θ-vacua, asymmetric Hawking radiation and PBH baryogenesis

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Key takeaways

- Black holes can carry new quantum hair associated with non-trivial topological structure of the Standard Model vacuum state (gauge theory θ -vacuum).

- This hair can mediate long-distance correlations resulting, in particular, in CP-asymmetric Hawking radiation (relevant for PBH baryogenesis)

Black holes

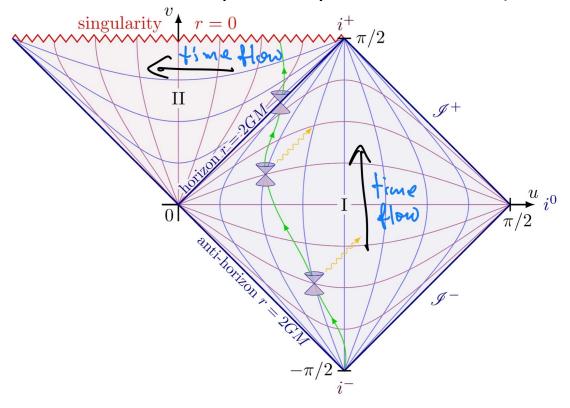
- Classical black holes are boringly simple

- Quantum black holes excitingly unmanageable

- Classical black holes in quantum environment are exciting and relatively simple

Black holes are bald (very few hairs)

- Any field configuration associated with the propagating degrees of freedom cannot be regular across the horizon, except they are trivial (aka vacuum conf.)



Israel 67', Carter 71', Wald 71' Ruffini, Wheeler 71'

Classical black hole hairs

 Classical (primary) hairs can probe features of the black hole associated with gauge redundancies (GR diffeomorphism, EM gauge invariance)

$$Q_{\chi} = \frac{1}{4\pi} \oint \nabla^{\mu} \chi^{\nu} dS_{\mu\nu} \text{ (mass, angular momentum, charge)}$$

$$q = \frac{1}{4\pi} \oint \nabla^i A^0 dS_i = \frac{1}{4\pi} \oint \vec{E} \cdot d\vec{S}$$

- Classical field configurations which are supported by primary hairs (not a new feature)

Quantum hairs

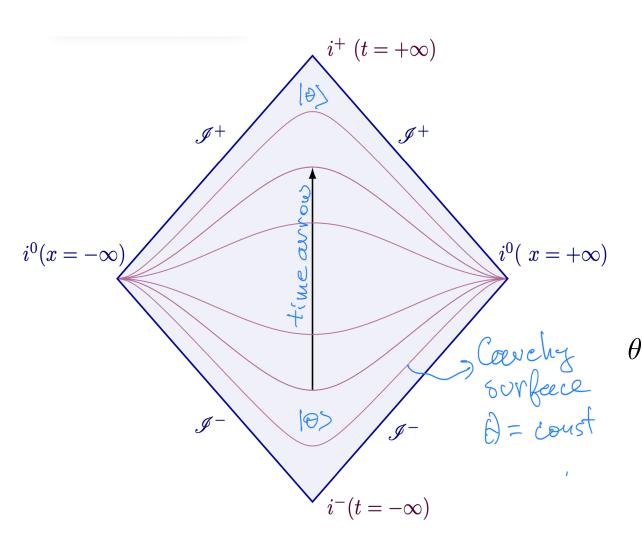
Stable field configurations carrying conserved topological charge, e.g., 2-form axion, skyrmion
 Luckock, Moss 86'
 Krauss and F. Wilczek 89'

 Topological charges may be related to some Noether charges via duality - skyrmion = baryon

Dvali, Gußmann 17'

 Topological structure of vacuum < - > anomalous global charges (chiral charge, B+L in the Standard Model)

θ-vacua in the World without gravity



Gauge invariance:

$$|\theta\rangle = \sum_{n} e^{in\theta} |n\rangle$$

$$\theta = const., \ \theta + 2\pi \sim \theta$$

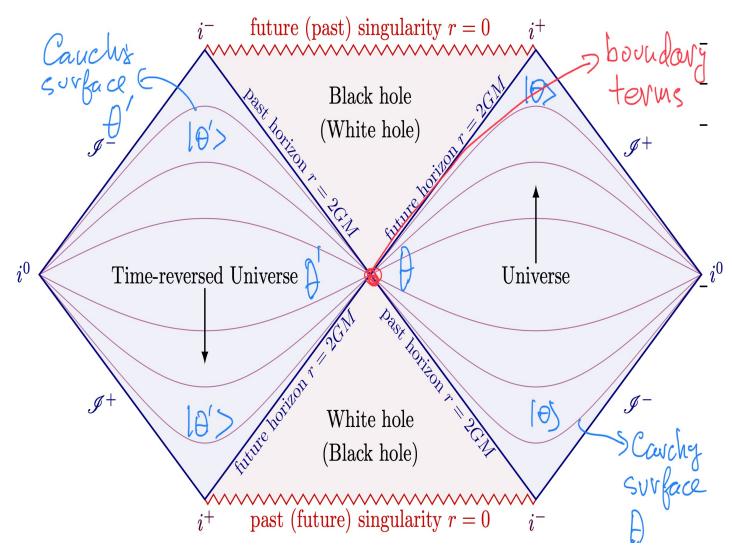
$$\frac{\text{QCD}}{\mathcal{L}_{QCD}} \propto \theta_{QCD} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

$$\theta_{QCD} < 10^{-10} \text{ (strong CP problem)}$$

EW
$$\mathcal{L}_{EW} \propto \theta_{EW} W_{\mu\nu} \tilde{W}^{\mu\nu}$$

 θ_{EW} – unphysical due to B + L
physical processes with $(B \neq L)$

θ-vacua in the World with a black hole



Allice observes θ-vacuum Bob observes θ'-vacuum Jump across the boundary (causally disconnected regions)

Any distant inertial observer in our Universe must observe physical effects of θ modulo the boundary contribution

Θ-term in the membrane paradigm for bl.h.

- Define (2+1)D time-like surface M just beyond the event horizon

Thorne, Price,

$$r_M = r_S + \epsilon$$
, $(\epsilon/r_s <<<1)$

Allice: observes physics outside M (modulo boundary effects)

Bob: observes phenomena inside M (modulo boundary effects)

$$S = S_A + S_B = (S_A + S_M) + (S_B - S_M)$$

Physics outside of the stretched horizon is completely standard

Macdonald 86'

Parikh, Wilczek

97'

Θ-term in the membrane paradigm for bl.h.

- Classical EoM: $\frac{\delta(S_A+S_M)}{\delta\phi}=0$

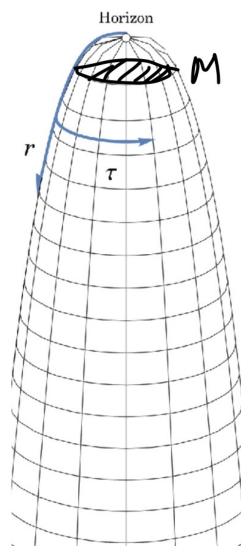
$$S_M = \int d^3x \sqrt{-h} J_M \phi, \quad J_M = n_\mu \frac{\partial \mathcal{L}_A}{\partial \nabla_\mu \phi} \Big|_M$$

- θ -term:

$$\int_{A} d^{4}x \sqrt{-g} \frac{\theta}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} + \int_{M} d^{3}x \sqrt{-h} \epsilon^{\mu\nu\rho\sigma} n_{\mu} F_{\nu\rho} A_{\sigma},$$

$$C_{\mu\nu\rho} = A_{[\mu} F_{\nu\rho]}$$

QFT in bl.h background = QFT at finite T



$$ds^2 = \left(1 - \frac{r_S}{r}\right)d\tau^2 + \left(1 - \frac{r_S}{r}\right)^{-1}dr^2 + r^2d\Omega^2$$

$$t \to -i\tau$$

Near horizon:

$$ds^2 = \frac{\rho^2}{4r_S^2}d\tau^2 + d\rho^2 + r_S^2d\Omega^2 \bigg| \rho = \sqrt{4r_S(r-r_S)}$$

$$\tau + \beta \sim \tau \longrightarrow T_H = 1/\beta = 1/4\pi r_S = 1/8\pi G_N M$$
 Hawking 77'

Instantons in the black hole background

- Topological classification of vacua $R_+ \times S^1_{\beta} \times S^2$ $(vs \ R^4 \ or \ S^1_{\beta} \times R^3)$

$$\pi_1(S^1_\beta) = Z, \quad H^2(S^2, \pi_1(S^1_\beta)) = Z$$

- Instantons
$$A_\mu^a = \left(\pm \omega_\mu^{0a} - \frac{1}{2}\epsilon^{abc}\omega_\mu^{bc}\right)$$
 Charap, Duff 77'

$$P = \frac{1}{64\pi^2} \int d^4x \sqrt{g} F_{\mu\nu} \tilde{F}^{\mu\nu} = \pm 1.$$

- Different regimes:

$$r_S < a < \beta$$

 $r_S < a < eta$ ~ pure YM instantons

$$a > \beta$$

3D static dipole

Fermions in the background of bl.h. instantons

$$Z = \int [dA_{\mu}][d\psi][d\psi^{+}] \exp\{-(S_{A}[A_{\mu}, \psi, \psi^{+}] + S_{M}[A_{\mu}, \psi, \psi^{+}])\}$$
$$= \int [dA_{\mu}]Det(\not D + M) \exp\{-(S_{A}[A_{\mu}] + S_{M}[A_{\mu}])\}$$

- Fermion zero modes

$$\left(D\!\!\!/+M\right)\phi=0 \ \longrightarrow \ Z[\theta]=0 \ \longrightarrow \Theta$$
 unobservable

- This is the case for the electroweak θ_{EW} in the Standard Model [Krasnikov, Rubakov, Tokarev 79']

Fermions in the background of bl.h. instantons

- In the black hole background the spectrum of the Dirac operator gets modified due to the boundary (stretched horizon)
- Proper boundary conditions must be imposed to keep the Dirac operator self-adjoint [Atiyah, Patodi, Singer 75']

$$\not\!\!D\phi_{\lambda} = \lambda \phi_{\lambda}, \quad \phi_{\lambda} = f_{\lambda}(r)\chi(y)$$

Fermions in the background of bl.h. instantons

$$\begin{cases} \partial_r^2 f_{\lambda} = (\lambda^2 - \alpha^2) f_{\lambda}, \\ \not D_M^2 \chi = \alpha \chi \end{cases}$$

$$f_{\lambda}^{(+)} = C^{(+)} \sin(\sqrt{\lambda^2 - \alpha^2} \ r) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$f_{\lambda}^{(-)} = C^{(-)} \left[(\sqrt{\lambda^2 - \alpha^2}) \cos(\sqrt{\lambda^2 - \alpha^2} \ r) + \alpha \sin(\sqrt{\lambda^2 - \alpha^2} \ r) \right] \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- No solutions with $|\lambda| = |\alpha|$
- No normalisable 0-modes (α =0) on the horizon
- While the bulk θ-term can be rotated away, the boundary term remains!

Physical implications: Long-range force

Non-zero boundary term can be viewed as a (B+L) accumulated on the horizon

- It is a source for 3-form gauge field. This gauge field does not carry any propagating dof, but can produce longrange Coulomb force [Lucher 78', Dvali 05']

F.T.
$$\langle T C_{\mu\nu\rho}(x) C_{\alpha\beta\gamma}(y) \rangle \propto \frac{1}{p^2}$$

- Signal in binary mergers?

Physical implications: Asymmetric Hawking radiation

- Black hole emitting null dust [Vaidya 51']

$$ds^{2} = -\left(1 - \frac{2GM(u)}{r}\right)du^{2} - 2dudr + r^{2}d\Omega^{2}$$

$$n_{\mu} = \left(-\sqrt{|M'(u)|}, 1/(2\sqrt{|M'(u)|}), 0, 0\right), |M'| \propto T_{\rm H}^2$$

$$\frac{\mu_{B+L}}{T_H} \propto \theta_{EW}$$

Physical implications: Baryogenesis

 PBHs are produced in the universe with initial non-zero B+L number (possibly generated in GUT or other processes)

- They store B+L charge on the horizon and shield it from the wash-out by sphalerons

- The active phase of evaporation starts after the sphaleron decoupling (details are work in progress)

Summary

- Even within a very standard physics (SM+GR) black holes may carry quantum hair which are associated with the topological structure of the SM ground state
- Physically it can be interpreted as an anomalous charge accumulated on the black hole horizon
- Long-range correlations are mediated by 3-form gauge fields
- In the case of (dynamical) evaporating black the horizon charge acts as a chemical potential and led to an asymmetric Hawking radiation (could be relevant for PBH baryogenesis).