

The 3-pion K-matrix at NLO in ChPT: all isospin channels

Lattice 2024, Liverpool

Mattias Sjö, CPT Marseille



Baeza-Ballesteros, Bijmens, Husek, Romero-López, Sharpe & **MS** "The isospin-3 three-particle K -matrix at NLO in ChPT"

JHEP, 2303.13206[hep-ph]

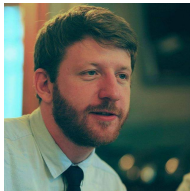
Baeza-Ballesteros, Bijmens, Husek, Romero-López, Sharpe & **MS** "The three-pion K -matrix at NLO in ChPT"

JHEP, 2401.14293[hep-ph]

The people



Hans Bijmens,
Lund U.



Tomáš Husek,
Birmingham U.



Mattias Sjö,
CPT Marseille



Stephen Sharpe,
U. of Washington



Fernando Romero-López,
MIT → Bern U.



Jorge Baeza-Ballesteros,
U. de València

Background

Finite-volume spectrum

Lattice QCD
(Fernando's talk, etc.)

Amplitude

ChPT or similar
Bijnens & Husek (2021)
Bijnens, Husek & **MS** (2022)

Athari's talk



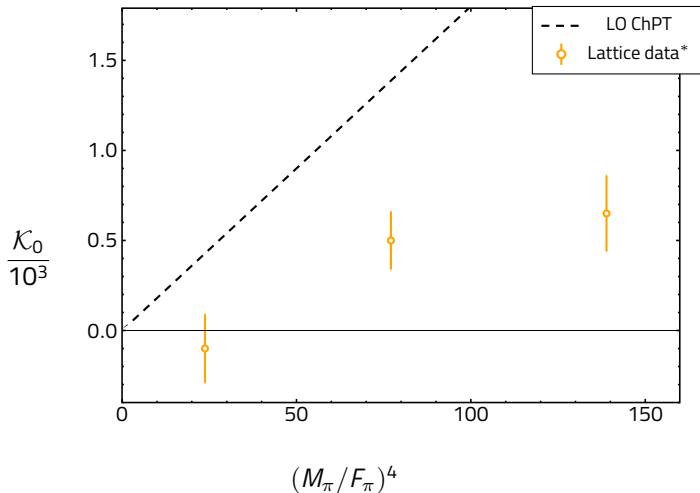
K-matrix

Scheme-dependent
Hansen & Sharpe (2019)

This talk

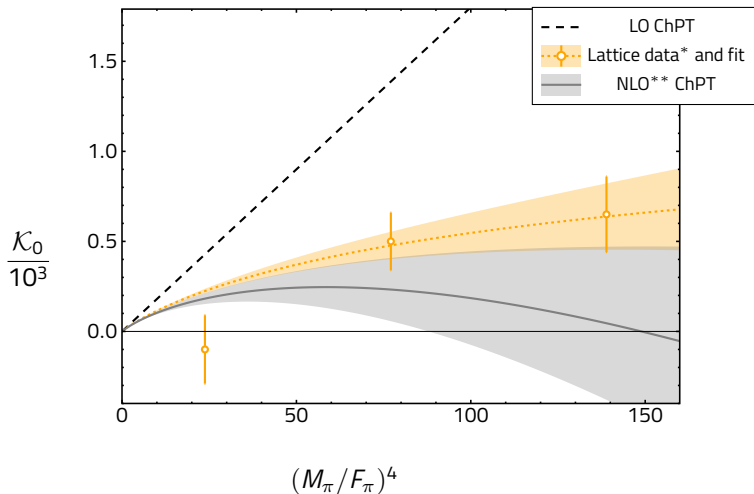


The tension that was



* Blanton, Hanlon, Hörz, Morningstar, Romero-López & Sharpe,
"Three-body interactions from the finite-volume QCD spectrum"

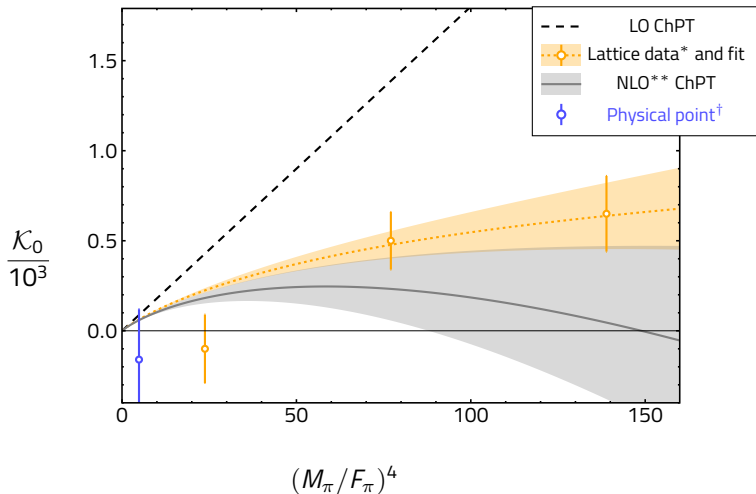
In the previous episode...



* Blanton, Hanlon, Hörz, Morningstar, Romero-López & Sharpe,
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** using LECs from FLAG and Colangelo, Gasser & Leutwyler, " $\pi\pi$ scattering"

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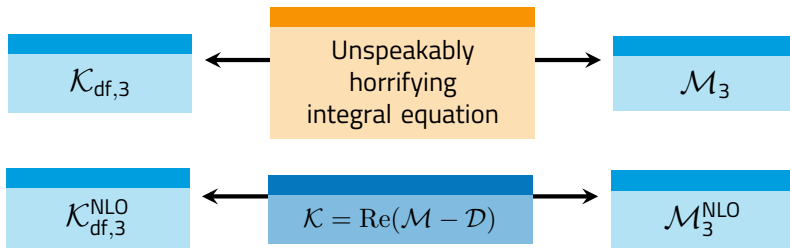
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† Preliminary (see Fernando's talk yesterday)

- ▶ Gain experience needed for $\pi\pi K$, etc.
- ▶ There will eventually be lattice data
- ▶ Fun to do

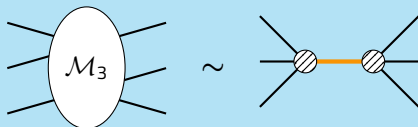
The K-matrix



Guiding properties of \mathcal{D}

- ▶ Exactly cancels divergences \rightarrow **df**
- ▶ All internal lines on-shell

s -channel one-particle exchange

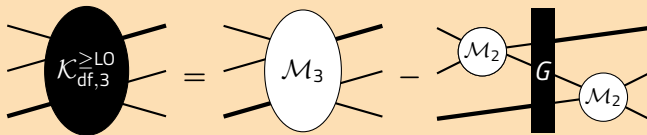


- ▶ Only present at **isospin 1**
- ▶ **No subtraction** needed since pole is sub-threshold

t -channel one-particle exchange

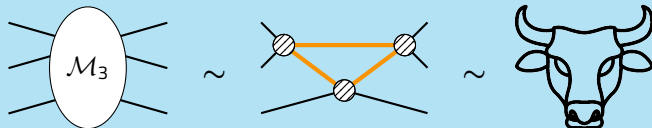


OPE subtraction

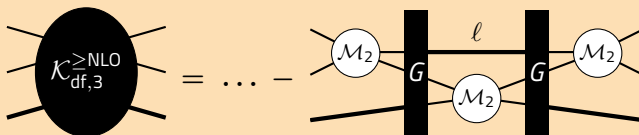


Note: G necessarily contains a cutoff function

Bull's head cut



Bull's head subtraction



The bull's head integral is **awful**:

- ▶ Triangle loop \Rightarrow complicated, pole-ridden integrand
- ▶ On-shell \Rightarrow no loop momentum shift
- ▶ Non-analytic \Rightarrow no Wick rotation, etc.

Different approaches

- ▶ Divide & conquer
Simple part with poles + complicated part (numerics-friendly)
- ▶ Subtract & conquer
Cancel divergences against \mathcal{M}_3 *before* evaluating
- ▶ Brute-force numerics
Because Tomáš is a Mathematica wizard
- ▶ Semi-analytic
Threshold-expand, then apply deep magic

Expansion parameters

$$\Delta \propto P^2 - (3M_\pi)^2 \quad (\text{system above-threshold-ness})$$

$$\Delta_i^{(I)} \propto (P - p_i^{(I)})^2 - (2M_\pi)^2 \quad (\text{pair above-threshold-ness})$$

$$\tilde{t}_{ij} \propto (p_i - p_j')^2 \quad (\text{spectator above-threshold-ness})$$

Compound parameters

$$\Delta_A = \sum (\Delta_i^2 + \Delta_i'^2) - \Delta^2 \quad \Delta_B = \sum \tilde{t}_{ij}^2 - \Delta^2$$

Maximum isospin threshold expansion

$$\mathcal{K}_{\text{df},3}^{[I=3]} = \mathcal{K}_0 + \mathcal{K}_1 \Delta + \mathcal{K}_2 \Delta^2 + \mathcal{K}_A \Delta_A + \mathcal{K}_B \Delta_B + \mathcal{O}(\Delta^3)$$

$l = 3$

Singlet

$l = 2$

Doublet

$l = 1$

Singlet

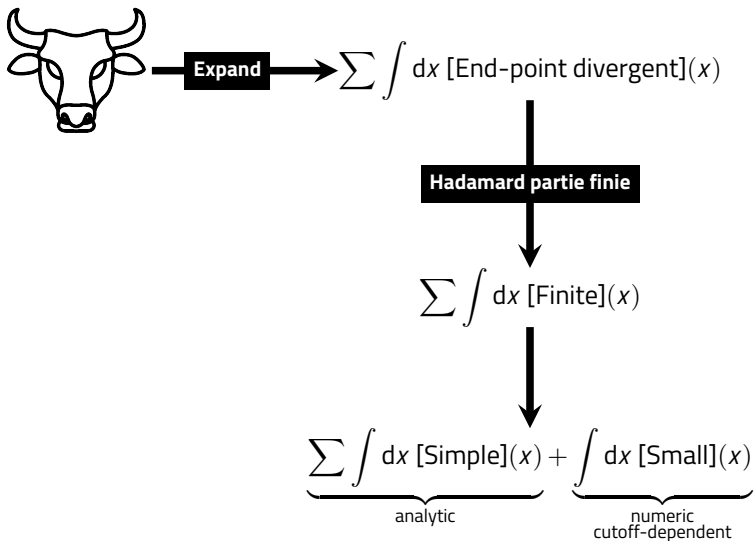
Doublet

$l = 0$

Antisymmetric singlet

Minimum isospin threshold expansion

$$\mathcal{K}_{\text{df},3}^{[l=0]} = \mathcal{K}_0^{\text{AS}} \sum \epsilon_{ijk} \epsilon_{lmn} \tilde{t}_{il} \tilde{t}_{jm} + \mathcal{O}(\Delta^3)$$

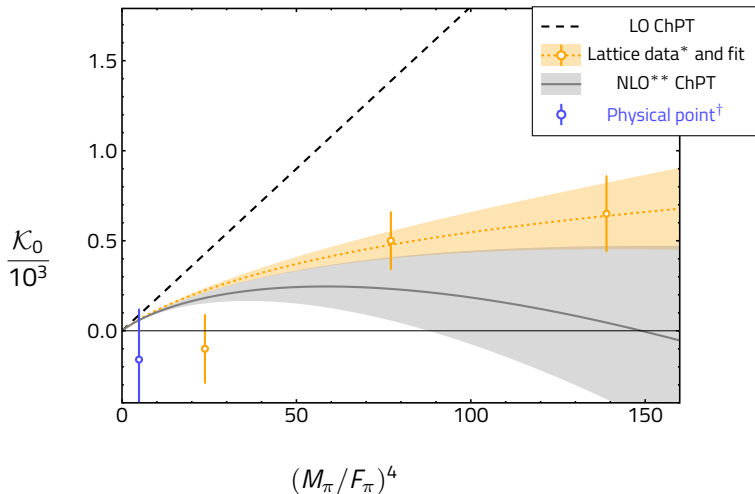


Costin & Friedman, "Foundational aspects of singular integrals"

J.Functional Analysis, 1401.7045[math.FA]

Results

Maximum isospin, again

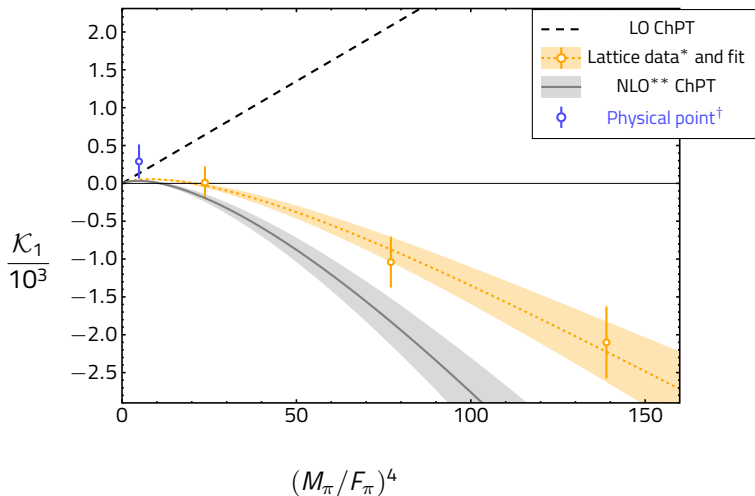


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Subleading order



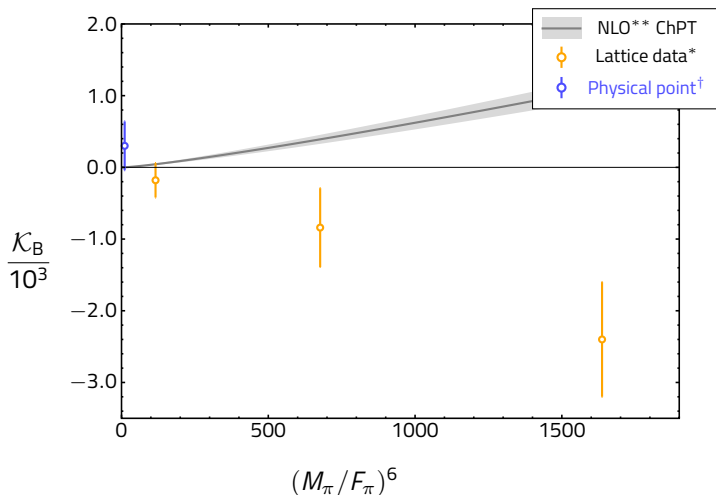
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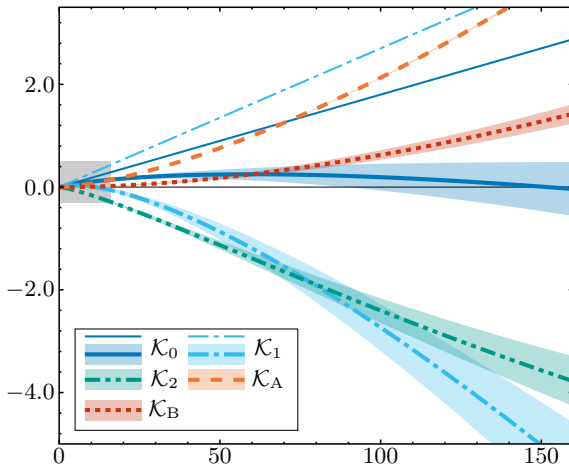


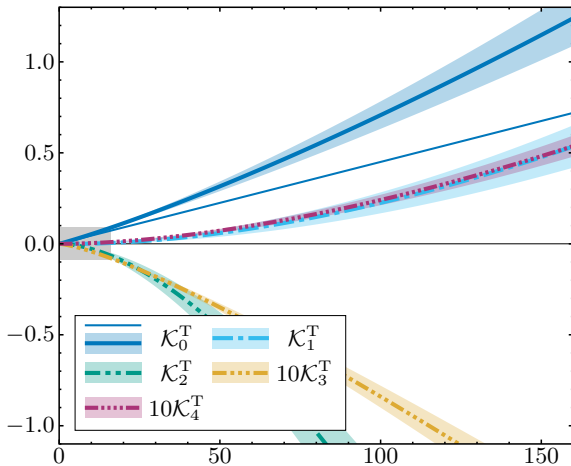
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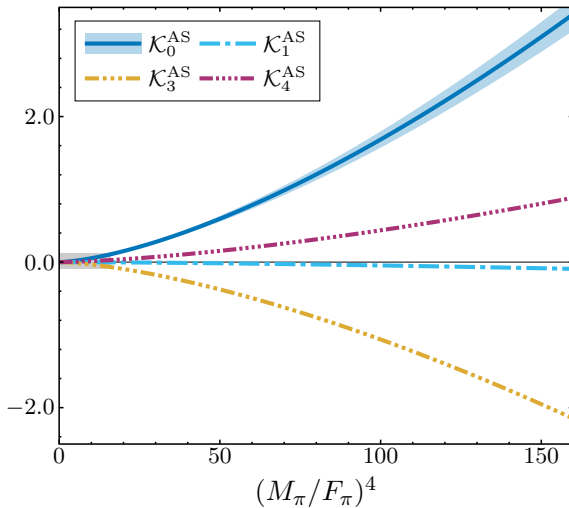
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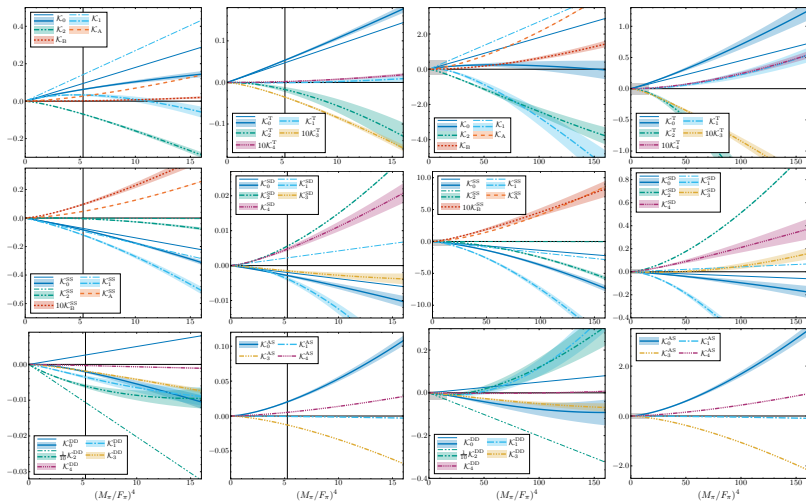
All maximum-isospin components





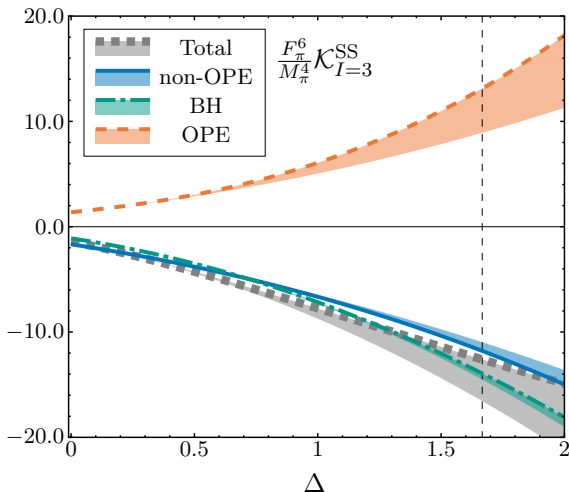


Awaiting more lattice results...

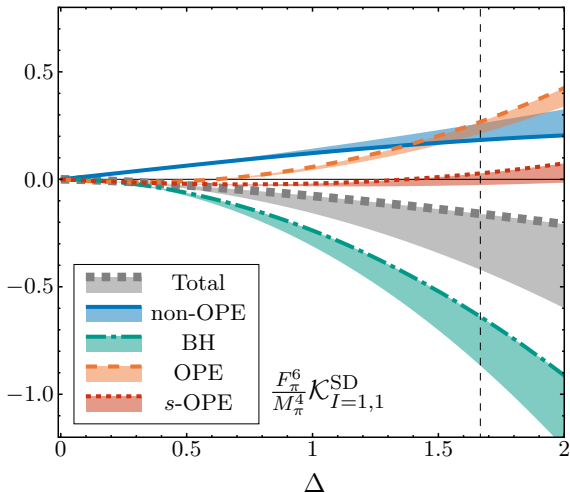


Convergence

The threshold expansion works



...better than it has to



- ▶ Large LO-NLO difference is troubling...
- ▶ ...but LO is very constrained
 - ⇒ **qualitative** difference expected
- ▶ Adding NNLO: **extremely difficult:**
 - Two-loop 6-point amplitude
 - Integral relation between \mathcal{M}_3 and $\mathcal{K}_{\text{df},3}$

Summary

- ▶ All three-pion channels covered
- ▶ Main tension resolved
(where lattice data are available)
- ▶ Next steps: see [Steve's talk yesterday](#)
- ▶ Pion-kaon case is underway on the amplitude side