Cutoff effects and scale determination in pure gauge theory

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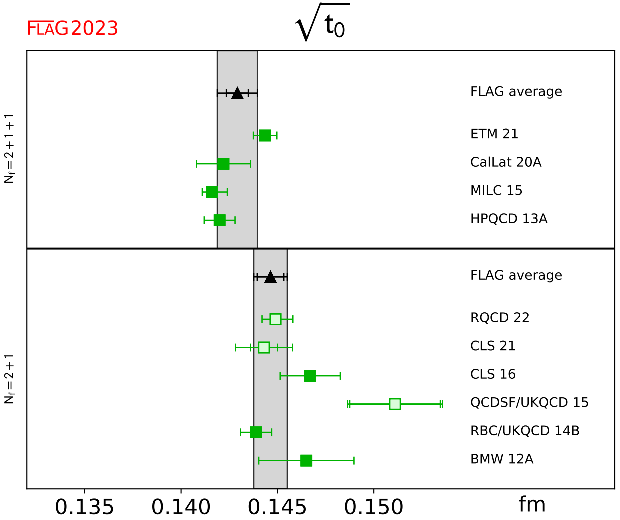
GENERALITAT VALENCIANA

Motivation: understanding scale setting

- t_0 Very precise, little systematic
- 1.5% in scale determinations
- Crucial for precision physics
- Results obtained with different actions
- $a \in [0.05 - 0.1]$ fm

Potential problems

- Continuum extrapolation
- Determination of physical quantity (i.e. $f_\pi, M_\Omega, \dots$)



How to understand cutoff effects?

Symanzik effective theory

- Any lattice action that we simulate $S_{\rm latt}$ can be described by an effective action

$$S_{\rm latt} \overset{a\to 0}{\sim} S_{\rm cont} + a^2 S_2 + \dots$$

- Spectral quantities computed on the lattice have an asymptotic expansion

$$\langle O \rangle_{\rm latt} \overset{a\to 0}{\sim} \langle O \rangle + a^2 \langle O S_2 \rangle_c + \dots$$

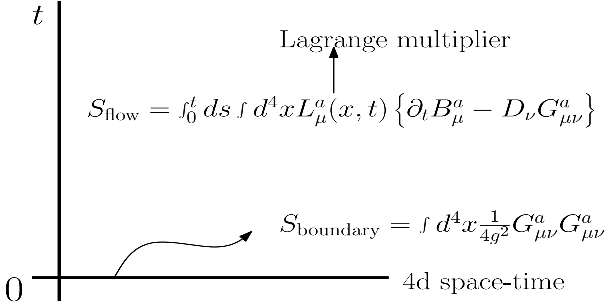
But they are difficult to compute (signal-to-noise, finding plateaus, ...)

Flow quantities as an alternative

- Symanzik expansion for flow quantities
- Lessons for QCD?

5d Local formulation [Lüscher, Weisz '11]

We can see the theory as a 5d local field theory [Zinn-Justin '86, Zinn-Justin, Zwanziger '88]



$$S_{\text{Total}} = S_{\text{flow}} + S_{\text{boundary}}$$

The important point

- No loops on the bulk $\Rightarrow$ "Classical theory" at $t > 0$

Symanzik effective theory for the gradient flow [A. Ramos, S. Sint '15]

Symanzik effective theory has several "parts"

$$S^{5d}_{\rm latt} \overset{a\to 0}{\sim} S^{5d}_{\rm cont} + a^2 S_{2,b} + a^2 S_{2,fl} + \ldots$$

- "Usual" corrections
- Affects all quantities (i.e. $m_p, g-2, t_0, \ldots$)
- Determined by the action that you simulate (i.e. Iwasaki/Wilson, Domain Wall/Clover)
- Affects only flow quantities
- Determined by *how you integrate the flow equations* (i.e. Wilson/Symanzik flow)

Symanzik expansion of a flow quantity $O \overset{a\to 0}{\sim} O_0 + a^2 O_2$

$$\langle O\rangle_{\rm latt} \overset{a\to 0}{\sim} \langle O_0\rangle + a^2 \left\{ \langle O_2\rangle + \langle O_0 S_{2,b}\rangle + \langle O_0 S_{2,fl}\rangle + c_b t^2 \frac{\rm d}{{\rm d}t}\Big|_{t_0} \langle O_0\rangle \right\}$$

Theory "classical" at $t > 0$: Non-perturbative result/all improvement

Use Zeuthen flow $\Longrightarrow$ $S_{2,fl} = 0$

Use Classically improved observables (i.e. $(4E_{\rm pl} - E_{\rm cl})/3)$) $\Longrightarrow$ $O_2 = 0$

Understanding $t_0^{\mathrm{pl}}/t_0^{\mathrm{cl}}$

Apply Symanzik expansion for t_0

$$t_0^{\mathrm{pl}} \overset{a\to 0}{\sim} t_0 - \frac{a^2}{D}\left\{ t_0^2\langle E(t_0) S_{2,b}\rangle + t_0^2\langle E(t_0) S_{2,fl}\rangle + t_0^2\langle E_2^{\mathrm{pl}}(t_0)\rangle + c_b \frac{\mathrm{d}}{\mathrm{d}t} t^2\langle E(t)\rangle \right\}$$

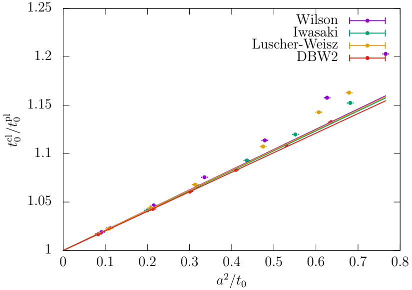
$$t_0^{\mathrm{cl}} \overset{a\to 0}{\sim} t_0 - \frac{a^2}{D}\left\{ t_0^2\langle E(t_0) S_{2,b}\rangle + t_0^2\langle E(t_0) S_{2,fl}\rangle + t_0^2\langle E_2^{\mathrm{cl}}(t_0)\rangle + c_b \frac{\mathrm{d}}{\mathrm{d}t} t^2\langle E(t)\rangle \right\}$$

The ratio/difference does not say anything useful

$$\frac{t_0^{\mathrm{pl}}}{t_0^{\mathrm{cl}}} \overset{t\to 0}{\sim} 1 - \frac{a^2}{D}\left\{ t_0^2\langle E_2^{\mathrm{pl}}(t_0)\rangle - t_0^2\langle E_2^{\mathrm{cl}}(t_0)\rangle \right\}$$

- Insensitive to $S_{2,b}$
- Only sensitive to something that can be made zero explicitly: Choose

$$E^{\mathrm{latt}}(t) = \frac{4}{3}E^{\mathrm{pl}}(t) - \frac{1}{3}E^{\mathrm{cl}}(t)$$



Flow scales

Most natural quantities

- Numerically very precise
- Little systematic (i.e. no signal to noise)

Two natural candidates

- t_0 - like scales [Luscher '10]

$$t^2 \langle E(t) \rangle \Big|_{t=t_c} = \begin{cases} 0.15 & (t_c = t_2) \\ 0.3 & (t_c = t_0) \\ 0.5 & (t_c = t_1) \end{cases}$$

- w_0 - like scales [BMW '10]: No discussed here, but similar conclusions

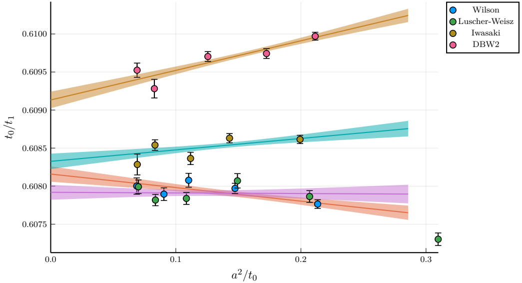
Testing ratios of flow scales

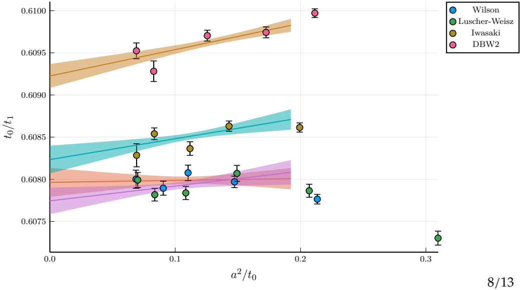
"Long distance" ratio t_0/t_2

- Good fits with $a < 0.08, 0.068$ fm:

$$\chi^2/\langle\chi^2\rangle = \begin{array}{ccc|l} 2.15 & / & 2.82 & \text{PL} \\ 3.44 & / & 2.97 & \text{LW} \\ 2.36 & / & 3.05 & \text{IW} \\ 1.78 & / & 3.00 & \text{DB} \end{array}$$

$$\chi^2/\langle\chi^2\rangle = \begin{array}{ccc|l} 1.19 & / & 1.86 & \text{PL} \\ 2.36 & / & 1.96 & \text{LW} \\ 2.20 & / & 2.00 & \text{IW} \\ 1.52 & / & 2.03 & \text{DB} \end{array}$$



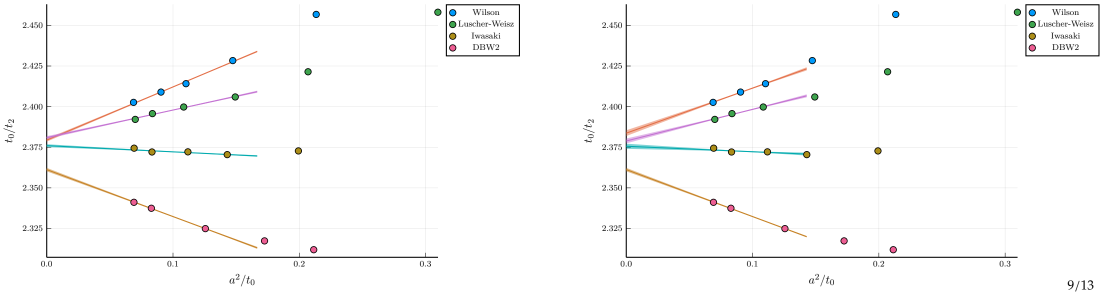


Testing ratios of flow scales

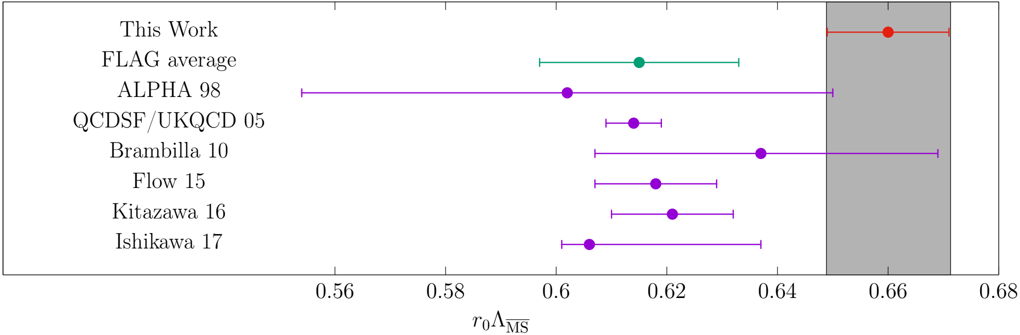
"Large distance" ratio t_0/t_1

- Good fits with $a < 0.068, 0.06$ fm:

$$\chi^2/\langle\chi^2\rangle = \begin{array}{ccc|l} 6.20 & / & 1.97 & \text{PL} \\ 2.44 & / & 2.04 & \text{LW} \\ 2.83 & / & 2.02 & \text{IW} \\ 0.03 & / & 0.97 & \text{DB} \end{array} \qquad \chi^2/\langle\chi^2\rangle = \begin{array}{ccc|l} 0.14 & / & 1.04 & \text{PL} \\ 1.08 & / & 1.02 & \text{LW} \\ 2.79 & / & 1.00 & \text{IW} \\ 0.03 & / & 0.97 & \text{DB} \end{array}$$



Motivation [Dalla Brida, Ramos '19]



String tension

- More precise results based on PT at the scale of the cutoff

DETERMINATION OF $\Lambda_{\overline{\rm MS}}$ FROM QUANTITIES AT THE CUTOFF

- Lattice connects with $\overline{\rm MS}$

$$g_0^2 \overset{g_0\to 0}{\sim} g^2_{\overline{\rm MS}}(\mu) + \sum_n c_n g^{2n}_{\overline{\rm MS}}(\mu)\,, \qquad (\mu = 1/a)$$

- Very poor convergence, but *tadpole improvement* helps a lot! **[Lepage, Mackenzie, Phys. Rev. D 48 (1993)]**

$$g^2_{\Box} = \frac{g_0^2}{\langle P\rangle} \overset{g_\Box\to 0}{\sim} g^2_{\overline{\rm MS}}(\mu) + \sum_n t_n g^{2n}_{\overline{\rm MS}}(\mu)\,, \qquad (\mu = 1/a)$$

- t_1 known for Wilson, Luscher-Weisz, Iwasaki actions
- t_2 known for Wilson action (used in previous works)

1. Get $\Lambda_{\overline{\rm MS}}$ in units of the lattice spacing. (Depends on perturbation theory)

$$a\Lambda_{\overline{\rm MS}} = \frac{\Lambda_{\overline{\rm MS}}}{\Lambda_\Box} \times \left[b_0 \bar g^2_\Box(1/a)\right]^{\frac{-b_1}{2b_0^2}} e^{-\frac{1}{2b_0\bar g^2_\Box(1/a)}} \exp\left\{-\int_0^{\bar g_\Box(1/a)} dx \left[\frac{1}{\beta^{\rm PT}_\Box(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0 x}\right]\right\}$$

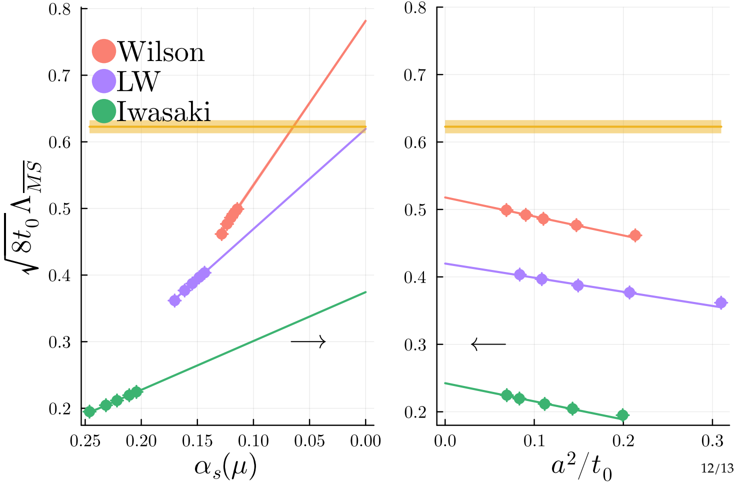
2. Get $\Lambda_{\overline{\rm MS}}$ in uits of t_0

$$\sqrt{8t_0}\Lambda_{\overline{\rm MS}} = \left(\frac{\sqrt{8t_0}}{a}\right) \times (a\Lambda_{\overline{\rm MS}}) + \ldots$$

The problem: cutoff vs perturbative corrections

$$\sqrt{8t_0}\Lambda_{\overline{MS}} = \left(\frac{\sqrt{8t_0}}{a}\right) \times (a\Lambda_{\overline{MS}}) \quad + \quad \mathcal{O}(a^2) + O(\alpha(1/a)^p)$$

- Both corrections eliminated $a \to 0$
- PT corrections depends on known orders in PT:
  - $p = 1$ for PL, LW, IW
  - $p = 2$ known for PL
- Extrapolations **needs assumptions**:
  - PT corrections negligible
  - a^2 corrections negligible
  - You name it...
- Assumptions change result by **197σ**
- Difference decreases with PT knowledge (but not fast enough)



The problem: cutoff vs perturbative corrections

$$\sqrt{8t_0}\Lambda_{\overline{MS}} = \left(\frac{\sqrt{8t_0}}{a}\right) \times (a\Lambda_{\overline{MS}}) \quad + \quad \mathcal{O}(a^2) + O(\alpha(1/a)^p)$$

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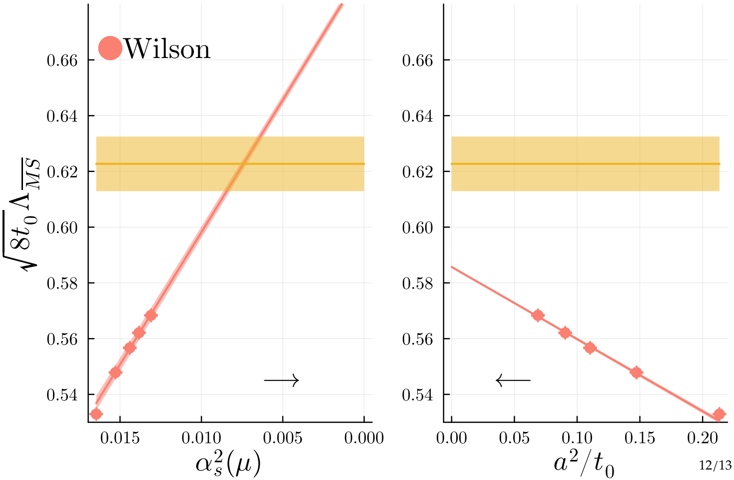

CONCLUSIONS

Scale setting

- Difference between continuum limit on different actions
  - Topology freezing?
  - Is g_0 small enough?
- Delicate continuum extrapolation for high precision **[Husung, Sommer]**
- Problems might not be clear from examining your data
- Looking at different "valence" discretizations
  - Misleading (i.e $t_o^{\mathrm{cl}}/t_0^{\mathrm{pl}}$)
- Benchmark for different actions?
  - t_c/t_0 with $t^2\langle E(t)\rangle = c$
  - Short distance $c = 0.15$
  - Long distance $c = 0.5$
- Better use improved ("Zeuthen") flow/Observable

Determination of α_s/Renormalization

- PT corrections difficult to evaluate
- Data not enough to differentiate $a^2, \alpha^p(1/a)$
- limit $\alpha \to 0$ has to be taken seriously
- Not only α_s
  - B_K, 4-fermion operators, etc...

Many thanks!