Baryon Form Factors and Dispersive Bounds

Beautiful and Charming Baryon Workshop –
Durham – 09/09/2024

Méril Reboud

Mostly based on:

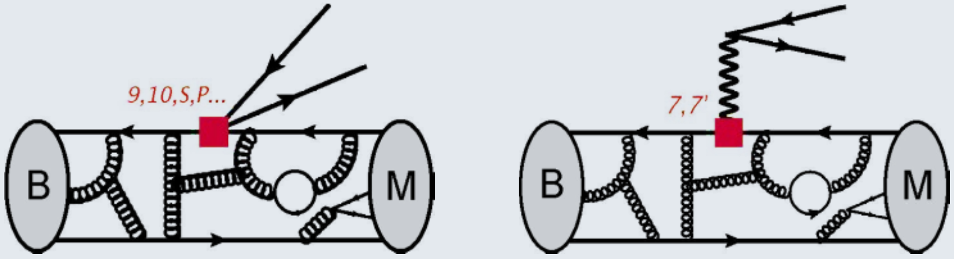
- Gubernari, MR, van Dyk, Virto 2206.03797, 2305.06301
- Amhis, Bordone, MR 2208.08937

Form factors in b → sℓℓ

$$\mathcal{H}(b \to s\ell\ell) = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)$$

$$\mathcal{O}_{9(10)} = \frac{e^2}{16\pi^2}\left(\bar{s}_L\gamma_\mu b_L\right)\left(\bar{\ell}\gamma^\mu(\gamma_5)\ell\right)$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2}\left(\bar{s}_L\sigma_{\mu\nu}b_R\right)F^{\mu\nu}$$



$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10})\mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2}\left[C_7 \mathcal{F}_\lambda^T(q^2) \right] \right\}$$

- $B \to K^{(*)}\, \mu\mu$
- $B_s \to \varphi\, \mu\mu$
- $\Lambda_b \to \Lambda^{(*)}\, \mu\mu$

Local form-factors,
involves e.g.

$$\mathcal{F}_\mu(k,q) = \langle \bar{M}(k)|\bar{s}\gamma_\mu b_L|\bar{B}(q+k)\rangle$$

Form factors in b → sℓℓ

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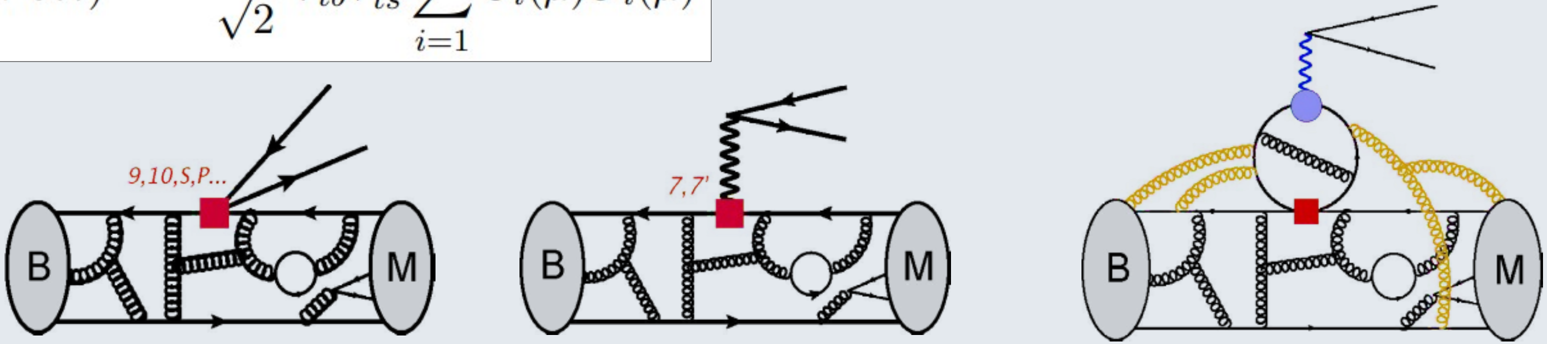

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$$\mathcal{H}_\mu(k,q) = i\int d^4x\, e^{iq\cdot x}\langle \bar{M}(k)|T\{\mathcal{J}_\mu^{\mathrm{em}}(x), \mathcal{C}_i\mathcal{O}_i\}|\bar{B}(q+k)\rangle$$

Non-local form-factors

→ Main contributions: the **“charm-loops”** $\mathcal{O}_{2(1)}^c = (\bar{s}_L\gamma_\mu(T^a)c_L)(\bar{c}_L\gamma^\mu(T^a)b_L)$

Form factors in b → sℓℓ

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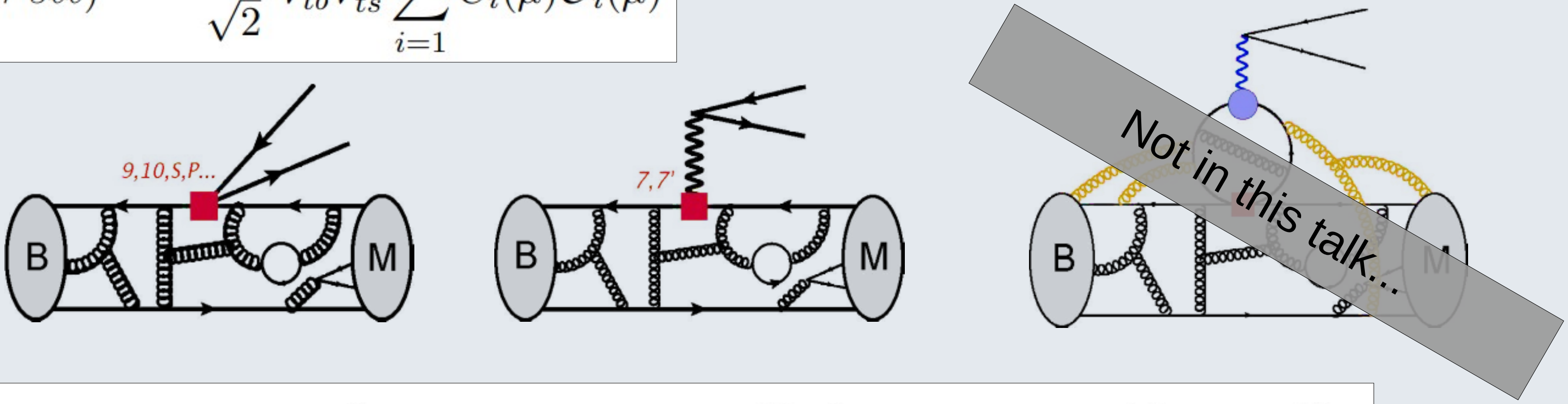

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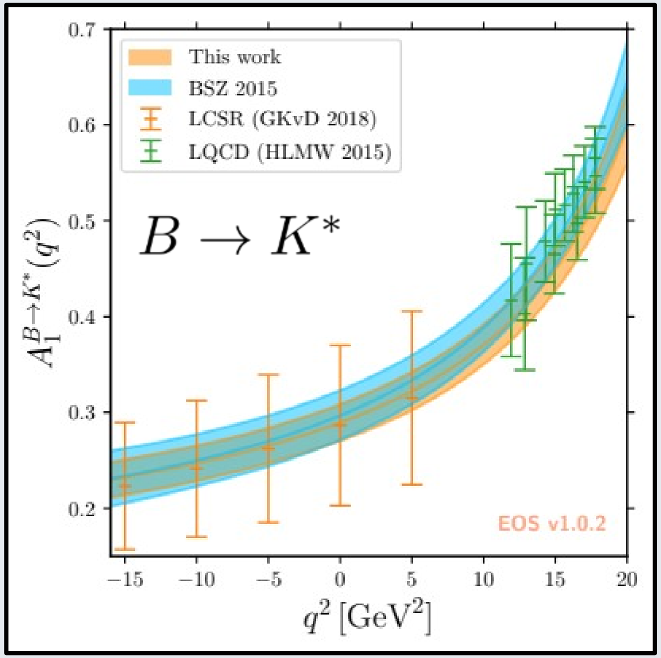
Local form factors

- **2 main approaches**
  - Lattice QCD → most feasible at **large q^2**
  - Light-cone sum rules → most feasible at **small q^2**

- 2 possible LCSRs:
  - Light meson LCDA [recent works: Bharrucha, Straub, Zwicky '15; Khodjamirian, Rusov '17]
  - Heavy meson LCDA [recent works: Khodjamirian, Mannel, Pivovarov, Wang '10; Gubernari, Kokulu, van Dyk '18, recent review Khodjamirian, Melic, Wang, '23]

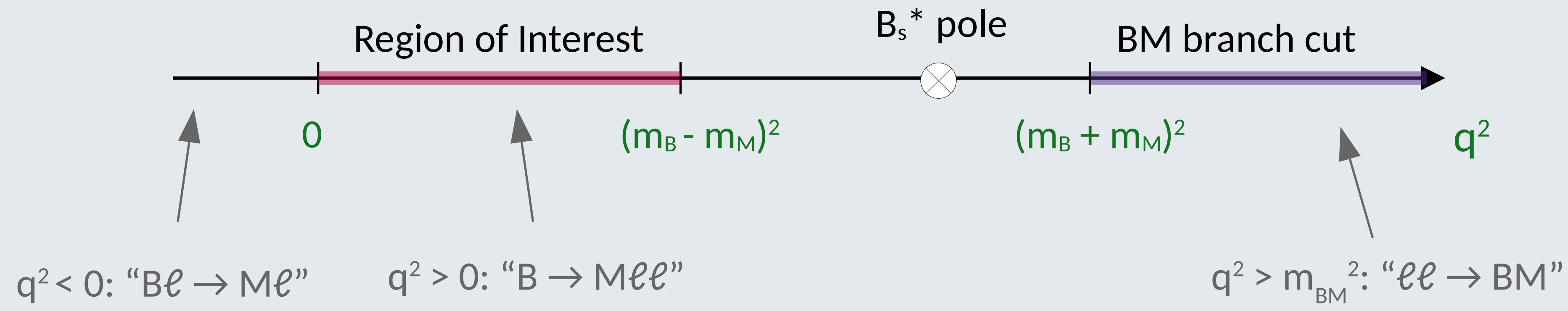


→ **Interpolation** in the physical range

→ **Problem #1**: we don't know much about baryon LCDAs [Wang, Shen, *et al* '09, Wang, Shen, '15]

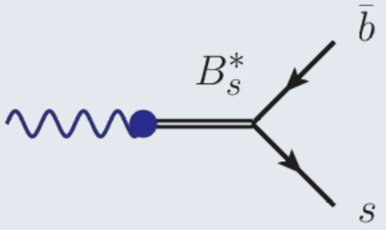
Form Factor Properties

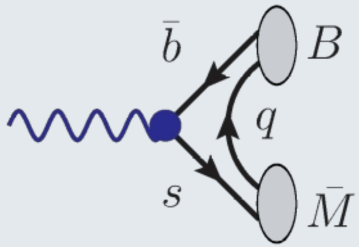
$$\mathcal{F}_\mu(k,q) = \langle \bar{M}(k) | \bar{s}\gamma_\mu b_L | \bar{B}(q+k) \rangle$$



Analytic properties of the form factors:

- Pole due to **$\bar{b}s$ bound state**
- **Branch cut** due to on-shell BM production





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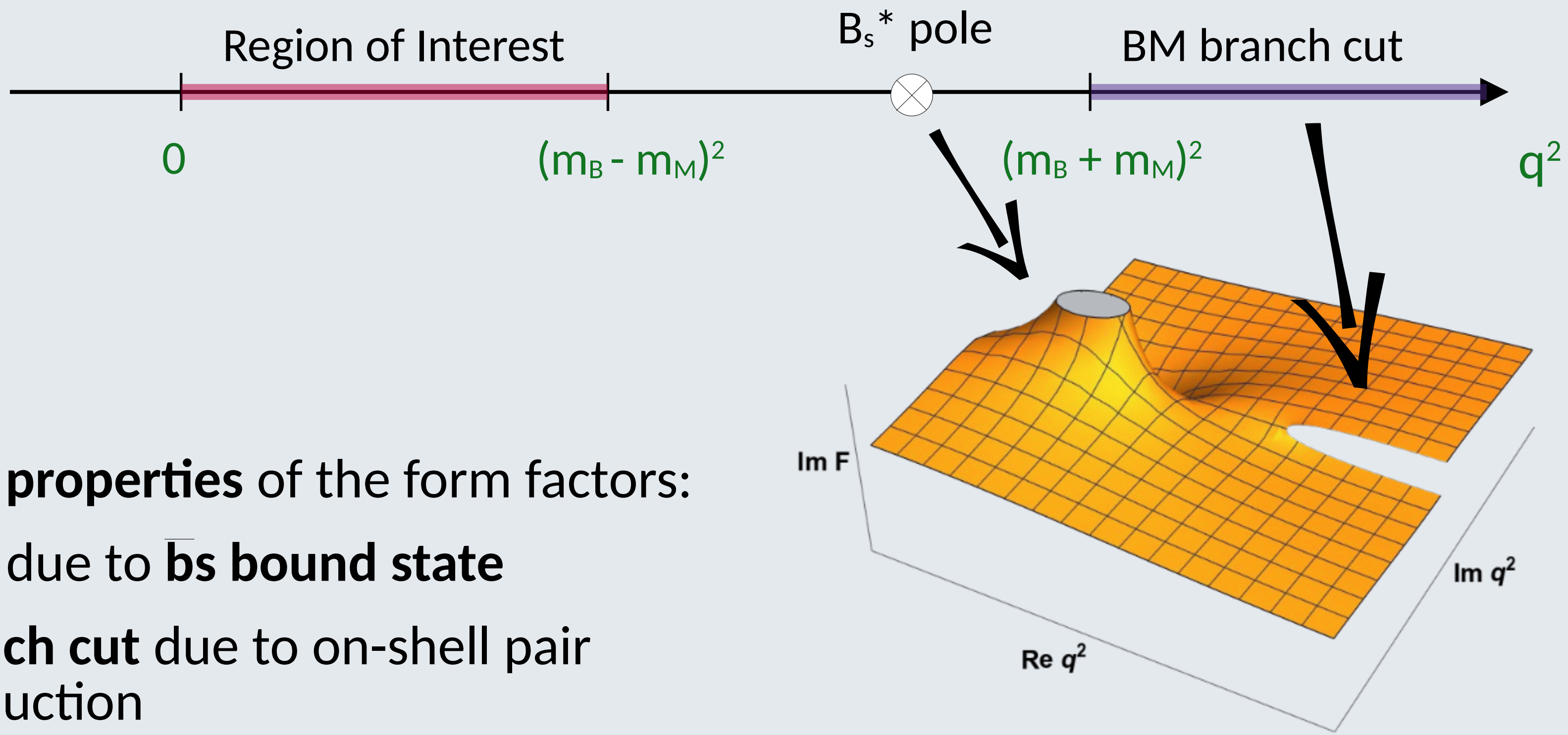

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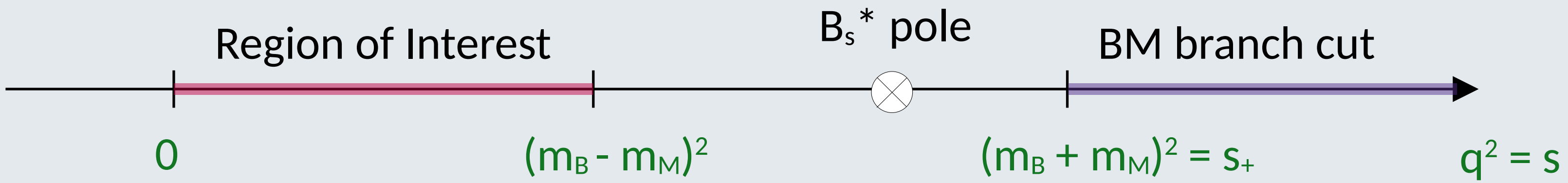
Form Factor Parametrization



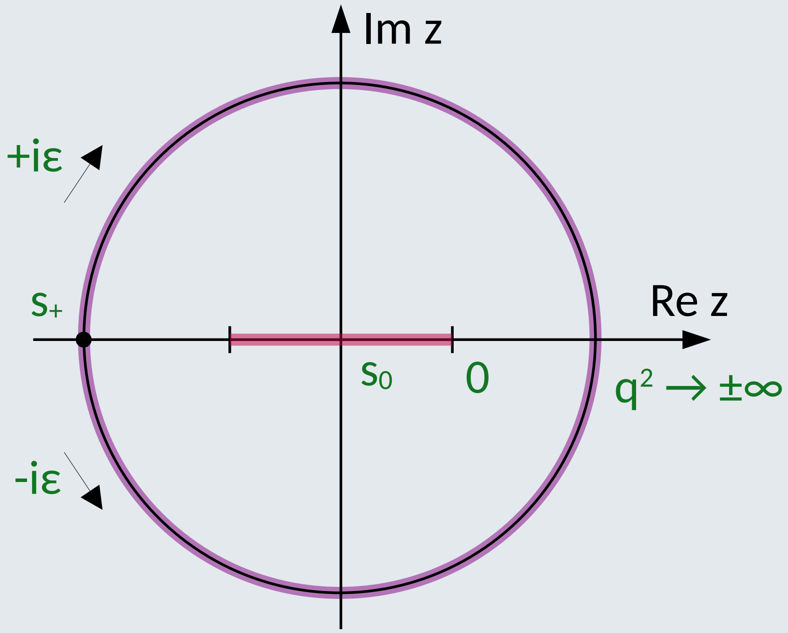
Conformal mapping [Boyd, Grinstein, Lebed '97]

$$z(s) \equiv \frac{\sqrt{s_+ - s} - \sqrt{s_+ - s_0}}{\sqrt{s_+ - s} + \sqrt{s_+ - s_0}}$$

Simplified Series expansion [Bourrely, Caprini, Lellouch, '08; Bharucha, Feldmann, Wick '10]

$$\mathcal{F}_\lambda^{(T)}(q^2) = \frac{1}{q^2 - m_{B_s^*}^2} \sum_{k=0}^{N} \alpha_{\lambda,k} z^k$$

N = 2 is usually enough to provide an **excellent description of the data** (p-values > 70%), but what about the *truncation error*?



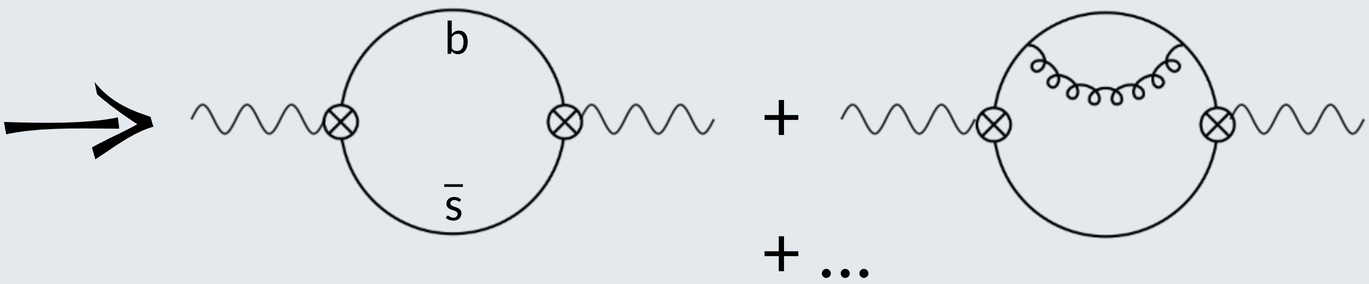
II. Dispersive bound

- **Main idea:** Compute the inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick '10]

$$\Pi^{\mu\nu}_{\Gamma}(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0|\mathcal{T}\left\{J^{\mu}_{\Gamma}(x) J^{\dagger,\nu}_{\Gamma}(0)\right\}|0\rangle$$

1) Partonic calculation

Insertion of a scalar, vector or tensor current



Dispersive bounds

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$$\Pi_\Gamma^{\mu\nu}(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0|\mathcal{T}\left\{ J_\Gamma^\mu(x) J_\Gamma^{\dagger,\nu}(0)\right\}|0\rangle$$

2) Relation to form factors

$$\mathrm{Im}\,\Pi_I^X(q^2) = \frac{1}{2}\sum_\Gamma \int d\rho_\Gamma\,(2\pi)^4\,\delta^4(q-p_\Gamma)\,P_I^{\mu\nu}\,\langle 0|\,j_\mu^X\,|\Gamma\rangle\langle\Gamma|\,j_\nu^{\dagger X}\,|0\rangle$$

Sum over all the $\bar{s}b$ states: $\bar{B}_s$, $\bar{B}K$, $\bar{B}K^*$, $\bar{B}K\pi$, baryons...

~ |form factor|^2

Dispersive bounds

- **Main idea:** Compute the inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick '10]

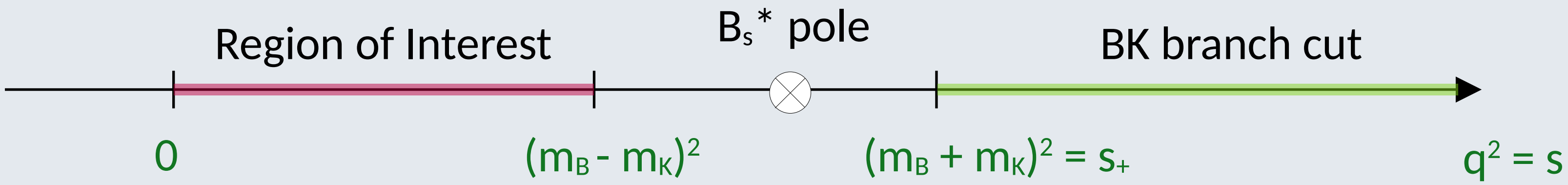
- Assuming global quark-hadron duality we have

$$\underbrace{\chi_\Gamma^{(\lambda)}\big|_{\mathrm{OPE}} = \chi_\Gamma^{(\lambda)}\big|_{\mathrm{1pt}}}_{\text{Known terms}} + \underbrace{\chi_\Gamma^{(\lambda)}\big|_{\bar{B}K} + \chi_\Gamma^{(\lambda)}\big|_{\bar{B}K^*} + \chi_\Gamma^{(\lambda)}\big|_{\bar{B}_s\phi} + \dots}_{\text{Sum of positive quantities}}\,.$$

Further contributions such as B → Kππ or $\Lambda_b \to \Lambda^{(*)}$.

Any new terms *strengthens* the bound.

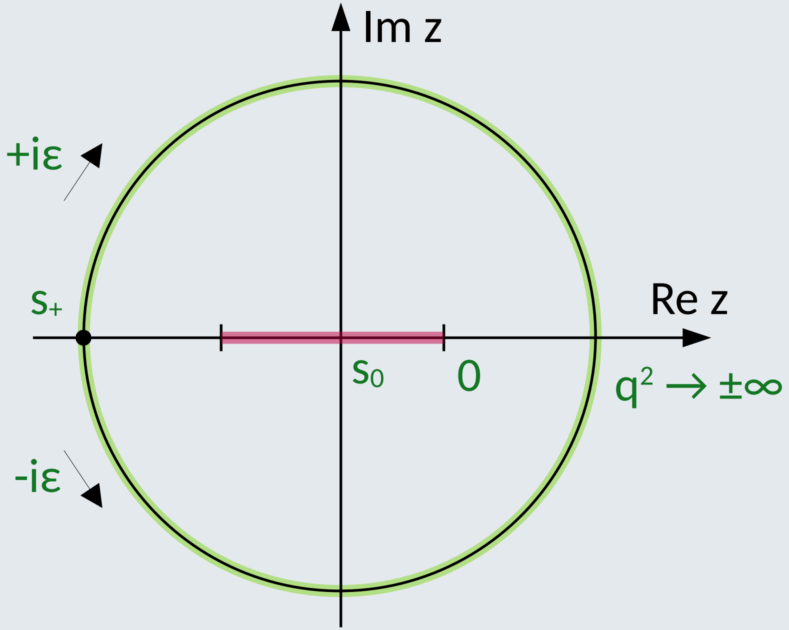
Simple case: B → K



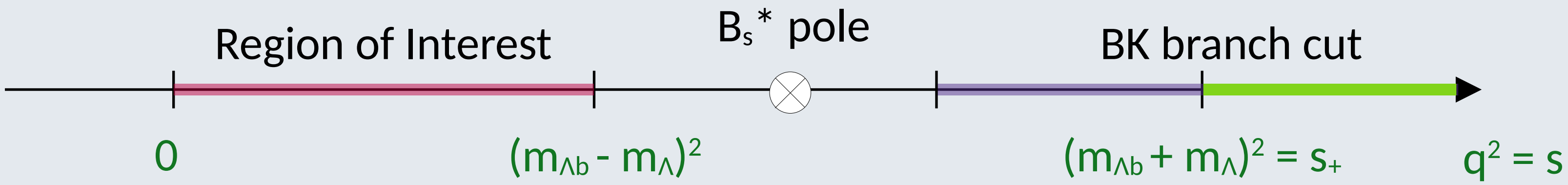
- The branch cut starts **at** the pair production threshold (neglecting $B_s\pi$)
- The monomial z^k are **orthogonal** on the unit circle

$$\mathcal{F}^{B\to K} = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{N} \alpha_k z^k$$

$$\chi_{\Gamma}^{(\lambda)}|_{\bar{B}K} = \sum_{k=0}^{N} |\alpha_k|^2$$



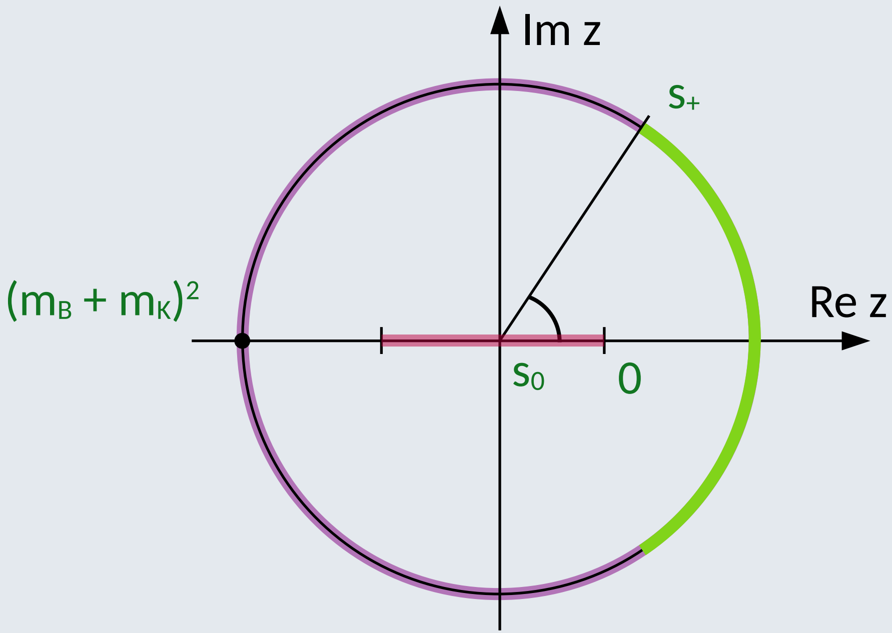
Less simple case, e.g. $\Lambda_b \to \Lambda$



- The first branch cut (BK) starts **before** the pair production threshold
- Introduce orthonormal polynomials of the **arc of the unit circle**

$$\mathcal{F}^{\Lambda_b \to \Lambda} = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{N} \alpha_k p_k(z)$$

- (Or still expand in z and deal with a more complicated bound [Flynn, Jüttner, Tsang '23])



IV. Numerical results for $\Lambda_b \to \Lambda(1520)$

Example with $\Lambda_b \to \Lambda(1520)\ell\ell$

- Inputs:
  - **LQCD** estimates at q^2 = 16.3 and 16.5 GeV^2 [Meinel, Rendon '21]
  - no LCSR available
    → use (loose) **SCET relations** [Descotes-Genon, M. Novoa-Brunet '19]

$$
\begin{aligned}
f_{\perp'}(0) &= 0 \pm 0.2\,, & g_{\perp'}(0) &= 0 \pm 0.2\,, & h_{\perp'}(0) &= 0 \pm 0.2\,,\\
\tilde{h}_{\perp'}(0) &= 0 \pm 0.2\,, & f_+(0)/f_\perp(0) &= 1 \pm 0.2\,, & f_\perp(0)/g_0(0) &= 1 \pm 0.2\,,\\
g_\perp(0)/g_+(0) &= 1 \pm 0.2\,, & h_+(0)/h_\perp(0) &= 1 \pm 0.2\,, & f_+(0)/h_+(0) &= 1 \pm 0.2\,,
\end{aligned}
$$

$O(\alpha_s/\pi, \Lambda_{QCD}/m_b)$

- 14 form factors: **17 parameters (N = 1), 31 parameters (N = 2)**

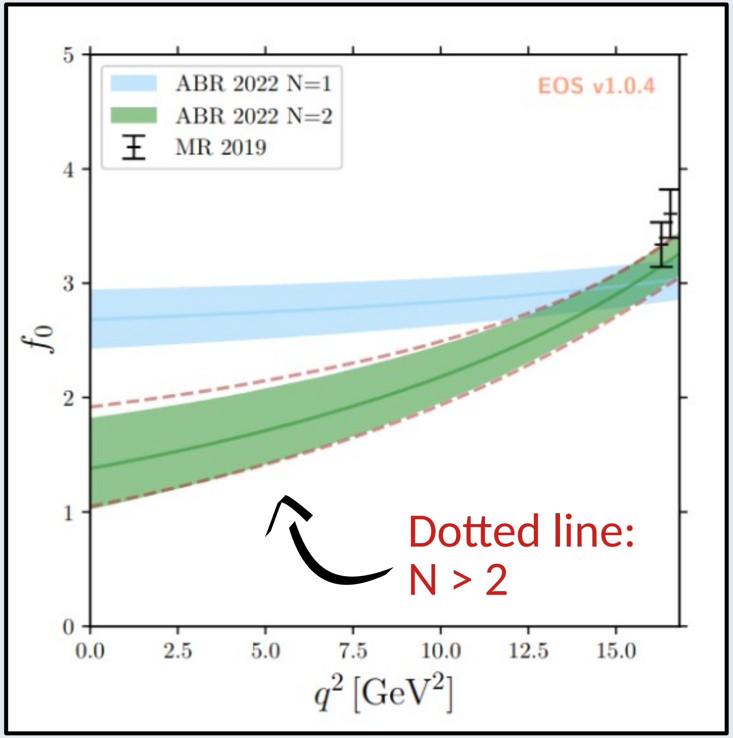
  21 LQCD inputs + 9 SCET relations: **30 constraints**

2 * 14 – 7 endpoint relations at q^2_{max}

Example with $\Lambda_b \to \Lambda(1520)\ell\ell$

- N = 1 does not give a good fit (p value ~ 0)
- Use an **under-constrained fit** (N>1) and allows for saturation of the dispersive bound

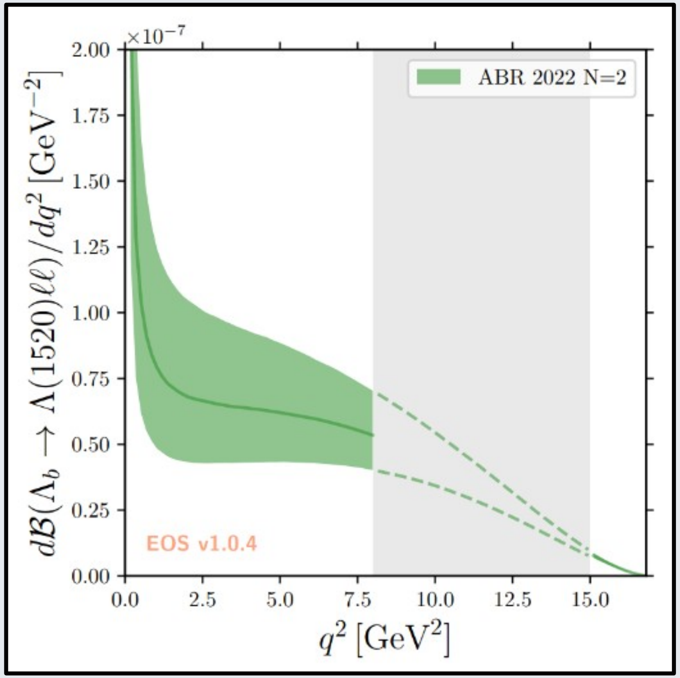
  → The uncertainties are truncation order independent: increasing the order does not change their size
- Same conclusions were found for $\Lambda_b \to \Lambda$ form factors [Blake, Meinel, Rahimi, van Dyk '22]

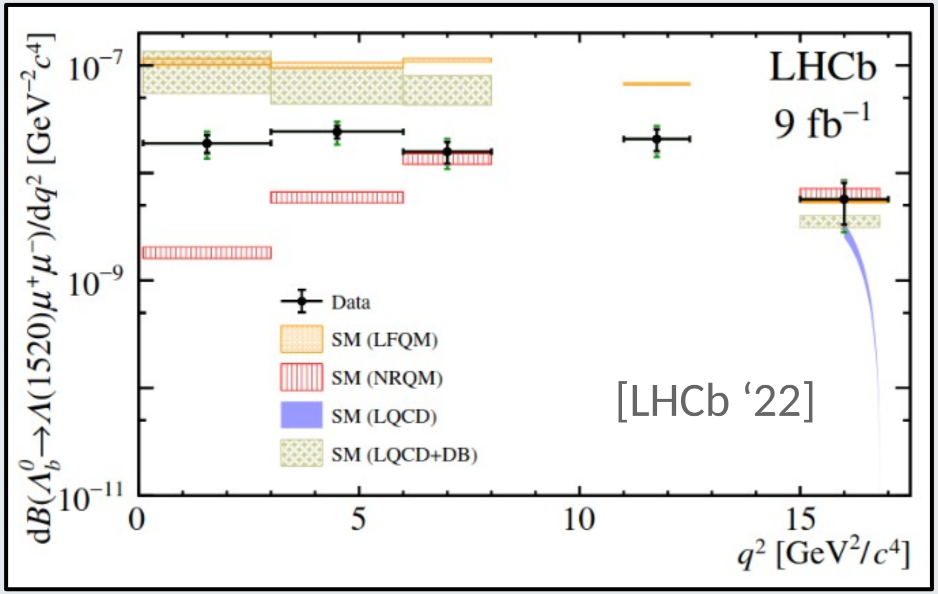


[Ahmis, MR, Bordone '22]

Phenomenology

- Uncertainties are large but **under control** and **systematically improvable**
- LHCb analysis confirmed the usual $b \to s\ell\ell$ tension at low q^2





[LHCb '22]

IV. Combined mesonic analysis

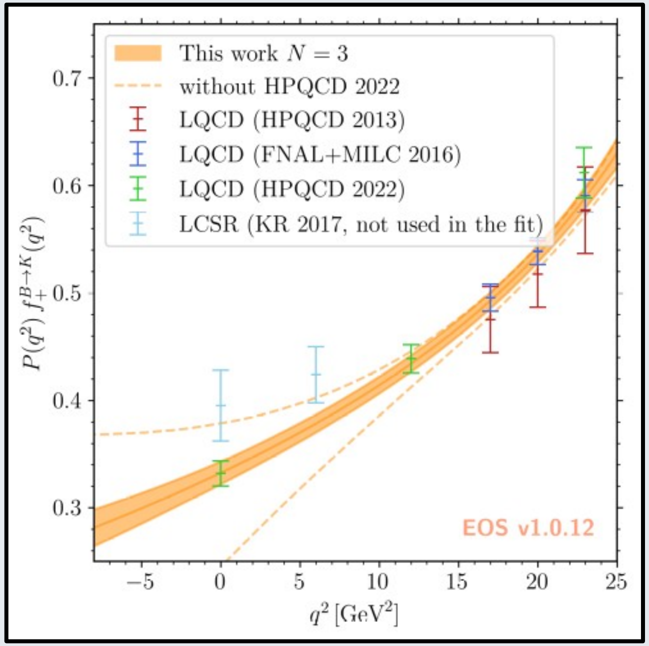
Local form factors fit

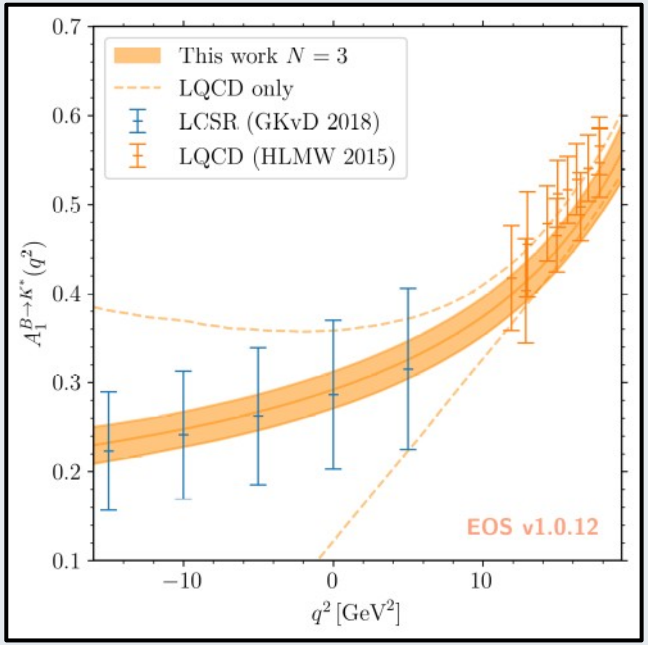
- With this framework we perform a **combined fit** of $B \to K$, $B \to K^*$ and $B_s \to \varphi$ LCSR and lattice QCD inputs:
  - $B \to K$:
    - [HPQCD ’13 and ’22; FNAL/MILC ’17]
    - ([Khodjamiriam, Rusov ’17]) → large uncertainties, not used in the fit
  - $B \to K^*$:
    - [Horgan, Liu, Meinel, Wingate ’15]
    - [Gubernari, Kokulu, van Dyk ’18] (B-meson LCSRs)
  - $B_s \to \varphi$:
    - [Horgan, Liu, Meinel, Wingate ’15]
    - [Gubernari, van Dyk, Virto ’20] (B-meson LCSRs)
- Adding $\Lambda_b \to \Lambda^{(*)}$ form factors is possible and desirable

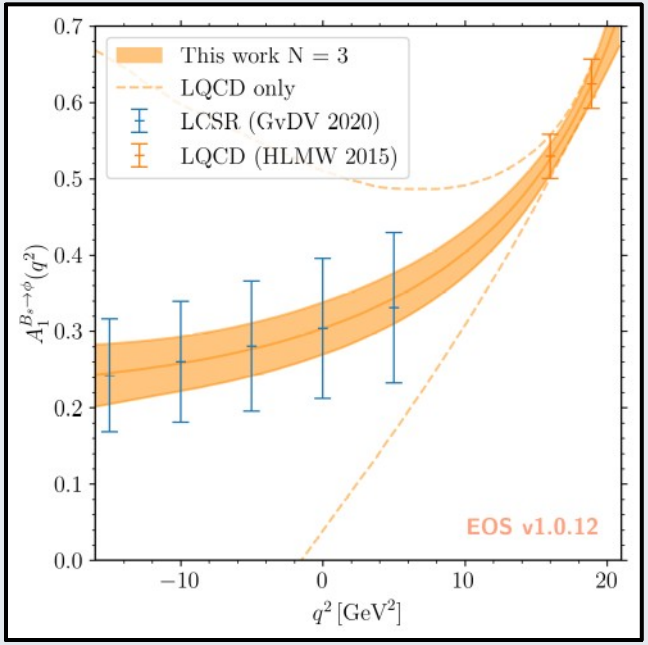
Results for mesonic form-factors

Main conclusions:

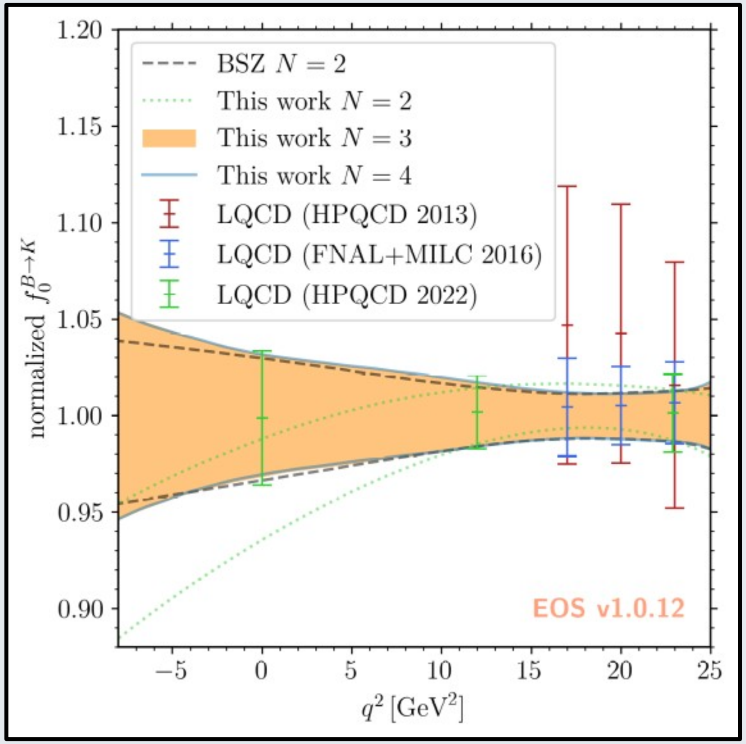
- Fits are very good already at N = 2 (p-values > 77%)
- LCSR and LQCD combine nicely and still dominate the uncertainties
- Progresses in LQCD will gradually replace LCSR

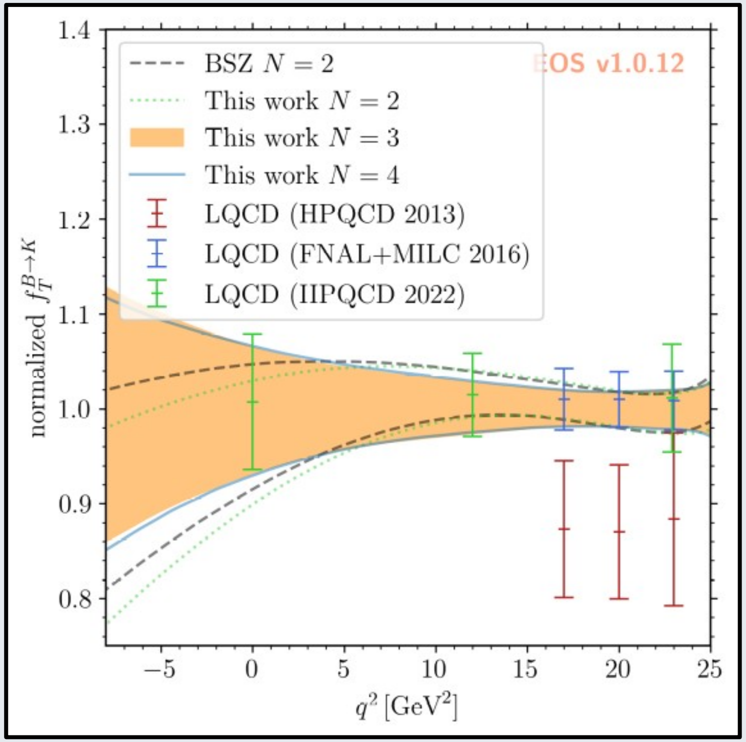






Comparison plots for B → K



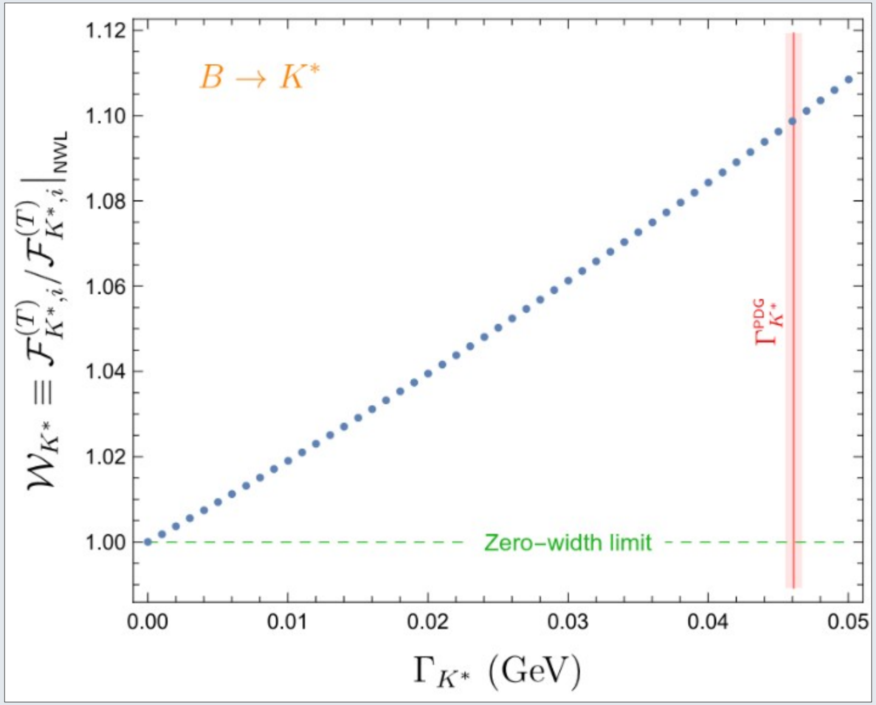


- Normalizing the form factors to the N = 3 best fit point allows for a model comparison
- All the plots are available here: https://doi.org/10.5281/zenodo.7919635

V. Beyond narrow-width approximation

Caveat: finite width effects in B → K*

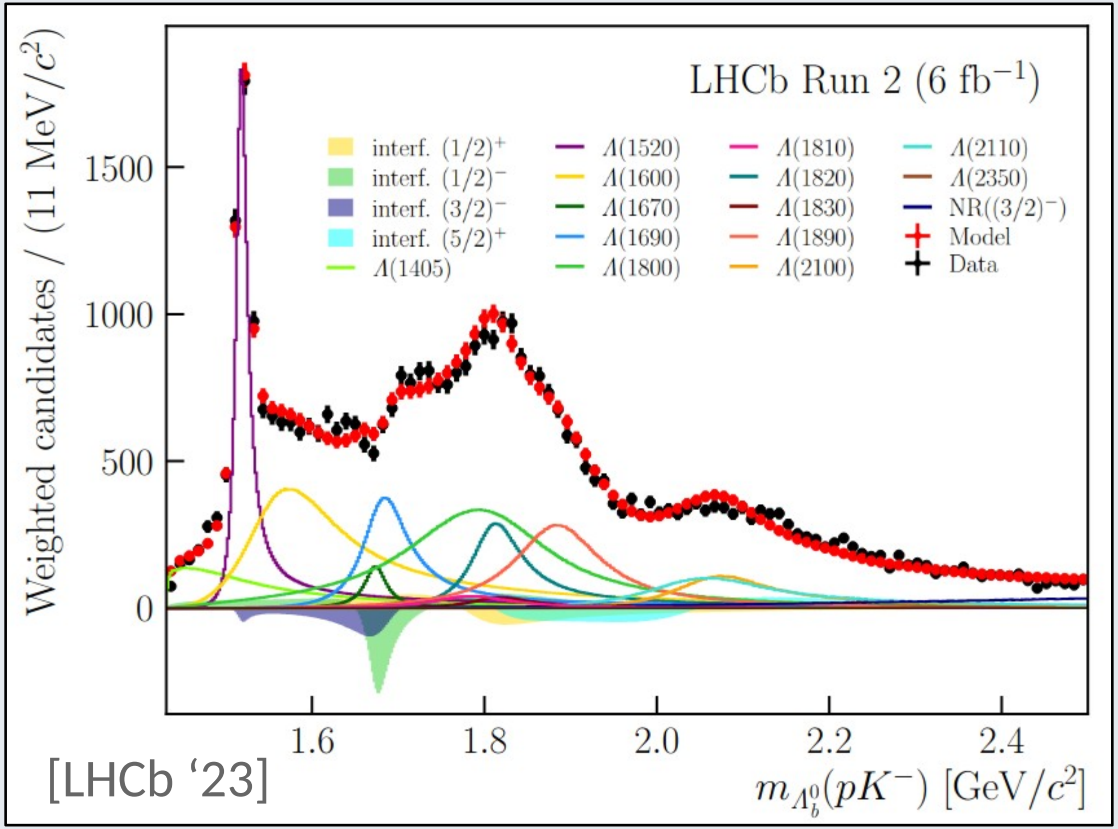
- $\Gamma_{K^*} / M_{K^*} \sim 5\%$ is not very small
- **Finite width effects** have to be accounted for in the LQCD and LCSR calculations
  - Universal 20% correction to the observables [Descotes-Genon, Khodjamirian, Virto '19]
  - Computable in LQCD [Leskovec '24]
- B → Kπμμ decays also have a large **S-wave component** [LHCb '16]
  - LCSR inputs for the S-wave are now available [Descotes-Genon, Khodjamirian, Virto, Vos '23]
- Need for a generic parametrization for B → Kπ form factors [Gustafson, Herren et al '23 (B → Dπ)]



What about the baryons?

- Width effects are ~10% for the K* (Γ_{K^*} / M_{K^*} ~5%)

- $\Gamma_{\Lambda(1520)}$ / $M_{\Lambda(1520)}$ ~ 1%
  - width effects probably **safely negligible**
  - Pollution from the other resonances
- The other pK resonances:
  - can hardly be isolated exp-erimentally
  - suffer from **large width ef-fects**



[LHCb '23]

3-body form factors

known Lorentz structures

- Generalized matrix elements

$$\langle p(k_1)K(k_2)|\mathcal{O}_i^{\mu}|\Lambda_b(q+k)\rangle = F_i(q^2, m_{pK}^2, \cos\theta_K)\boxed{S_\mu^i}$$

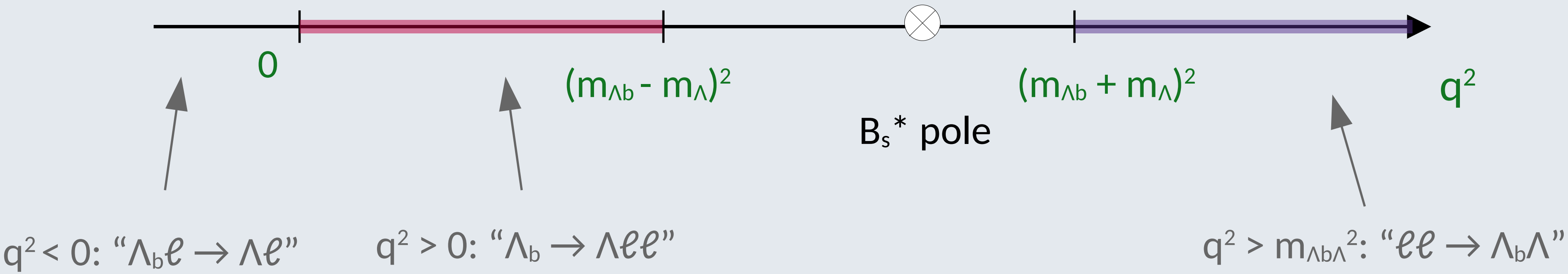
- Partial-wave expansion

$$F_i(q^2, m_{pK}^2, \cos\theta_K) = \sum_{\ell=0}^{\infty}\sqrt{2\ell+1}\,\boxed{F_i^{(\ell)}(q^2, m_{pK}^2)}\,\boxed{P_\ell(\cos\theta_K)}$$

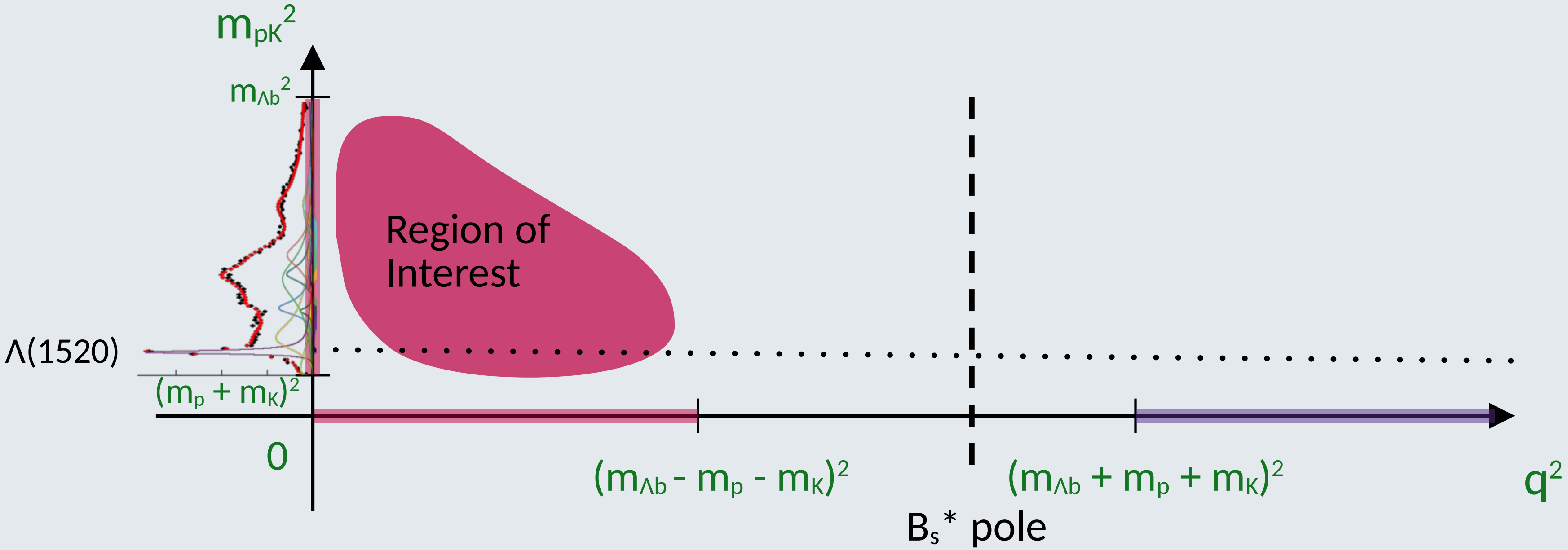
3-body form-factors

Legendre polynomials

Analytic structure

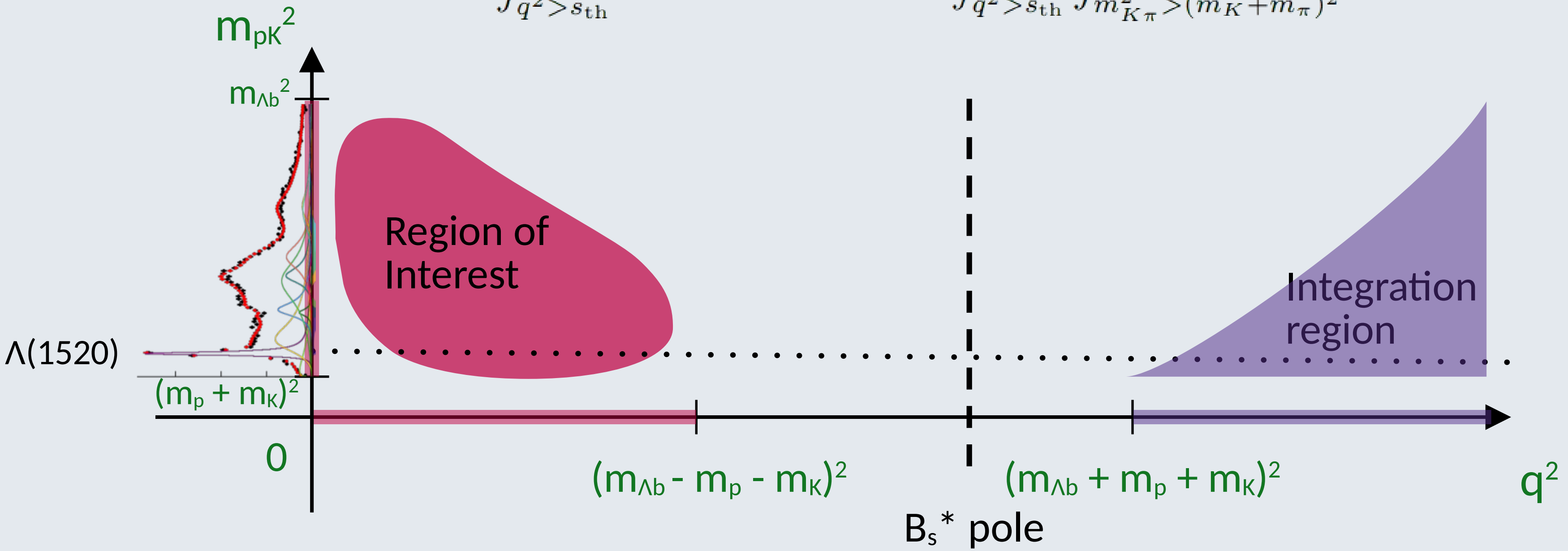


Analytic structure



Dispersive bound

$$\int_{q^2>s_{\rm th}} |\mathcal{F}(q^2)|^2 < 1 \quad \Longrightarrow \quad \int_{q^2>s_{\rm th}} \int_{m_{K\pi}^2>(m_K+m_\pi)^2}^{\left(\sqrt{q^2}-m_{\Lambda_b}\right)^2} |\mathcal{F}(q^2, m_{K\pi}^2)|^2 < 1$$



Dispersive bound

- How should we parametrize $F_i^{(\ell)}(q^2, m_{pK}^2)$?
  - Analyticity suggests a **double z-expansion**: simple but convergence is not ensured
  - Model + z-expansion:
    - Muskhelishvili-Omnès [Gustafson, Herren et al '23 (B → Dπ)]
    - K matrix?

- **Problem #2**: Dispersive bounds *don't take a simple form*

- Factorization ansatz:

$$F_i^{(\ell)}(q^2, m_{pK}^2) = \hat{F}_i^{(\ell)}(q^2, m_{pK}^2)\,\hat{g}(m_{pK}^2) = \left(\hat{F}_i^{(\ell)}(q^2) + \mathcal{O}\left(\frac{m_{pK}^2 - m_R^2}{m_B^2}\right)\right)\hat{g}(m_{pK}^2)$$

Dispersive bound, finally

$$F_i^{(\ell)}(q^2, m_{pK}^2) = \hat{F}_i^{(\ell)}(q^2, m_{pK}^2)\,\hat{g}(m_{pK}^2) = \left(\hat{F}_i^{(\ell)}(q^2) + \mathcal{O}\left(\frac{m_{pK}^2 - m_R^2}{m_B^2}\right)\right)\hat{g}(m_{pK}^2)$$

- Dispersive bound now takes the form

$$\hat{F}_i^{(\ell)}(q^2) = \frac{1}{\phi_i(z)\mathcal{B}_i(z)} \sum_{n>0} \alpha_n^{i\ell} p_n(z)\Big|_{z=z(q^2)} \Longrightarrow \sum_{n>0} \left|\alpha_n^{i\ell}\right|^2 < 1$$

Outer function, includes some normalization

Inner function = Blaschke factor, accounts for the $b\bar{s}$ poles

Conclusion

- I focused on $b \to s\ell\ell$ but all of this is also valid for all other quark transitions e.g. $\Lambda_c \to p\mu^+\mu^-$ [LHCb '24]

- Analyticity constraints are **great to reduce (extrapolation) uncertainties**…

  … but their complexity increases exponentially (number of parameters, analytic continuations, …)

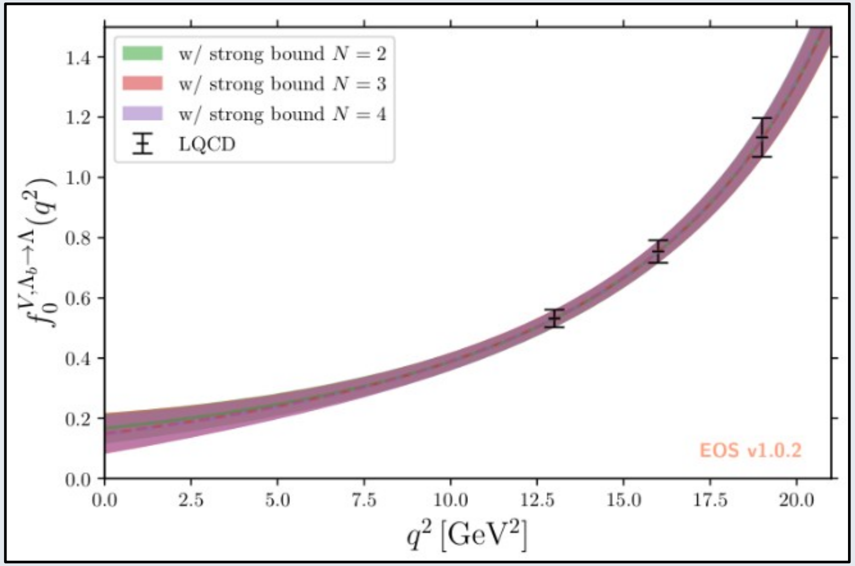
  → Recent improvements in fitting/sampling techniques already allowed us to go one step further w.r.t to ~20 years ago

  → More than ever, we are going to need a solid collaboration between experimentalists, theorists and (Bayesian ;)) statisticians

Back-up

Another example $\Lambda_b \to \Lambda\ell\ell$ [Blake, Meinel, *et al* '23]

- 10 form factors: **25 parameters (N = 2), 35 parameters (N = 3), 45 parameters (N = 4)**
- 25 constraints from LQCD [Detmold, Meinel, '16]
- Excellent p-values for N > 2
- Clear impact on the extrapolation:



No bound:

$$f_\perp^T(q^2=0)\big|_{[14]} = 0.166 \pm 0.072$$

With bound:

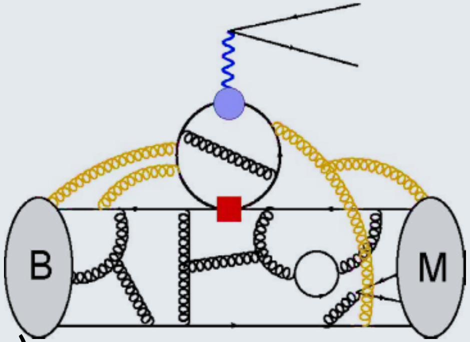
$$f_\perp^T(q^2=0)\big|_{N=2} = 0.190 \pm 0.043\,,$$
$$f_\perp^T(q^2=0)\big|_{N=3} = 0.173 \pm 0.053\,,$$
$$f_\perp^T(q^2=0)\big|_{N=4} = 0.166 \pm 0.049\,.$$

Non-local contributions

Non-local form factors

$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10})\mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

$$\mathcal{H}_\mu(k,q) = i \int d^4x \, e^{iq\cdot x} \langle \bar{M}(k) | T\{\mathcal{J}_\mu^{\mathrm{em}}(x), \mathcal{C}_i \mathcal{O}_i\} | \bar{B}(q+k) \rangle$$



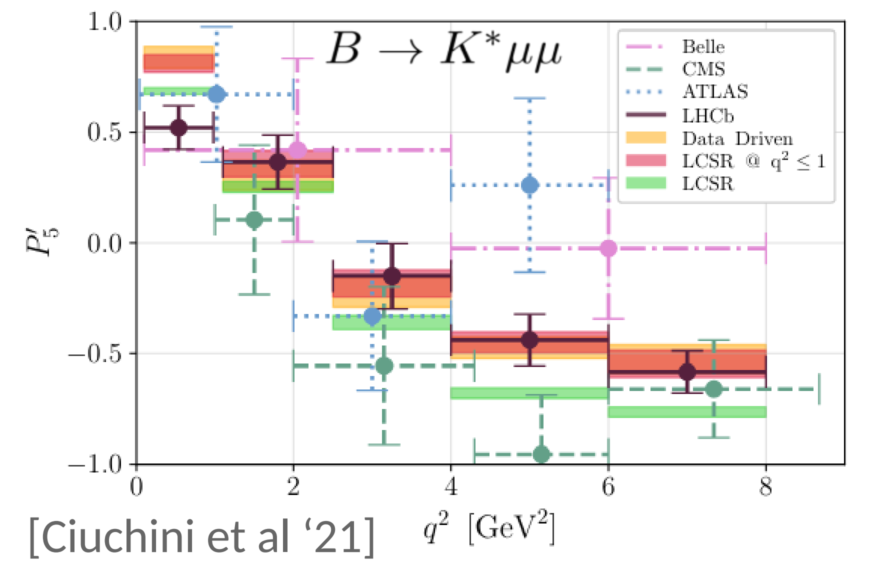
- Problematic because **they can mimic a BSM signal!**
  - $\mathcal{H}_\lambda$ can be interpreted as a shift to C_9 and C_7
  - This shift is lepton-flavour universal (as now seen in the data)
- Notably **harder to estimate**, no lattice computation so far
- Dominated by O_1^c and O_2^c : “charm loop” [Khodjamirian, Mannel, Wang, ‘12]
- **Different parametrizations** are suggested

q^2 parametrization

- **Simple q^2 expansion** [Jäger, Camalich '12; Ciuchini et al. '15]

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_\lambda^{\text{QCDF}}(q^2) + h_\lambda(0) + \frac{q^2}{m_B^2} h'_\lambda(0) + \dots$$

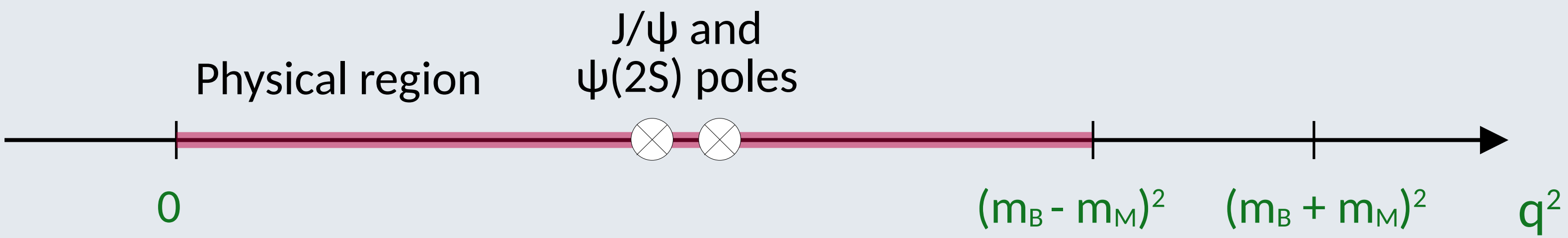
Computed in [Beneke, Feldman, Seidel '01]



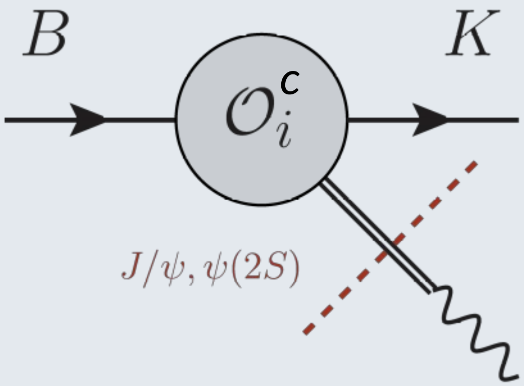
[Ciuchini et al '21]

- The h_λ terms can be fitted or varied
- Fitting the h_λ terms on data gives a satisfactory but uninformative result
- This parametrization **cannot account** for the analyticity properties of $\mathcal{H}_\lambda$

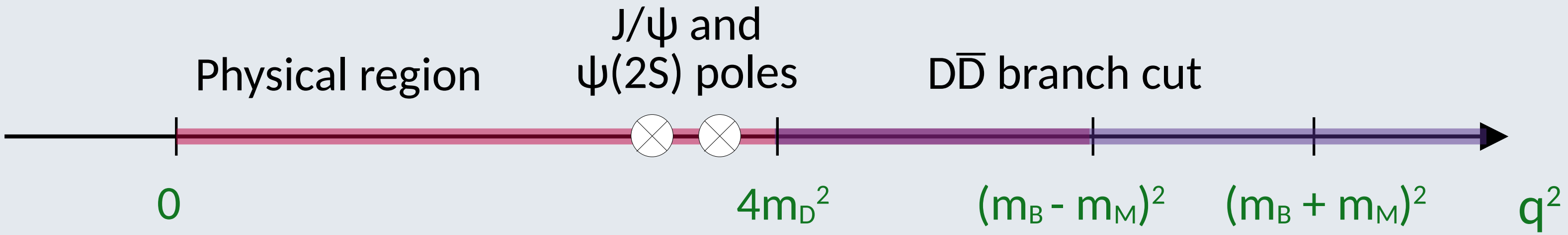
Analyticity properties of H_μ



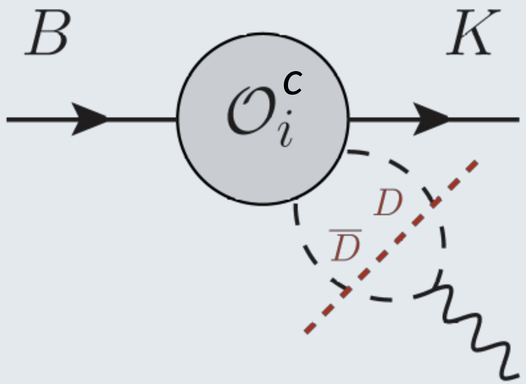
- Poles due to the narrow charmonium resonances



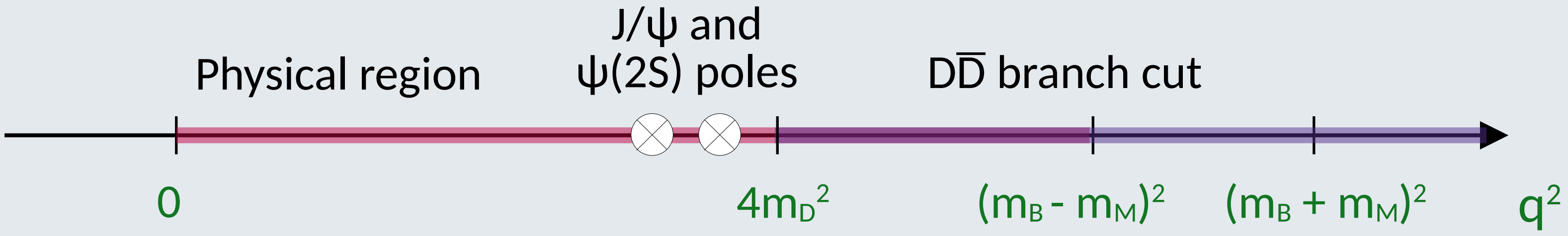
Analyticity properties of H_μ



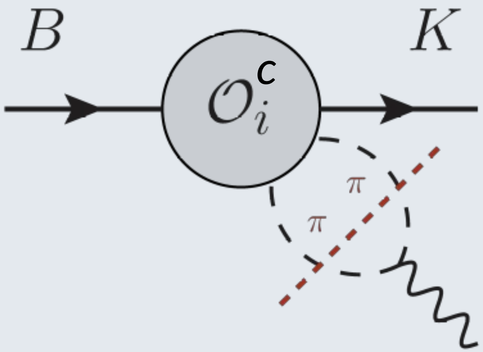
- Poles due to the narrow charmonium resonances
- Branch-cut starting at $4m_D^2$



Analyticity properties of H_μ



- Poles due to the narrow charmonium resonances
- Branch-cut starting at $4m_D^2$
- Branch-cut starting at $4m_\pi^2 \rightarrow$ negligible (OZI suppressed)



Anatomy of H_μ in the SM

$C_1(\mu_b)$	$C_2(\mu_b)$	$C_3(\mu_b)$	$C_4(\mu_b)$	$C_5(\mu_b)$	$C_6(\mu_b)$	$C_7(\mu_b)$	$C_8(\mu_b)$	$C_9(\mu_b)$	$C_{10}(\mu_b)$
-0.2906	1.010	-0.0062	-0.0873	0.0004	0.0011	-0.3373	-0.1829	4.2734	-4.1661

- The contribution of O_8 is **negligible** [Khodjamirian, Mannel, Wang, ‘12]
- The contributions of $O_{3,\,4,\,5,\,6}$ are suppressed by **small Wilson coefficients**

$$\mathcal{O}_3 = (\bar{s}_L \gamma_\mu b_L) \sum_p (\bar{p}\gamma^\mu p)\,, \qquad \mathcal{O}_4 = (\bar{s}_L \gamma_\mu T^a b_L) \sum_p (\bar{p}\gamma^\mu T^a p)\,,$$

$$\mathcal{O}_5 = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho b_L) \sum_p (\bar{p}\gamma^\mu \gamma^\nu \gamma^\rho p)\,, \qquad \mathcal{O}_6 = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho T^a b_L) \sum_p (\bar{p}\gamma^\mu \gamma^\nu \gamma^\rho T^a p)\,,$$

Anatomy of H_μ in the SM

$$\mathcal{O}_1^q = (\bar{s}_L \gamma_\mu T^a q_L)(\bar{q}_L \gamma^\mu T^a b_L)\,, \qquad \mathcal{O}_2^q = (\bar{s}_L \gamma_\mu q_L)(\bar{q}_L \gamma^\mu b_L)$$

- Light-quark loops are CKM suppressed → **small contributions** even at the resonances [Khodjamirian, Mannel, Wang, '12]

Vector meson	ρ	ω	ϕ	J/ψ	$\psi(2S)$
f_V	221^{+1}_{-1}	195^{+3}_{-4}	228^{+2}_{-2}	416^{+5}_{-6}	297^{+3}_{-2}
$\lvert A_{\bar{B}^0 V \bar{K}^0}\rvert$	$1.3^{+0.1}_{-0.1}$	$1.4^{+0.1}_{-0.1}$	$1.8^{+0.1}_{-0.1}$	$33.9^{+0.7}_{-0.7}$	$44.4^{+2.2}_{-2.2}$
$\lvert A_{B^- V K^-}\rvert$	$1.2^{+0.1}_{-0.1}$	$1.5^{+0.1}_{-0.1}$	$1.8^{+0.1}_{-0.1}$	$35.6^{+0.6}_{-0.6}$	$42.0^{+1.2}_{-1.2}$

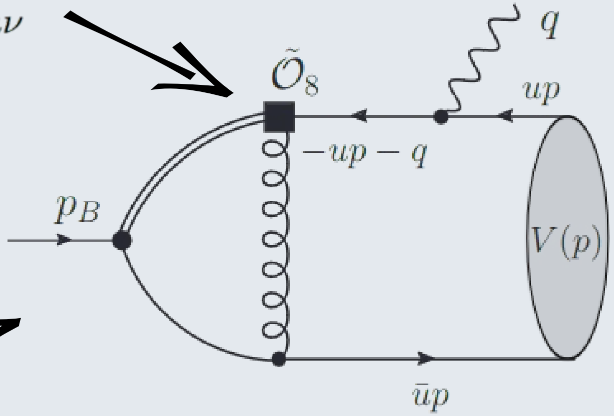
→ The main contribution comes from $\mathbf{O_1^c}$ and $\mathbf{O_2^c}$: **"charm loop"**

Anatomy of H_μ in the SM

$C_1(\mu_b)$	$C_2(\mu_b)$	$C_3(\mu_b)$	$C_4(\mu_b)$	$C_5(\mu_b)$	$C_6(\mu_b)$	$C_7(\mu_b)$	$C_8(\mu_b)$	$C_9(\mu_b)$	$C_{10}(\mu_b)$
-0.2906	1.010	-0.0062	-0.0873	0.0004	0.0011	-0.3373	-0.1829	4.2734	-4.1661

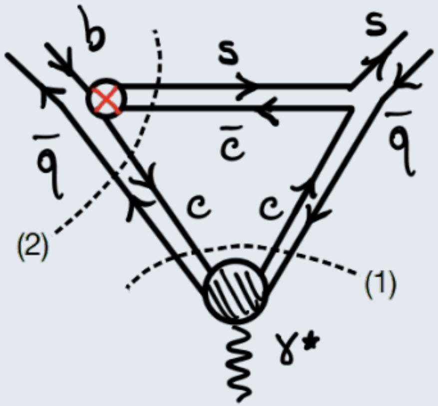
- The contribution of O_8 is **negligible** [Khodjamirian, Mannel, Wang, '12; Dimou, Lyon, Zwicky '12]

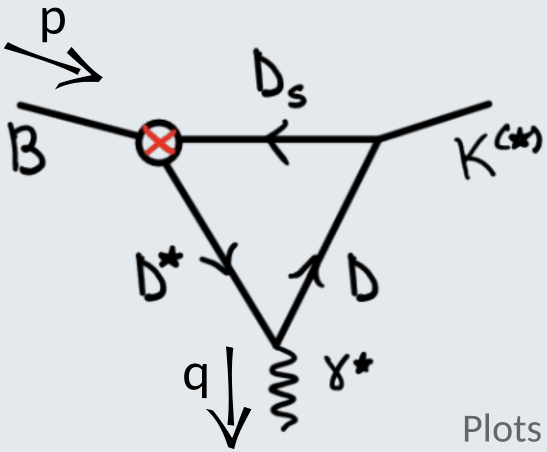
$$\mathcal{O}_8 = \frac{g_s}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$



One of the non-factorizable contributions

More involved analytic structure?





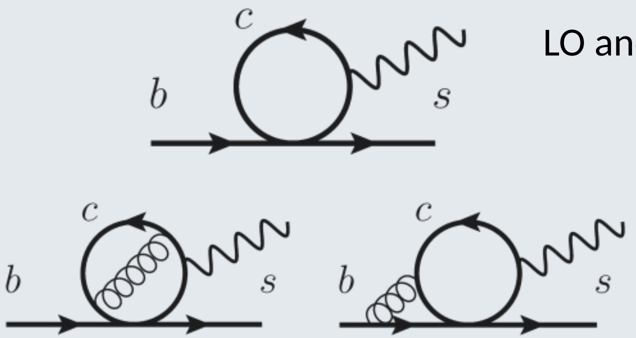
Plots from [Ciuchini et al. '22]

- $M_B > M_{D^*} + M_{D_s}$ → The function $H_\lambda(p^2,q^2)$ has a branch cut in p^2 and the physical decay takes place on this branch cut: **H_λ is complex-valued!**
- Triangle diagrams are known to create *anomalous* branch cuts in q^2 [e.g. Lucha, Melikhov, Simula '06] → Does this also apply here? We have no Lagrangian nor power counting!
- The presence and the impact of such a branch cut in our approach is under investigation

Theory inputs

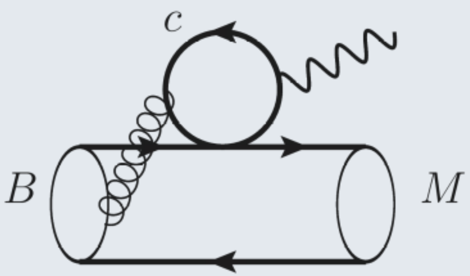
$\mathcal{H}_\lambda$ can still be calculated in **two kinematics regions**:

- **Local** OPE $|q|^2 \gtrsim m_b^2$ [Grinstein, Piryol ‘04; Beylich, Buchalla, Feldmann ‘11]
- **Light Cone** OPE $q^2 \ll 4m_c^2$ [Khodjamirian, Mannel, Pivovarov, Wang ‘10]



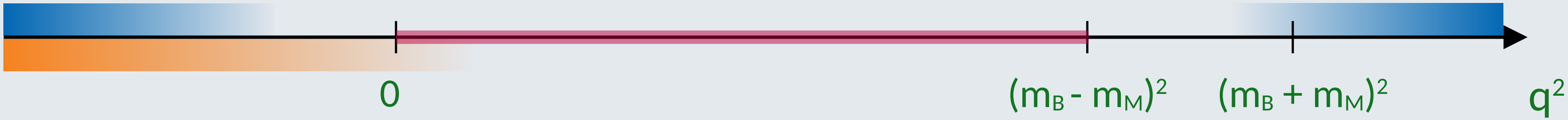
LO and α_s corrections

[Asatarian, Greub, Virto ‘19]



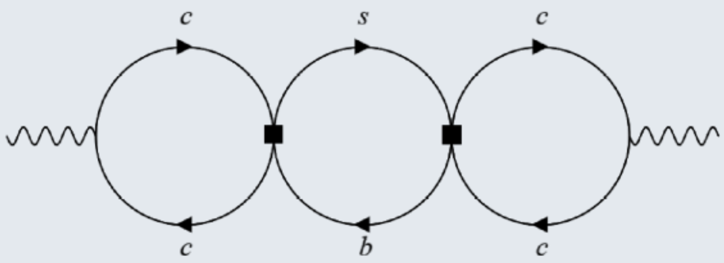
Non-perturbative soft gluon corrections

[Gubernari, van Dyk, Virto ‘20]



Dispersive bound

- **Main idea:** Compute the charm-loop induced, inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to $\mathcal{H}_\lambda$ [Gubernari, van Dyk, Virto '20]



+ other diagrams…

- The optical theorem gives a **shared bound** for **all the b → s processes**:

$$1 > 2 \int_{(m_B+m_K)^2}^{\infty} \left|\hat{\mathcal{H}}_0^{B\to K}(t)\right|^2 dt + \sum_\lambda \left[2 \int_{(m_B+m_{K^*})^2}^{\infty} \left|\hat{\mathcal{H}}_\lambda^{B\to K^*}(t)\right|^2 dt + \int_{(m_{B_s}+m_\phi)^2}^{\infty} \left|\hat{\mathcal{H}}_\lambda^{B_s\to \phi}(t)\right|^2 dt \right]$$

$$+ \Lambda_b \to \Lambda^{(*)} \ldots$$

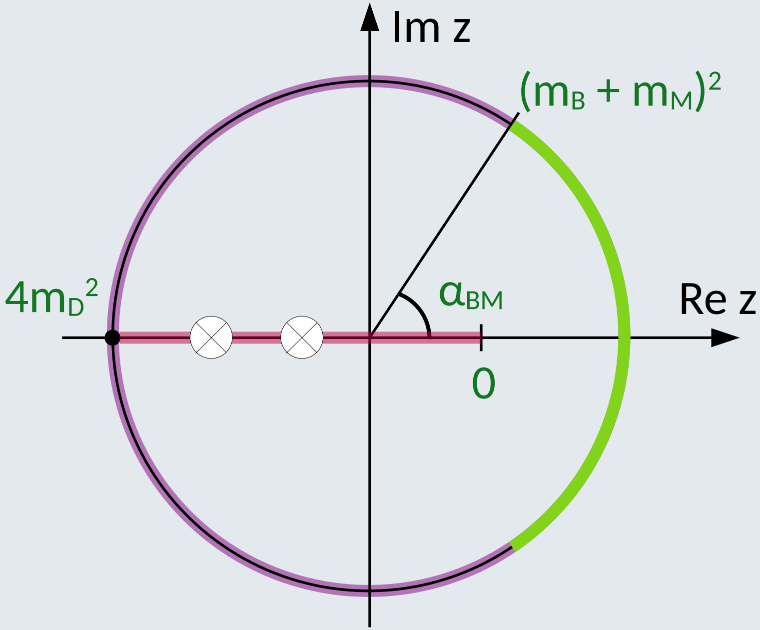
$$\text{known functions} \times \mathcal{H}_0^{B\to K}(t)$$

GRvDV parametrization

- The bound can be "**diagonalized**" with **orthonormal polynomials** of the arc of the unit circle [Gubernari, van Dyk, Virto '20]

$$z(s) = \frac{\sqrt{4m_D^2 - s} - \sqrt{4m_D^2 - s_0}}{\sqrt{4m_D^2 - s} + \sqrt{4m_D^2 - s_0}}$$

$$\mathcal{H}_\lambda(z) = \frac{1}{\phi(z)\mathcal{P}(z)} \sum_{k=0}^{N} a_{\lambda,k}\, p_k(z)$$



- The coefficients respect the **simple bound**:

$$\sum_{n=0}^{\infty} \left\{ 2\left|a_{0,n}^{B\to K}\right|^2 + \sum_{\lambda=\perp,\parallel,0} \left[2\left|a_{\lambda,n}^{B\to K^*}\right|^2 + \left|a_{\lambda,n}^{B_s\to\phi}\right|^2 \right] \right\} < 1$$

Numerical analysis

- The parametrization is fitted to

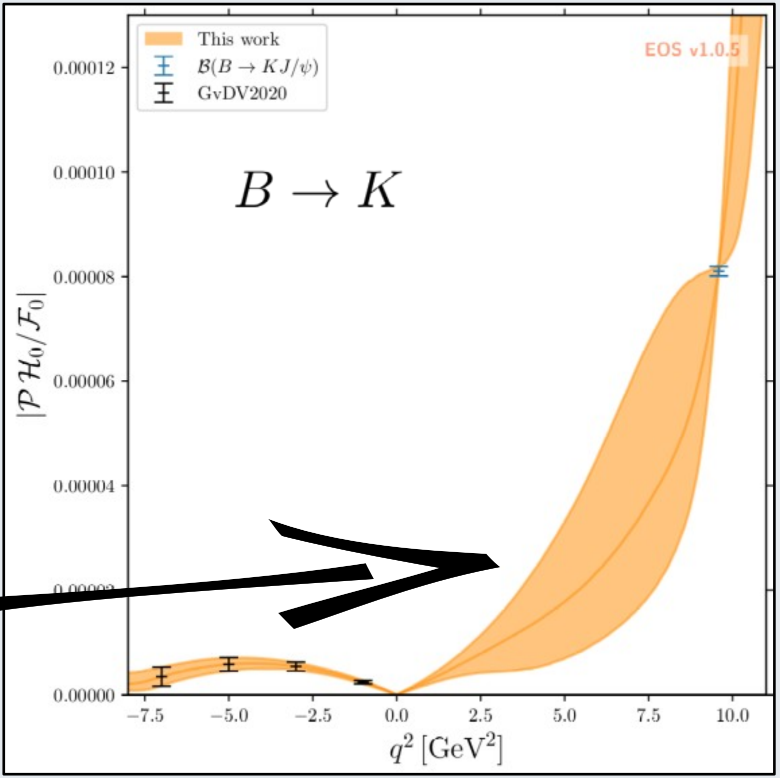
  $B \to K$, $B \to K^*$, $B_s \to \varphi$

  using:
  - 4 theory point at negative q^2 from the light cone OPE
  - Experimental results at the J/ψ
  - Use an **under-constrained fit** and allow for **saturation of the dispersive bound**

→ The uncertainties are **truncation order-independent**, i.e., increasing the expansion order does not change their size

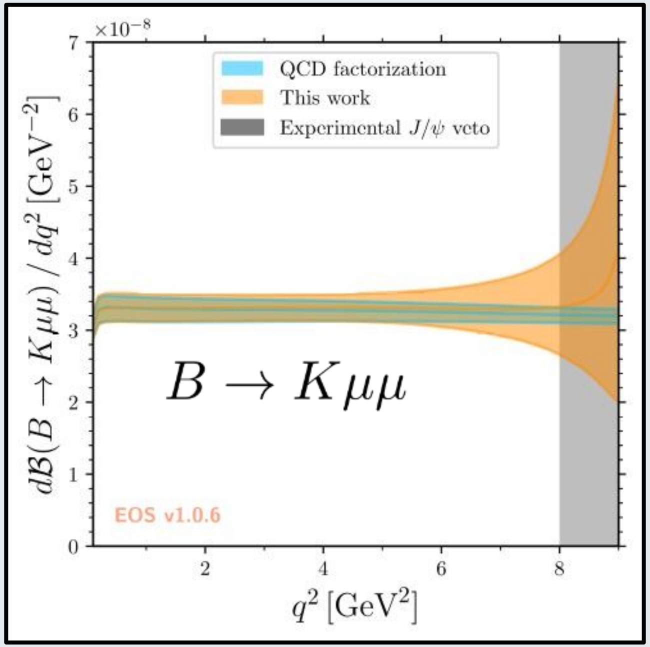
→ All p-values are larger than 11%

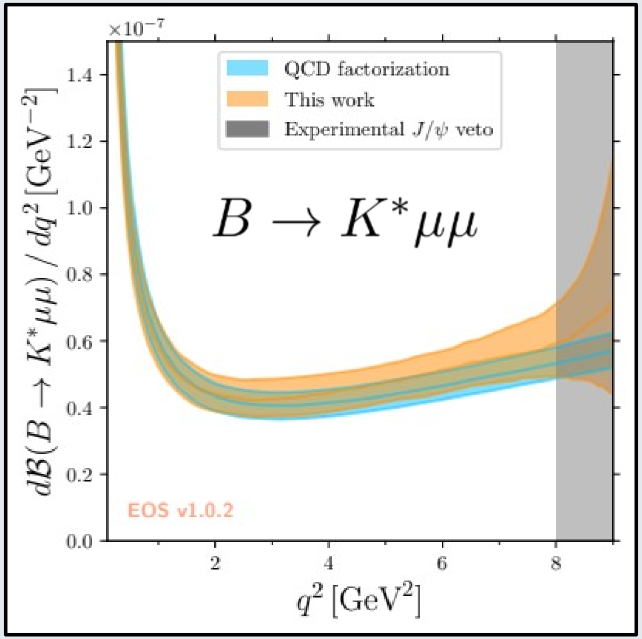
[Gubernari, MR, van Dyk, Virto '22]

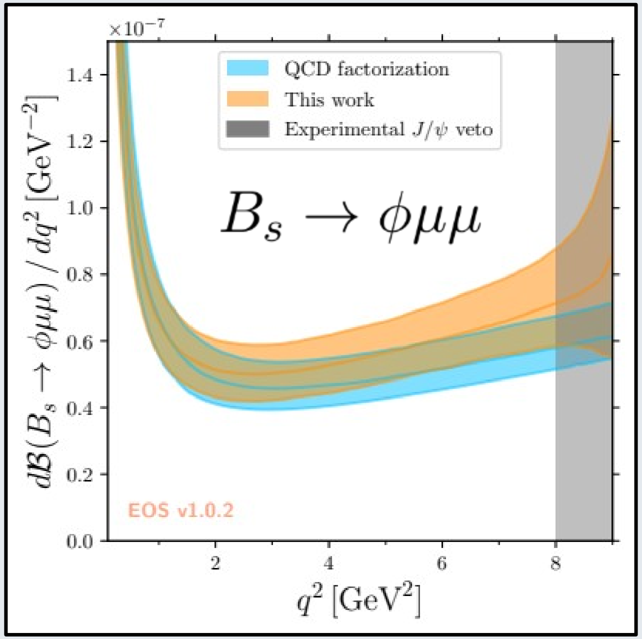


SM predictions

- **Good overall agreement** with previous theoretical approaches
  - Small deviation in the slope of $B_s \to \phi\mu\mu$
- **Larger** but **controlled** uncertainties especially near the J/ψ
  - The approach is **systematically improvable** (new channels, ψ(2S) data…)



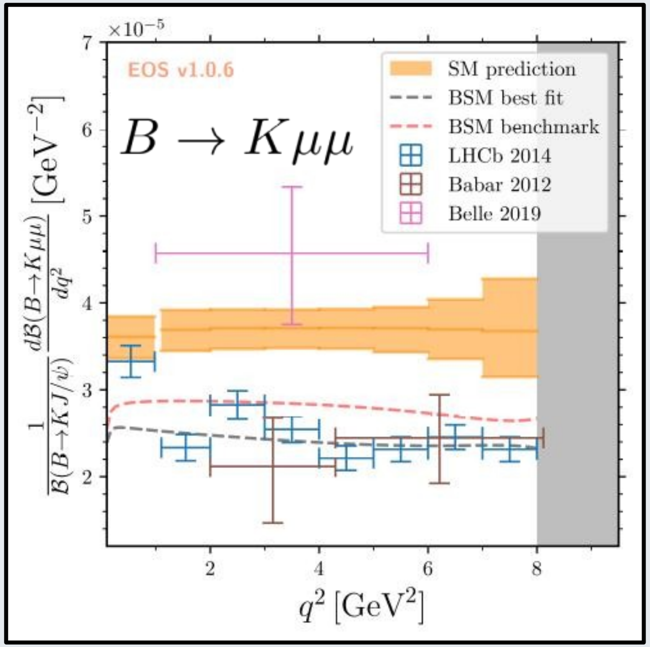


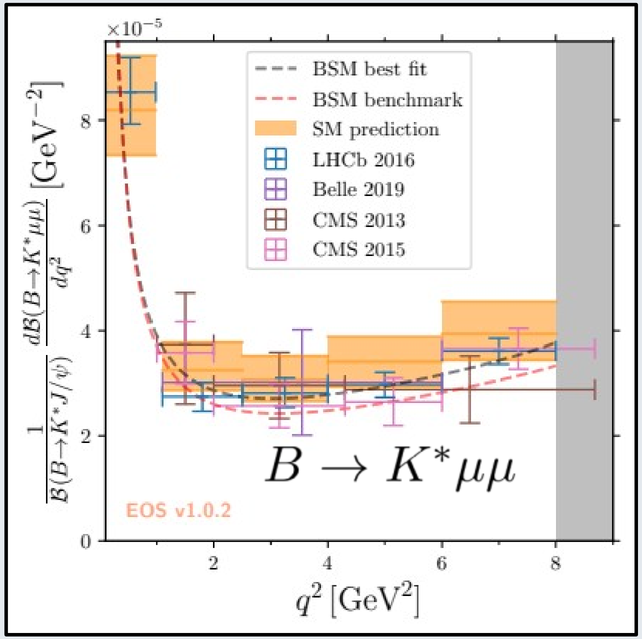


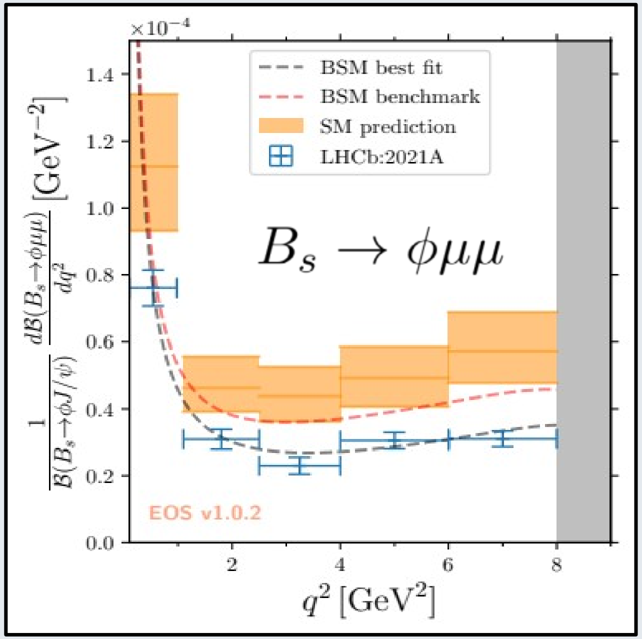
Confrontation with data

- This approach of the non-local form factors **does not solve the “B anomalies”**.
- In this approach, the greatest source of theoretical uncertainty now comes from **local form factors**.

Experimental results:
[Babar: 1204.3933; Belle: 1908.01848, 1904.02440; ATLAS: 1805.04000, CMS: 1308.3409, 1507.08126, 2010.13968, LHCb: 1403.8044, 2012.13241, 2003.04831, 1606.04731, 2107.13428]



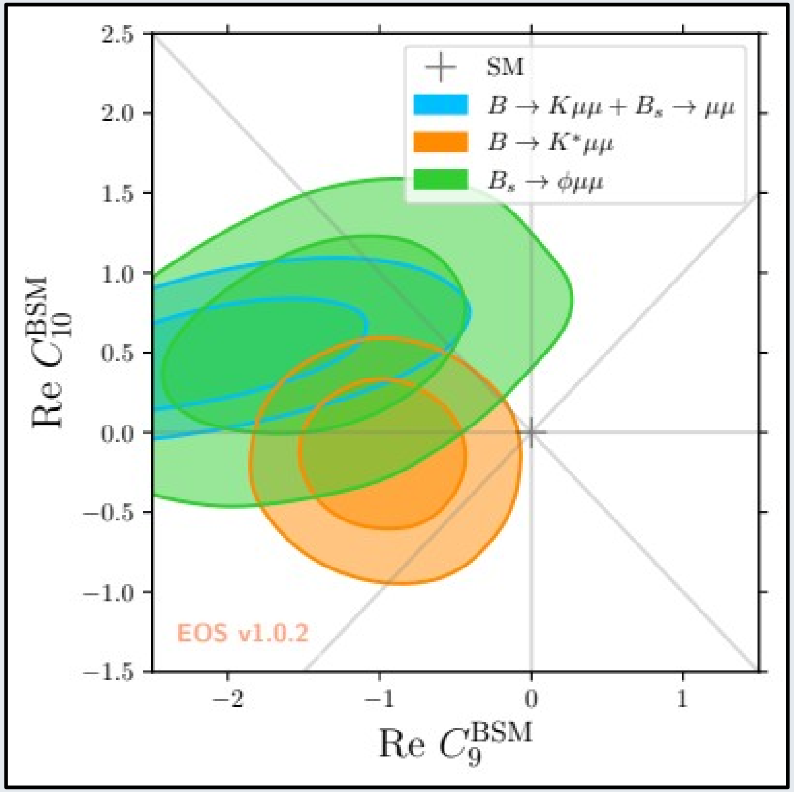




Additional plots can be found in the paper: 2206.03797

BSM analysis

- A combined BSM analysis would be **very CPU expensive** (130 correlated, non-Gaussian, nuisance parameters!)
- Fit **separately** C_9 and C_{10} for the three channels:
  - $B \to K\mu^+\mu^- + B_s \to \mu^+\mu^-$ (*)
  - $B \to K^*\mu^+\mu^-$
  - $B_s \to \varphi\mu^+\mu^-$



(*) CMS recently updated their $B_s \to \mu^+\mu^-$ measurement [2212.10311]