The scales r_0 & r_1 in $N_f = 2+1$ QCD

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FOR 5269

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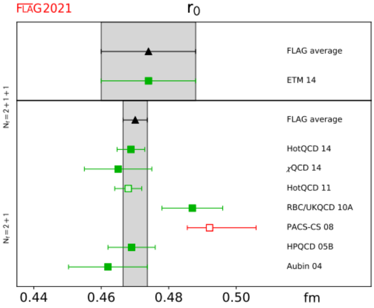
Motivation

- r_0 as reference scale
  - Weak quark mass dependence
  - Reasonable statistical precision
  - Mild lattice artifacts
- **Goal:** To add to existing measurements with state of the art simulated data



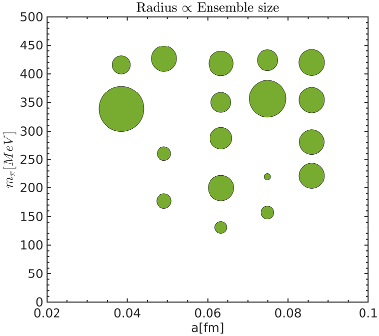
[FLAG Review 2021, 2111.09849]

Coordinated Lattice Simulations ($N_f = 2+1$)

"Simulation of QCD with $N_f = 2+1$ flavors of non-perturbatively improved Wilson fermions"

[M.Bruno et al, (2014) 1411.3982]

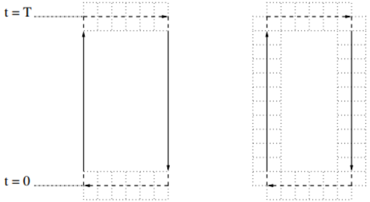
- 2+1 flavors of O(a) improved Wilson fermions using Lüscher-Weisz gauge action with open boundary conditions in time.
- Chiral trajectory with $tr(M) = m_u + m_d + m_s = const$ while making sure that $m_\pi L > 4$
- 5 lattice spacings from 0.085 fm to 0.037 fm
- m_π from 430 MeV to 134 MeV

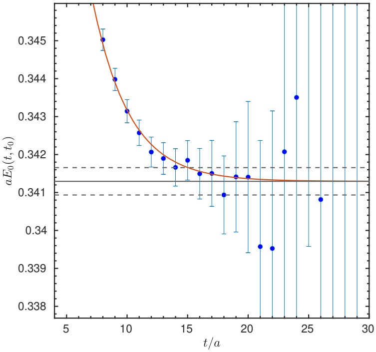


Wilson loops

"Determination of the Static Potential with Dynamical Fermions" [M.Donnellan, F.Knechtli, B.Leder, R.Sommer, 1012.3037]

- Operator basis with different levels of HYP smearing.





- HYP2 static quark action

Finding r_0 and r_1

We are using an improved definition of the force:
$F(r_I) = [V(r) - V(r-a)]/a$
where $r_I = r - a/2 + \mathcal{O}(a^2)$

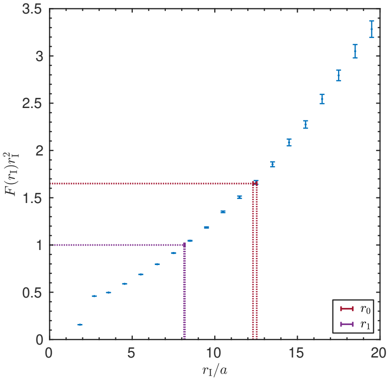
[S. Necco and R. Sommer, hep-lat/0108008]

Using that, we find the following scales:
$r_I^2 F(r_I)|_{r_I=r_0} = 1.65$

[R. Sommer, hep-lat/9310022]

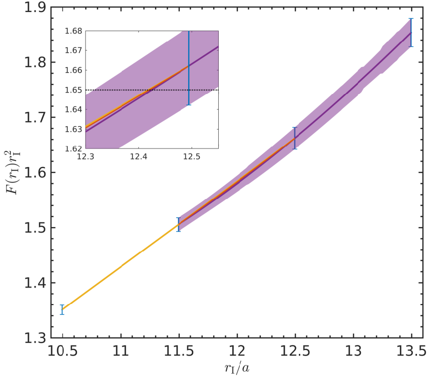
$r_I^2 F(r_I)|_{r_I=r_1} = 1$

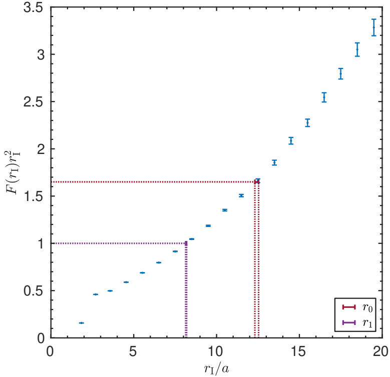
[C.Bernard et al, (2000)]



Finding r_0 and r_1

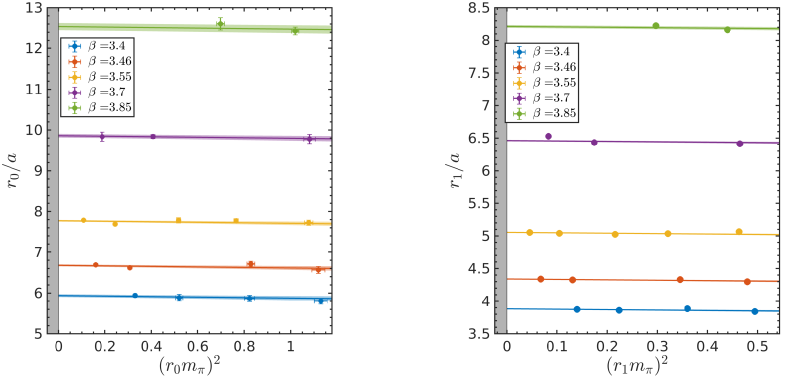
Zooming in at the r_0 interpolation: 2-points (red), 3-points early (yellow), and 3-point late (purple)





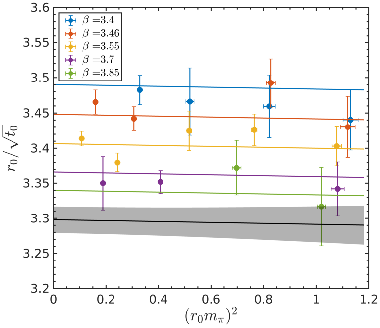
Chiral extrapolations in lattice units

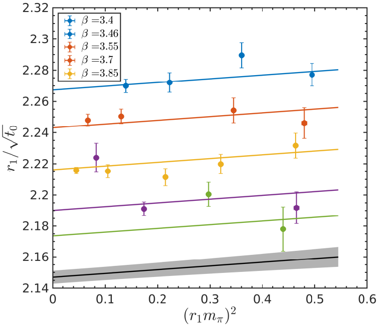
Global fit using: $\frac{r_i}{a} = c_1|_\beta + c_2(r_i m_\pi)^2$



Continuum & chiral extrapolation using a global fit

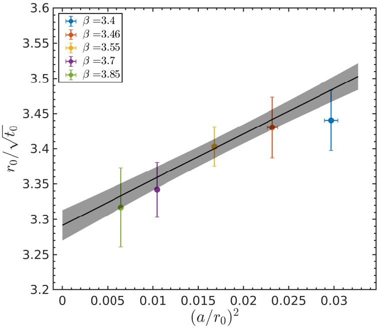
Global fit using: $\frac{r_i}{\sqrt{t_0}} = c_1 + c_2 \left(\frac{a}{r_{0,sym}} \right)^2 + c_3 (r_0 m_\pi)^2$

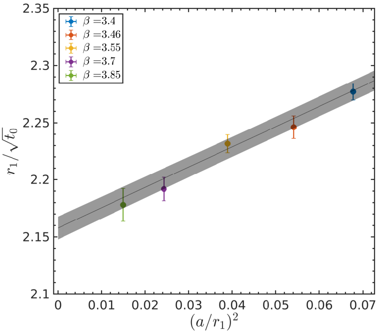




At the symmetric point

Using the parameters from previous fit results





Different Fits

$$\frac{r_0}{\sqrt{t_0}} = F_{fit}(a, m_u, m_d, m_s)$$
$$F_1 : c_1 + c_2 \left(\frac{a}{r_{0,sym}}\right)^2 + c_3 (r_0 m_\pi)^2$$
$$F_2 : c_1 + c_2 \left(\frac{a}{r_{0,sym}}\right)^2 + c_3 (r_0 m_\pi)^2 + c_4 (m_\pi a)^2$$
$$F_3 : c_1 + c_2 \left(\frac{a}{r_{0,sym}}\right)^2 + c_3 \phi_2 + c_4 (1.098 - \phi_4)$$
$$F_4 : c_1 + c_2 \left(\frac{a}{r_{0,sym}}\right)^2 + c_3 (t_0 m_\pi^2) + c_4 (1.098 - \phi_4),$$

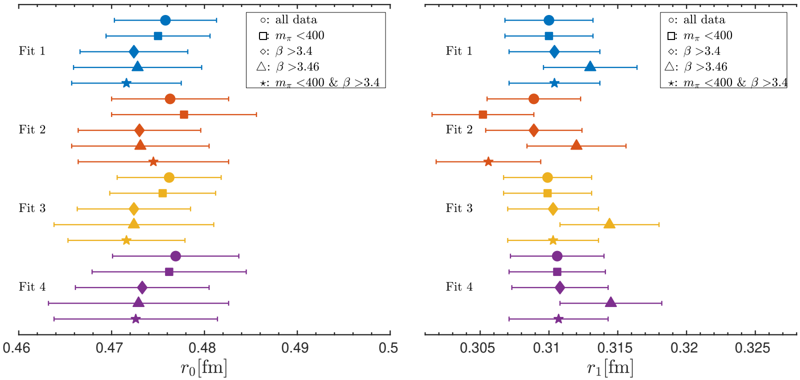
$(1.098 - \phi_4)$ takes mistuning into account

[B. Straßberger et al, (2021) hep-lat/2112.06696]

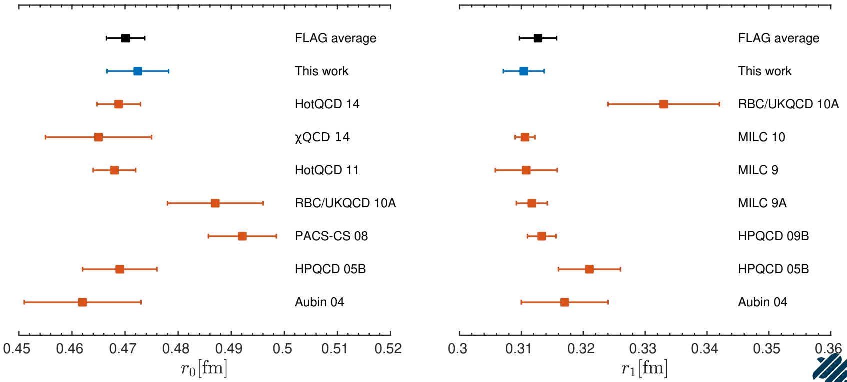
$\phi_2 \sim m_u + m_d, \quad \phi_4 \sim m_u + m_d + m_s$

Comparison between the fits with datacuts

A cut in the symmetrical mass (□), 1-2 coarse lattice spacings(◇,△), and a combination (⋆)



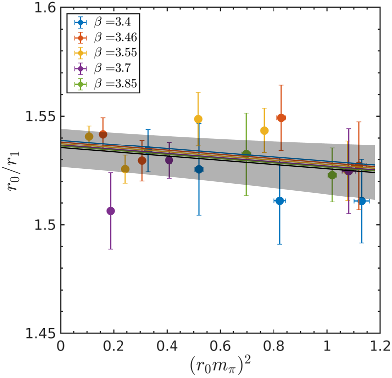
The physical value

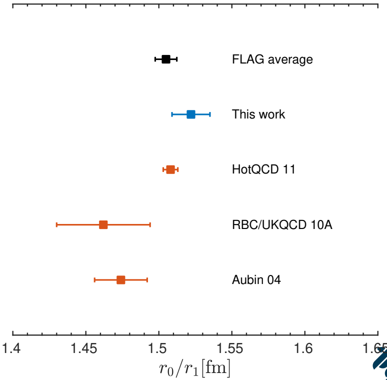


$$r_0^{\mathrm{Phys}} = 0.4724(58)[\mathrm{fm}]$$

$$r_1^{\mathrm{Phys}} = 0.3104(33)[\mathrm{fm}]$$

r_0/r_1





$$\frac{r_0}{r_1}^{\text{Phys}} = 1.522(13)[\text{fm}]$$

Structure of the static potential

Through the motivation of the Cornell potential:

$V = \sigma r - \frac{K}{r}$

$K = 0.52$ [E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane and T.M. Yan, Phys. Rev. D21 (1980) 203]

Building following quantity

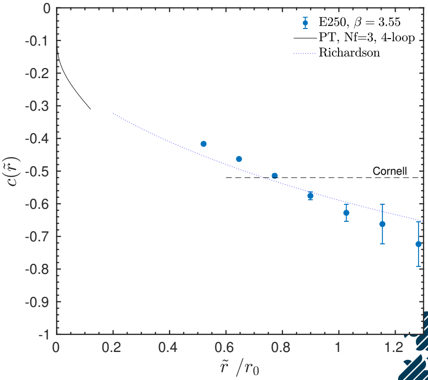
$c(\tilde{r}) = \frac{1}{2}\tilde{r}^3 F'(r)$

with $\tilde{r} = r + \mathcal{O}(a^2)$ such that

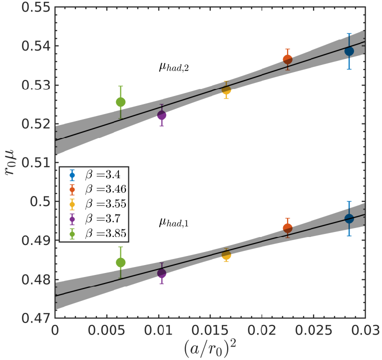
$c_{tree}(\tilde{r}) = -C_F \frac{g_0^2}{4\pi}$

[M. Lüscher and P. Weisz, hep-lat/0207003]

[J.L. Richardson, Phys. Lett. B82 (1979) 272]



Lambda parameter



Nonperturbative determination of $\beta(\bar{g})$ between μ_{had} and μ_{PT}

[M.Bruno et al, (2017) 1706.03821]

- $\mu_{had,1}$: scale where $\bar{g}^2_{GF} = 11.31$
  $\mu_{had,2}$: scale where $\bar{g}^2_{GF} = 10.20$
- $\frac{\Lambda^{(3)}_{\overline{\rm MS}}}{\mu_{had,1}} = 1.729(57)$
  $\frac{\Lambda^{(3)}_{\overline{\rm MS}}}{\mu_{had,2}} = 1.593(53)$

We combine our $\frac{r_0}{a}$ with $a\mu_{had}$

- $r_0\Lambda^{(3)}_{\overline{\rm MS}} = 0.820(28)$
- Flag: $r_0\Lambda^{(3)}_{\overline{\rm MS}} = 0.808(29)$

Conclusion

- The results were shown to be stable under alterations of different cuts, fits, as well as the determination of the effective masses
- Detailed analysis with masses down to the pion mass and lattice spacing down to 0.037 fm giving:
  - $r_0 = 0.4724(58)\mathrm{fm}$
  - $r_0 = 0.3104(33)\mathrm{fm}$
  - $\frac{r_0}{r_1} = 1.537(5)$
  - $r_0\Lambda^{(3)}_{\overline{\mathrm{MS}}} = 0.820(28)$

That's it, questions?

The difficulties of E300