B dependence of the QED chiral condensate induced by an external magnetic field.

D. K. Sinclair and J. B. Kogut

- Introduction
- Electrons in an external magnetic field
- QED in an external magnetic field
- Preliminary results of RHMC simulations with $\alpha = 1/5$ and $eB = 2\pi \times 6/18^2 = 2\pi \times 24/36^2$
- Discussions and Conclusions

Introduction

QED in (strong) external magnetic fields breaks chiral symmetry non-perturbatively.

This was predicted by Schwinger-Dyson methods, and confirmed by us (J. B. Kogut and D. K. Sinclair, Phys. Rev. 109, 034511 (2024)), using lattice QED (RHMC) simulations.

Our simulations used $eB = 2\pi \times 100/36^2 = 0.4848...$ at bare coupling $\alpha = 1/5$ in the limit $m \to 0$.

We are now repeating our simulations at a much smaller B $eB = 2\pi \times 24/36^2 = 0.1163....$ This will enable us to check the prediction $\langle\bar{\psi}\psi\rangle \propto (eB)^{3/2}$ and possibly that the electron gains a dynamical mass $m_{dyn} \propto \sqrt{eB}$.

Electrons in an external magnetic field

Consider electrons in an applied constant homogeneous magnetic field B in the z direction.

Classically the electrons traverse helical orbits around magnetic field lines. The projection of these orbits on the xy plane is a circle, while the motion in the z direction is free.

Quantum mechanically, since the motion in the xy plane is compact, it is quantized.

These quantized energy levels – the Landau levels have energies

$$E_n(p_z) = \pm\sqrt{m^2 + 2eBn + p_z^2}$$

where $n = 0, 1, 2, ...$

Without the effects of QED the chiral condensate $\langle\bar{\psi}\psi\rangle$ vanishes in the limit $m \to 0$.

QED in an external magnetic field

We now consider what happens when we add the effects of QED to the dynamics of electrons in a strong external magnetic field.

The dynamics is expected to be dominated by the lowest Landau levels (LLL) i.e. those with $n = 0$.

The attractive forces between electrons and positrons confined to the LLLs with radii $\approx 1/\sqrt{eB}$ are expected to yield a finite chiral condensate in the $m \to 0$ limit.

We employ staggered fermions for our RHMC simulations, using rational approximations to the fractional powers of the quadratic fermion operators required to restrict our simulations to a single electron "flavour".

As $m \to 0$, chiral symmetry breaking is expected to be associated with near-zero modes.

This means that we can expect to need to perform finite-size scaling analyses.

However, domination by LLLs with radii $\approx 1/\sqrt{eB} \approx 2$, means that our choice of $N_x = N_y = 18$ or $N_x = N_y = 36$ should be adequate and not need to be increased.

Hence we only need to increase $N_z = N_t$ for finite-size scaling analyses.

Preliminary results of RHMC simulations with $\alpha = 1/5$ and $eB = 2\pi \times 6/18^2 = 2\pi \times 24/36^2$

We perform RHMC simulations on lattices with either $N_x = N_y = 36$ or $N_x = N_y = 18$ with $\alpha = 1/5$ and $eB = 2\pi \times 24/36^2$.

We simulate over a range of (small) electron masses $0.0005 \leq m \leq 0.05$. At each mass we start with $N_z = N_t = 36$ and increase it until the chiral condensate ceases to increase with increasing $N_z = N_t$ (finite size scaling).

At the smallest masses this requires $N_z = N_t \geq 160$.

Figures 1,2,3,4 show the preliminary results. Also shown is the prediction of the $m = 0$ limit obtained from our simulations at $eB = 2\pi \times 100/36^2$ assuming $\langle\bar{\psi}\psi\rangle \propto (eB)^{3/2}$, at $m = 0$.

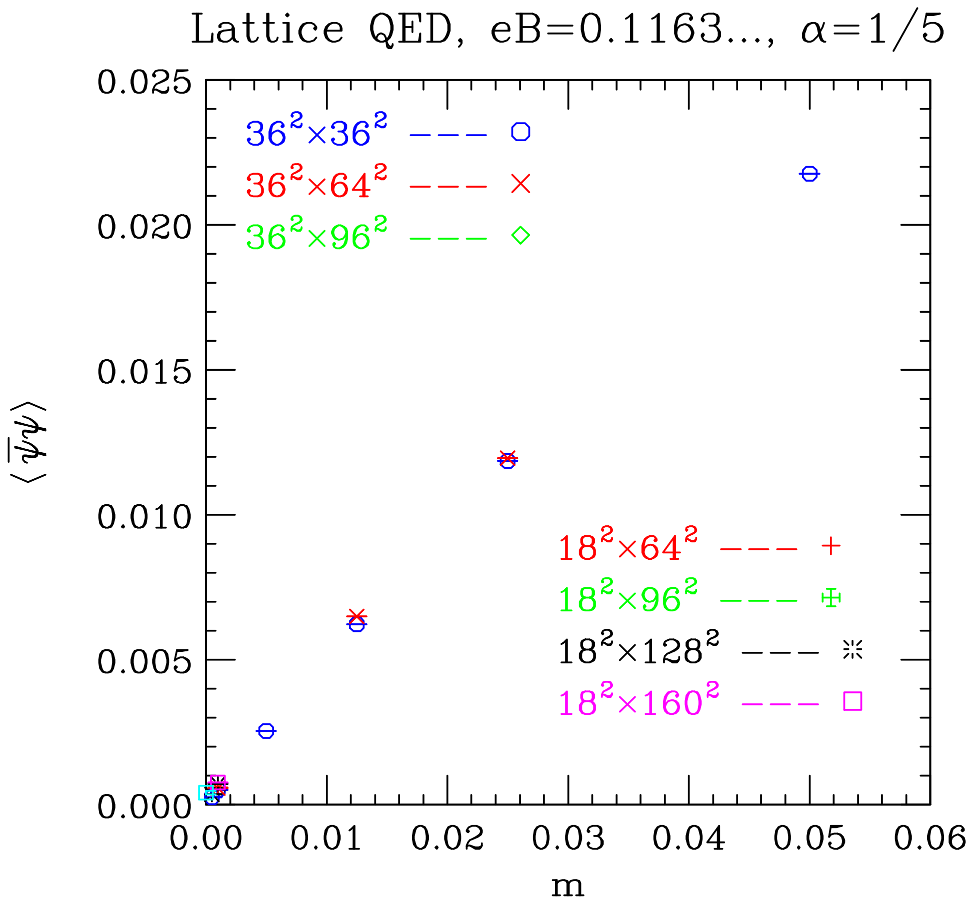

Figure 1: $\langle\bar\psi\psi\rangle$ as a function of mass at $eB = 2\pi \times 24/36^2$,showing dependence on lattice size in the z and t directions.

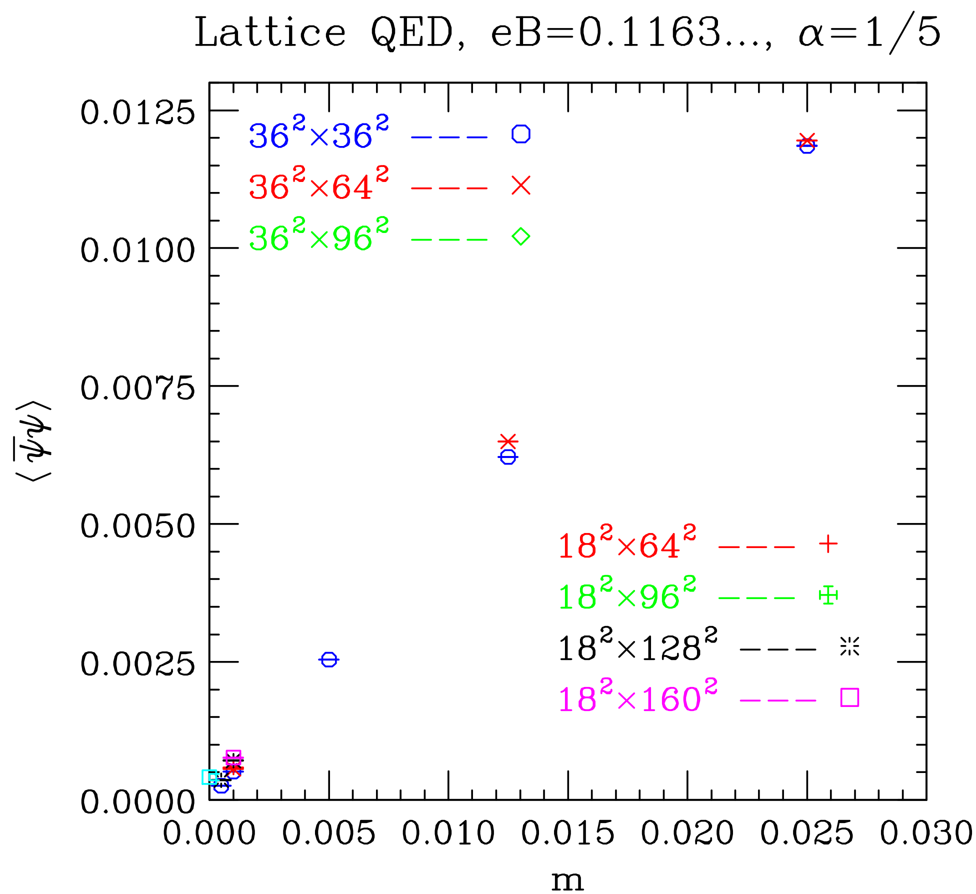

Figure 2: As in figure 1, but on an expanded scale

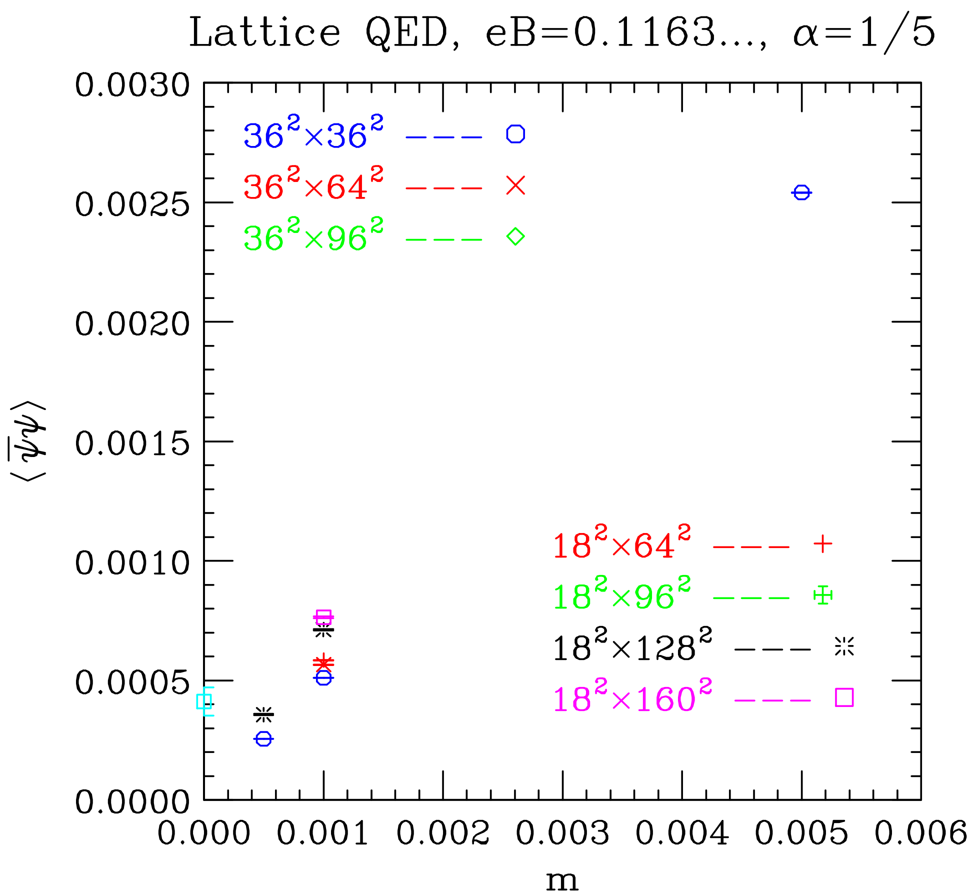

Figure 3: As in figure 2, but on an expanded scale

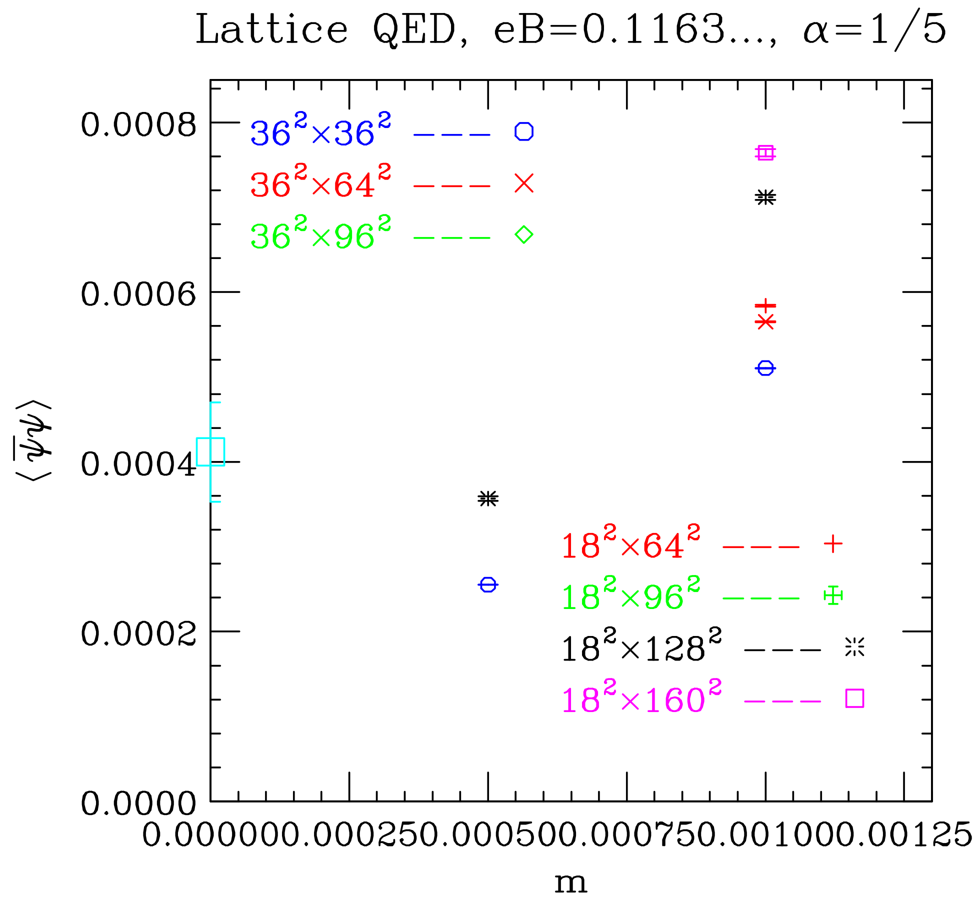

Figure 4: As in figure 3, but on an expanded scale. The cyan strip at $m = 0$ is the prediction extrapolated from the $eB = 2\pi \times 100/36^2$ simulations.

What these suggest is that the measured values of the chiral condensate will be less than or equal to the prediction based on the extrapolation from the larger magnetic fields. It is possible that the chiral condensate at this B value will be indistinguishable from zero.

However, the fact that the chiral condensate has strong dependence on $N_z = N_t$ up to large values indicates the existence of low lying modes contributing to it. This suggests that chiral symmetry will remain broken at $m = 0$.

The strong dependence on $N_z = N_t$ compared to the earlier measurements at $eB = 2\pi \times 100/36^2$ suggests that we need to increase $N_z = N_t$ even further to get accurate results. In fact, dimensional analysis would suggest that we might need lattices with $N_z = N_t$ about twice as large at $m = 0.0005$ and $eB = 2\pi \times 24/36^2$ as was needed for $m = 0.001$ and $eB = 2\pi \times 100/36^2$.

The most important next measurement to make is that of the chiral condensate with $m = 0.0005$ on an $18^2 \times 160^2$ lattice. Preliminary measurements suggest that this will show a significant increase over the chiral condensate on an $18^2 \times 128^2$ lattice.

Discussions and Conclusions

- We continue our studies of chiral symmetry breaking in (lattice) QED catalyzed by strong magnetic fields.
- For simplicity we consider the effect of constant, strong homogeneous magnetic fields oriented in the z direction.
- We simulate on lattices with bare lattice fine structure constant $\alpha = 1/5$, so that $\alpha(\pi^2) = 1/5$ compared with $\alpha(m_e^2) \approx 1/137$.
- One loop running of the electric charge gives an estimate of the momentum cutoff

$$\log(\Lambda_{1/5}/m_e)/\log(\Lambda_\infty/m_e) \approx (137-5)/137,$$

 where Λ_∞ is the momentum cutoff at the Landau pole. So, the momentum scales and hence the magnetic fields we consider, are so large as to be of only theoretical interest.
- So far, our runs at the smaller $eB = 0.1163...$ (our previous project was at $eB = 0.4848...$) have yet to show direct evidence of a non-zero chiral condensate in the $m \to 0$ limit.

- If chiral symmetry were unbroken as $m \to 0$, one would expect little dependence of the chiral condensate on lattice size since chiral symmetry breaking would be an ultraviolet effect with $\langle\bar{\psi}\psi\rangle \propto m\Lambda^2$ up to logarithms.
- The fact that for small input electron masses m the chiral condensate shows significant increase with increasing $N_z = N_t$ can be considered as indirect evidence that the chiral condensate remains finite as $m \to 0$.
- Simulations at larger $N_z = N_t$ for small masses will be needed to obtain direct evidence that chiral symmetry is broken in the $m \to 0$ limit at the lower eB.
- Eventually we hope to test if our simulations show that the chiral condensate at $m = 0$ is proportional to $(eB)^{3/2}$

These simulations are being performed on the Improv and Bebop clusters at Argonne's LCRC, on Stampede3 at TACC, Expanse at SDSC and Bridges2 at PSC under an ACCESS(NSF) allocation and on Perlmutter at NERSC under an ERCAP(DOE) allocation.