{PC}}$.

# RI/MOM for Clover fermion

## vector and axial-vector currents



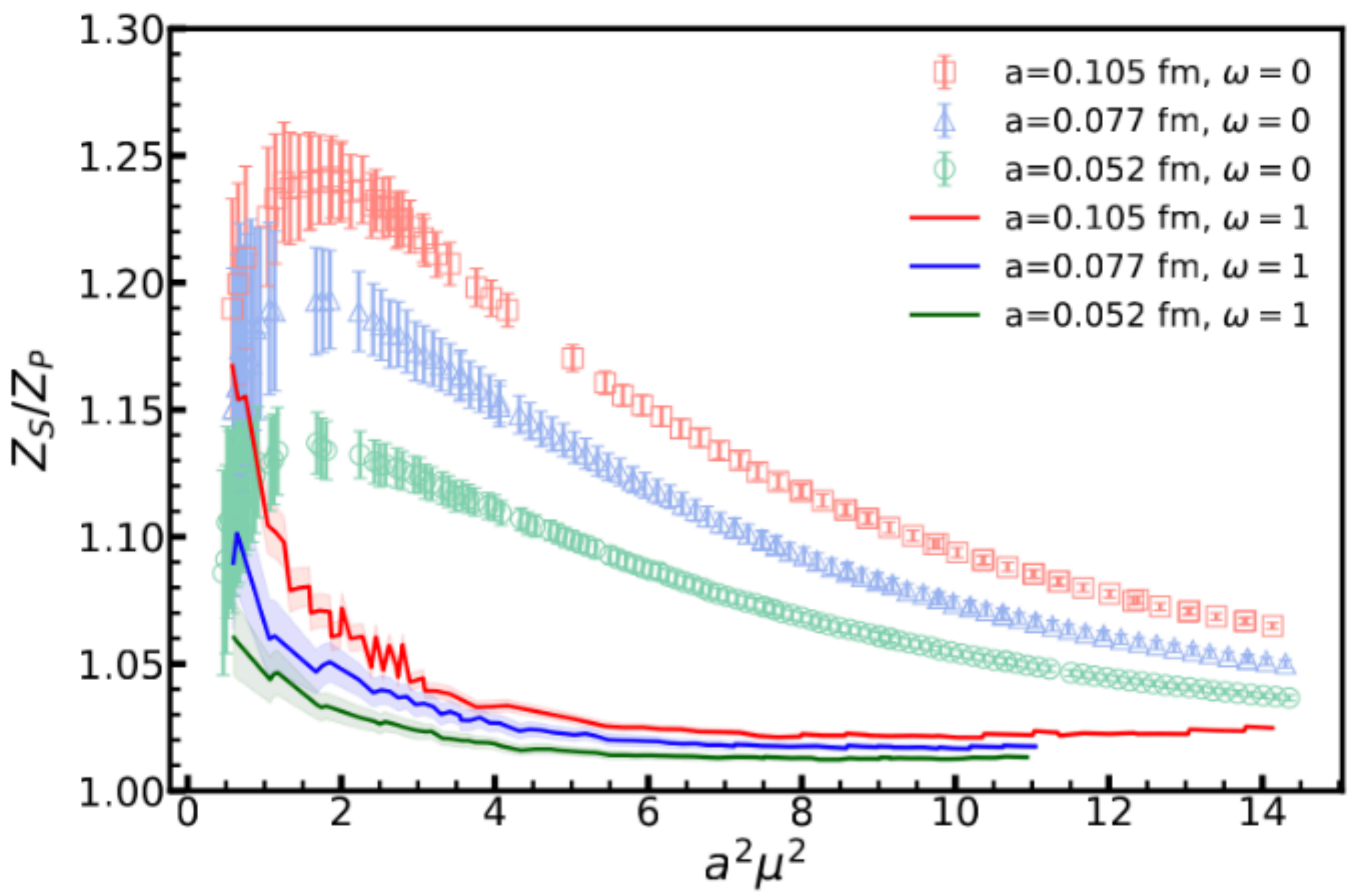

- Clover fermion shows additional chiral symmetry breaking between $Z_V$ and $Z_A$;
- Such a breaking is necessary to reproduce the correct $f_{\pi,K}$ after the continuum extrapolation;
- Continuum extrapolation also eliminate the difference between MOM ($\omega = 0$) and SMOM ($\omega = 1$) schemes.




Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, 2310.00814

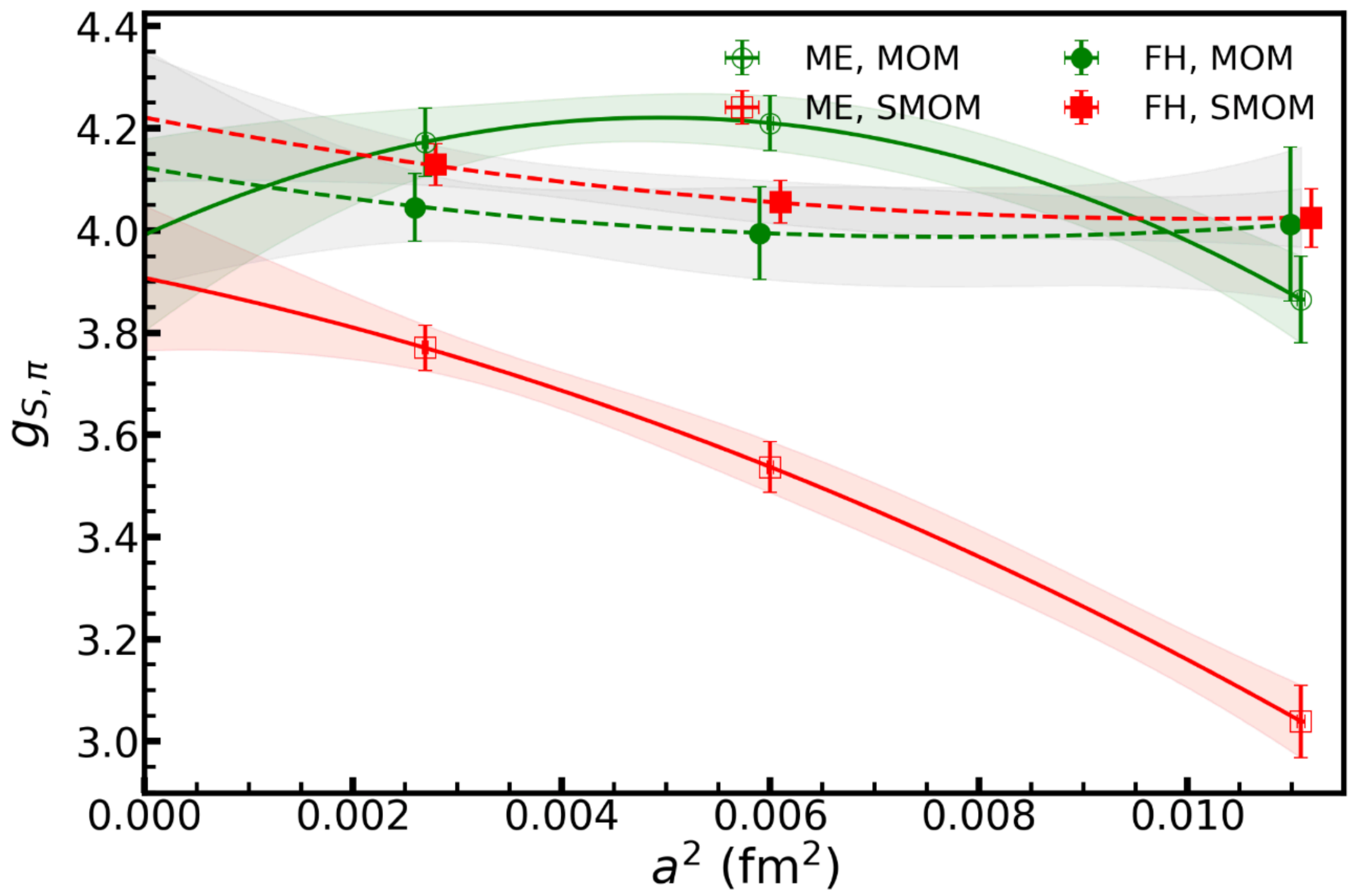
# RI/MOM for Clover fermion

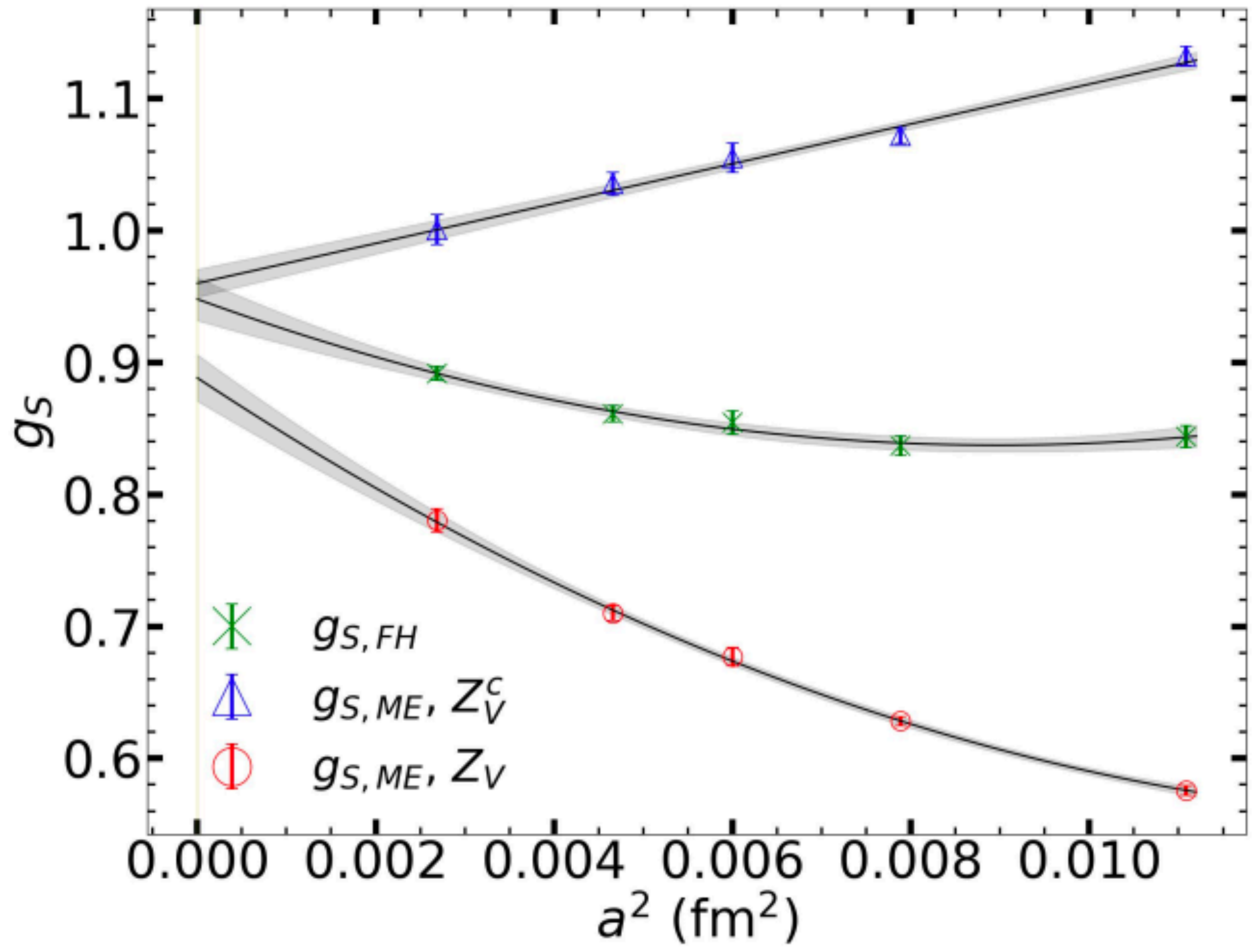
## scalar and pseudo-scalar currents




- Clover fermion also shows additional chiral symmetry breaking between $Z_S$ and $Z_P$;

- Light quark scalar matrix element (ME) from the direct calculation $Z_S\langle\pi\,|\,\bar{q}q\,|\,\pi\rangle$ and Feynman-Hellman (FH) theorem $Z_P/Z_A\dfrac{\partial m_\pi}{\partial m_q^{\rm PC}}$ are consistent after the renormalization;



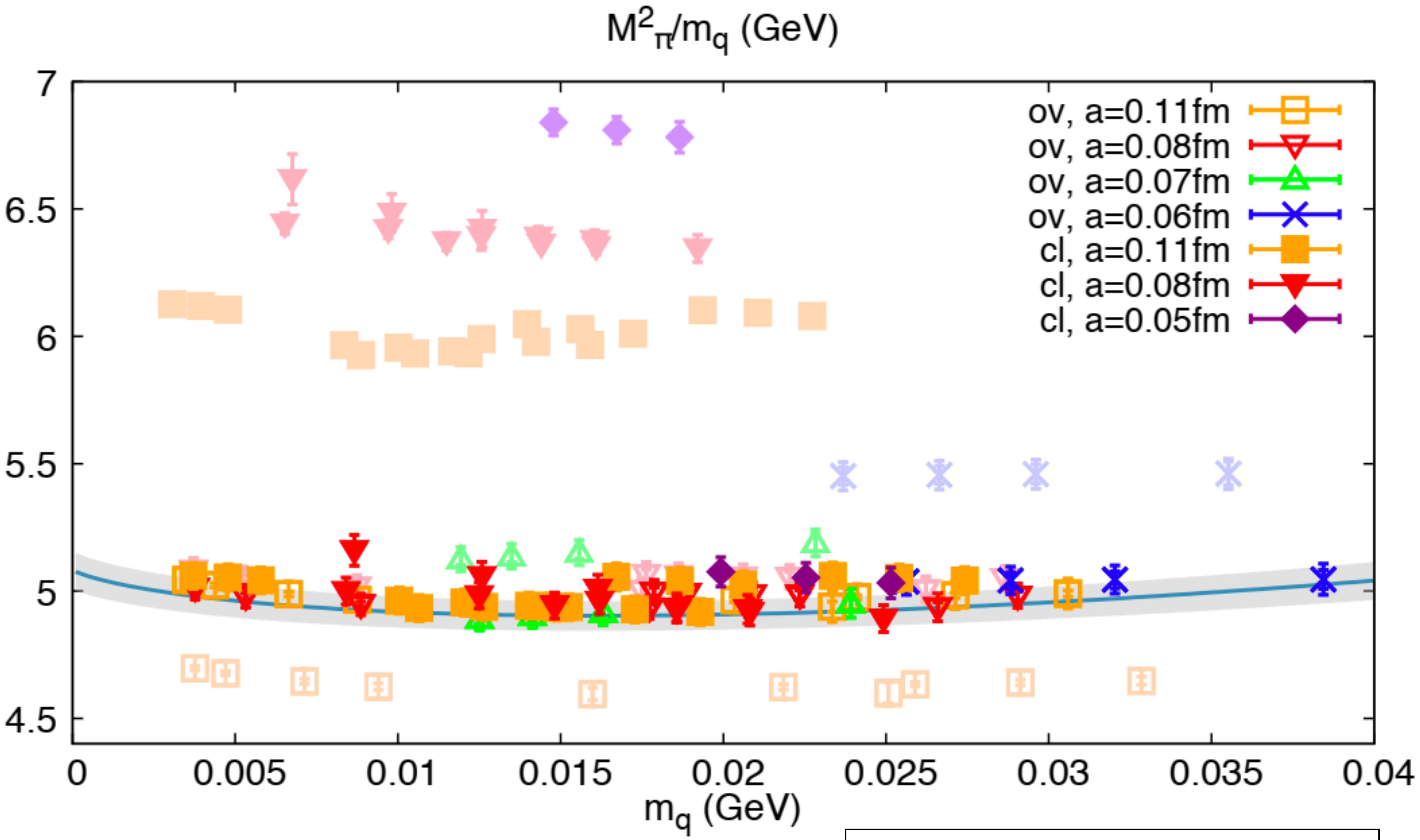

- Charm quark scalar ME from the direct calculation and FH theorem are also consistent after the charm quark improved normalization applied.

Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, 2310.00814

Hai-Yang Du, B.L. Hu, et. al., CLQCD, in preparation

# Renormalized quark masses

## Impact of the renormalization




Dian-Jun Zhao, et.al., $\chi$QCD, in preparation

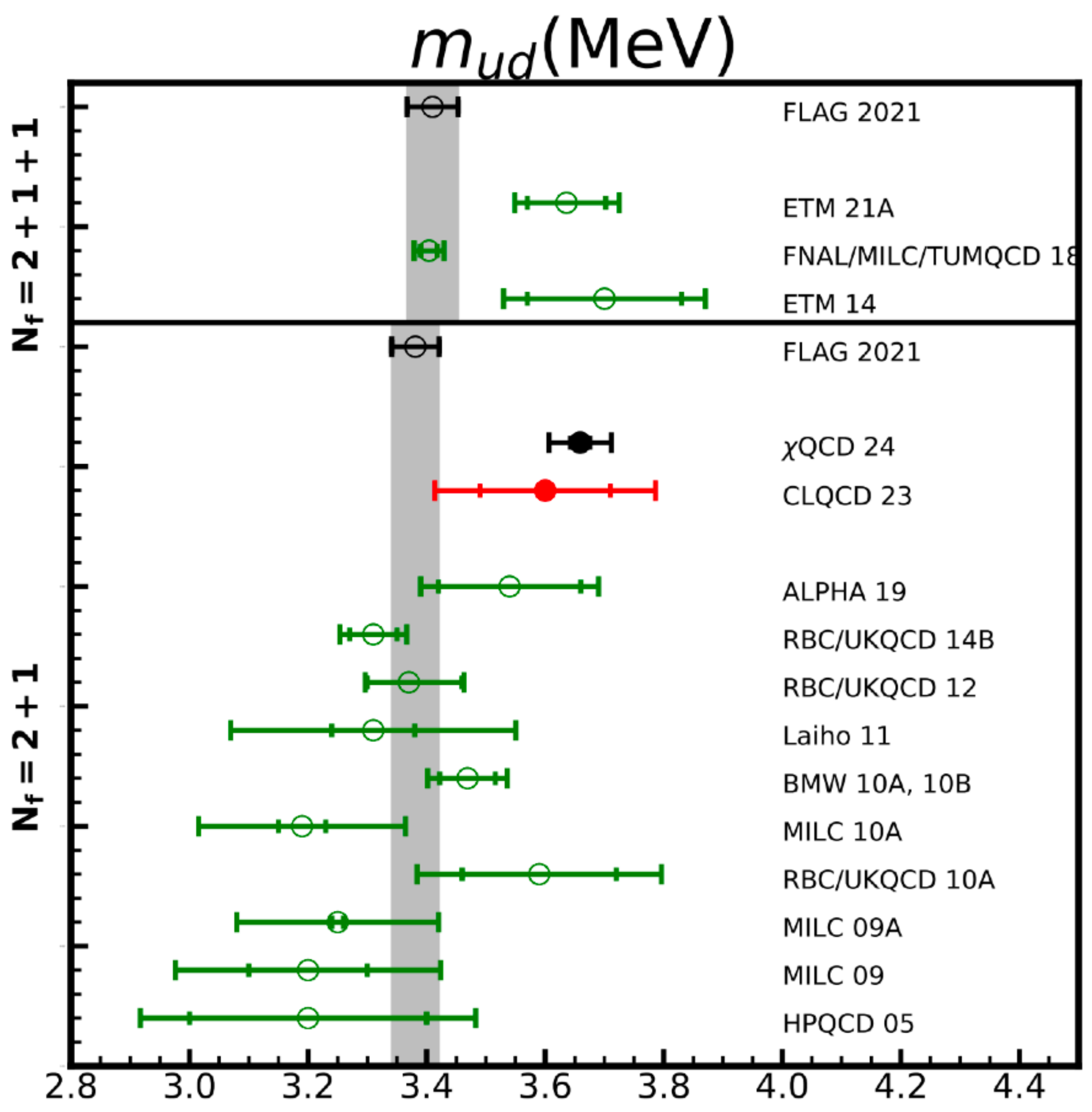
- $m_\pi^2/m_q \sim \Sigma/F^2$ which is insensitive to the quark mass, with the partially quenching effect subtracted;
- The PCAC mass $m_q^{\rm PC} = \frac{\langle 0|\partial_4 A_4|{\rm PS}\rangle}{2\langle 0|P|{\rm PS}\rangle}$ has obvious 1/a and action dependences:

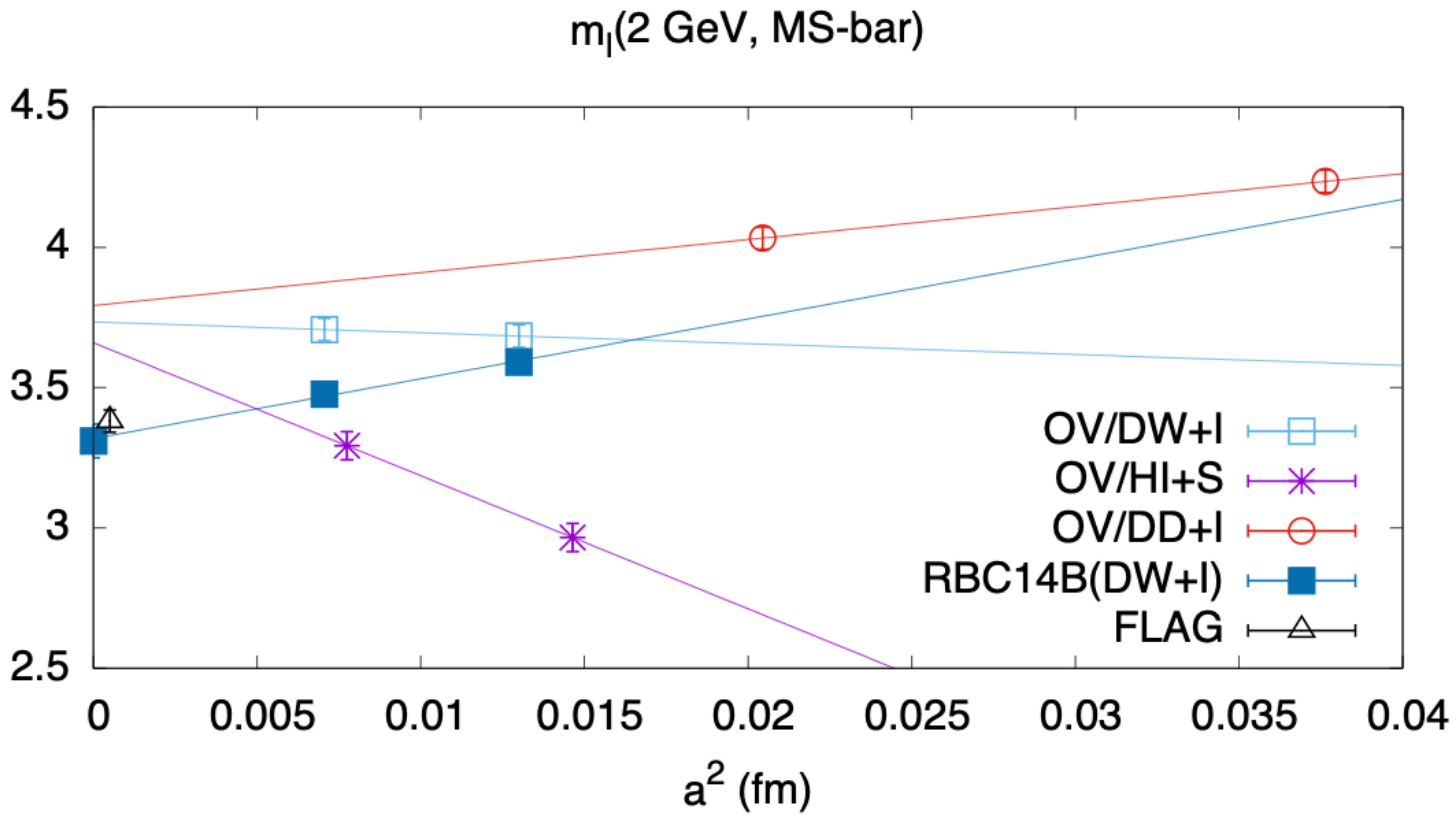
1. Smaller with large intrinsic scale 1/a;
2. Very sensitive to the fermion action.

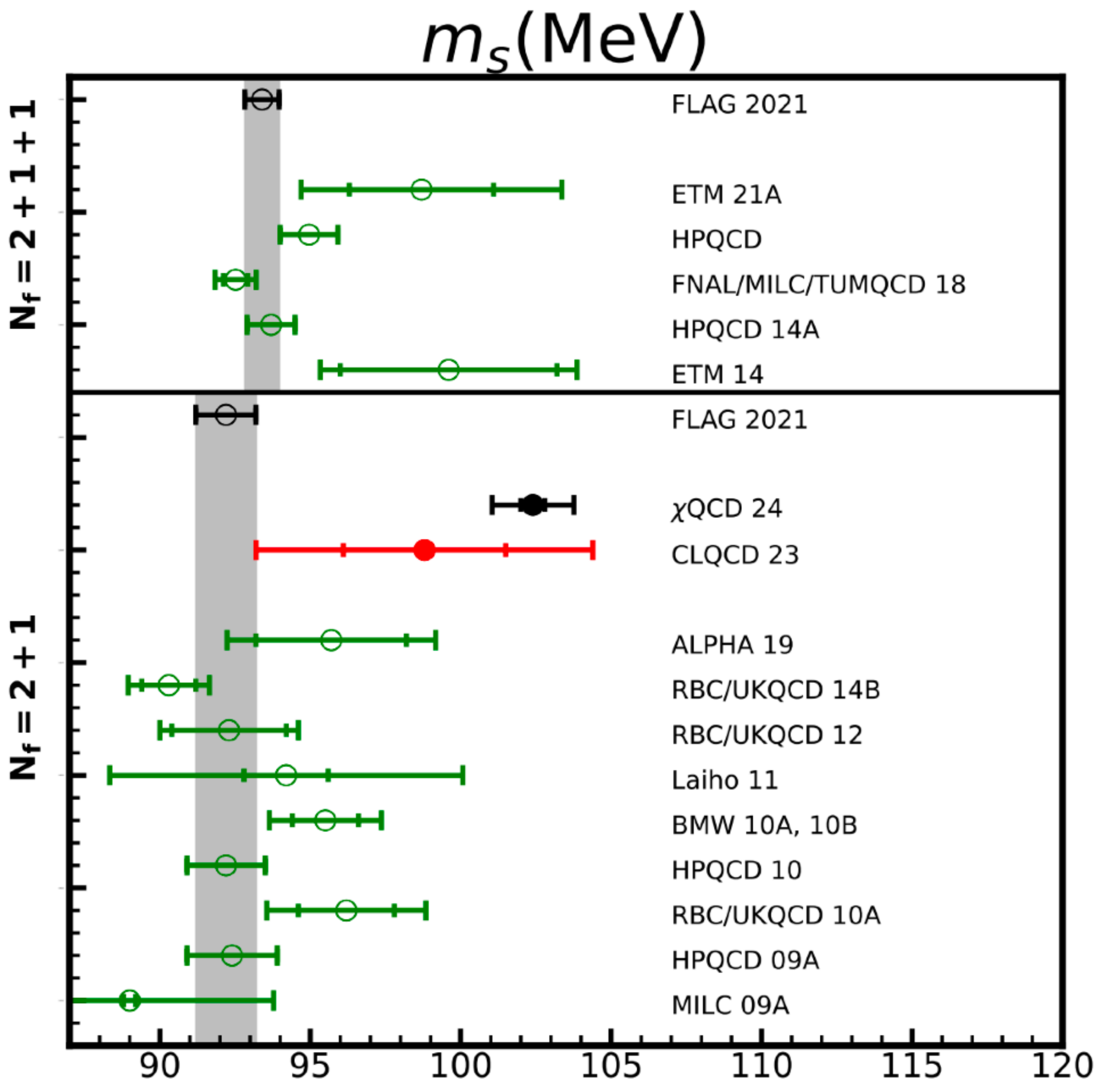
- RI/MOM renormalization eliminates both the dependences and makes $m_\pi^2/m_q^{\overline{\rm MS}}$ of all the ensembles on a similar curve.

# Renormalized quark masses

## light and strange quark mass



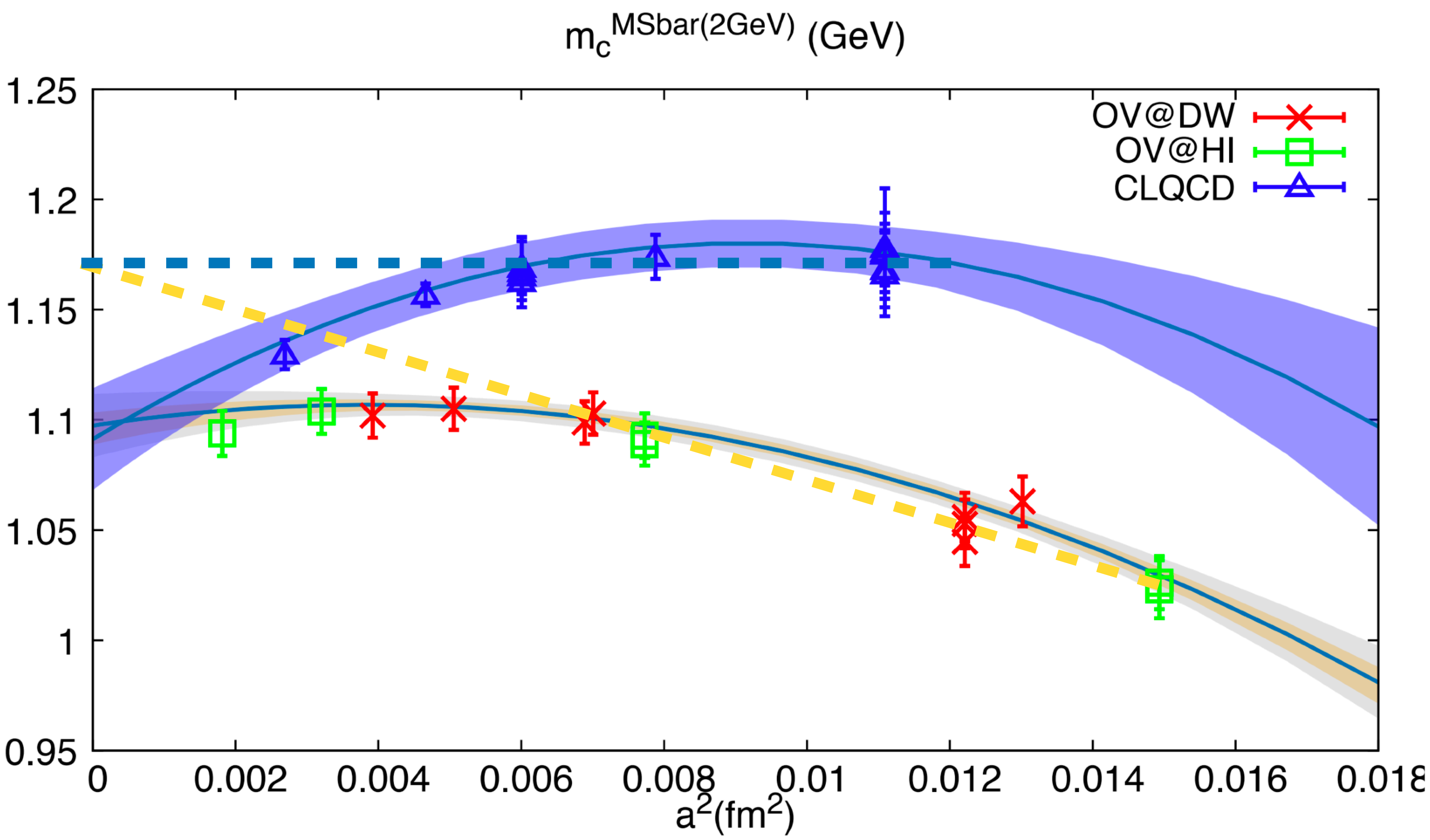




- Based on the RI/MOM renormalization with 2-step matching, the renormalized light and strange quark masses are ~10(2)% higher than the previous precise DWF and HISQ results using the SMOM scheme;
- Results on three kinds of ensembles have the continuum limit using the linear $a^2$ extrapolation.

Dian-Jun Zhao, et.al., $\chi$QCD, in preparation

# Renormalized quark masses

## Charm quark mass




Dian-Jun Zhao, et.al., $\chi$QCD, in preparation

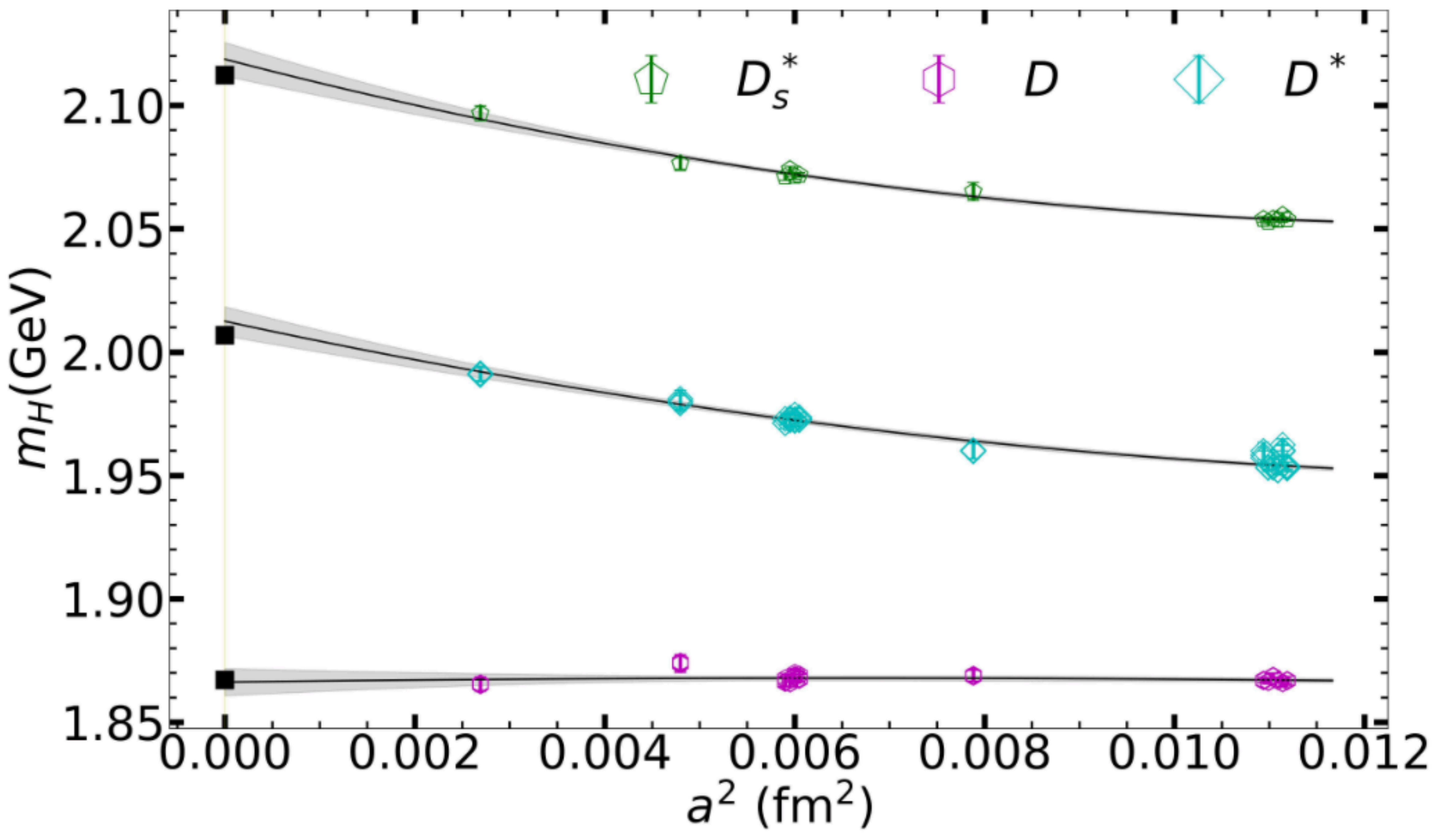
Hai-Yang Du, B.L. Hu, et. al., CLQCD, in preparation

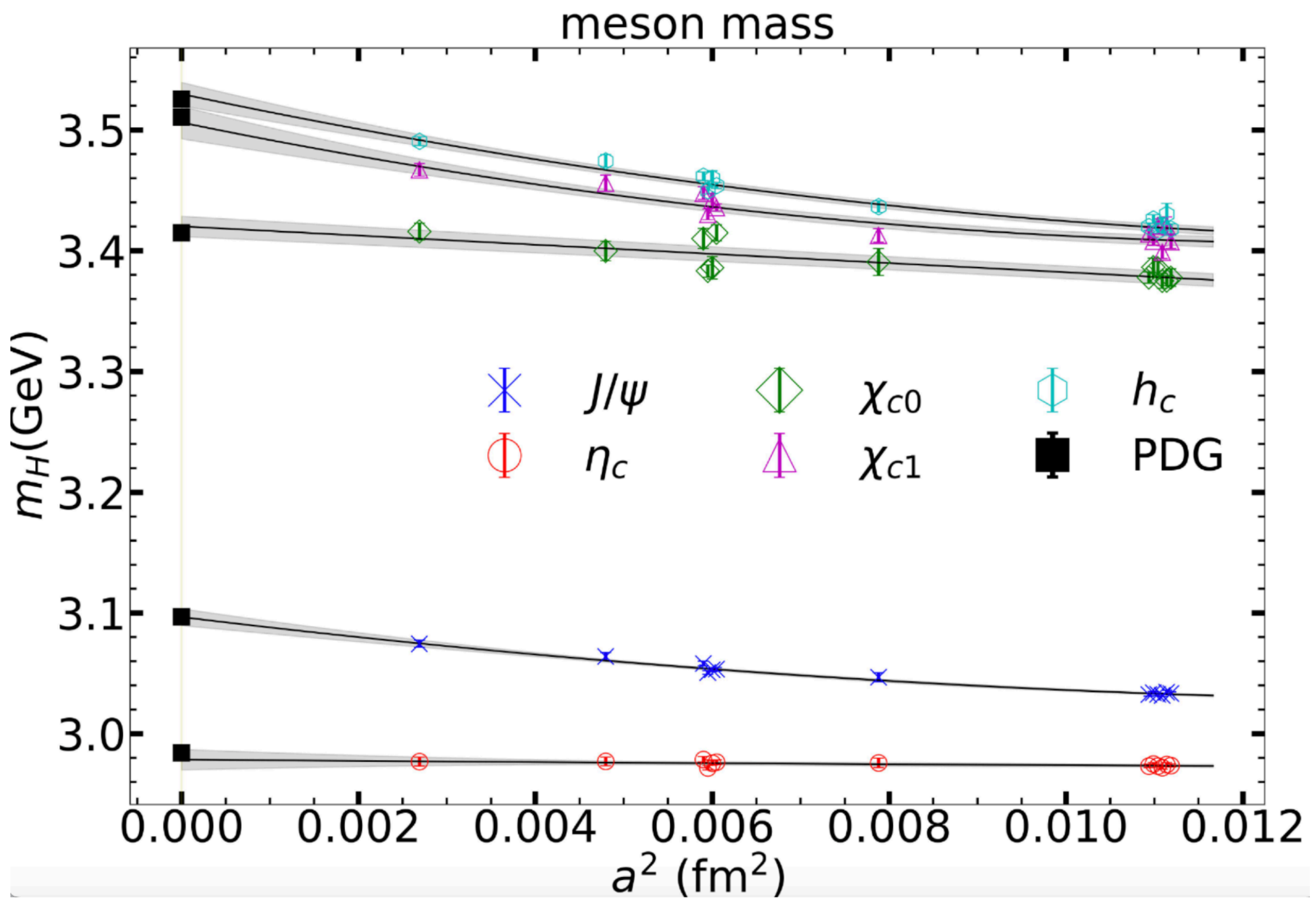
- Charm quark mass can be accessed at much smaller lattice spacing with affordable cost;
- Based on the $a^2 + a^4$ extrapolation, the renormalized quark mass is similar to the FLAG value within ~1%;
- While the linear $a^2$ extrapolation at large lattice spacings can lead to a significantly heavier value.
- Similar small lattice spacing study would also be necessary for the light quark case.

# Charmed meson spectrum

## using clover fermion

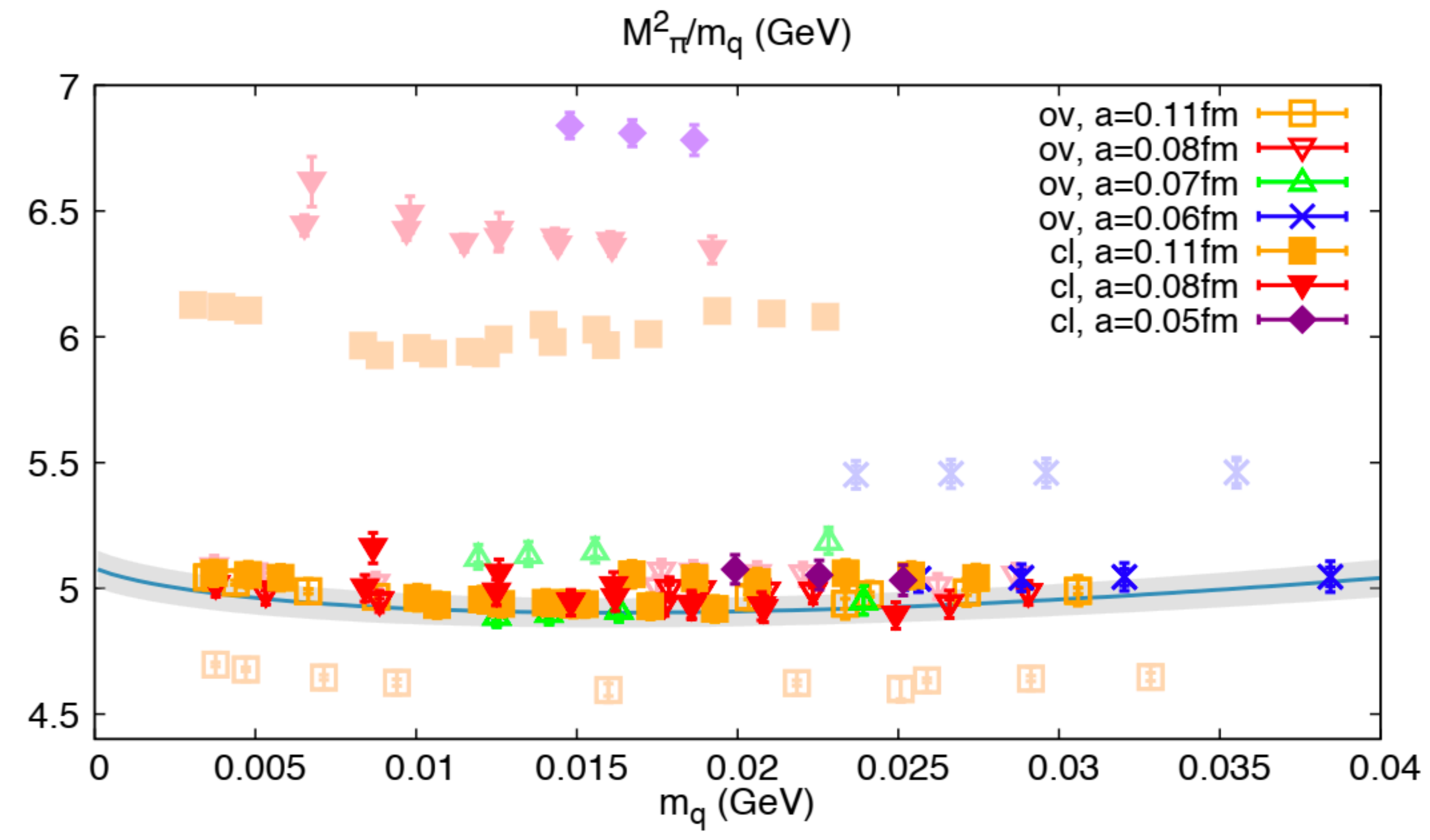
- Determine valence strange and charm quark masses using $\eta_s$ and $D_s$ respectively;
- Residual light and strange quark masses effects are eliminated using the joint fit;
- Open charm meson and charmonium spectrum agree with the experimental results well.

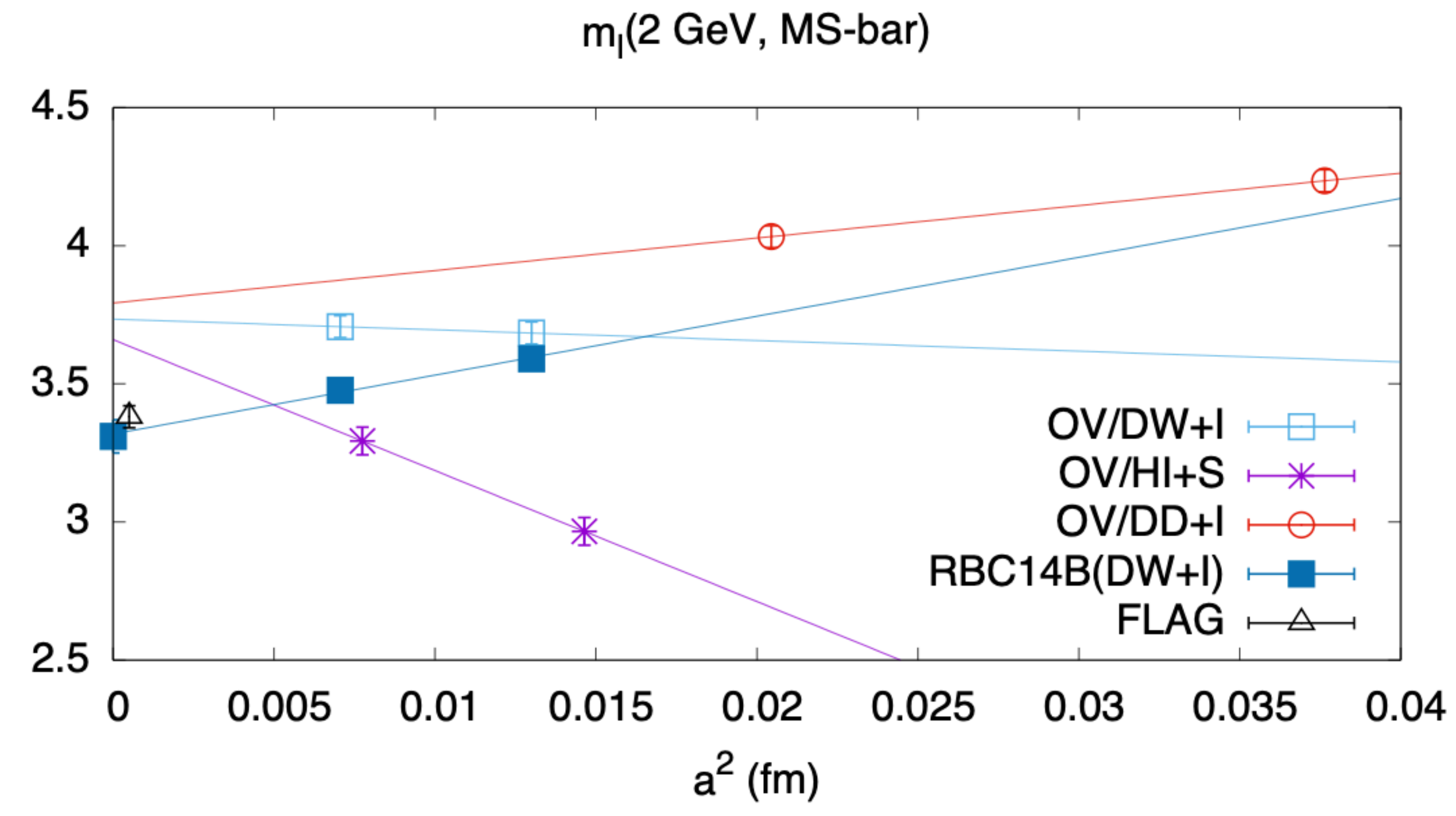


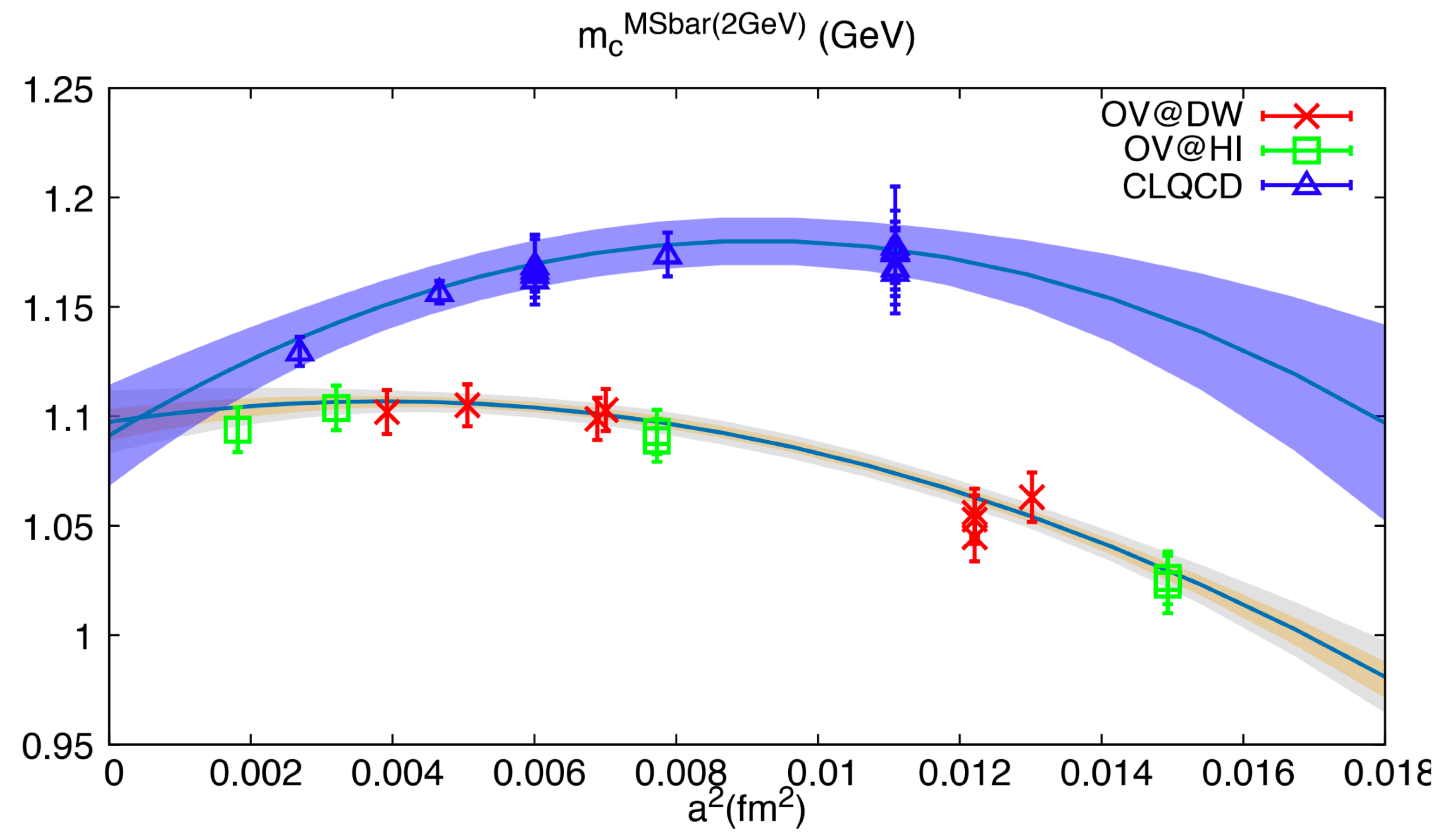




Hai-Yang Du, B.L. Hu, et. al., CLQCD, in preparation

# Summary




- RI/MOM renormalization with 2-step matching can be provide precise agreement between the renormalized light quark masses using overlap or clover fermion at different lattice spacings;

- Current prediction of the light and strange quark masses are still ~10% higher than RBC result using DWF.

- The charm quark mass has much better consistency with the FLAG values using the the $a^2 + a^4$ extrapolation, thus similar test for the light/strange quark would also be needed.