Beautiful and Charming Baryon Workshop, IPPP, 2024

Status of and Prospects for Baryon Form Factors from Lattice QCD

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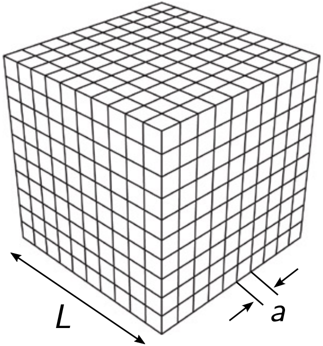
1. Beautiful and charming baryon form factors from lattice QCD: methods and challenges

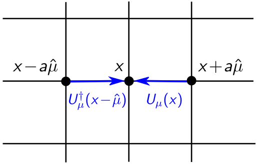
2. $\Xi_c \to \Xi \ell^+ \nu$

Lattice QCD

Lattice gauge theory allows us to nonperturbatively compute imaginary-time ($t = -i\tau$, $\tau \in \mathbb{R}$), a.k.a. Euclidean, QCD correlation functions in a finite volume:

$$\langle O_n(\mathbf{x}_n, \tau_n) \ldots O_1(\mathbf{x}_1, \tau_1) \rangle = \frac{1}{Z} \int \mathcal{D}[\psi, \overline{\psi}, U]\, O_n(\mathbf{x}_n, \tau_n) \ldots O_1(\mathbf{x}_1, \tau_1)\, e^{-S_E[\psi, \overline{\psi}, U]}$$





$$U_\mu(x) \sim \exp\left(iag_s A_\mu(x)\right)$$

The parameters are the bare quark masses and the bare gauge coupling. Taking the gauge coupling to 0 is equivalent to taking the lattice spacing to 0. One also needs to keep L large enough.

Lattice actions

Gluon actions: discretization errors start at order a^2 or higher for all of them; generally under good control.

Commonly used fermion actions suitable for **light quarks** ($ma \ll 1$):

- *Staggered (e.g. asqtad, HISQ):* has a chiral symmetry that prevents additive mass renormalization and yields automatic order-a improvement, but has the fermion doubling problem (e.g. multiple tastes of pions with different masses, requiring root of fermion determinant) and complicated symmetry properties. Poorly suited for baryons.
- *Wilson/clover:* no fermion doubling, but no chiral symmetry. Order-a improvement requires tuning.
- *Twisted-mass:* no fermion doubling, automatic order-a improvement, but breaks flavor and parity symmetry.
- *Overlap or domain-wall (DWF):* no fermion doubling, continuum-like symmetries, automatic order-a improvement, most expensive.

In the continuum limit, full QCD with the correct continuum symmetries is obtained with all of these actions.

Lattice actions

Common treatments of **heavy quarks on the lattice:**

- *Lattice HQET:* Discretization of continuum HQET with $\mathbf{v} = 0$. Continuum limit is possible when treating $1/m_Q$ corrections as insertions in correlation functions.
- *Lattice NRQCD:* Discretization of continuum NRQCD. Computations must be done with $am_Q > 1$.
- *Fermilab method/RHQ(Columbia/Tsukuba) action/Oktay-Kronfeld action:* Wilson-like action with one or more coefficients tuned to remove heavy-quark discretization errors. Can be used for any am_Q.
- *Use the same action as for the light quarks:* requires very fine lattices to keep $m_Q a < 1$ and typically an extrapolation in $1/m_Q$ to reach the physical bottom mass. Simplifies renormalization of currents.

Hadron interpolating fields

Examples:

$$O_B(\mathbf{x},\tau) \;=\; \bar{b}^a_\alpha(\mathbf{x},\tau)(\gamma_5)_{\alpha\beta}\, d^a_\beta(\mathbf{x},\tau)$$

$$O_{\Lambda_b}(\mathbf{x},\tau)_\gamma \;=\; \epsilon^{abc}\,(C\gamma_5)_{\alpha\beta}\; d^a_\alpha(\mathbf{x},\tau)\; u^b_\beta(\mathbf{x},\tau)\; b^c_\gamma(\mathbf{x},\tau) \;-\; (d \leftrightarrow u)$$

Often, "smearing" is used for the quark fields. Example: Gaussian smearing

$$q_{\text{smeared}} = \left(1 + \frac{\sigma^2}{4N}\nabla^2\right)^N q$$

Correlation functions from gauge configurations

The path integral over the quark fields is done symbolically,

$$\frac{1}{Z}\int \mathcal{D}[\psi,\overline{\psi},U]\; O_n ... O_1\; e^{-S_E[\psi,\overline{\psi},U]} = \frac{1}{Z}\int \mathcal{D}[U]\; f_n[U]\; \left(\prod_F \det D^F[U]\right)\, e^{-S_E^{\rm gluon}[U]}.$$

Any correlation functions can be computed as averages over previously generated random gauge-link configurations U with probability density

$$\rho[U] = \frac{1}{Z}\left(\prod_F \det D^F[U]\right)\, e^{-S_E^{\rm gluon}[U]},$$

where $D^F[U]$ is the lattice Dirac operator for quark flavor F for a given U. Typically, $F = u, d, s$ or $F = u, d, s, c$ are included here.

For example, the two-point function with $O_1 = O_2 = \bar{b}\gamma_5 d$ is given by

$$C_2(\mathbf{p},\tau) = \lim_{N_{\rm cfg}\to\infty} \frac{1}{N_{\rm cfg}} \sum_{n=1}^{N_{\rm cfg}} a^3 \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}_{\rm src})} {\rm Tr}\left[\gamma_5\, (D^d)^{-1}_{(\mathbf{x},\tau_{\rm src}+\tau),(\mathbf{x}_{\rm src},\tau_{\rm src})}[U_n]\; \gamma_5\, (D^b)^{-1}_{(\mathbf{x}_{\rm src},\tau_{\rm src}),(\mathbf{x},\tau_{\rm src}+\tau)}[U_n]\right].$$

Because we have $N_{\rm cfg} < \infty$ [typically $O(10^2) - O(10^3)$], there is a statistical uncertainty. This uncertainty can also be reduced by averaging over multiple $(\tau_{\rm src}, \mathbf{x}_{\rm src})$.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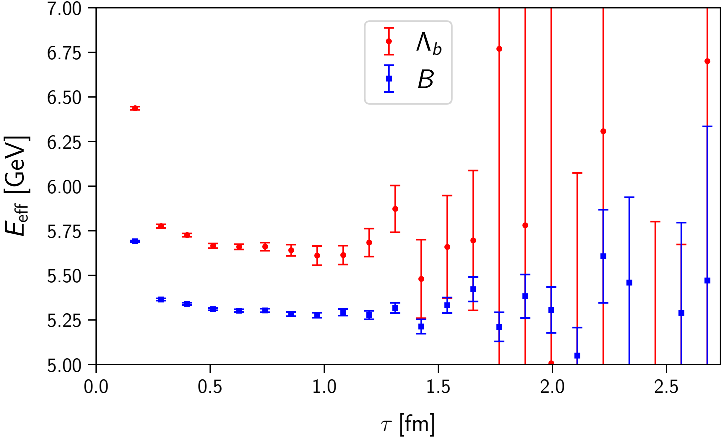
Meson and baryon two-point functions

Showing $E_{\text{eff}}(\tau) = a^{-1}\ln[C(\tau)/C(\tau+a)]$. From a $(5.5\text{ fm})^3 \times (11\text{ fm})$ lattice with spacing $a \approx 0.114$ fm and approximately physical quark masses:



Computing form factors in lattice QCD

Three-point function

$$C_3(\mathbf{p}', \mathbf{p}, \tau_{\text{snk}}, \tau_J, \tau_{\text{src}}) = \sum_{\mathbf{z}} \sum_{\mathbf{y}} e^{-i\mathbf{p}'\cdot(\mathbf{z}-\mathbf{y})} e^{-i\mathbf{p}\cdot(\mathbf{y}-\mathbf{x})} \langle O'(\mathbf{z}, \tau_{\text{snk}})\, J(\mathbf{y}, \tau_J)\, O^\dagger(\mathbf{x}, \tau_{\text{src}}) \rangle$$

- O: interpolating field for initial-state hadron
- O': interpolating field for final-state hadron
- J: the quark current from the weak effective Hamiltonian

Inserting complete sets of states shows that

$$\begin{aligned} C_3(\mathbf{p}', \mathbf{p}, \tau_{\text{snk}}, \tau_J, \tau_{\text{src}}) \quad = \quad & \sum_m \sum_n e^{-E_m(\tau_{\text{snk}}-\tau_J)} e^{-E_n(\tau_J-\tau_{\text{src}})} \\ & \times \langle \Omega | O'(0) | m(\mathbf{p}') \rangle \langle m(\mathbf{p}') | J(0) | n(\mathbf{p}) \rangle \langle n(\mathbf{p}) | O(0) | \Omega \rangle \end{aligned}$$

For large $\tau_{\text{snk}} - \tau_J$ and $\tau_J - \tau_{\text{src}}$, the lowest-energy states dominate (excited states decay exponentially faster).

Computing form factors in lattice QCD

The energies and unwanted overlap factors factors can be obtained from two-point functions:

$$\begin{aligned} C_2(\mathbf{p}, \tau_{\rm snk}, \tau_{\rm src}) &= \sum_{\mathbf{z}} e^{-i\mathbf{p}\cdot(\mathbf{z}-\mathbf{x})} \langle O(\mathbf{z}, \tau_{\rm snk})\, O^\dagger(\mathbf{x}, \tau_{\rm src})\rangle \\ &= \sum_n e^{-E_n(\tau_{\rm snk}-\tau_{\rm src})} |\langle n(\mathbf{p})|O^\dagger(0)|\Omega\rangle|^2 \end{aligned}$$

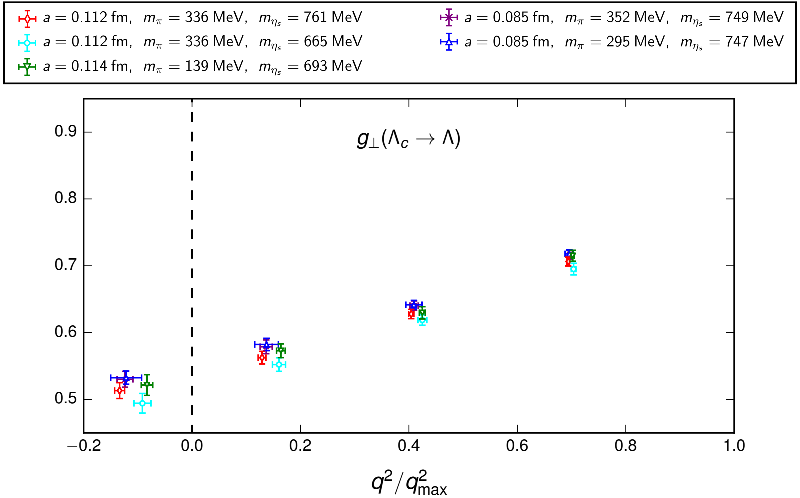
$$\begin{aligned} C_2'(\mathbf{p}', \tau_{\rm snk}, \tau_{\rm src}) &= \sum_{\mathbf{z}} e^{-i\mathbf{p}'\cdot(\mathbf{z}-\mathbf{x})} \langle O'(\mathbf{z}, \tau_{\rm snk})\, O'^\dagger(\mathbf{x}, \tau_{\rm src})\rangle \\ &= \sum_m e^{-E_m(\tau_{\rm snk}-\tau_{\rm src})} |\langle m(\mathbf{p}')|O'^\dagger(0)|\Omega\rangle|^2 \end{aligned}$$

One can construct appropriate ratios containing three-point and two-point functions such that for large $\tau_{\rm snk} - \tau_J$ and $\tau_J - \tau_{\rm src}$ one obtains the desired

$$\langle H'(\mathbf{p}')|J(0)|H(\mathbf{p})\rangle$$

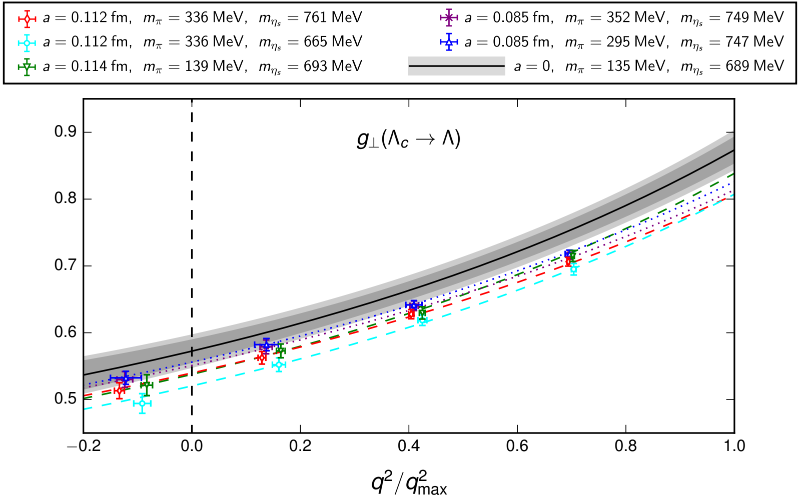
Computing form factors in lattice QCD

Here are example lattice results for the form factor $g_\perp(\Lambda_c \to \Lambda)$, computed for various lattice spacings and quark masses, at $\mathbf{p} = 0$ and various $\mathbf{p}' \neq 0$ (quantized due to periodic boundary conditions).



Computing form factors in lattice QCD

The final step is to fit the dependence on q^2, on the quark masses, and on the lattice spacing, and to estimate systematic uncertainties.



Baryon angular momentum on the lattice

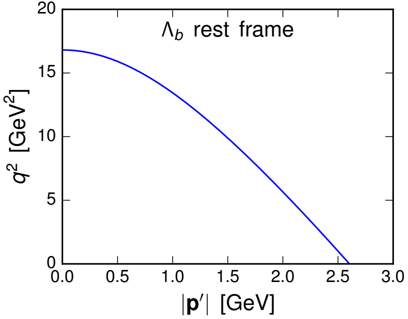
Momentum $[\frac{2\pi}{L}]$	Irrep	Dimension	J^P content
$(0,0,0)$	G_1^g	2	$1/2^+$, $7/2^+$, ...
	H^g	4	$3/2^+$, $5/2^+$, ...
	G_2^g	2	$5/2^+$, $7/2^+$, ...
	G_1^u	2	$1/2^-$, $7/2^-$, ...
	H^u	4	$3/2^-$, $5/2^-$, ...
	G_2^u	2	$5/2^-$, $7/2^-$, ...
$(0,0,1)$	G_1	2	$1/2^\pm$, $3/2^\pm$, ...
	G_2	2	$3/2^\pm$, $5/2^\pm$, ...
$(0,1,1)$	$(2)G$	2	$1/2^\pm$, $3/2^\pm$, ...
$(1,1,1)$	G	2	$1/2^\pm$, $3/2^\pm$, ...
	F_1	1	$3/2^\pm$, $5/2^\pm$, ...
	F_2	1	$3/2^\pm$, $5/2^\pm$, ...

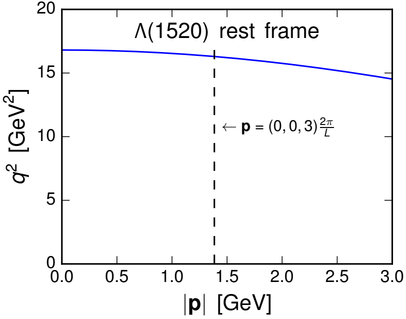
Not a problem for the ground-state $\frac{1}{2}^+$ baryons (always the lowest energy level).

For $\frac{1}{2}^-$ and $\frac{3}{2}^-$ baryons, it is easiest to use zero momentum only.

q^2 coverage for bottom-to-light decays

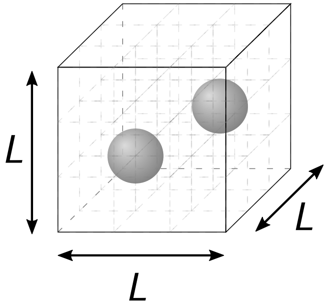
Example: $\Lambda_b \to \Lambda^*(1520)$ [in narrow-width approximation]:

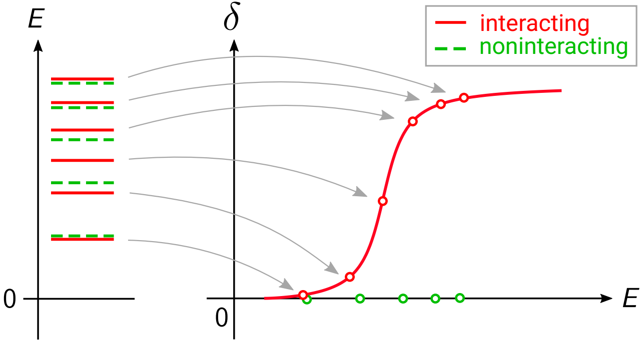




Multi-hadron states and resonances

Discrete spectrum of finite-volume energy eigenstates affected by hadron-hadron interactions





Energy shifts related to infinite-volume scattering phase shifts through Lüscher quantization condition, and matrix elements related through Lellouch-Lüscher relation. Figure shows simple single-channel case.

Overview: b and c baryon semileptonic form factors from lattice QCD

Early lattice studies of $\Lambda_b \to \Lambda_c$ (quenched, focused on Isgur-Wise function):

K. C. Boweler *et al.* (UKQCD Collaboration), hep-lat/9709028/PRD 1998

S. Gottlieb and S. Tamhankar, hep-lat/0301022/Lattice 2002

Our calculations, using RBC/UKQCD gauge-field configurations, $N_f = 2+1$ domain-wall, RHQ for m_Q phys.

Transition	m_Q	a [fm]	m_π [MeV]	Reference
$\Lambda_b \to \Lambda$	∞	0.08, 0.11	230–360	WD, DL, SM, MW, 1212.4827/PRD 2013
$\Lambda_b \to p$	∞	0.08, 0.11	230–360	WD, DL, SM, MW, 1306.0446/PRD 2013
$\Lambda_b \to p$	phys.	0.08, 0.11	230–360	WD, CL, SM, 1503.01421/PRD 2015
$\Lambda_b \to \Lambda_c$	phys.	0.08, 0.11	230–360	WD, CL, SM, 1503.01421/PRD 2015; AD, SK, SM, AR, 1702.02243/JHEP 2017
$\Lambda_b \to \Lambda$	phys.	0.08, 0.11	230–360	WD, SM, 1602.01399/PRD 2016
$\Lambda_b \to \Lambda^*(1520)$	phys.	0.08, 0.11	300–430	SM, GR, 2009.09313/PRD 2021; 2107.13140/PRD 2022
$\Lambda_b \to \Lambda_c^*(2595)$	phys.	0.08, 0.11	300–430	SM, GR, 2103.08775/PRD 2021; 2107.13140/PRD 2022
$\Lambda_b \to \Lambda_c^*(2625)$	phys.	0.08, 0.11	300–430	SM, GR, 2103.08775/PRD 2021; 2107.13140/PRD 2022
$\Lambda_c \to \Lambda$	phys.	0.08, 0.11	140–360	SM, 1611.09696/PRL 2017
$\Lambda_c \to n$	phys.	0.08, 0.11	230–360	SM, 1712.05783/PRD 2018
$\Lambda_c \to \Lambda^*(1520)$	phys.	0.08, 0.11	300–430	SM, GR, 2107.13140/PRD 2022; 2107.13084/PRD 2022

+work in progress: $\Xi_c \to \Xi$: CF, SM, 2309.08107; Next-gen $\Lambda_b \to p, \Lambda, \Lambda_c$: SM, 2309.01821

Recent lattice calculations by other collaborations:

$\Xi_c \to \Xi$: $N_f = 2+1$ clover, $a \approx 0.08, 0.11$ fm, $m_\pi \approx 290, 300$ MeV, Q.-A. Zhang, J. Hua, F. Huang, R. Li, Y. Li, C.-D. Lu, P. Sun, W. Sun, W. Wang, Y.-B. Yang, 2103.07064/CPC 2022

$\Lambda_c \to \Lambda$: $N_f = 2$ clover, $a \approx 0.16$ fm, $m_\pi \approx 550$ MeV, H. Bahtiyar, 2107.13909/Turk.J.Phys. 2021

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Recent experimental progress

$$\frac{\mathcal{B}(\Xi_c^0 \to \Xi^- e^+ \nu_e)}{\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+)} = \begin{cases} (0.730 \pm 0.021 \pm 0.033)\,\%, & \text{Belle, 2103.06496/PRL 2021} \\ (1.38 \pm 0.14 \pm 0.22)\,\%, & \text{ALICE, 2105.05187/PRL 2021} \end{cases}$$

$$\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+) = (1.80 \pm 0.50 \pm 0.14)\,\%, \quad \text{Belle, 1811.09738/PRL 2019}$$

$$\Rightarrow \quad \mathcal{B}(\Xi_c^0 \to \Xi^- e^+ \nu_e) = \begin{cases} (1.31 \pm 0.04 \pm 0.07 \pm 0.38)\,\%, & \text{Belle} \\ (2.48 \pm 0.25 \pm 0.40 \pm 0.72)\,\%, & \text{ALICE \& Belle} \end{cases}$$

$$\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+) = (1.43 \pm 0.27)\,\%, \quad \text{PDG 2024}$$

$$\mathcal{B}(\Xi_c^0 \to \Xi^- e^+ \nu_e) = (1.05 \pm 0.20)\,\%, \quad \text{PDG 2024}$$

Flavor SU(3) symmetry predicts a much higher $\mathcal{B}(\Xi_c^0 \to \Xi^- e^+ \nu_e) \approx \frac{3}{2} \frac{\tau_{\Xi_c^0}}{\tau_{\Lambda_c}} \mathcal{B}(\Lambda_c \to \Lambda e^+ \nu_e) \approx 4\,\%$ using $\mathcal{B}(\Lambda_c \to \Lambda e^+ \nu_e)$ from BESIII [2207.14149/PRL 2022]

Standard-Model predictions

Reference	Method	$\mathcal{B}(\Xi_c^0 \to \Xi^- e^+ \nu_e)$ [%]
Zhang *et al.*, 2103.07064/CPC 2022	Lattice QCD	$2.38 \pm 0.30 \pm 0.33$
Zhao *et al.*, 2103.09436/PRD 2023	QCD sum rules	1.83 ± 0.45
He *et al.*, 2110.04179/PLB 2021	Flavor SU(3)	4.10 ± 0.46
Geng *et al.*, 2012.04147/PRD 2021	Light-front quark model	3.49 ± 0.95
Faustov and Galkin, 1905.08652/EPJC 2019	Relativistic quark model*	2.38
Geng *et al.*, 1901.05610/PLB 2019	Flavor SU(3)*	3.0 ± 0.3
Zhao, 1803.02292/CPC 2018	Light-front quark model*	1.35
Geng *et al.*, 1801.03276/PRD 2018	Flavor SU(3)*	4.87 ± 1.74
Geng *et al.*, 1709.00808/JHEP 2017	Flavor SU(3)*	3.0 ± 0.5
Liu and Huang, 1102.4245/JPG 2010	QCD sum rules*	2.4

*using pre-LHCb $\tau_{\Xi_c^0} = (112 \pm 13)$ fs

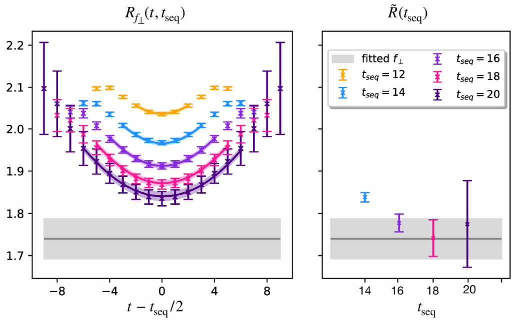
PDG 2024 including LHCb: $\tau_{\Xi_c^0} = (150.4 \pm 2.8)$ fs

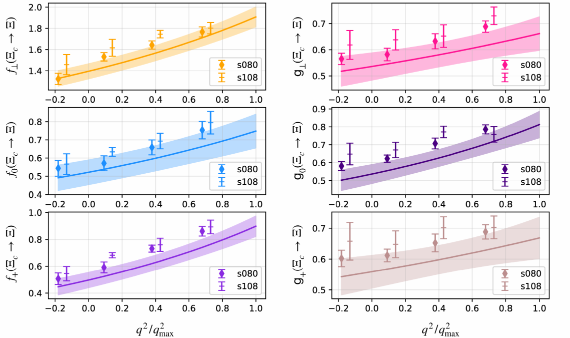
Zhang *et al.* lattice calculation [2103.07064/CPC 2022]

TABLE I. Parameters of the 2+1 flavor clover fermion ensembles used in this calculation. The π/η_s masses and the lattice spacings are given in units of MeV, and fm, respectively.

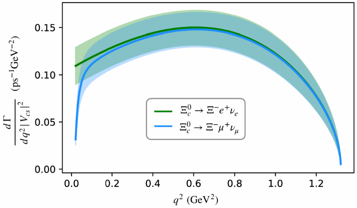
	$\beta = \frac{10}{g^2}$	$L^3 \times T$	a	$c_{\rm sw}$	κ_l	m_π	κ_s	m_{η_s}
s108	6.20	$24^3 \times 72$	0.108	1.161	0.1343	290	0.1330	640
s080	6.41	$32^3 \times 96$	0.080	1.141	0.1326	300	0.1318	650

Valence charm also implemented using clover, tuned using J/ψ rest mass





↑ Curves incorrectly plotted vs q^2 in GeV^2 while data plotted vs $q^2/q^2_{\rm max}$



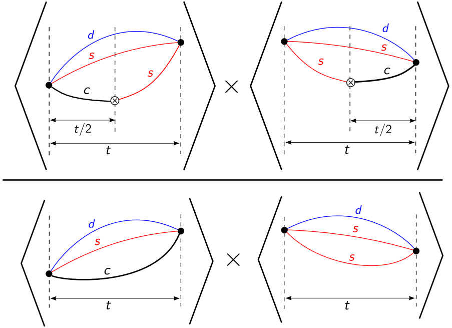
Our lattice calculation [Callum Farrell and SM, in preparation]

(2+1)-flavor domain-wall RBC/UKQCD ensembles

Label	$N_s^3 \times N_t \times N_5$	a [fm]	$am_{u,d}$	$am_s^{(\mathrm{sea})}$	$am_s^{(\mathrm{val})}$	m_π [MeV]
C01	$24^3 \times 64 \times 16$	≈ 0.111	0.01	0.04	0.0323	≈ 420
C005	$24^3 \times 64 \times 16$	≈ 0.111	0.005	0.04	0.0323	≈ 340
F004	$32^3 \times 64 \times 16$	≈ 0.083	0.004	0.03	0.0248	≈ 300
F1M	$48^3 \times 96 \times 12$	≈ 0.073	0.002144	0.02144	0.02217	≈ 230

Valence charm using three-parameter heavy-quark action (tuned using D_s dispersion relation and hyperfine splitting [SM, 2309.01821])

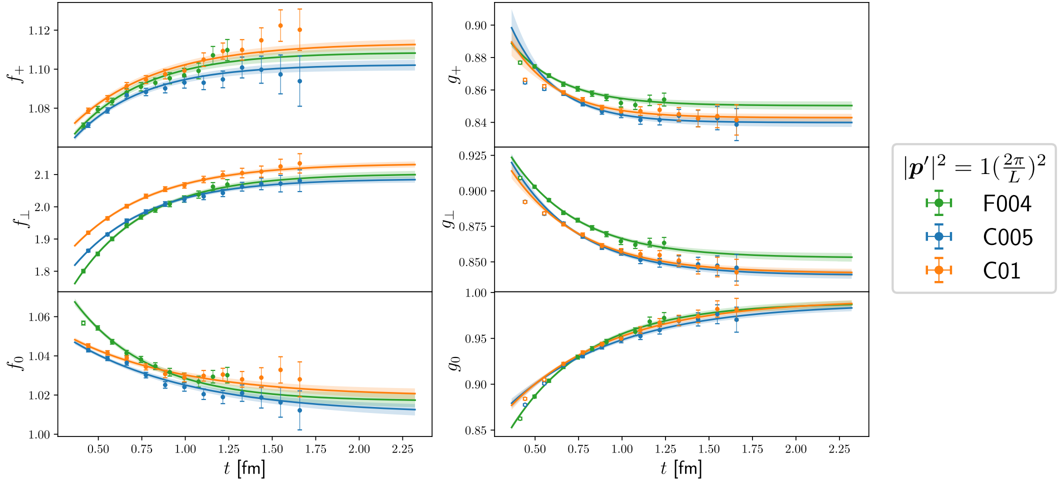
$$R_f(t) = \sqrt{(\text{kinematic factors}) \times \frac{\langle \cdots \rangle \times \langle \cdots \rangle}{\langle \cdots \rangle \times \langle \cdots \rangle}}$$



* Here we denote Euclidean time by t

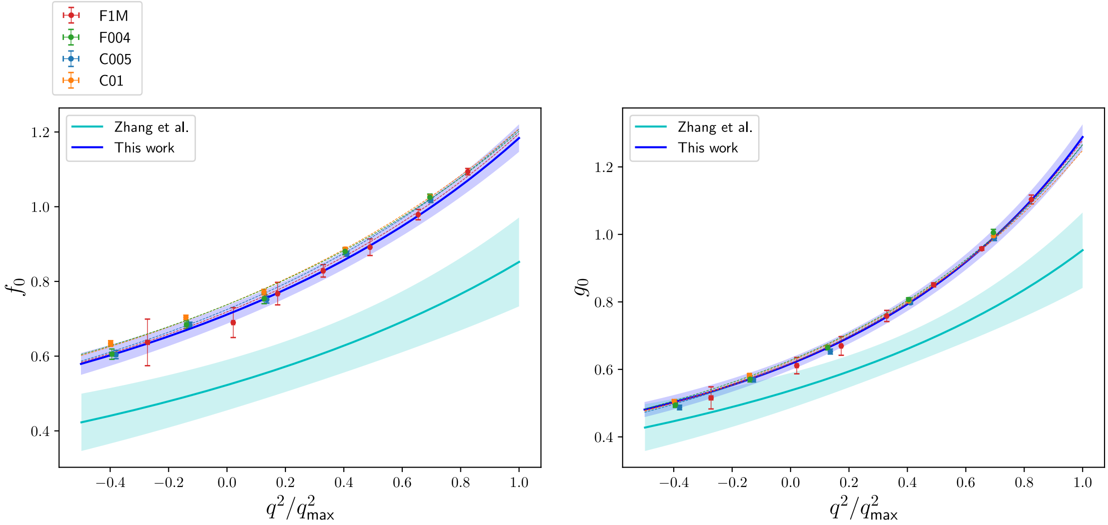
preliminary

$R_f(t) \to f$ for large t



We are now using a generalized version of the Akaike information criterion to average over fits with different $t_{\min}$ [W. Jay and E. Neil, 2008.01069/PRD 2021]

preliminary



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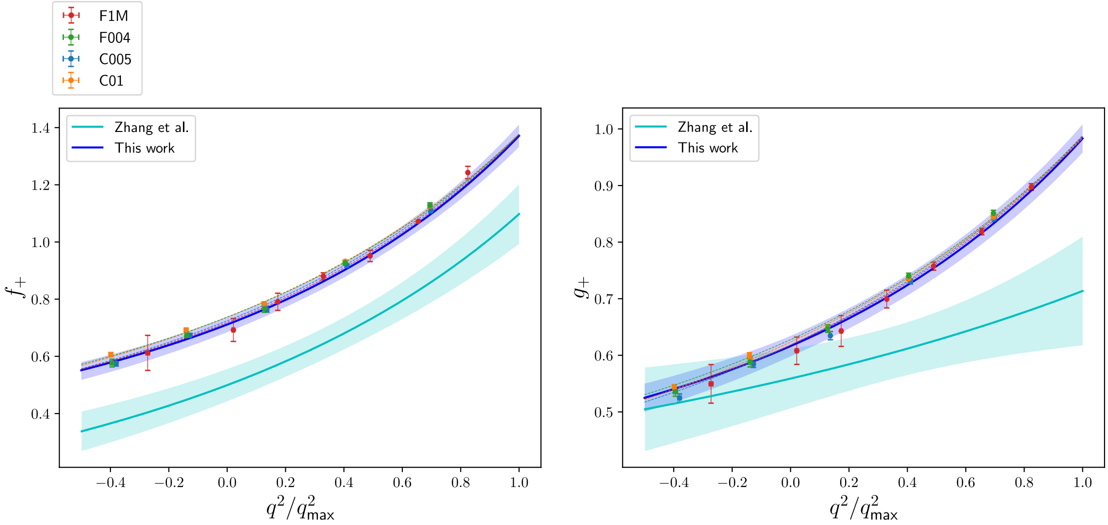

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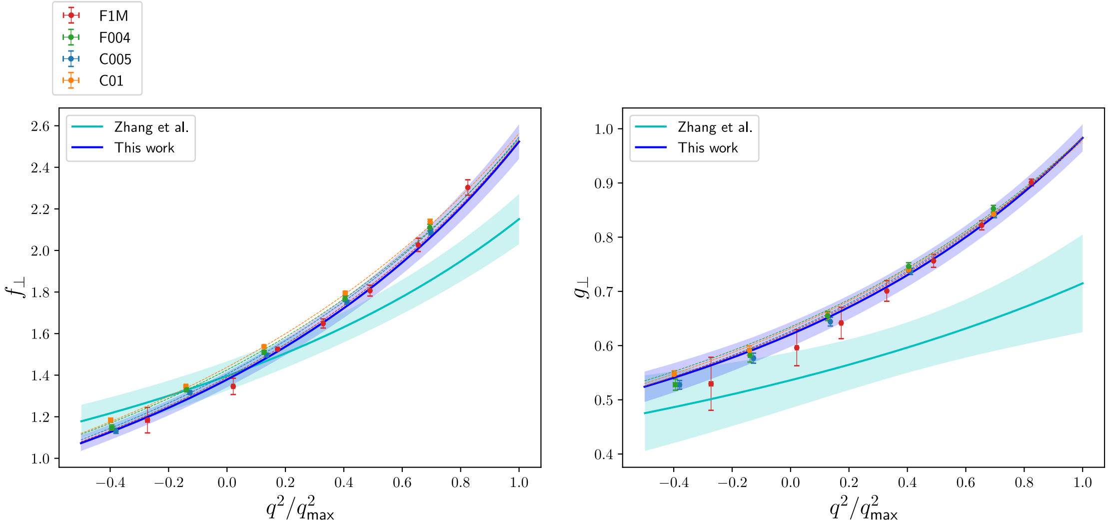

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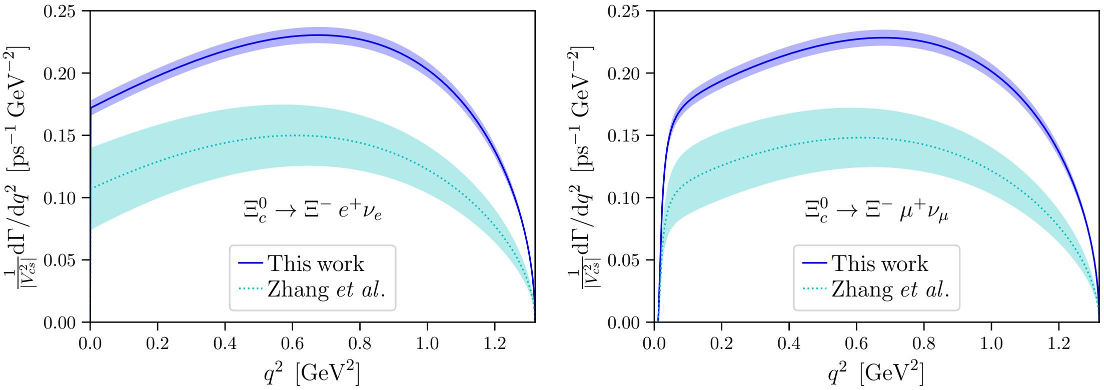


We predict $\mathcal{B}(\Xi_c^0 \to \Xi^- e^+\nu_e) \approx 3.7\%$, which is about 3.5 times higher than PDG and close to the expectation from $SU(3)$ flavor symmetry.

Lattice prospects for other processes

In progress [Callum Farrell and SM]:

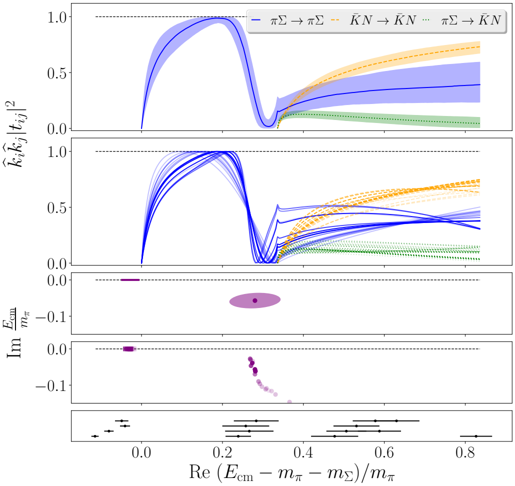
- $\Xi_b \to \Xi \, \ell^+ \ell^-$

How interesting are the following?

- $\Xi_b \to \Xi_c \, \ell^- \bar{\nu}_\ell$
- $\Omega_b \to \Omega_c \, \ell^- \bar{\nu}_\ell$
- $\Omega_c \to \Omega \, \ell^+ \nu_\ell$
- $\Omega_b \to \Omega \, \ell^+ \ell^-$

The following could in principle be studied for the lowest partial wave(s) and low enough $m_{pK}, m_{\Sigma\pi}$, but it would be very challenging and very expensive. Needs coupled-channel fits.

- $\Lambda_c \to \{p\,K,\, \Sigma\,\pi\}\ell^+ \nu_\ell$
- $\Lambda_b \to \{p\,K,\, \Sigma\,\pi\}\ell^+ \ell^-$



[J. Bulava *et al.*, 2307.10413/PRL 2024] ($m_\pi \approx 200$ MeV)