Non-singlet axial current improvement for massless and massive sea quarks

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Relevance for further improvement and physics

- exp. Wilson-clover fermion framework
- massive $\widehat{=}$ at $N_\mathrm{f} = 3$ symmetric point

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- improvement and renormalisation:
  - c_V, c_T, Z_A
  - no c_A $\Rightarrow$ no improvement of other channels

Determination of c_A

- Schrödinger functional boundary conditions
- similar to quenched [hep-lat/9609035], $N_\mathrm{f} = 2$ [hep-lat/0503003] and std. Wilson-Clover $N_\mathrm{f} = 3$ [1502.04999, hep-lat/0703006]
- derive from PCAC mass

$$m_\mathrm{PCAC} = \frac{\partial_0 f_\mathrm{A}}{2 f_\mathrm{P}} + c_\mathrm{A} \; a \frac{\partial_0^2 f_\mathrm{P}}{2 f_\mathrm{P}} = r + c_\mathrm{A} \; as$$

$$m_\mathrm{PCAC}^{(0)} = m_\mathrm{PCAC}^{(1)} \quad \Leftrightarrow \quad c_\mathrm{A} = -\frac{r^{(1)} - r^{(0)}}{s^{(1)} - s^{(0)}}$$

- states (0) and (1) are the PS ground and first excited state in our setup
- PCAC relation holds for both

The wavefunction method

- construct pseudoscalar states
  - H-like basis wavefunctions: $\omega_1 = e^{-r/a_0}\ , \quad \omega_2 = r\ e^{-r/a_0}\ , \quad \omega_3 = e^{-r/(2a_0)}$
  - also include $\omega_4 = \mathrm{cons.}\ , \quad \omega_5 = -r^2\ e^{-r/a_0}$ with $r = |\vec{y} - \vec{x}|$

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- evaluate $c_\mathrm{A}(x_0)$ with projected correlation functions
- later: choice of x_0 and wavefunction basis is part of the improvement condition

Ensembles

$T = L \approx 3\,\mathrm{fm}$ Schrödinger-Functional ensembles, exp. Wilson-Clover fermions

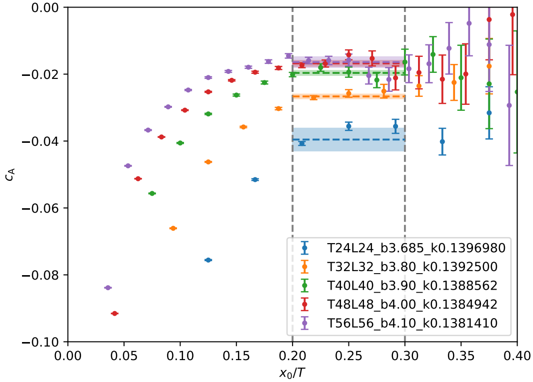
L/a	β	$\kappa_1 \approx \kappa_{\rm cr}$	κ_2	$\kappa_3 \approx \kappa_{\rm sym}$	$\approx a$ [fm]
24	3.685	0.1396980	0.1395500	0.1394400	0.120
32	3.80	0.1392500	—	0.1389630	0.095
40	3.90	0.1388562	0.1386148	0.1386030	0.080
48	4.00	0.1384942	0.1384880	0.1382720	0.064
56	4.10	0.1381410	0.1380000	0.1379450	0.055
96	4.37	—	—	—	0.035

- interested in 2 LCPs: chiral and $N_{\rm f} = 3$ sym. point
- matching sym. point of OpenLat [2201.03874]

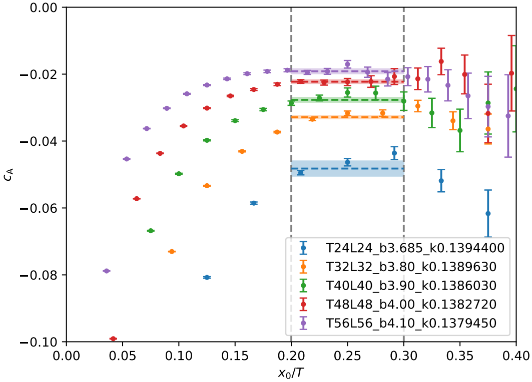
Improvement of the axial-vector current

c_A estimators

Critical point ensembles



Symmetric point ensembles



Interpolation

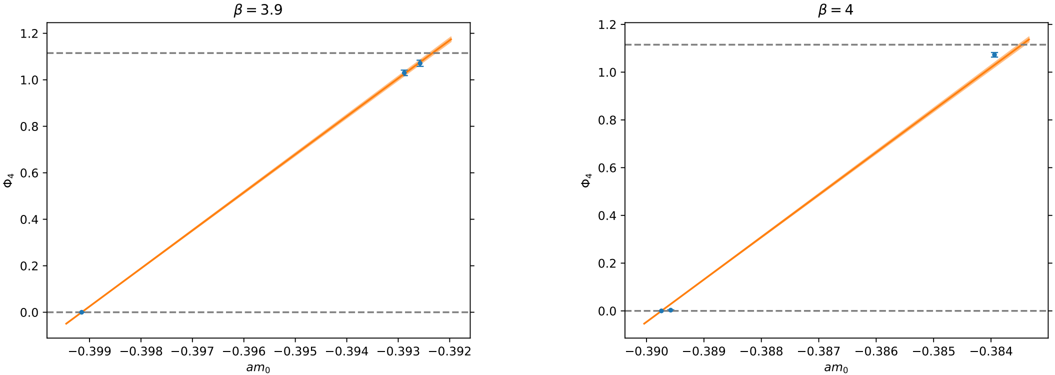
... to the symmetric and critical point

- ensembles not exactly tuned
- able to interpolate to the desired points due to 2 or 3 ensembles per β
- determine points of interest as in OpenLat ensembles [2201.03874]
- define:

$$\Phi_4^{\mathrm{SF}} = \frac{3}{2} 8t_0 \left|m_{\mathrm{eff}}\right| m_{\mathrm{eff}} \quad \Rightarrow \quad \Phi_4^{\mathrm{SF}} \Big|_{m_{0,\mathrm{cr}}} = 0 \,, \; \Phi_4^{\mathrm{SF}} \Big|_{m_{0,\mathrm{sym}}} = 1.115$$

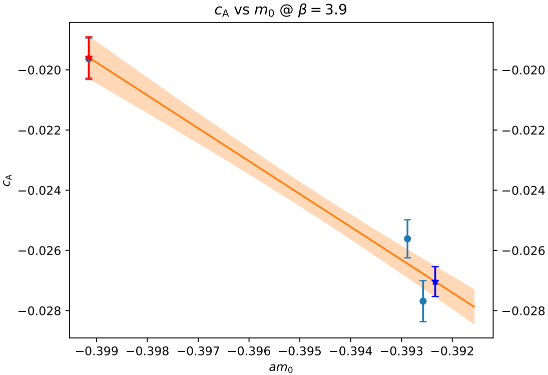
Improvement of the axial-vector current

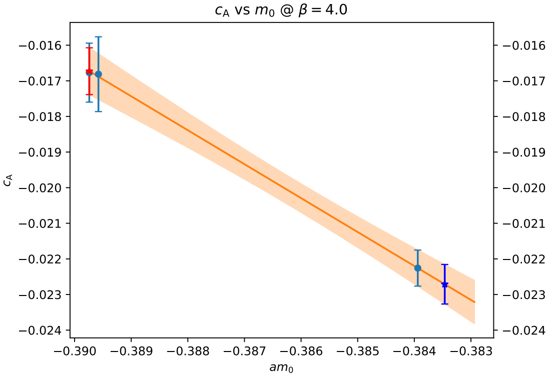
Finding the symmetric and chiral point



Improvement of the axial-vector current

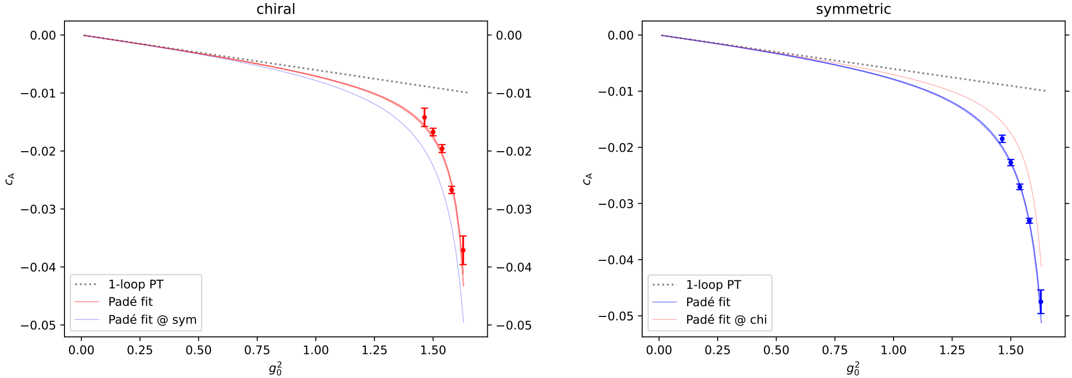
Interpolations in c_A





Improvement of the axial-vector current

Interpolations in g_0^2



First scaling test of improvement

- Example: Calculate $f_{\pi K}$ with stabilised Wilson fermions
- symmetric point OpenLat ensembles
- improve with $c_\mathrm{A} = 0$ vs $c_\mathrm{A}(g_0^2)|_\mathrm{chi}$ vs $c_\mathrm{A}(g_0^2)|_\mathrm{sym}$

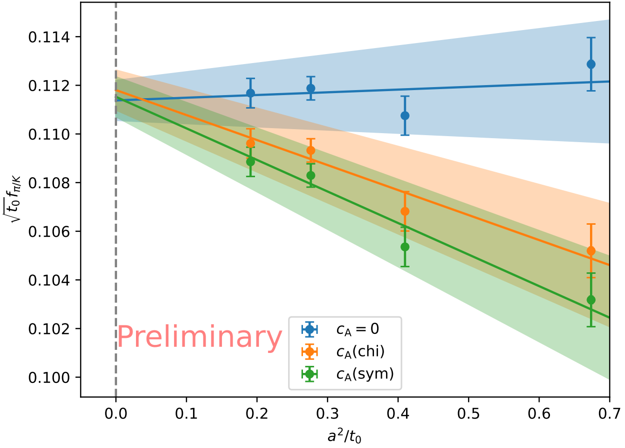
$$f_{\pi K}^\mathrm{RI} = Z_\mathrm{A}(1 + b_\mathrm{A} a m_\mathrm{q} + \bar{b}_\mathrm{A} a \mathrm{Tr}[M_\mathrm{q}]) \frac{\sqrt{2}\mathcal{A}_{\mathrm{A_0 P}}}{\sqrt{\mathcal{A}_\mathrm{PP} m_\pi}}$$

with $C_{XX} \propto \mathcal{A}_{XX} \mathrm{e}^{-m x_0}$

- renormalisation: Z_A preliminary, b_A from pert. theory, $\bar{b}_\mathrm{A}$ neglected

First scaling test of improvement

Results

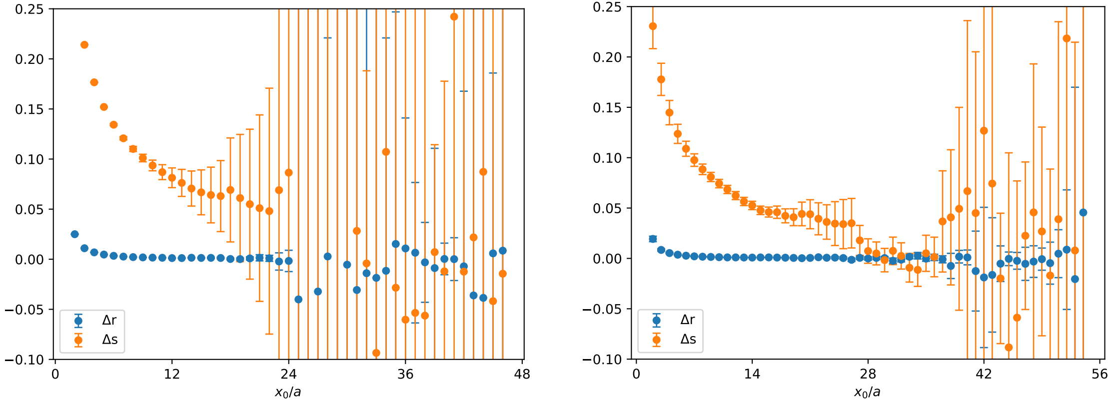


Outlook

- finish Z_A, Z_V, b_V and $\bar{b}_\mathrm{V}$ through SF
- further improvement and renormalisation currently in the works:
  - vector and tensor current improvement (c_V, c_T)
  - current quark mass renormalisation ($b_\mathrm{A} - b_\mathrm{P}$, b_m, Z)
  - determination of Z_A, Z_V, $Z_\mathrm{S}/Z_\mathrm{P}$ through χSF

Backup

Δr and Δs behaviour



Backup

Behaviour of f_{P} projected